EUROPEAN TRANSACTIONS ON TELECOMMUNICATIONS
Eur. Trans. Telecomms. 2008; **19**:67–75
Published online 23 April 2007 in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/ett.1214

Transmission Systems

Design and performance analysis of a novel trellis coded space–time–frequency OFDM transmission scheme†

Onur Oğuz[1]*, Ümit Aygölü[2] and Erdal Panayırcı[3]

[1] *Işık University, Electronics Engineering Department, Istanbul, Turkey*
[2] *Istanbul Technical University, Electronics and Communication Engineering Department, Istanbul, Turkey*
[3] *Department of Electrical and Electronics Engineering, Kadir Has University, Istanbul, Turkey*

SUMMARY

In this paper a new transmit diversity technique is proposed for wireless communications over frequency selective fading channels. The proposed technique utilises orthogonal frequency division multiplexing (OFDM) to transform a frequency selective fading channel into multiple flat fading subchannels on which first space–frequency coding and then additional space–time coding are applied resulting in a space–time–frequency OFDM (STF-OFDM) scheme. This scheme provides a quasi-orthogonal 4×4 transmission matrix but relax the constraint on channel fading parameters to be constant over four time or frequency slots into two time and two frequency slots. The pairwise error probability of the new scheme is evaluated and new code design criteria are obtained to improve the error performance. 64-, 128- and 256-state 4-PSK trellis codes for STF transmission scheme are generated based on these criteria and their frame error performances in an OFDM system are evaluated by computer simulations. Copyright © 2007 John Wiley & Sons, Ltd.

1. INTRODUCTION

Spatial diversity is a well-known technique for combating the detrimental effects of multipath fading. Traditionally, spatial diversity has been implemented at the receiver end, requiring multiple antennas and RF front-end circuits at the receivers. This multiplicity of receiver hardware is a major drawback, especially for portable receivers where physical size and current drain are important constraints.

In recent years, transmit diversity has received strong interest. The main advantage of transmit diversity is that diversity gain can be achieved by transmitting from multiple spatially separated antennas without significantly increasing the size or complexity of the receivers.

A number of orthogonal space–time transmit diversity techniques have been proposed recently [2–4], but unfortunately, the large delay spreads in frequency selective fading channels destroy the orthogonality of the received signals, which is critical to the operation of the diversity systems. Consequently, these techniques are often only effective over flat fading channels, such as indoor wireless networks or low data rate systems. Space–time coded OFDM (ST-OFDM) systems [5] have been proposed recently for delay spread channels. In Reference [6] it was shown that OFDM modulation with cyclic prefix can be used to transform frequency selective fading channels into multiple flat fading channels so that orthogonal space–time transmit diversity can be applied, even for channels with large delay spreads. The use of OFDM also offers the possibility of additional coding in the frequency dimension in a form of space–frequency OFDM (SF-OFDM) transmit diversity, which has also been suggested in References [7, 8].

This paper is concerned with a transmit diversity technique for wireless communications over frequency selective fading channels. The technique utilises orthogonal frequency division multiplexing (OFDM) to transform a

* Correspondence to: Onur Oğuz, (TELE-UCL), 2, place du levant, 1348 Louvain-la-Neuve, Belgium. E-mail: oguz@tele.ucl.ac.be.
† An earlier version of this paper has been presented at the *XI National Symposium of Radio Science* [1].

Received 27 August 2005
Revised 16 May 2006
Accepted 22 December 2006

frequency selective fading channel into multiple flat fading sub-channels. Then space–frequency coding and additional space–time coding are applied resulting in a space–time–frequency (STF-OFDM) transmission scheme. In order to improve the performance of the system, a convenient trellis coding scheme is investigated and design criteria are established based on the pairwise error probability analysis. Performance of the proposed STF-OFDM transmit diversity technique has been examined by computer simulations.

The organisation of the paper is as follows; Section 2 provides a detailed information about the new STF-OFDM transmission scheme. Section 3 investigates the performance analysis and design criteria for the trellis codes in fast fading channels when combined with the proposed STF-OFDM scheme. In Section 4 we explain how to design optimal trellis codes for the STF-OFDM scheme. We present simulation results in Section 5 and provide some concluding remarks in Section 6.

2. STF-OFDM TRANSMIT DIVERSITY

The proposed four-branch STF-OFDM transmit diversity scheme utilises Alamouti's [2] simple orthogonal transmit diversity matrix defined as (1)

$$\begin{array}{c} \\ \frac{\text{Time}}{\text{Frequency}} \downarrow \end{array} \begin{array}{c} \rightarrow \text{Antenna} \\ \begin{pmatrix} x_0 & x_1 \\ -x_1^* & x_0^* \end{pmatrix} \end{array} \tag{1}$$

over frequency (sub-carriers) and then time slots consecutively. In Equation (1), x_0 and x_1 are the transmitted symbols in the first time or frequency slot, from the first and second antennas, respectively. Similarly $-x_1^*$ and x_0^* are the transmitted symbols in the second time or frequency slot, from the first and second antennas, respectively. The OFDM modulation also allows the transmit diversity technique to work in a frequency selective channel. A functional block diagram of the proposed STF-OFDM system is shown in Figure 1.

As illustrated in Figure 1, the binary data from information source is applied to a trellis encoder that transforms it into limited length sequences of two dimensional signal points, via a predefined trellis structure. Then Serial-to-Parallel converter (S/P) groups symbol sequences into K symbols long blocks during each block interval. Here K is the number of available OFDM sub-carriers and each block interval is as long as $K \times T_s$, where T_s denotes the sampling interval.

