Iterative Channel Estimation and Decoding of Turbo Coded SFBC-OFDM Systems

Hakan Doğan, *Student Member, IEEE,* Hakan A. Çırpan, *Member, IEEE,* and Erdal Panayırcı, *Fellow, IEEE*

***Abstract*— We consider the design of turbo receiver structures for space-frequency block coded orthogonal frequency division multiplexing (SFBC-OFDM) systems in the presence of unknown frequency and time selective fading channels. The Turbo receiver structures for SFBC-OFDM systems under consideration consists of an iterative MAP Expectation/Maximization (EM) channel estimation algorithm, soft MMSE-SFBC decoder and a soft MAP outer-channel-code decoder. MAP-EM employs iterative channel estimation and it improves receiver performance by re-estimating the channel after each decoder iteration. Moreover, the MAP-EM approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. The optimal truncation property of the KL expansion can reduce computational load on the iterative estimation approach. The performance of the proposed approaches are studied in terms of mean square error and bit-error rate. Through computer simulations, the effect of a pilot spacing on the channel estimator performance and sensitivity of turbo receiver structures on channel estimation error are studied. Simulation results illustrate that receivers with turbo coding are very sensitive to channel estimation errors compared to receivers with convolutional codes. Moreover, superiority of the turbo coded SFBC-OFDM systems over the turbo coded STBC-OFDM systems is observed especially for high Doppler frequencies.**

***Index Terms*— EM algorithm, MAP channel estimation, OFDM systems, space-frequency coding, turbo receiver.**

I. Introduction

THE goal of the developments for the future generations of broadband wireless mobile systems is to provide a wide range of high quality enhanced and integrated services with high data rates. Several key enabling techniques capable of achieving the highest possible spectrum efficiency are therefore currently being investigated. An important area that has to be focused on to make this goal accomplished is related to spectrally efficient and flexible modulation and coding techniques. Specifically, the combined application of orthogonal frequency division multiplexing (OFDM) and transmit antenna diversity appears to be capable of enabling the types of capacities and data rates needed for broadband wireless services.

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H. Doğan and H. A. Çırpan are with the Department of Electrical and Electronics Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey (e-mail: {hdogan,hcirpan}@istanbul.edu.tr).

E. Panayırcı is with the Department of Electronics Engineering, Kadir Has University, Cibali 34083, Istanbul, Turkey (e-mail: eepanay@khas.edu.tr).

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Transmit antenna diversity has been exploited recently to develop high-performance space-time/frequency codes and simple maximum likelihood (ML) decoders for transmission over flat-fading channels [1]–[3]. Unfortunately, their practical application can present a real challenge to channel estimation algorithms, especially when the signal suffers from frequency selective multipath channels. One of the solutions alleviating the frequency selectivity is the use of OFDM together with transmit diversity which combats long channel impulse response by transmitting parallel symbols over many orthogonal subcarriers yielding a unique reduced-complexity physical layer capabilities [4].

The continued increase in demand for all types of services further necessitates the need for higher capacity and data rates. In this context, emerging technology that improves the wireless systems spectrum efficiency is error control coding. Recent trends in coding favor parallel and/or serially concatenated coding and probabilistic soft-decision iterative (turbo-style) decoding. Such codes are able to exhibit near-Shannon-limit performance with reasonable complexities in many cases and are of significant interest for communications applications that require moderate error rates. An outer channel code is therefore applied in addition to transmit diversity to further improve the receiver performance. We therefore consider the combination of turbo codes with the transmit diversity OFDM systems. Especially we address the design of iterative channel estimation approach for transmit diversity OFDM systems employing an outer channel code.

Channel estimation for transmit diversity OFDM systems has attracted much attention with pioneering works by Li [5], [6]. However, most of the early work on channel estimation for transmit diversity OFDM systems focused on uncoded systems. Since most practical systems use error control coding, more recent work have addressed the coded transmit diversity OFDM systems. Among many other techniques, an iterative procedures based on Expectation-Maximization (EM) algorithm was also applied to channel estimation problem in the context of space-time block-coding (STBC) [7], [8] as well as transmit diversity OFDM systems with or without outer channel coding (e.g. convolutional code or Turbo code) [10]–[13]. In [10], maximum a *posteriori* (MAP) EM based iterative receivers for STBC-OFDM systems with Turbo code are proposed to directly detect transmitted symbols under the assumption that fading processes remain constant across several OFDM symbols contained in one STBC code-word.

An EM approach proposed for the general estimation of the superimposed signals [9] is applied to the channel estimation for transmit diversity OFDM systems with outer channel code (convolutional code) and compared with the SAGE version presented in [12]. Moreover in [13], a modified version of [12] is proposed for the STBC-OFDM and space-frequency block-coding (SFBC)-OFDM systems. Unlike the EM approaches treated in [10]–[13], we propose in the paper a new Turbo receiver based on MAP-EM channel estimation algorithm for SFBC-OFDM systems employing outer channel coding. The Turbo receiver scheme under consideration employs iterative channel estimation and it improves receiver performance by re-estimating the channel after each decoder iteration. The paper has several major novelties and contributions. The main contribution of the paper mainly comes from the fact that the channel estimation technique presented in our work is an EM based *non-data-aided* approach as opposed to the existing works in the literature which are mostly assumed either the data is known at the receiver through a training sequence or a joint data detection and the channel estimation. Note that very small number of pilots used in our approach is necessary only for initialization of the EM algorithm leading to channel estimation. Although, the joint data and channel estimation technique with EM algorithm seems to be attractive in practice, it is known that the convergency of the algorithm is much slower, it is more sensitive to the initial selection of the parameters and the algorithm is more computationally complex than the techniques that deal with only channel estimation. As it is known in the estimation literature, non data-aided estimation techniques are more challenging mainly due to a data averaging process which must be performed prior to optimization step. Most of the time this may not lead to a simple analytical expression for the estimates. Thanks to the orthogonal space/frequency coding techniques which made possible to derive exact and simple analytical expressions for the unknown channel parameters in our work.

Another significant contribution of the paper comes from the fact that the channel parameter estimation technique proposed in our paper is for the SFBC-OFDM transmitter diversity systems with outer channel coding. The estimation algorithm performs an iterative estimation of the fading channel parameters in frequency domain according to the maximum a posteriori criterion (MAP) as opposed to the ML approaches adopted in many publications appeared in the literature. Furthermore, our approach is based on a novel representation of the fading channel by means of the Karhunen-Loeve (KL) expansion and the application of this expansion to the turbo receiver structures for SFBC-OFDM systems. Note that, KL orthogonal expansion together with space-frequency coded system based on the Alamouti orthogonal design enable us to estimate the channel in a very simple way without taking inverse of large dimensional matrices, yielding a computationally efficient iterative analytical expressions [15]–[17]. Moreover, optimal truncation property of the KL expansion is exploited in our paper resulting in a further reduction in computational load on the channel estimation algorithm. In order to explore the performance of the proposed turbo receivers, we first investigate the effect of a pilot spacing on the turbo receiver performance by considering average MSE as well as bit-error-rate (BER). We also analyze the sensitivity of turbo receiver structures on channel estimation errors.

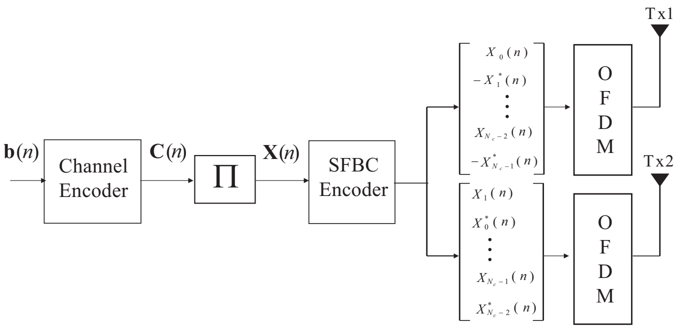

Fig. 1. Transmitter structure for turbo coded SFBC-OFDM systems.

The rest of the paper is organized as follows. In Section II, a system model for turbo coded SFBC-OFDM is introduced. In Section III, multipath channel and its orthogonal series representation based on the Karhunen-Loeve expansion is presented. Vector representation of the received signal is formulated in Section IV, while an EM based MAP channel estimation algorithm is developed in Section V. In addition to iterative MAP channel estimation approach, iterative channel equalization and decoding structures are proposed in Section VI. Some computer simulation are provided in Section VII. Finally, conclusions are drawn in Section VIII.

II. Turbo Coded SFBC-OFDM System Model

We consider a SFBC-OFDM system with outer channel coding. Turbo code is applied, in addition to a SFBC system, to further improve the error performance of the SFBC-OFDM system. A block diagram of a transmitter structure for a turbo coded two-branch SFBC transmitter diversity OFDM system is shown in Fig. 1.

A. Turbo Encoder

Turbo codes are a class of powerful error correction codes that enable reliable communications with power efficiencies close to the theoretical Shannon channel capacity limit. In particular, a turbo code is formed from the parallel or serial concatenation of codes separated by an interleaver. In general, Turbo codes are low-rate codes which require considerable bandwidth expansion for high rate data transmission. In order to improve spectral efficiency, it is necessary to combine turbo codes with a bandwidth efficient transmit diversity systems. Thus combinations of implicit (turbo coding) and external (i.e. multiple transmit antenna) diversity can be used to improve the performance of the communication system in fading environments.

As illustrated in Fig. 1, the block of binary data bits of length $N_c/2$, $\mathbf{b}(n) = [b_0(n), b_1(n), ..., b_{\frac{N_c}{2}-1}(n)]^T$ at time n are encoded by an 1/2 rate outer-channel-encoder, resulting in a BPSK-coded symbol stream $\mathbf{C}(n) = [C_0(n), C_1(n), ..., C_{N_c-1}(n)]^T$ of length N_c . The coded symbols are then interleaved by a random permutation resulting in a stream of independent symbols(of length N_c), denoted by $\{\mathbf{X}(n)\}$. A code-bit interleaver reduces probability of burst error bursts and removes correlation in the coded symbol stream. Finally, the modulated BPSK symbols are encoded by a SFBC encoder and transmitted from two transmit antennas on corresponding OFDM subcarriers.

B. SFBC-OFDM Encoder

In this paper, we consider a transmitter diversity OFDM scheme in conjunction with inner channel coding. In order to compensate for the reduced data rate of turbo codes, some space-time codes having data rates greater than one could be employed. However it is well known from literature that the Alamouti antenna modulation configuration is the only scheme which retain orthogonality and full rate when for the complex-valued data as well as the low complexity. As will be seen shortly, orthogonality property is essential and required condition for the channel estimation algorithm in our paper. Moreover, orthogonality structure of Alamouti allows decoupling of the channel and reduces the equalizer complexity. Furthermore, the Alamouti‘s schemes has been adopted in several wireless standards such as WCDMA and CDMA2000. It imposes an orthogonal spatio-temporal structure on the transmitted symbols that guarantees full (i.e., order 2) spatial diversity. In addition to the spatial level, to realize multipath diversity gains over frequency selective channels, the Alamouti block coding scheme is implemented at a block level in frequency domain. Thus the use of OFDM in transmitter diversity systems also offers the possibility of coding in a form of space-frequency OFDM [18], [19]. Under the assumption that the channel responses are known or can be estimated accurately at the receiver, it was shown that the SFBC-OFDM system has the same performance as a previously reported STBC-OFDM scheme in slow fading environments but shows better performance in the more difficult fast fading environments [18]. Also, since SFBC-OFDM transmitter diversity scheme performs decoding within one OFDM block, it requires only half of the decoder memory needed for the STBC-OFDM system of the same block size. Similarly, the decoder latency for SFBC-OFDM is also half of the STBC-OFDM implementation. In SFBC-OFDM systems, the OFDM subchannels are divided into certain number of groups. This subchannel grouping with appropriate system parameters does preserve diversity gain while simplifying not only the code construction but decoding algorithm significantly as well [18].

Adopting the notation of [18], let N_c turbo coded, interleaved and BPSK modulated symbols, taking values $\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}$, be represented by a vector $\mathbf{X}(n)$= $[X(nN_c), X(nN_c+1), \cdots, X(nN_c+N_c-1)]^T$, where $X_k(n) = X(nN_c+k)$ denotes the kth forward polyphase component of the serial data symbols, for $k = 0, \cdots, N_c-1$. Polyphase component $X_k(n)$ can also be viewed as the coded symbol to be transmitted on the kth tone during the block instant n. The coded symbol vector $\mathbf{X}(n)$ can therefore be expressed as $\mathbf{X}(n)$= $[X_0(n), X_1(n), \cdots, X_{N_c-1}]^T$. Resorting subchannel grouping, $\mathbf{X}(n)$ is coded into two vectors $\mathbf{X_e}(n)$ and $\mathbf{X_o}(n)$ by the space-frequency encoder as

$$\begin{aligned}\mathbf{X_e}(n) &= [X_0(n), X_2(n), \cdots, X_{N_c-4}(n), X_{N_c-2}(n)]^T, \\ \mathbf{X_o}(n) &= [X_1(n), X_3(n), \cdots, X_{N_c-3}(n), X_{N_c-1}(n)]^T, \end{aligned} \quad (1)$$

where $\mathbf{X_e}(n)$ and $\mathbf{X_o}(n)$ actually corresponds to even and odd polyphase component vectors of $\mathbf{X}(n)$,respectively. Then the space-frequency block-coded transmission matrix may be represented by

$$\begin{array}{cc} & \text{frequency} \rightarrow \\ \text{space} \downarrow & \left[\begin{array}{cc} \mathbf{X_e}(n) & -\mathbf{X_o}^*(n) \\ \mathbf{X_o}(n) & \mathbf{X_e}^*(n) \end{array}\right], \end{array} \quad (2)$$

where * stands for complex conjugation.

III. Channel: KL-Basis Expansion Model

Dispersive fading channels are modeled widely by the block fading channel model[20]. According to this model, the channel is assumed to remain constant over a block of a given size and successive blocks may be correlated or independent. This is an approximate model that would be applied to some of the practical communication systems such as OFDM, frequency-hopped spread-spectrum (FHSS) and time-division multiple access (TDMA).

In this paper, it is assumed that the channel is frequency selective during each OFDM symbol [21]and exhibits time selectivity over the OFDM symbols according to Doppler frequency. We consider the Alamouti transmitter diversity coding scheme, employed in an OFDM system utilizing N_c subcarrier per antenna transmissions. Note that N_c is chosen as an even integer. The fading channel between the μth transmit antenna and the receive antenna is described by the baseband equivalent discrete frequency response $\mathbf{H}_\mu(n)$ at the nth time slot.

In wireless mobile communications, channel variations arise mainly due to multipath effect. Consequently, these variations evolve in a progressive fashion and hence fit in some evolution models. It appears that a basis expansion approach would be a natural way of modelling the channel variation [22]. Fourier and Taylor series expansions as well as the polynomial expansion have played a prominent role in deterministic modelling. In contrast, a convenient choice for bases expansion of random processes is Karhunen-Loeve (KL) series. Moreover, the KL expansion methodology has been also used for efficient simulation of the multipath fading environments [16]. Prompted by the general applicability of the KL expansion, we consider in this paper the parameters of $\mathbf{H}_\mu(n)$ to be expressed by a linear combination of orthonormal bases.

An orthonormal expansion of the vector $\mathbf{H}_\mu(n)$ involves expressing the $\mathbf{H}_\mu(n)$ as a linear combination of the orthonormal basis vectors as follows:

$$\mathbf{H}_\mu(n) = \mathbf{\Psi}\mathbf{G}_\mu(n), \quad (3)$$

where $\mathbf{\Psi} = [\boldsymbol{\psi}_0, \boldsymbol{\psi}_1, \cdots, \boldsymbol{\psi}_{N_c-1}]$, $\boldsymbol{\psi}_i$'s are the orthonormal basis vectors, and $\mathbf{G}_\mu(n) = [G_{\mu,0}(n), \cdots, G_{\mu,N_c-1}(n)]^T$ is the vector representing the weights of the expansion. By using different basis functions $\mathbf{\Psi}$, we can generate sets of coefficients with different properties. The autocorrelation matrix $\mathbf{C}_{\mathbf{H}_\mu} = E\left[\mathbf{H}_\mu \mathbf{H}_\mu^\dagger\right]$ can be decomposed as

$$\mathbf{C}_{\mathbf{H}_\mu} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^\dagger, \quad (4)$$

where $\mathbf{\Lambda} = E\{\mathbf{G}_\mu \mathbf{G}_\mu^\dagger\}$ and $\dagger$ denotes the complex transpose. The KL expansion yields $\mathbf{\Lambda}$ in (4) to be a diagonal matrix (i.e., the coefficients are uncorrelated). Then (4) represents an *eigendecomposition* of $\mathbf{C}_{\mathbf{H}_\mu}$. As a result, diagonalization of $\mathbf{C}_{\mathbf{H}_\mu}$ leads to a desirable property that the KL coefficients are uncorrelated. Furthermore, in the Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem becomes equivalent to estimating the i.i.d. Gaussian vector $\mathbf{G}_\mu$, whose coefficients are the KL expansion coefficients.

As mentioned earlier, the channels between transmitter and receiver in this paper are assumed to be doubly-selective where, $\mathbf{H}_\mu(n)$'s have exponentially decaying power delay profiles, described by $\theta(\tau_\mu) = C\exp(-\tau_\mu/\tau_{rms})$. The delays τ_μ are uniformly and independently distributed over the length of the cyclic prefix. τ_{rms} determines the decay of the power-delay profile and C is the normalizing constant. Note that the normalized discrete channel-correlations for different subcarriers and blocks of this channel model were presented in [23] as follows,

$$r_2(k,k') = \frac{1-\exp\left[-L\left(\frac{1}{\tau_{rms}}+\frac{2\pi j(k-k')}{N_c}\right)\right]}{\tau_{rms}(1-\exp(-\frac{L}{\tau_{rms}}))\left(\frac{1}{\tau_{rms}}+\frac{j2\pi(k-k')}{N_c}\right)}, \quad (5)$$

$$r_1(n,n') = J_0(2\pi(n-n')f_dT_s), \quad (6)$$

where, (k,k') denotes different subcarriers, L is the cyclix prefix, N_c is the total number of subcarriers. Also in (6) (n,n') denotes the discrete times for the different OFDM symbols, $J_0(.)$ is the zeroth-order Bessel function of the first kind and f_d is the Doppler frequency.