Let

$$\mathbf{A}(n) = [a_0(n), a_1(n), \ldots, a_{K-1}(n)]^T \tag{2}$$

be the symbol vector for the nth block interval where $a_k(n)$, represents the kth symbol to be transmitted during nth block interval. As shown in Figure 1, the space–frequency encoder employs the Alamouti matrix over neighbouring symbols of $\mathbf{A}(n)$ vector and produces space–frequency encoded symbol

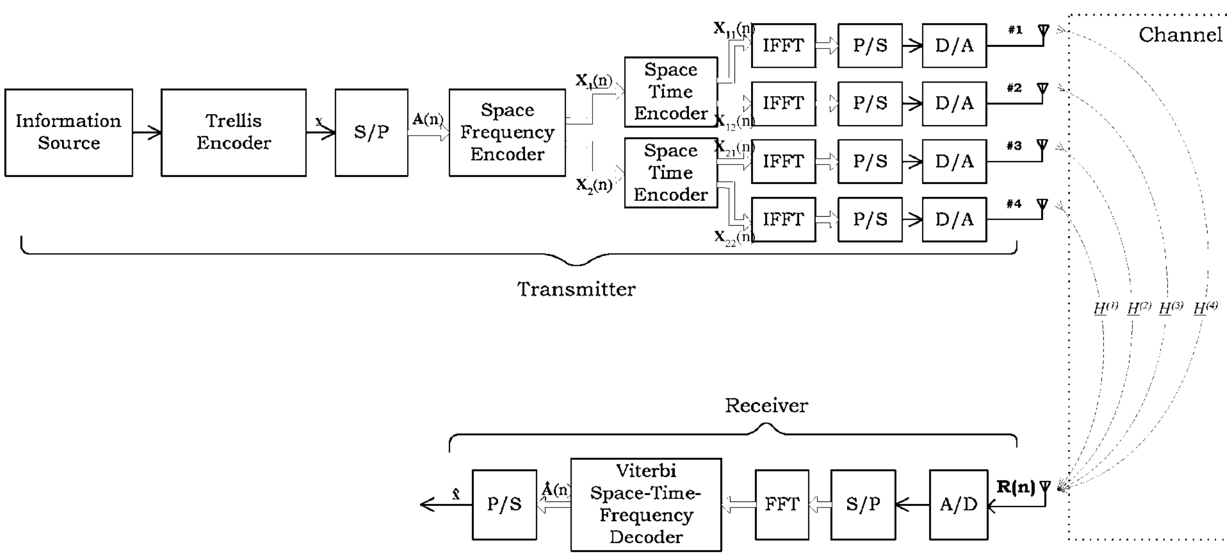

Figure 1. Block diagram of the proposed four-branch space–time–frequency OFDM (STF-OFDM) transmit diversity system.

vectors,

$$\boldsymbol{X}_1(n) = [a_0(n), -a_1^*(n), \ldots, a_{K-2}(n), -a_{K-1}^*(n)]^T,$$
$$\boldsymbol{X}_2(n) = [a_1(n), a_0^*(n), \ldots, a_{K-1}(n), a_{K-2}^*(n)]^T \quad (3)$$

to be transmitted during the nth block interval.

Successively encoded symbol vectors $\boldsymbol{X}_1(2n)$, $\boldsymbol{X}_2(2n)$ and $\boldsymbol{X}_1(2n+1)$, $\boldsymbol{X}_2(2n+1)$ generated by the space–frequency Encoder during the $2n$th and $2n+1$th block intervals are then re-encoded over consecutive block intervals by space–time encoder using again the Alamouti matrix as;

$$\begin{array}{ccc} \text{Block Interval} & \text{1st antenna} & \text{2nd antenna} \\ \begin{array}{c} 2n \\ 2n+1 \end{array} & \multicolumn{2}{c}{\begin{pmatrix} \boldsymbol{X}_1(2n) & \boldsymbol{X}_1(2n+1) \\ -\boldsymbol{X}_1^*(2n+1) & \boldsymbol{X}_1^*(2n) \end{pmatrix}} \\ & \text{3rd antenna} & \text{4th antenna} \\ \begin{array}{c} 2n \\ 2n+1 \end{array} & \multicolumn{2}{c}{\begin{pmatrix} \boldsymbol{X}_2(2n) & \boldsymbol{X}_2(2n+1) \\ -\boldsymbol{X}_2^*(2n+1) & \boldsymbol{X}_2^*(2n) \end{pmatrix}} \end{array} \quad . \quad (4)$$

After these two encoding operations, the resulting STF encoded four symbol vectors at the output of the space–time encoder are transmitted from four transmit antennas. The STF coded transmission scheme can simply be represented as in Table 1, or via the equivalent STF block code transmission matrix $\mathbf{G}_k(n)$ as,

$$\mathbf{G}_k(n) = \begin{pmatrix} a_{2k}(2n) & a_{2k}(2n+1) & a_{2k+1}(2n) & a_{2k+1}(2n+1) \\ -a_{2k+1}^*(2n) & -a_{2k+1}^*(2n+1) & a_{2k}^*(2n) & a_{2k}^*(2n+1) \\ -a_{2k}^*(2n+1) & a_{2k}^*(2n) & -a_{2k+1}^*(2n+1) & a_{2k+1}^*(2n) \\ a_{2k+1}(2n+1) & -a_{2k+1}(2n) & -a_{2k}(2n+1) & a_{2k}(2n) \end{pmatrix} \quad (5)$$

Table 1. Antenna excitation table of space–time–frequency (STF) block coding.