IV. Received Signal Model

At receiver, after matched filtering and symbol rate sampling, the discrete Fourier transform is applied to the received discrete time signal to obtain $\mathbf{R}(n)$. If $\mathbf{R}(n)$ is parsed into even and odd blocks of $N_c/2$ tones each as , $\mathbf{R}_e(n) = [R_0(n), R_2(n), \cdots, R_{N_c-2}(n)]^T$ and $\mathbf{R}_o(n) = [R_1(n), R_3(n), \cdots, R_{N_c-1}(n)]^T$, the received signal can be expressed in vector form as follows.

$$\begin{aligned}\mathbf{R}_e(n) &= \mathcal{X}_e(n)\mathbf{H}_{1,e}(n) + \mathcal{X}_o(n)\mathbf{H}_{2,e}(n) + \mathbf{W}_e(n) \quad (7)\\ \mathbf{R}_o(n) &= -\mathcal{X}_o^\dagger(n)\mathbf{H}_{1,o}(n) + \mathcal{X}_e^\dagger(n)\mathbf{H}_{2,o}(n) + \mathbf{W}_o(n),\end{aligned}$$

where $\mathcal{X}_e(n)$ and $\mathcal{X}_o(n)$ are $N_c/2 \times N_c/2$ diagonal matrices whose elements are $\mathbf{X_e}(n)$ and $\mathbf{X_o}(n)$, respectively. $\mathbf{H}_{\mu,e}(n) = [H_{\mu,0}(n), H_{\mu,2}(n), \cdots, H_{\mu,N_c-2}(n)]^T$ and $\mathbf{H}_{\mu,o}(n) = [H_{\mu,1}(n), H_{\mu,3}(n) \cdots, H_{\mu,N_c-1}(n)]^T$ are $N_c/2$ length vectors denoting the even and odd component vectors of the channel coefficients between the μth transmitter and the receiver. Finally, $\mathbf{W}_e(n)$ and $\mathbf{W}_o(n)$ are an $N_c/2 \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive Gaussian noise in the N_c tones, with variance σ^2 per dimension.

Equation (7) shows that the information symbols $\mathcal{X}_e(n)$ and $\mathcal{X}_o(n)$ are transmitted twice in two consecutive adjacent subchannel groups through two different channels. In order to estimate the channels and decode $\mathbf{X}(n)$ with the embedded diversity gain through repeated transmission, for each n, we can write the following from (7), assuming the complex channel gains between adjacent subcarriers are approximately constant, i.e., $\mathbf{H}_{1,e}(n) \approx \mathbf{H}_{1,o}(n)$ and $\mathbf{H}_{2,e}(n) \approx \mathbf{H}_{2,o}(n)$.

$$\begin{bmatrix}\mathbf{R}_e(n)\\ \mathbf{R}_o(n)\end{bmatrix} = \begin{bmatrix}\mathcal{X}_e(n) & \mathcal{X}_o(n)\\ -\mathcal{X}_o^\dagger(n) & \mathcal{X}_e^\dagger(n)\end{bmatrix}\begin{bmatrix}\mathbf{H}_{1,e}(n)\\ \mathbf{H}_{2,e}(n)\end{bmatrix} + \begin{bmatrix}\mathbf{W}_e(n)\\ \mathbf{W}_o(n)\end{bmatrix}. \quad (8)$$

The effect of this assumption allows us to omit dependencies between $\mathbf{H}_{1,e}(n)$ and $\mathbf{H}_{2,e}(n)$ on even channel components. Using (8) and dropping subscript "$_e$" in $\mathbf{H}_{1,e}(n)$ and $\mathbf{H}_{2,e}(n)$, we have

$$\begin{bmatrix}\mathbf{R}_e(n)\\ \mathbf{R}_o(n)\end{bmatrix} = \begin{bmatrix}\mathcal{X}_e(n) & \mathcal{X}_o(n)\\ -\mathcal{X}_o^\dagger(n) & \mathcal{X}_e^\dagger(n)\end{bmatrix}\begin{bmatrix}\mathbf{H}_1(n)\\ \mathbf{H}_2(n)\end{bmatrix} + \begin{bmatrix}\mathbf{W}_e(n)\\ \mathbf{W}_o(n)\end{bmatrix}. \quad (9)$$

Finally, (9) can be expressed in a more succinct form as

$$\mathbf{R}(n) = \mathcal{X}(n)\mathbf{H}(n) + \mathbf{W}(n) \;. \quad (10)$$

V. Iterative Channel Estimation

In recent years, inspired by the development of turbo coding, various types of iterative channel estimation, detection and decoding schemes have been proposed in the literature. These approaches have shown that iterative receivers can offer significant performance improvements over the noniterative counterparts. We therefore consider an EM based MAP iterative channel estimation technique in frequency domain for turbo coded SFBC-OFDM systems. Frequency domain estimator presented in this paper was inspired by the conclusions in [24]-[25], where it has been shown time domain channel estimators based on a Discrete Fourier Transform (DFT) approach for non sample-spaced channels cause aliased spectral leakage and result in an error floor.

Details of the algorithm will not be presented here since the EM algorithm has been studied and applied to a number of problems in communications over the years. The reader is suggested to refer [27], [28] for a general exposition to EM algorithm and [17] for its applications to the estimation problem related to the work herein. Basically, this algorithm inductively reestimate $\mathbf{G}$ so that a monotonic increase in the *a posteriori* conditional pdf $p(\mathbf{R}|\mathbf{G})$ is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q(\mathbf{G}|\mathbf{G}^{(q)}) = \sum_{\mathcal{X}} p(\mathbf{R},\mathcal{X},\mathbf{G}^{(q)})\log p(\mathbf{R},\mathcal{X},\mathbf{G}), \quad (11)$$

where $\mathbf{G}^{(q)}$ is the estimation of $\mathbf{G}$ at the qth iteration.

Note that, the term $\log p(\mathbf{R},\mathcal{X},\mathbf{G})$ in (11) can be expressed as [35],

$$\log p(\mathbf{R},\mathcal{X},\mathbf{G}) = \log p(\mathcal{X}\,|\,\mathbf{G}) + \log p(\mathbf{R}\mid\mathcal{X},\mathbf{G}) + \log p(\mathbf{G}). \quad (12)$$

The first term on the right hand side of (12) is constant, since, the data sequence $\mathcal{X} = \{X_k(n)\}$ and $\mathbf{G}$ are independent of each other and $\mathcal{X}$ have equal *a priori* probability. Moreover, the *a priori* PDF of the KL expansion coefficients $\mathbf{G}$ can be expressed as $p(\mathbf{G}) \sim \exp(-\mathbf{G}^\dagger\widetilde{\mathbf{\Lambda}}^{-1}\mathbf{G})$ where $\mathbf{G} = [\mathbf{G}_1^T, \mathbf{G}_2^T]^T$, $\widetilde{\mathbf{\Lambda}} = \mathrm{diag}(\mathbf{\Lambda}\;\;\mathbf{\Lambda})$. Also, since the noise samples are independent, it follows from (7) that the second and third terms on the right hand side of (12) can be written as

$$\log p(\mathbf{R}|\mathcal{X},\mathbf{G}) \sim -\left[\mathbf{R}_e(n)-\mathcal{X}_e(n)\mathbf{H}_1-\mathcal{X}_o(n)\mathbf{H}_2\right]^\dagger \times\mathbf{\Sigma}^{-1}\left[\mathbf{R}_e(n)-\mathcal{X}_e(n)\mathbf{H}_1-\mathcal{X}_o(n)\mathbf{H}_2\right] -\left[\mathbf{R}_o(n)+\mathcal{X}_o^\dagger(n)\mathbf{H}_1-\mathcal{X}_e^\dagger(n)\mathbf{H}_2\right]^\dagger \times\mathbf{\Sigma}^{-1}\left[\mathbf{R}_o(n)+\mathcal{X}_o^\dagger(n)\mathbf{H}_1-\mathcal{X}_e^\dagger(n)\mathbf{H}_2\right],$$

$$\log p(\mathbf{G}) \sim -\mathbf{G}_1^\dagger\mathbf{\Lambda}^{-1}\mathbf{G}_1-\mathbf{G}_2^\dagger\mathbf{\Lambda}^{-1}\mathbf{G}_2\ , \qquad (13)$$

where $\mathbf{\Sigma}$ is an $N_c/2\times N_c/2$ diagonal matrix with $\mathbf{\Sigma}[k,k]=\sigma^2$, for $k=0,1,\cdots,N_c/2-1$.