Antenna	Sub-carrier	Block interval $2n$	Block interval $2n+1$
# 1	$2k$	$a_{2k}(2n)$	$-a_{2k}^*(2n+1)$
	$2k+1$	$-a_{2k+1}^*(2n)$	$a_{2k+1}(2n+1)$
# 2	$2k$	$a_{2k}(2n+1)$	$a_{2k}^*(2n)$
	$2k+1$	$-a_{2k+1}^*(2n+1)$	$-a_{2k+1}(2n)$
# 3	$2k$	$a_{2k+1}(2n)$	$-a_{2k+1}^*(2n+1)$
	$2k+1$	$a_{2k}^*(2n)$	$-a_{2k}(2n+1)$
# 4	$2k$	$a_{2k+1}(2n+1)$	$a_{2k+1}^*(2n)$
	$2k+1$	$a_{2k}^*(2n+1)$	$a_{2k}(2n)$

where the indices k and n, are defined in the range,

$$0 \leqslant k \leqslant K/2 - 1, \quad 0 \leqslant n \leqslant N/2 - 1 \quad (6)$$

In Equation (5), all the symbols in the same row of $\mathbf{G}_k(n)$ are transmitted simultaneously from the four different transmit antennas, and all the symbols in the same column of $\mathbf{G}_k(n)$ are transmitted from the same antenna in couples over two successive OFDM sub-carriers $(2k, 2k+1)$ and, over two consecutive time slots $(2n, 2n+1)$. Note that STF block code transmission matrix $\mathbf{G}_k(n)$ in Equation (5) has a quasi-orthogonal structure [4].

Let the frequency selective fading channel transfer function $H^{\mu}(f, t)$, between the μth $(\mu = 1, 2, 3, 4)$ transmit antenna and the receiver antenna is known and represented via its sampled complex channel gains $H_k^{(\mu)}(n)$, corresponding to the kth tone and nth time slot. Assuming $H_k^{(\mu)}(n)$'s are constant over two adjacent sub-carriers and time instants, and vary independently from one time-frequency (TF) block to another, we can define a TF block complex channel gain $H_k^{(\mu)}(n)$ for the μth transmit antenna as,

$$H_k^{(\mu)}(n) \doteq H_{2k}^{(\mu)}(2n) = H_{2k+1}^{(\mu)}(2n) = H_{2k}^{(\mu)}(2n+1) = H_{2k+1}^{(\mu)}(2n+1) \quad (7)$$

and the STF channel vector $\boldsymbol{h}_k(n)$,

$$\boldsymbol{h}_k(n) = \left[H_k^{(1)}(n), H_k^{(2)}(n), H_k^{(3)}(n), H_k^{(4)}(n)\right]^T \quad (8)$$

where the pair (k, n) is defined as in Equation (6). As a result the received signal can be expressed as

$$\mathbf{R}_k(n) = \mathbf{G}_k(n)\boldsymbol{h}_k(n) + \mathbf{W}_k(n) \quad (9)$$

Here $\mathbf{W}_k(n)$ stands for the additive complex white Gaussian noise component with a power spectral density $N_0/2$ per dimension, and it is independent for each k and n.

3. PERFORMANCE ANALYSIS AND DESIGN CRITERIA

We now investigate design criteria for trellis codes employing STF block coding scheme defined above, based on the pairwise error probability analysis, in fast fading channels.

DOI: 10.1002/ett

Consider the probability that the maximum-likelihood receiver decides erroneously in favour of the sequence,

$$\hat{x} = \left[\hat{\mathbf{A}}^T(0), \hat{\mathbf{A}}^T(1), \ldots, \hat{\mathbf{A}}^T(N-1)\right]^T$$

instead of the transmitted sequence

$$x = \left[\mathbf{A}^T(0), \mathbf{A}^T(1), \ldots, \mathbf{A}^T(N-1)\right]^T$$

where $\hat{x}$ and x are composed of successive $\mathbf{A}(n)$'s from Equation (2), at time instants $n = 0, \ldots, N-1$. Assuming an ideal channel state information (CSI) at the receiver, the probability of erroneous decision of $\hat{x}$ instead of x, so-called pairwise error probability, is well approximated by Reference [9]

$$P(x \to \hat{x}|\boldsymbol{h}_k(n)) \leqslant \exp\left(-d^2(x, \hat{x})\frac{E_\mathrm{s}}{4N_0}\right) \tag{10}$$

Here $N_0/2$ is the noise variance per dimension and,

$$d^2(x, \hat{x}) = \sum_n \sum_k \left|(\mathbf{G}_k(n) - \hat{\mathbf{G}}_k(n))\boldsymbol{h}_k(n)\right|^2 \tag{11}$$

where $\mathbf{G}_k(n)$ and $\hat{\mathbf{G}}_k(n)$ are the corresponding STF block code transmission matrices for x and $\hat{x}$, respectively, as given in Equation (5) and $\boldsymbol{h}_k(n)$ is given by Equation (8). Let us define a difference matrix $\mathbf{D}_k(n)$, such that $\mathbf{D}_k(n) = (\mathbf{G}_k(n) - \hat{\mathbf{G}}_k(n))$. It can be easily shown that $\mathbf{D}_k(n)$ has the same quasi-orthogonal form of Equation (5). Then Equation (11) takes the form,

$$d^2(x, \hat{x}) = \sum_n \sum_k \left(\boldsymbol{h}_k^\dagger(n) \underbrace{\mathbf{D}_k^\dagger(n)\mathbf{D}_k(n)}_{\mathbf{C}_k(n)} \boldsymbol{h}_k(n)\right) \tag{12}$$

where $\dagger$ denotes the complex conjugate operation. Note that, $\mathbf{C}_k(n) = \mathbf{D}_k^\dagger(n)\mathbf{D}_k(n)$ is an Hermitian matrix with real eigenvalues, $\lambda_k^{(\mu)}$, $\mu = 1, 2, 3, 4$, in the following form,

$$\mathbf{C}_k(n) = \begin{pmatrix} \zeta_k^n & 0 & 0 & \xi_k^n \\ 0 & \zeta_k^n & -\xi_k^n & 0 \\ 0 & -\xi_k^n & \zeta_k^n & 0 \\ \xi_k^n & 0 & 0 & \zeta_k^n \end{pmatrix} \tag{13}$$

$$\lambda_k^{(1)}(n) = \lambda_k^{(2)}(n) = \zeta_k^n + \xi_k^n \tag{14}$$

$$\lambda_k^{(3)}(n) = \lambda_k^{(4)}(n) = \zeta_k^n - \xi_k^n \tag{15}$$

where

$$\zeta_k^n = \sum_{i=1}^{4} \left|D_k(n)[1, i]\right|^2 \tag{16}$$

$$\xi_k^n = 2\Re \begin{Bmatrix} D_k^*(n)[1, 1]D_k(n)[1, 4] \\ -D_k^*(n)[1, 2]D_k(n)[1, 3] \end{Bmatrix} \tag{17}$$

In Equations (16) and (17), $D_k(n)[i, j]$, denotes the ith row, jth column entry of the difference matrix $\mathbf{D}_k(n)$. Since $\mathbf{C}_k(n)$ is Hermitian, a unitary matrix $\mathbf{V}_k(n)$ and a diagonal matrix $\Lambda_k(n)$ can be found, resulting in $\mathbf{C}_k(n) = \mathbf{V}_k(n)\Lambda_k(n)\mathbf{V}_k^\dagger(n)$ where the diagonal elements of $\Lambda_k(n)$ are the corresponding eigenvalues of $\mathbf{C}_k(n)$ given in Equation (14). If we define $\beta_k^{(\mu)}(n)$ as,

$$\left[\beta_k^{(1)}(n), \ldots, \beta_k^{(4)}(n)\right] = \boldsymbol{h}_k^\dagger(n)\mathbf{V}_k(n) \tag{18}$$

then for complex channel gains, $\boldsymbol{h}_k(n)$, and $\beta_k^{(\mu)}(n)$s are independent complex Gaussian variables with zero mean and variance equal to 0.5 per dimension. Thus, similarly to Reference [9], we can write

$$\boldsymbol{h}_k^\dagger(n)\mathbf{C}_k(n)\boldsymbol{h}_k(n) = \sum_{\mu=1}^{4} \left|\beta_k^{(\mu)}(n)\right|^2 \lambda_k^{(\mu)}(n) \tag{19}$$

If Equations (10), (12) and (19) are combined, the resulting expression for pairwise error probability can be expressed as,

$$P(x \to \hat{x}|\underline{\beta}) \leqslant \exp\left\{-\frac{E_\mathrm{s}}{4N_0} \sum_n \sum_k \sum_{\mu=1}^{4} \left|\beta_k^{(\mu)}(n)\right|^2 \lambda_k^{(\mu)}(n)\right\} \tag{20}$$

where $\underline{\beta} = [\beta_k^{(1)}(n), \ldots, \beta_k^{(4)}(n)]$. Averaging Equation (20) with respect to Rayleigh distributed and independent $|\beta_k^{(\mu)}(n)|$ samples as,

$$E_{\underline{\beta}}\left[P(x \to \hat{x}|\underline{\beta})\right] = \prod_n \prod_k \prod_{\mu=1}^{4} \left(1 + \lambda_k^{(\mu)}(n)\frac{E_\mathrm{s}}{4N_0}\right)^{-1} \tag{21}$$

we end up with an upper bound for pairwise error probability independent of the CSI,

$$P(x \to \hat{x}) \leqslant \prod_{n,k} \left(1 + \lambda_k^{(1)} \frac{E_s}{4N_0}\right)^{-2} \left(1 + \lambda_k^{(3)} \frac{E_s}{4N_0}\right)^{-2}$$
$$= \prod_{n,k} \left(1 + (\zeta_k^n + \xi_k^n) \frac{E_s}{4N_0}\right)^{-2}$$
$$\times \left(1 + (\zeta_k^n - \xi_k^n) \frac{E_s}{4N_0}\right)^{-2} \quad (22)$$

From the definition given in Equations (16) and (17), ζ_k^n and ξ_k^n are functions of the elements of the difference matrix $\mathbf{D}_k(n)$. Thus, if for any particular (n_i, k_j) pair the difference matrix $\mathbf{D}_k(n)$ becomes a zero matrix, then the corresponding factor in Equation (22) becomes equal to one and has no influence on the upper bound for pairwise error probability. So defining a set η such that η contains only (n, k) pairs for which $\mathbf{D}_k(n) \neq \underline{\mathbf{0}}$, we can rewrite Equation (22) as,

$$P(x \to \hat{x}) \leqslant \prod_{k,n\in\eta} \left(1 + 2\zeta_k^n \frac{E_s}{4N_0} + \left((\zeta_k^n)^2 - (\xi_k^n)^2\right) \left(\frac{E_s}{4N_0}\right)^2\right)^{-2} \quad (23)$$

The pairwise error probability given in Equation (23) can be analysed for both high and low signal to noise ratio (SNR) cases as follows;