Taking derivatives in (11) with respect to $\mathbf{G}_1$ and $\mathbf{G}_2$, along with the fact that $\|\mathcal{X}_e(n)\|^2=\|\mathcal{X}_o(n)\|^2=\frac{1}{2}\mathbf{I}$, and equating the resulting equations to zero, the expression of the reestimate $\hat{\mathbf{G}}_\mu^{(q+1)}$ $(\mu=1,2)$ for SFBC-OFDM can be obtained as follows:

$$\hat{\mathbf{G}}_1^{(q+1)}=(\mathbf{I}+\mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{\Psi}^\dagger\left[\hat{\mathcal{X}}_e^{\dagger(q)}\mathbf{R}_e(n)-\hat{\mathcal{X}}_o^{(q)}\mathbf{R}_o(n)\right]$$
$$\hat{\mathbf{G}}_2^{(q+1)}=(\mathbf{I}+\mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{\Psi}^\dagger\left[\hat{\mathcal{X}}_o^{\dagger(q)}\mathbf{R}_e(n)+\hat{\mathcal{X}}_e^{(q)}\mathbf{R}_o(n)\right] \quad (14)$$

It can be easily seen that $(\mathbf{I}+\mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}=\mathrm{diag}([(1+\sigma^2/\lambda_0)^{-1},\cdots,(1+\sigma^2/\lambda_{\frac{N_c}{2}-1})^{-1}])$ and $\hat{\mathcal{X}}_e^{(q)}$, $\hat{\mathcal{X}}_o^{(q)}$ in (14) are an $\frac{N_c}{2}\times\frac{N_c}{2}$ dimensional diagonal matrices whose diagonal elements are estimated values of the coded symbols $\hat{\mathbf{X}}_e^{(q)}$, $\hat{\mathbf{X}}_o^{(q)}$ obtained at the qth iteration step.

Initialization: For initialization of the EM algorithm leading to channel estimation, a small number of pilot symbols are inserted in each OFDM frame, known by the receiver. Corresponding to pilot symbols, we focus on an under-sampled signal model and employ the linear minimum mean-square error (LMMSE) estimate to obtain the under-sampled channel coefficients. Then the complete initial channel coefficients are easily determined using an interpolation technique, i.e., Lagrange interpolation algorithm. Finally, the initial values for the $\mathbf{G}_\mu^{(0)}$ are used in the iterative EM algorithm to avoid divergence. The details of the initialization process is presented in [15], [23].

Truncation property: The truncated basis vector $\mathbf{G}_{\mu,r}$ can be formed by selecting r orthonormal basis vectors among all basis vectors that satisfy $\mathbf{C}_{\mathbf{H}_\mu}\mathbf{\Psi}=\mathbf{\Psi}\mathbf{\Lambda}$. The optimal solution that yields the smallest average mean-squared truncation error $\frac{1}{N_c/2}\,E[\boldsymbol{\epsilon}_r^\dagger\boldsymbol{\epsilon}_r]$ is the one expanded with the orthonormal basis vectors associated with the first largest r eigenvalues as given by

$$\frac{1}{N_c/2-r}\,E[\boldsymbol{\epsilon}_r^\dagger\boldsymbol{\epsilon}_r]=\frac{1}{N_c/2-r}\sum_{i=r}^{N_c/2-1}\lambda_i, \qquad (15)$$

where $\boldsymbol{\epsilon}_r=\mathbf{G}_\mu-\mathbf{G}_{\mu,r}$. For the problem at hand, truncation property of the KL expansion results in a low-rank approximation as well. Thus, a rank-r approximation of $\mathbf{\Lambda}$ can be defined as $\mathbf{\Lambda}_r=diag\{\lambda_0,\lambda_1,\cdots,\lambda_{r-1}\}$ by ignoring the trailing $N_c/2-r$ variances $\{\lambda_l\}_{l=r}^{N_c/2-1}$, since they are very small compared to the leading r variances $\{\lambda_l\}_{l=0}^{r-1}$. Actually, the pattern of eigenvalues for $\mathbf{\Lambda}$ typically splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious.

Complexity: Based on the approach presented in [23], the traditional LMMSE estimation for $\mathbf{H}_\mu$ can be easily expressed as

$$\hat{\mathbf{H}}_\mu=\underbrace{\mathbf{C}_{\mathbf{H}_\mu}(\mathbf{\Sigma}+\mathbf{C}_{\mathbf{H}_\mu})^{-1}}_{Precomputed}\mathbf{P}_\mu, \qquad \mu=1,2$$

where $\mathbf{P_1}=\hat{\mathcal{X}}_e^{\dagger(q)}\mathbf{R}_e(n)-\hat{\mathcal{X}}_o^{(q)}\mathbf{R}_o(n)$ and $\mathbf{P_2}=\hat{\mathcal{X}}_o^{\dagger(q)}\mathbf{R}_e(n)+\hat{\mathcal{X}}_e^{(q)}\mathbf{R}_o(n)$. Since $\mathbf{C}_{\mathbf{H}_\mu}(\mathbf{\Sigma}+\mathbf{C}_{\mathbf{H}_\mu})^{-1}$ does not change with data symbols, its inverse can be pre-computed and stored during each OFDM block. Since $\mathbf{C}_{\mathbf{H}_\mu}$ and $\mathbf{\Sigma}$ are assumed to be known at the receiver, the estimation algorithm in (16) requires $N_c^2/4$ complex multiplications [1] after precomputation. However, this direct approach has high computational complexity due to required large-scale matrix inversion [2] of the precomputation matrix. Moreover, the error caused by the small fluctuations in $\mathbf{C}_{\mathbf{H}_\mu}$ and $\mathbf{\Sigma}$ have an amplified effect on the channel estimation due to the matrix inversion. Furthermore, this effect becomes more severe as the dimension of the matrix, to be inverted, increases [26]. Therefore, the KL based approach is need to avoid matrix inversion. Using the equations (3) and (14), the iterative estimate of $\mathbf{H}_\mu$ with KL expansion can be obtained as

$$\hat{\mathbf{H}}_\mu^{(q+1)}=\mathbf{\Psi}((\mathbf{I}+\mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1})\mathbf{\Psi}^\dagger\mathbf{P}_\mu. \qquad (16)$$

To reduce the complexity of the estimator further, we proceed with the low-rank approximations by considering only r column vectors of $\mathbf{\Psi}$ corresponding to the r largest eigenvalues of $\mathbf{\Lambda}$.

$$\hat{\mathbf{H}}_\mu^{(q+1)} = \mathbf{\Psi}_r\underbrace{((\mathbf{I}+\mathbf{\Sigma}_r\mathbf{\Lambda}_r^{-1})^{-1})\mathbf{\Psi}_r^\dagger}_{Precomputed}\mathbf{P}_\mu, \qquad (17)$$

where $((\mathbf{I}+\mathbf{\Sigma}_r\mathbf{\Lambda}_r^{-1})^{-1})=diag(\frac{\lambda_0}{\lambda_0+\sigma^2},\cdots,\frac{\lambda_{r-1}}{\lambda_{r-1}+\sigma^2})$. $\mathbf{\Sigma}_r$ in (17) is a $r\times r$ diagonal matrix whose elements are equal to σ^2 and $\mathbf{\Psi}_r$ is an $N_c/2\times r$ matrix which can be formed by omitting the last $N_c/2-r$ columns of $\mathbf{\Psi}$. The low-rank estimator is shown to require N_cr complex multiplications [3]. In comparison with the estimator (Traditional) the number of multiplications has been reduced from $N_c/4$ to r per tone.

VI. Iterative Channel Equalization and Decoding

We now consider the SFBC-OFDM decoding algorithm and the MAP outer channel code decoding to complete the description of the Turbo receiver.

A. SFBC-OFDM Decoding Algorithm

Since the channel vectors or equivalently the KL-expansion coefficients are estimated through EM based iterative approach, it is possible to decode $\mathbf{R}$ with diversity gains by

[1]Multiplication of $N_c/2\times N_c/2$ precomputation matrix with $N_c/2\times 1$ P_μ vector.

[2]The computational complexity of an $N_c/2\times N_c/2$ matrix inversion, using Gaussian elimination is $O((N_c/2)^3)$.

[3]First, multiplication of precomputation matrix with P_μ, has $\frac{N_cr}{2}$ complex multiplications and then multiplication with $\mathbf{\Psi}_r$ has $\frac{N_cr}{2}$ complex multiplication which totally requires N_cr complex multiplication.