For high SNR case, if we denote the number of n, k pairs, where $(\zeta_k^n)^2 - (\xi_k^n)^2 \neq 0$ in η by $l_\eta^{(1)}$, and the number of remaining terms where $(\zeta_k^n)^2 - (\xi_k^n)^2 = 0$ by $l_\eta^{(2)}$ then pairwise error probability expression in Equation (23) becomes,

$$P_{\mathrm{HI}}(x \to \hat{x}) \leqslant \frac{1}{(\frac{E_s}{4N_0})^{l_\eta} \prod_{k,n\in\eta} \psi(\zeta_k^n, \xi_k^n)} \quad (24)$$

where, $\psi(\zeta_k^n, \xi_k^n)$ in Equation (24) is defined as *modified sum distance*, such that,

$$\psi(\zeta_k^n, \xi_k^n) = \begin{cases} 4(\zeta_k^n)^2, & (\zeta_k^n)^2 - (\xi_k^n)^2 = 0 \\ \left((\zeta_k^n)^2 - (\xi_k^n)^2\right)^2, & (\zeta_k^n)^2 - (\xi_k^n)^2 \neq 0 \end{cases} \quad (25)$$

and l_η is the diversity gain defined as

$$l_\eta = 4l_\eta^{(1)} + 2l_\eta^{(2)} \quad (26)$$

For low SNR values, Equation (23) becomes,

$$P_{\mathrm{LO}}(x \to \hat{x}) \leqslant \left(1 + \frac{E_s}{2N_0} \sum_{k,n\in\eta} \zeta_k^n + o\left(\frac{E_s}{2N_0}\right)\right)^{-2} \quad (27)$$

where $o(\star)$ denotes the summation of all the terms including higher order quantities of $\star$, which can be neglected.

Using the pairwise error probability upper bound, an upper bound for the error event probability P_e, then can be obtained by means of the union bound,

$$P_e \leqslant \sum_x \sum_{\hat{x}\neq x} P(x) P(x \to \hat{x}) \quad (28)$$

where $P(x)$ is the probability of transmitting the sequence x. It can be shown that the total number of possible transmitting sequences, x, is $M^{RKN} = 2^{RKN \log_2 M}$, where R is the code rate and M is the constellation size. Due to the fact that the data bits are generated with equal probability, the transmit sequences, x, are equally likely. Thus, if we define *modified product-sum distance* as,

$$\Psi_{x,\hat{x}} = \prod_{n,k\in\eta} \psi(\zeta_k^n, \xi_k^n) \quad (29)$$

for high SNR values, Equation (28) can be rewritten as follows,

$$P_e \leqslant M^{-RKN} \sum_x \sum_{\hat{x}\neq x} \frac{1}{(\frac{E_s}{4N_0})^{l_\eta} \Psi_{x,\hat{x}}}. \quad (30)$$

It can be easily observed that, the term in Equation (30) having effective length $L_\eta = \min(l_\eta)$ dominates the error event probability P_e for high SNR values. Therefore P_e can be approximated as,

$$P_e \approx \mathcal{C}\left[\left(\frac{E_s}{4N_0}\right)^{L_\eta} \overline{\Psi}\right]^{-1} \quad (31)$$

where $\overline{\Psi}$ is the averaged $\Psi_{x,\hat{x}}$ over $x, \hat{x}$ pairs having $L_\eta = \min(4l_\eta^{(1)} + 2l_\eta^{(2)})$. From Equation (31), it is obvious that, for high SNR values, P_e asymptotically decreases with L_ηth power of SNR, thus L_η sets the minimum achievable diversity order of the proposed scheme.

For the STF coded system, it follows from Equation (31) that, L_η can be chosen as the prime design parameter as P_e asymptotically decreases with L_ηth power of SNR. From the definition of L_η, it is obvious that we should maximise

DOI: 10.1002/ett

$l_\eta^{(1)}$ while minimising $l_\eta^{(2)}$ for a constant $l_\eta^{(1)} + l_\eta^{(2)}$ value, but for a full rate trellis structure, for each and every possible transmitted sequence x we can find an erroneous sequence $\hat{x}$ such that $L_\eta = \min(2l_\eta^{(2)})$, due to quasi-orthogonality of $\mathbf{G}_k(n)$, which introduces an important tradeoff between rate and diversity gain. Therefore the design criteria for a possible trellis structure involves:

- maximisation of L_η, and
- maximisation of minimum modified product-sum distance $\overline{\Psi}$.

Note that these code design criteria are a specialisation of the design criteria for fast fading channels given in Reference [9] and are different from that of the STF trellis coded modulation (TCM) design criteria presented earlier in literature [10–13]. So new optimal codes can be constructed using these criteria.

If we examine the low SNR case, from Equation (27) the sum of quartet Euclidian distances clearly becomes the leading factor. Thus for low SNR values, ζ_k^n as given in Equation (16), instead of L_η and $\overline{\Psi}$ becomes the dominant factor.

4. CODE DESIGN

In this section, we design some trellis structures having STF block code transmission matrix $\mathbf{G}_k(n)$ given by Equation (5), based on the high SNR design criteria stated in Section 3. The elements, $a_k(n)$, structuring the transmission matrix $\mathbf{G}_k(n)$ (5) are selected from a four-PSK constellation via parameter $\theta \in \{0,\ 1,\ 2,\ 3\}$ generating the symbols $\exp\{j(\theta\frac{\pi}{2} + \phi)\}$, where ϕ is the constellation rotation phase. Thus $4^4 = 256$ different arrangements for $\mathbf{G}_k(n)$,denoted by $G^{(i)}$, employing $\Theta^{(i)} = \theta_1\theta_2\theta_3\theta_4$, $i = 0\ldots255$, are available. In order to maximise L_η, we should avoid, if possible, or minimise couplings of $\mathbf{G}^{(i)}, \mathbf{G}^{(j)}, i \neq j$ where corresponding value is zero, for a minimum error path length. Once the codes maximising L_η are identified, via an exhaustive computer search algorithm, we choose the one that maximises $\overline{\Psi}$, as the surviving code.

Based on the design criteria derived, rate 1.5 bps/Hz trellis structures employing four-PSK signalling are obtained with different state numbers thus effective lengths. Table 2 shows the set partitioning for the resulting codes. Note that the four-digit numbers given in Table 2 indicate the $\Theta^{(i)}$ values for a given $G^{(i)}$, and the columns labelled with S_* indicate the subsets.

The $\Theta^{(i)}$ values are grouped into four subsets, as indicated in Table 2. Each subset contains 64 elements, in three different arrangements, indicated by S_1, S_2, S_3, S_4, S_α, S_β, S_γ, S_δ, S_a, S_b, S_c, S_d. As this would immediately set L_η to its minimum value, the parallel transitions between any state pair are not permitted. This also achieves a lower bound on the minimum number of states in a trellis structure which is equal to 64. Based on this way, we have designed three STF trellis codes with the number of states being 64, 128 and 256, all having the rate 1.5 bps/Hz, labelled as Code I, Code II and Code III, respectively. For all of these codes, one

Table 2. Set partitioning.

S_1				S_2				S_3				S_4			
S_α		S_β		S_α		S_β		S_γ		S_δ		S_γ		S_δ	
S_a	S_b	S_c	S_d	S_a	S_b	S_c	S_d	S_a	S_b	S_c	S_d	S_a	S_b	S_c	S_d
0222	2313	3032	1030	3102	0130	1110	1031	0023	0021	0013	0231	2202	2123	2103	0100
2310	0322	1221	0020	2012	0003	0101	1122	1232	1012	0120	3013	0301	2330	2132	0211
3122	0213	1201	0123	0233	2220	3312	1001	0230	2222	1312	3123	1323	3003	2010	1002
0312	2331	0112	0122	0200	0331	3302	3313	2323	0223	3001	2233	2031	0232	1321	1010
2002	1003	0030	2120	1331	2322	3133	0102	1023	0103	3101	0000	1132	1133	0133	1131
0221	2302	1210	0012	0001	2000	3321	3301	2111	1112	1000	3300	0332	2300	2113	1212
3232	3203	3112	1220	0313	3222	0321	1313	0121	3023	2030	3200	1121	3011	2001	2020
2032	0210	1302	1322	2121	2320	3220	3012	3221	0011	1332	2212	3131	2013	0031	2102
0333	2301	3121	0303	1103	2130	2011	0220	3322	1330	2131	1333	1301	0201	1021	2210
2022	2110	3213	2122	1223	2332	2200	2033	3130	1033	1102	1100	3000	3120	3103	2303
3333	2231	1303	3223	3021	1300	1200	1222	2101	3332	2213	0132	3320	0131	1113	2321
1320	0311	1233	3310	2021	0203	2230	1013	3132	1011	3210	3002	1311	1032	3331	3100
3111	2201	2333	3201	1202	3211	0212	1203	3030	1020	2100	1213	0320	3212	1130	3231
3020	0302	3010	0300	1310	3202	2003	1230	2203	0002	2112	3110	2311	0202	3113	3233
1120	1022	1231	3022	0110	1111	0113	0032	1123	3311	2023	3031	2211	2221	0111	3303
0010	2232	0310	2133	1211	3033	0022	2223	3230	0033	1101	2312	3323	0330	0323	3330

DOI: 10.1002/ett

Table 3. Effective length L_η, averaged minimum modified product-sum distance $\overline{\Psi}$ and their probabilities $\Xi(L_\eta, \overline{\Psi})$.

Code	State	L_η	$\overline{\Psi(\eta)}$	$\Xi(L_\eta, \overline{\Psi})$
I	64	4	4.8277e + 4	8.3124e − 4
II	128	4	4.5249e + 4	8.3148e − 4
III	256	4	5.0613e + 4	6.6719e − 4

of the subsets S_1, S_2, S_3, S_4 is attributed to each departure states of the trellis structure. The codes differ in their arrival state subsets. Code I employs the arrival subsets S_a, S_b, S_c, S_d, while Code II uses S_α, S_β, S_γ, S_δ. Code III uses S_1, S_2, S_3, S_4 for both departure and arrival subsets.

The effective length L_η and the averaged minimum modified product-sum distance $\overline{\Psi}$ as well as their occurrence rates among first error events $\Xi(L_\eta, \overline{\Psi})$ for the resulting schemes are as given in Table 3. Note that all the codes have equal effective length L_η, and different $\overline{\Psi}$ for given L_η. In order to achieve larger L_η values either the state number must be increased or the rate must be decreased.

5. SIMULATION RESULTS

In this section, we provide the simulation results for the codes given in Section 4. As stated earlier the codes utilise trellis structures with four-PSK signalling. The code performances are described by means of frame error rate (FER). The performance curves are plotted against SNR per transmit antenna.

As we stated earlier, the system employs four transmit antennas and one receive antenna. In all the simulations, an OFDM frame consists of 128 information bearing symbols, transmitted over $K = 64$ OFDM sub-carriers in two succeeding time slots. Thus the 32, 4×4 STF transmission blocks are transmitted through a frame. The simulation results are obtained for COST-207 'Typical Urban' (TU) and 'Bad Urban' (BU) channels, which are described in the literature [14, 15]. In order to model an appropriate fast fading, frequency selective channel, we choose normalised Doppler frequency $T_s f_d$ to be in the range of $\sim 10^{-2}$.