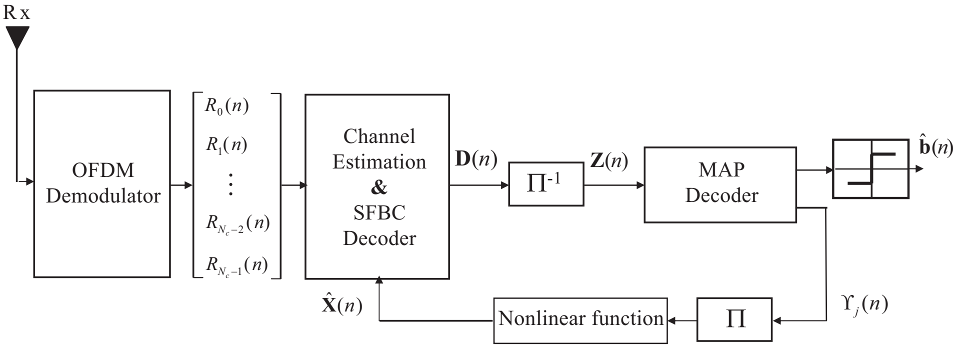

Fig. 2. Turbo receiver structure for SFBC-OFDM systems.

a simple matrix multiplication. Before dealing with how we resolve decoding, let us first re-express the received signal model (7) as follows.

$$\tilde{\mathbf{R}}(n) = \mathcal{H}(n)\tilde{\mathbf{X}}(n) + \tilde{\mathbf{W}}(n), \tag{18}$$

where $\tilde{\mathbf{R}}(n) = [\mathbf{R}_e^T(n), \mathbf{R}_o^\dagger(n)]^T$, $\tilde{\mathbf{X}}(n) = [\mathbf{X}_e^T(n), \mathbf{X}_o^T(n)]^T$, $\tilde{\mathbf{W}}_k(n) = [\mathbf{W}_e^T(n),\ \mathbf{W}_o^\dagger(n)]^T$ and

$$\mathcal{H}(n) = \begin{bmatrix} \mathcal{H}_{1,e}(n) & \mathcal{H}_{2,e}(n) \\ \mathcal{H}_{2,o}^\dagger(n) & -\mathcal{H}_{1,o}^\dagger(n) \end{bmatrix}. \tag{19}$$

Here, $\mathcal{H}_{\mu,e}(n)$ and $\mathcal{H}_{\mu,o}(n)$ $\mu = 1,2$ are $N_c/2 \times N_c/2$ diagonal matrices whose elements are $\mathbf{H}_{\mu,e}(n) = [H_{\mu,0}(n), H_{\mu,2}(n), \cdots,\ H_{\mu,N_c-2}(n)]^T$ and $\mathbf{H}_{\mu,o}(n) = [H_{\mu,1}(n), H_{\mu,3}(n) \cdots,\ H_{\mu,N_c-1}(n)]^T$ respectively.

Depending on complexity versus performance tradeoffs, any linear equalizer can be applied to retrieve $\tilde{\mathbf{X}}(n)$ from (18). In this paper, we consider a linear equalizer where the parameters are updated using the MMSE criterion. Given the observation $\tilde{\mathbf{R}}(n)$, the linear MMSE estimate $\mathbf{D}(n)$ of $\tilde{\mathbf{X}}(n)$ is given as follows [29].

$$\begin{aligned} \mathbf{D}(n) &= \bar{\tilde{\mathbf{X}}} + \mathbf{C}_{\tilde{\mathbf{X}}}\mathcal{H}(\mathbf{n})^\dagger(\mathcal{H}(\mathbf{n})^\dagger\mathbf{C}_{\tilde{\mathbf{X}}}\mathcal{H}(\mathbf{n}) + \mathbf{C}_{\tilde{\mathbf{W}}})^{-\mathbf{1}} \\ &\times \ (\tilde{\mathbf{R}}(n) - \mathcal{H}(n)\bar{\tilde{\mathbf{X}}}(\mathbf{n})), \end{aligned} \tag{20}$$

where $\bar{\tilde{\mathbf{X}}}$ and $\mathbf{C}_{\tilde{\mathbf{X}}}$ are the mean and covariance of $\tilde{\mathbf{X}}(n)$, respectively. $\mathbf{C}_{\tilde{\mathbf{W}}}$ is the covariance of $\tilde{\mathbf{W}}(n)$.

With a scaled unitary matrix $\mathcal{H}(n)$ and approximately constant complex channel gains with $\mathcal{H}_{1,e,k}^2 + \mathcal{H}_{2,e,k}^2 \approx 1$ assumptions, $\mathcal{H}^\dagger(n)\mathcal{H}(n)$ can be simplified as

$$\mathcal{H}(n)^\dagger\mathcal{H}(n) = \begin{bmatrix} \mathcal{H}_{1,e}^2 + \mathcal{H}_{2,e}^2 & \mathbf{0} \\ \mathbf{0} & \mathcal{H}_{1,e}^2 + \mathcal{H}_{2,e}^2 \end{bmatrix} = \mathbf{I}_{N_c \times N_c}, \tag{21}$$

where $\mathbf{I}_{N_c \times N_c}$ is the $N_c \times N_c$ identity matrix. Moreover, following the assumptions used in [29], $\bar{\tilde{\mathbf{X}}} = \mathbf{0}$ and $\mathbf{C}_{\tilde{\mathbf{X}}} = \frac{1}{2}\mathbf{I}$, then (20) becomes

$$\mathbf{D}(n) = (\mathbf{I} + 2\sigma_n^2\mathbf{I})^{-1}\mathcal{H}^\dagger\tilde{\mathbf{R}}(n). \tag{22}$$

If we set $\mathbf{C}_{\tilde{\mathbf{W}}} = \mathbf{0}$ in (20), a further simplified form of the linear equalizer is obtained as follows.

$$\mathbf{D}(n) = \mathcal{H}^\dagger(n)\tilde{\mathbf{R}}(n) = \mathcal{H}^\dagger(n)\mathcal{H}(n)\tilde{\mathbf{X}}(n) + \boldsymbol{\eta}(n), \tag{23}$$

where $\boldsymbol{\eta}(n) = \mathcal{H}^\dagger(n)\tilde{\mathbf{W}}(n)$.

The Turbo receiver structure proposed in this paper for SFBC-OFDM systems consists of three submodules: (i) an iterative MAP-EM channel estimator, (ii) SFBC decoder and (iii) a soft MAP outer-channel-code decoder. As shown in Fig. 2 [34], first the EM based channel estimator computes the channel coefficients by means of the pilot symbols as described in the initialization step to use in the SFBC demodulator (23). Then, the equalized symbol sequence $\{\mathbf{D}(n)\}$ is passed through a channel deinterleaver, resulting in a deinterleaved equalized symbols sequence $\{\mathbf{Z}(n)\}$. Finally, $\{\mathbf{Z}(\mathrm{n})\}$ is applied to the MAP decoder submodule along with the deinterleaved estimated channel gains where the log-likelihood ratio's (LLRs) of the posteriori probabilities are computed based on the coded symbols and the uncoded bits[31], [32]. In the next iteration step, the LLRs of coded symbols $\{\Upsilon_j\}$ are deinterleaved and passed through a nonlinearity (see Appendix II for details.) yielding a soft estimate of $\mathbf{X}(n)$ as $\hat{\mathbf{X}}(n)$ as shown in Fig. 2. $\hat{\mathbf{X}}(n)$ is used in the form of $\hat{\mathcal{X}}_e^{(q)}$ and $\hat{\mathcal{X}}_o^{(q)}$ in (17) for the $(q+1)$th iteration. Thus, the MAP-EM channel estimator iteratively generates the channel estimates by taking the received signals from receiver antennas and the interleaved soft values of the LLRs which are computed by the outer channel code decoder at the previous iteration. Then, SFBC-OFDM decoder takes the channel estimates together with the received signals and computes the equalized symbol sequence for the next turbo iteration. Iterative operation is realized among these three submodules.

B. MAP Outer Channel Code Decoder

Maximum a Posterior (MAP) outer channel decoder takes the deinterleaved equalized symbol sequence and the corresponding fading amplitude values [32] as input and compute the extrinsic LLRs of the outer channel code bits as well as the hard decisions of the information bits $\{\hat{\mathbf{b}}_i\}$, at the last turbo iteration [31].

1) Details of The MAP Algorithm : The MAP algorithm provides the conditional probability of each coded symbol, C_k, taking values $-a$ and $+a$, given that the deinterleaved equalized symbol sequence is $\mathbf{Z}_0^{N_c-1} = [Z_0, \cdots, Z_k, \cdots, Z_{N_c-1}]^T$. Then, the LLR of these probabilities is calculated as

$$L(C_k) = \log\frac{P(C_k = -a|\mathbf{Z}_0^{N_c-1})}{P(C_k = a|\mathbf{Z}_0^{N_c-1})}. \tag{24}$$

Let S_k and C_k be the encoder state at time k and the encoded symbol, associated with the transition from step $k-1$ to step k. And, let trellis states at step $k-1$ and step k be indexed by the integer m' and m , respectively. Then, (24) can be expressed as

$$L(C_k)=\log \frac{\sum_{m'}\sum_{m} P(C_k=-a, S_{k-1}=m', S_k=m, \mathbf{Z}_0^{N_c-1})}{\sum_{m'}\sum_{m} P(C_k=a, S_{k-1}=m', S_k=m, \mathbf{Z}_0^{N_c-1})}. \quad (25)$$

Using the Bayes' rule and the fact that the channel is assumed memoryless, the joint probability $P(C_k, S_{k-1} = m', S_k = m, \mathbf{Z}_0^{N_c-1})$ may be written as a product of three independent probabilities

$$\begin{aligned} &= P(C_k, S_{k-1}=m', S_k=m, \mathbf{Z}_0^{k-1}, Z_k, \mathbf{Z}_{k+1}^{N_c-1}) \quad (26)\\ &= \underbrace{P(\mathbf{Z}_{k+1}^{N_c-1}|S_k=m, Z_k, S_{k-1}=m', \mathbf{Z}_0^{k-1}, C_k)}_{P(\mathbf{Z}_{k+1}^{N_c-1}|S_k=m, C_k)}\\ &\quad\times \underbrace{P(S_k=m, Z_k|S_{k-1}=m', \mathbf{Z}_0^{k-1}, C_k)}_{P(S_k=m, Z_k|S_{k-1}=m', C_k)}\\ &\quad\times P(S_{k-1}=m', \mathbf{Z}_0^{k-1}, C_k)\ . \end{aligned}$$

Applying now the Markovian property, (26) may be simplified as follows.

$$\begin{aligned} &P(C_k,S_{k-1}=m',S_k=m,\mathbf{Z}_0^{N_c-1})=P(\mathbf{Z}_{k+1}^{N_c-1}|S_k=m, C_k) \quad (27)\\ &\times P(S_k=m,Z_k|S_{k-1}=m',C_k)P(S_{k-1}=m',\mathbf{Z}_0^{k-1}, C_k). \end{aligned}$$

The joint likelihood function in (27) can be recursively computed by means of the forward and backward variables $\alpha_k(m)$, $\beta_k(m)$ and the transition probabilities $\gamma_k(m', m)$, which are defined as

$$\begin{aligned} \alpha_k(m) &\triangleq P(S_k=m, \mathbf{Z}_0^k, C_{k+1}) \quad (28)\\ \beta_k(m) &\triangleq P(\mathbf{Z}_{k+1}^{N_c-1}|S_k=m, C_k)\\ \gamma_k(m',m) &\triangleq P(S_k=m, Z_k|S_{k-1}=m', C_k)\ . \end{aligned}$$

Substituting $\alpha_k(m)$, $\beta_k(m)$ and $\gamma_k(m', m)$ in (24), the conditional LLRs of C_k, given the received sequence $\mathbf{Z}_0^{N_c-1}$, can be rewritten

$$L(C_k)=\log \frac{\sum_{m}\sum_{m'}^{C_k=-a} \alpha_{k-1}(m')\ \gamma(m',m)\ \beta_k(m)}{\sum_{m}\sum_{m'}^{C_k=+a} \alpha_{k-1}(m')\ \gamma(m',m)\ \beta_k(m)}\ . \quad (29)$$

Finally, the recursive computations of $\alpha_k(m)$, $\beta_k(m)$ are given by

$$\begin{aligned} \alpha_k(m) &= \sum_{m'}\alpha_{k-1}(m')\gamma_k(m',m) \qquad k=0,1,...,N_c-1\\ \beta_k(m) &= \sum_{m'}\beta_{k+1}(m')\gamma_{k+1}(m,m') \quad k=N_c-1,...,0 \quad (30) \end{aligned}$$

and $\gamma_k(m', m)$ can be computed from

$$\begin{aligned} \gamma_k(m',m) &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(Z_k-(\hat{H}_k^\dagger \hat{H}_k)C_k(m',m))^2}{2\sigma^2}\right]\\ &\times \quad P(S_k=m|S_{k-1}=m'), \quad (31) \end{aligned}$$

where C_k, Z_k and $\hat{H}_k$ are the encoded symbol, deinterleaved equalized codeword and the estimated channel coefficient, respectively. σ^2 is the Gaussian noise variance.

2) Log-MAP: The equations for the MAP algorithm presented above may be written in the logarithmic domain. Firstly, we express $\alpha_k(m)$, $\beta_k(m)$ and $\gamma_k(m', m)$ in the logarithmic domain as

$$\begin{aligned} \mathrm{A}_k(m) &= \log \alpha_k(m) \quad (32)\\ \mathrm{B}_k(m) &= \log \beta_k(m)\\ \Gamma_k(m',m) &= \log \gamma_k(m',m). \end{aligned}$$

Then the recursive equations can be derived for $\mathrm{A}_k(m)$ and $\mathrm{B}_k(m)$ easily as follows.

$$\begin{aligned} \mathrm{A}_k(m) &= \log \sum_{m'} e^{\mathrm{A}_{k-1}(m')+\Gamma_k(m',m)} \quad (33)\\ \mathrm{B}_k(m) &= \log \sum_{m'} e^{\mathrm{B}_{k+1}(m')+\Gamma_{k+1}(m,m')}\ . \end{aligned}$$

Substituting (33) in (29) yields,

$$\begin{aligned} L(C_k) &= \underbrace{\log \sum_{m'}\sum_{m} e^{\Gamma_k(m',m)+\mathrm{A}_{k-1}(m')+\mathrm{B}_k(m)}}_{for\quad C_k=-a}\\ &\quad - \underbrace{\log \sum_{m'}\sum_{m} e^{\Gamma_k(m',m)+\mathrm{A}_{k-1}(m')+\mathrm{B}_k(m)}}_{for\quad C_k=a}\ . \quad (34) \end{aligned}$$

If we define the $\mathbb{E}$ operator as $\underset{m}{\mathbb{E}}(x_m) = \log_m \sum e^{x_n}$ then, $L(C_k)$ can be expressed as,

$$\begin{aligned} L(C_k) = &\underset{\substack{m',m\\C_k=-a}}{\mathbb{E}} [\Gamma_k(m',m)+\mathrm{A}_{k-1}(m')+\mathrm{B}_k(m)]\\ &- \underset{\substack{m',m\\C_k=a}}{\mathbb{E}} [\Gamma_k(m',m)+\mathrm{A}_{k-1}(m')+\mathrm{B}_k(m)]\ . \quad (35) \end{aligned}$$

Calculation of the LLRs according to equation (35) for encoded bits with generator matrices g(5,7) and g(1,5/7) are demonstrated in Appendix III

VII. Computer Simulations

In this section, computer simulations are carried out to evaluate the performance of the proposed turbo receiver structure for SFBC-OFDM systems. To understand the behavior of different channel encoders, we simulated both turbo and convolutionally coded SFBC-OFDM systems. In case of Turbo Encoder, two identical recursive systematic convolutional component codes (RSC) with generator ($1,5_8/7_8$) concatenated in parallel via a pseudorandom interleaver formed the encoder [31], [32]. For the convolutionally coded system, a (5_8,7_8) code with rate 1/2 code was used.

The scenario for SFBC-OFDM simulation study is as follows: A BPSK modulation format is employed and the number of transmitting and receiving antennas are chosen as $T = 2$ and $R = 1$ respectively. The system has a 2.4 MHz bandwidth (for the pulse roll-off factor $\alpha = 0.2$) and is divided into $N_c = 512$ tones with a total period T_s of 136 μs, of which 8 μs constitute the cyclic prefix ($L = 32$). The data rate is 1.9 Mbit/s. We assume that the rms value of the multipath width is $\tau_{rms} = 1$ sample (0.25 μs) for the power-delay profile. In all simulations, three iterations are employed.

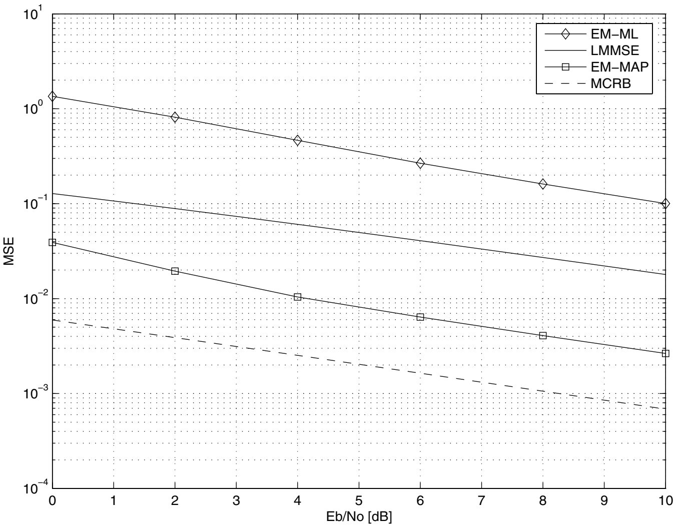

Fig. 3. MSE performance of SFBC-Turbo-OFDM turbo receivers, PIR=1:8.

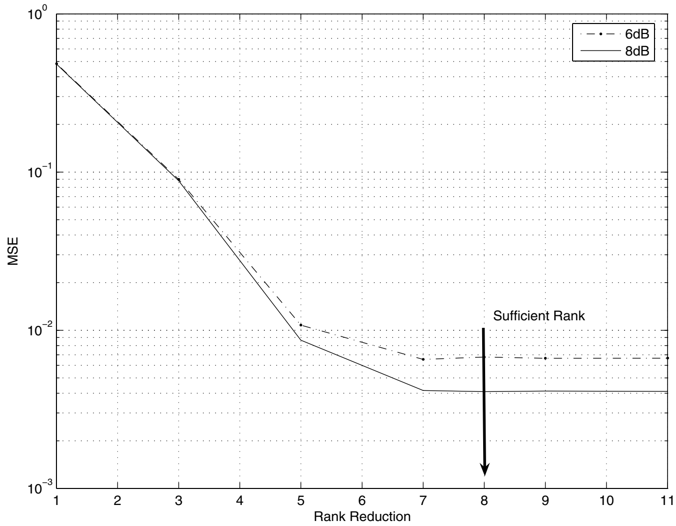

Fig. 5. MSE performance of SFBC-Turbo-OFDM Systems according to number of used KL coefficients, PIR=1:8.