The code performances are evaluated in both TU and BU channel conditions, and in addition we obtained the performance curves with the use of a random interleaver of length $2K$, that is twice the number of OFDM sub-carriers. The interleaving is performed on a SF block basis. Figure 2 shows the FER performance of the codes over the TU channel and Figure 3 over the BU channel.

Note that while the codes perform better in TU channel, when interleaver is introduced in the system, performance over the BU channel improves and results in better FER values than over TU channel. This is because the channel acts more like uncorrelated in case of interleaver over BU channel as the complex channel gains of BU channel involves more rapid changes and more suitable for our

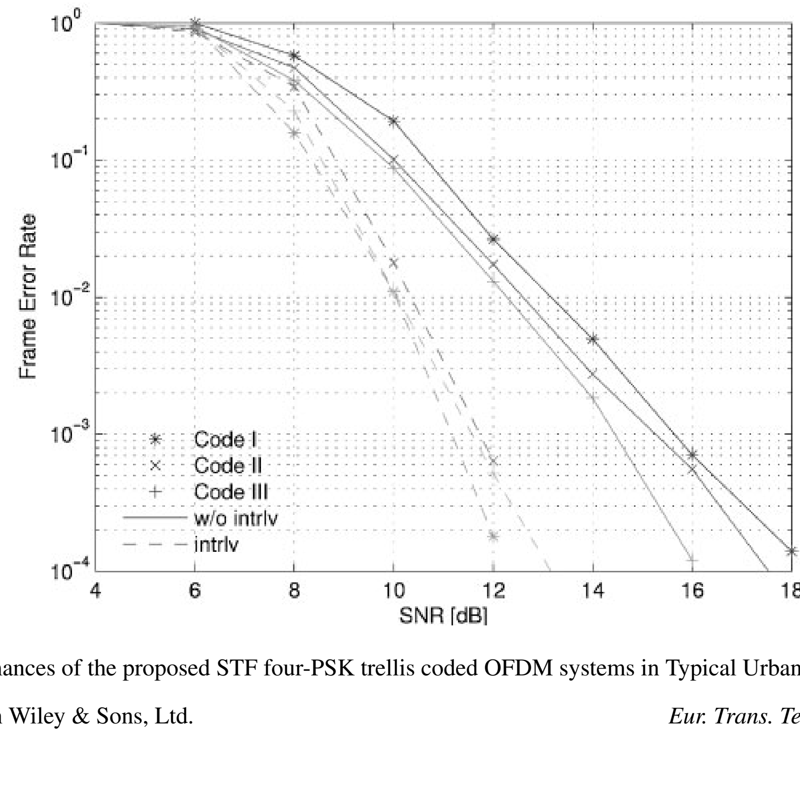

Figure 2. FER performances of the proposed STF four-PSK trellis coded OFDM systems in Typical Urban channel.

DOI: 10.1002/ett

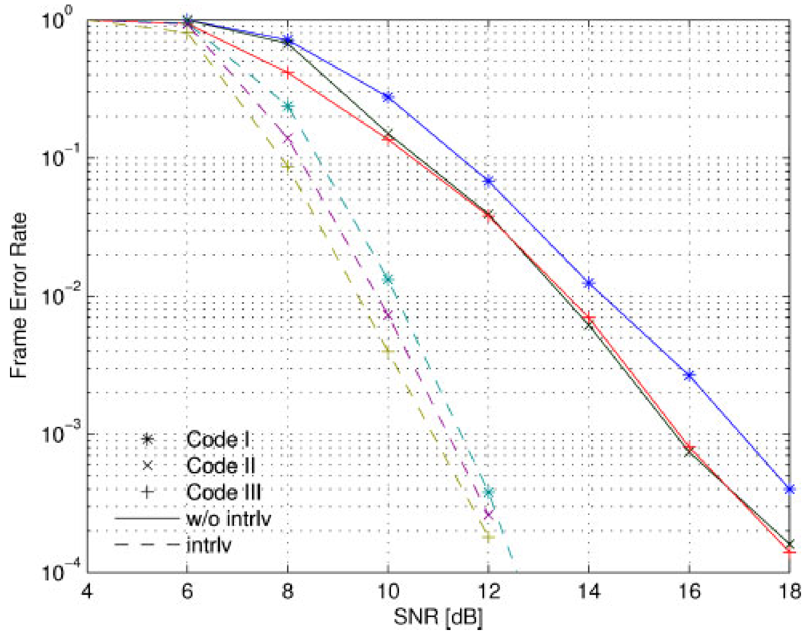

Figure 3. FER performances of the proposed STF four-PSK trellis coded OFDM systems in Bad Urban channel.

channel assumption. Another improvement introduced by interleaving is a significant diversity gain, which can cause SNR advantage as much as 6 dB.

As seen from the FER curves shown in Figures 2 and 3, codes perform closely in either channel condition, with or without interleaving. Without interleaving, influences of L_η and $\overline{\Psi}$ over performance changes according to the channel but when interleaving is introduced, performance depends mostly on L_η, that is why Code I can outperform the other ones in some channel conditions.

6. CONCLUSIONS

In this paper, we have proposed a new STF-OFDM transmission scheme in which each coding step, shares four symbols to be transmitted into two time and two frequency (sub-carriers) slots. This scheme provides a quasi-orthogonal 4×4 transmission matrix but relax the constraint on channel fading parameters to be constant over four time or frequency slots into two time and two frequency slots. New design criteria for trellis codes employing the proposed transmission technique at their branches are derived based on the pairwise error probability analysis. These criteria are different from the conventional design criteria given in the literature for the concatenation of TCM schemes and Alamouti transmit diversity schemes, and can be employed to design new optimal trellis code structures.