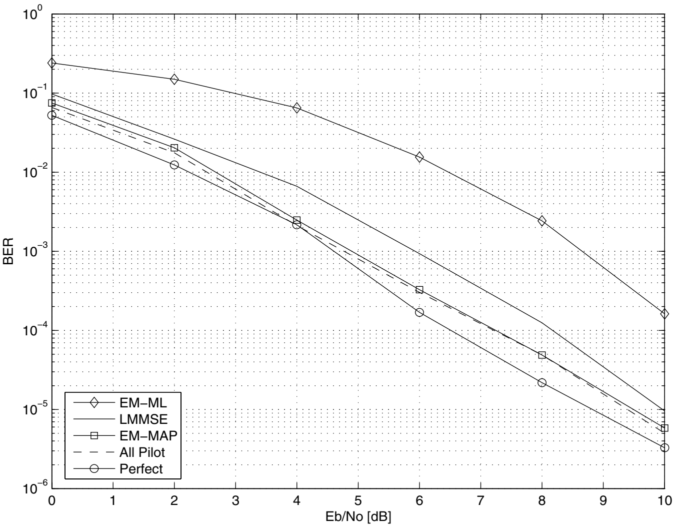

Fig. 4. BER performance of SFBC-Turbo-OFDM turbo receivers, PIR=1:8.

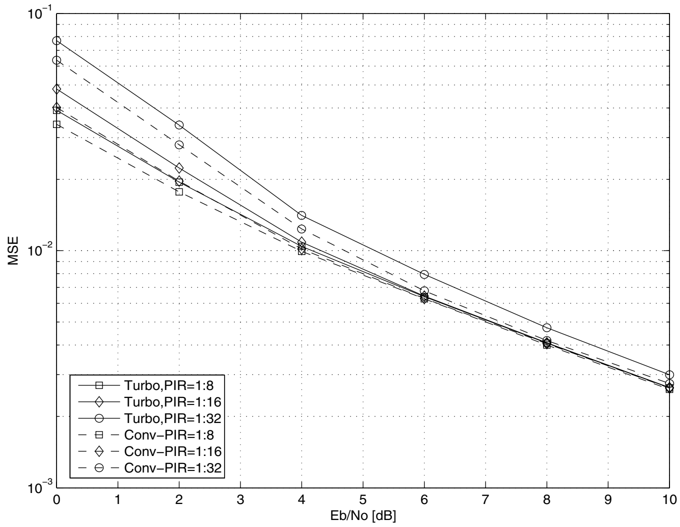

Fig. 6. MSE performance of the EM algorithms as a function of PIRs.

Fig. 3 compares the MSE performance of the EM-MAP channel estimation approach with EM-ML [13] and a widely used LMMSE pilot symbol assisted modulation (LMMSE-PSAM)schemes [14], as well as with the Modified Cramer Rao Bound (MCRB) for turbo coded SFBC-OFDM systems. Pilot Insertion Rate was chosen as (PIR) =1:8. That is one pilot is inserted for every 8 data symbols. It is observed that the proposed EM-MAP significantly outperforms the EM-ML as well as PSAM techniques and approaches the MCRB for higher Eb/No values. Moreover, in Fig. 4, the BER performance of proposed system is compared with the all-pilot and perfect channel cases for the turbo coded and the convolutionally coded systems.

The optimal truncation property of KL expansion minimizes the amount of information required to represent the statistically dependent data. Thus, this property can further reduce computational load on the channel estimation algorithm. If the number of parameters in the expansion include dominant eigenvalues (Rank=8), it is possible to obtain an excellent approximation with a relatively small number of KL coefficients. The optimal truncation property of the KL expansion is exploited in Fig. 5 and the simulation results are presented for the MSE performance.

It is clear that good channel codes are more sensitive to the poorly estimated channel. With high correlation between the coded bits, a well designed channel code is more sensitive to channel estimation errors which might cause severe error propagation in the decoding process. Therefore, it is expected that BER performance of turbo coded structures degrades more than convolutionally coded structures. Thus, lower pilot insertion rates provide poor initial estimates, resulting in a more BER performance degradation in turbo coded systems as compared to convolutionally encoded systems. This effect is demonstrated in Fig. 6 by the MSE performance curves for different values of PIR=1:8,1:16,1:32. Although a slight MSE performance difference is observed in Fig. 6,for chosen different pilot densities, its effect on the BER performance is more obvious especially at higher SNR values as seen

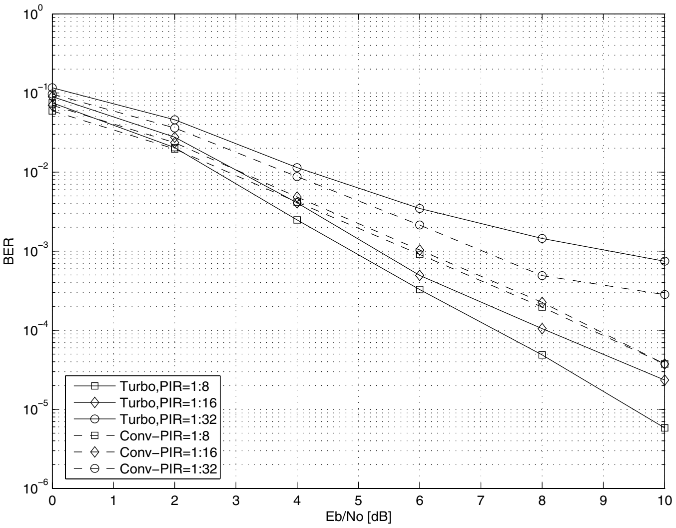

Fig. 7. BER performance of the EM algorithms as a function of PIRs.

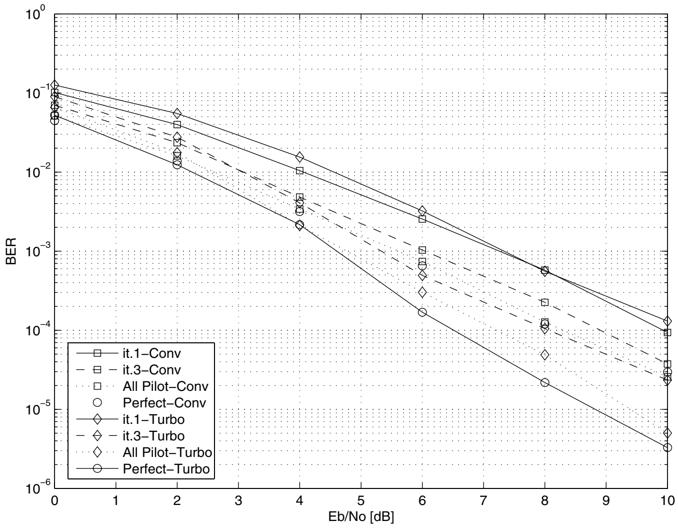

Fig. 9. BER performance of the turbo receiver structures as a function of average Eb/No, PIR=1:16.

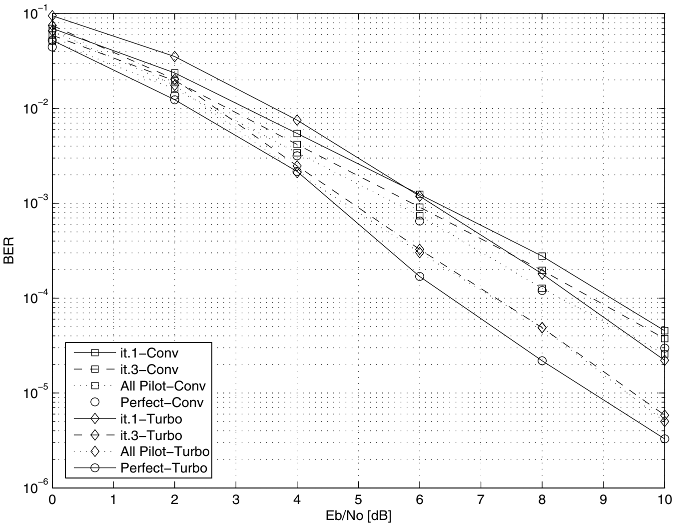

Fig. 8. BER performance of the turbo receiver structures as a function of average Eb/No, PIR=1:8.