ACKNOWLEDGEMENT

This work is part of the joint research activities of the NEWCOM Network of Excellence supported by the European Commission Sixth Frame Programme.

REFERENCES

1. Oğuz O, Aygölü Ü, Panayırcı E. A novel trellis coded space-time-frequency transmission scheme for OFDM. In *Proceedings of XI. National Symposium of Radio Sciences (URSI'05)*, Poznan, Poland, 7–8 April 2005.
2. Alamouti SM. A simple transmit diversity technique for wireless communications. *IEEE Journal on Selected Areas in Communications* 1998; **16**(8):1451–1458.
3. Tarokh V, Jafarkhani H, Calderbank AR. Space-time block codes from orthogonal designs. *IEEE Transactions on Information Theory* 1999; **45**(5):1456–1467.
4. Jafarkhani H. A quasi-orthogonal space-time block code. *IEEE Transactions on Communications* 2001; **49**(1):1–4.
5. Lu B, Wang X. Space-time code design in ofdm systems. In *Proceedings of GLOBECOM'00*, vol. 2, pp. 1000–1004, IEEE, November–December 2000.
6. Bingham JAC. Multicarrier modulation for data transmission: an idea whose time has come. *IEEE Communication Magazine* 1990; **28**(**5**):5–14.

7. Lee KF, Williams DB. A space-frequency transmitter diversity technique for OFDM systems. In *Proceedings of GLOBECOM 2000—IEEE Global Telecommunications Conference*, vol. 1, pp. 1473–1477, November 2000.
8. Gong Y, Letaief KB. An efficient space-frequency coded OFDM system for broadband wireless communications. *IEEE Transactions on Communications* 2003; **51**(11):2019–2029.
9. Tarokh V, Seshadri N, Calderbank AR. Space-time codes for high data rate wireless communication: performance criterion and code construction. *IEEE Transactions on Information Theory* 1998; **44**(2):744–765.
10. Liu Z, Xin Y, Giannakis GB. Space-time-frequency coded OFDM over frequency-selective fading channels. *IEEE Transactions on Signal Proceedings* 2002; **50**(10):2465–2476.
11. Divsalar D, Simon MK. Multiple trellis coded modulation (MTCM). *IEEE Transactions on Communications* 1988; **36**(4):410–419.
12. Liu Z, Xin Y, Giannakis GB. Space-time-frequency trellis coding for frequency-selective fading channels. In *Proceedings of Vehicular Technology Conference*, vol. 1, pp. 145–149, Spring 2002.
13. Gong Y, Letaief KB. Analysis and design of trellis coded modulation with transmit diversity for wireless communications. In *Proceedings of Wireless Communications and Networking Conference, 2000*, vol. 3, pp. 1356–1361, IEEE, Spring 2000.
14. Hoeher P. A statistical discrete-time model for WSSUS multipath channel. *IEEE Transactions on Vehicular Technology* 1992; **41**(2):461–468.
15. Pätzold M. *Mobile Fading Channels*. John Wiley & Sons, Ltd: Baffins Lane, Chichester, West Sussex, PO19 1UD, England, 2002.

AUTHORS' BIOGRAPHIES

Onur Oğuz received his B.S and M.S. degrees in Electronics Engineering, from Işık University, Istanbul, Turkey, in 2001 and 2004, respectively. He was a Research Assistant from 2001 to 2006 at the Electronics Engineering department of the same university. Currently, he is a Ph.D. student at Istanbul Technical University, Istanbul, Turkey, in Electronics and Communication Engineering. Since 2006 he is working as a research assistant at the Communications and Remote Sensing Laboratory of Université catholique de Louvain, Louvain-la-Neuve, Belgium. His research interests are in the fields of communication theory and applications, especially, wireless communications systems, cooperative and opportunistic communications, multiple access communications, channel coding techniques, diversity techniques and channel modelling.

Ümit Aygölü received his B.S., M.S. and Ph.D. degrees, all in Electrical Engineering, from Istanbul Technical University, Istanbul, Turkey, in 1978, 1984 and 1989, respectively. He was a Research Assistant from 1980 to 1986 and a Lecturer from 1986 to 1989 at Yıldız Technical University, Istanbul, Turkey. In 1989, he was an Assistant Professor at Istanbul Technical University, where he became an Associate Professor and Professor, in 1992 and 1999, respectively. His current research interests include the theory and applications of combined coding modulation systems, MIMO, space-time coding and cooperative communication.

Erdal Panayırcı received his Diploma Engineering degree in Electrical Engineering from Istanbul Technical University, Istanbul, Turkey and his Ph.D. degree in Electrical Engineering and System Science from Michigan State University, East Lansing Michigan, USA. Until 1998 he has been with the Faculty of Electrical and Electronics Engineering at the Istanbul Technical University, where he was a Professor and Head of the Telecommunications Chair. Currently, he is Head of the Electronics Engineering Department at Kadir Has University, Istanbul, Turkey. He is the head of the Turkish Scientific Commission on Communication Systems and Signal Processing of URSI (International Union of Radio Science). His research interests include communication theory, synchronisation and equalisation, multicarrier systems, coded modulation and interference cancellation with array processing and space-time coded systems. He published extensively in leading scientific journals and international conferences. Erdal Panayirci is an IEEE Fellow.

DOI: 10.1002/ett