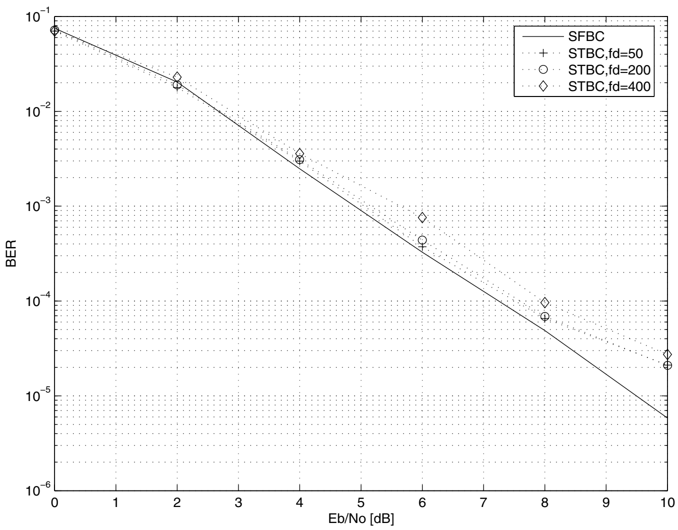

Fig. 10. BER performance of Transmit Diversity Coded Turbo-OFDM Systems, PIR=1:8.

in Fig. 7. Moreover, a slightly better MSE performance is observed in convolutional codes compared to the turbo code due to punctured structure of turbo codes.

The BER performances of the proposed systems are presented together with initial values for $PIR = 1:8$ and $PIR = 1:16$ in Fig. 8 and Fig. 9 respectively. All pilot and perfect channel cases for the turbo and convolutionally coded systems are included as well. Due to its structure, the turbo decoding evidently provides more reliable priori information for iterative processing compared to the convolutional decoding. Therefore, as seen in Fig. 8, the BER performance improvement observed due to iterative turbo coding structure is 1.3 dB for $Pe = 10^{-4}$, meanwhile it is 0.3 dB for convolutionally coded systems.

Although overall performance degrades by decreasing pilot insertion rate, iterative decoding provides better performance improvements on each iteration for the lower pilot insertion rate (e.g. $PIR = 1:16$). From simulation studies, it is observed in Fig. 9 that iterative performance gain is increased for PIR=1:16 as 2.3 dB and 1 dB, for turbo and convolutionally coded systems, respectively. As previously mentioned, turbo coding is more sensitive to channel estimation performance than the convolutional coding. The iterative performance gain increment for turbo coded structure is therefore lower than the convolutionally coded systems.

In space-frequency coding, detection is accomplished completely within a single OFDM block. Therefore the Doppler shift will not affect SFBC-OFDM system for the channel model we consider in this work. In Fig. 10, the computer simulation results are presented for the BER performances of both space-time and space-frequency block coded turbo aided receivers in the presence of different Doppler frequencies. We observe in Fig. 10 that at higher Doppler frequencies, the space-frequency block coded structure is superior and an error floor effect is observed in the BER performance of the space-time structure. Thus, we conclude that the space-frequency coded OFDM turbo receiver structure is a good candidate for doubly selective block fading channels.

VIII. CONCLUSIONS

We consider the design of turbo receiver structures for SFBC-OFDM systems in unknown frequency selective fading channels. By using extrinsic information of the transmitted symbols, both the channel estimation and the decoding process can be improved. The turbo structure performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing BPSK modulation scheme. Moreover, the MAP-EM approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. The optimal truncation property of the KL expansion can reduce computational load on the iterative estimation approach. The performance merits of our channel estimation algorithm is confirmed by corroborating computer simulations. It is observed that the proposed EM-MAP significantly outperforms the EM-ML as well as PSAM techniques. Furthermore, sensitivity to channel estimation errors of turbo receivers are investigated for turbo coded and convolutionally coded SFBC-OFDM systems. One of the important conclusions of the paper is that turbo codes are more sensitive to channel parameter estimation errors than the convolutional coded systems. It has been shown that receiver with turbo codes performs well over the convolutional receiver structure when the quality of the channel estimation performance is high. Moreover, superiority of the turbo coded SFBC-OFDM systems over the turbo coded STBC-OFDM systems is observed especially for high Doppler frequencies.

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APPENDIX I
MODIFIED CRAMER-RAO BOUND

The modified Fisher information matrix (FIM) can be obtained by a straightforward modification of FIM as[15],

$$\mathbf{J}_M(\mathbf{G}) \triangleq \underbrace{-E[\frac{\partial^2 \ln p(\mathbf{R}|\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T}]}_{\mathbf{J}(\mathbf{G})} \underbrace{-E[\frac{\partial^2 \ln p(\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T}]}_{\mathbf{J}_P(\mathbf{G})}, \tag{36}$$

where $\mathbf{J}_P(\mathbf{G})$ represents the *a priori* information.

Under the assumption that $\mathbf{G}$ and $\mathbf{W}(n)$ are independent of each other and $\mathbf{W}(n)$ is a zero-mean, the transmitted symbols become uncorrelated due to the channel interleaver. The conditional PDF $\mathbf{R}$ given $\mathbf{G}$ is given as

$$\begin{aligned} p(\mathbf{R}|\mathbf{G}) &= E_{\mathcal{X}}\left\{p(\mathbf{R}|\mathcal{X},\mathbf{G})\right\} \\ &\sim \frac{1}{\sigma^2} E_{\mathcal{X}}\Big\{(\mathbf{R}-\mathcal{X}\widetilde{\mathbf{\Psi}}\mathbf{G})^\dagger(\mathbf{R}-\mathcal{X}\widetilde{\mathbf{\Psi}}\mathbf{G})\Big\}, \end{aligned} \tag{37}$$

where $\widetilde{\mathbf{\Psi}} = \mathrm{diag}(\mathbf{\Psi}\ \mathbf{\Psi})$ From (37), the derivatives can be taken as follows.

$$\frac{\partial \ln p(\mathbf{R}|\mathbf{G})}{\partial \mathbf{G}^T} = \frac{1}{\sigma^2}(\mathbf{R}-\mathcal{X}\widetilde{\mathbf{\Psi}}\mathbf{G})^\dagger \mathcal{X}\widetilde{\mathbf{\Psi}} \tag{38}$$

$$\frac{\partial^2 \ln p(\tilde{\mathbf{R}}|\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T} = -\frac{1}{\sigma^2}\widetilde{\mathbf{\Psi}}^\dagger \mathcal{X}^\dagger \mathcal{X}\widetilde{\mathbf{\Psi}}\ . \tag{39}$$

Since, space frequency transition matrix provide $\mathcal{X}^\dagger\mathcal{X} = \mathbf{I}$ and using $\widetilde{\mathbf{\Psi}}^\dagger\widetilde{\mathbf{\Psi}} = \mathbf{I}$ and taking the expected values yields the following simple form:

$$\mathbf{J}(\mathbf{G}) = \frac{1}{\sigma^2}\mathbf{I}\ . \tag{40}$$

Second term in (36) is easily obtained as follows. Consider the prior PDF $p(\mathbf{G}) \sim \exp(-\mathbf{G}^\dagger\widetilde{\mathbf{\Lambda}}^{-1}\mathbf{G})$. The respective derivatives are found as

$$\frac{\partial \ln p(\mathbf{G})}{\partial \mathbf{G}^T} = -\mathbf{G}^\dagger\widetilde{\mathbf{\Lambda}}^{-1}, \qquad \frac{\partial^2 \ln p(\mathbf{G})}{\partial \mathbf{G}^*\partial \mathbf{G}^T} = -\widetilde{\mathbf{\Lambda}}^{-1}\ . \tag{41}$$

Upon taking the negative expectations, the second term in (41) becomes $\mathbf{J}_P(\mathbf{G}) = \widetilde{\mathbf{\Lambda}}^{-1}$. Substituting $\mathbf{J}(\mathbf{G})$ and $\mathbf{J}_P(\mathbf{G})$ in (36) produces for the modified FIM as follows

$$\mathbf{J}_M(\mathbf{G}) = \frac{1}{\sigma^2}\mathbf{I} + \widetilde{\mathbf{\Lambda}}^{-1}. \tag{42}$$

Inverting the matrix $\mathbf{J}_M(\mathbf{G})$ yields $MCRB(\hat{\mathbf{G}}) = \mathbf{J}_M^{-1}(\mathbf{G})$. $MCRB(\hat{\mathbf{G}})$ is a diagonal matrix with the elements on the main diagonal equaling the reciprocal of those $\mathbf{J}(\mathbf{G})$ matrix.

APPENDIX II
NONLINEAR FUNCTION

To use soft decision of the LLRs in the turbo receiver structure [34], let us find firstly $P(X_k = a)$ and $P(X_k = -a)$ in terms of LLRs ($a = 1/\sqrt{2}$).

$$L(X_k) = \log_e\left[\frac{P(X_k=-a)}{P(X_k=+a)}\right] = \log_e\left[\frac{P(X_k=-a)}{1-P(X_k=-a)}\right]. \tag{43}$$

From (43), $P(X_k = \pm a)$ can be determined as

$$P(X_k = -a) = \frac{e^{L(X_k)}}{1+e^{L(X_k)}} \tag{44}$$

$$P(X_k = a) = 1 - \frac{e^{L(X_k)}}{1+e^{L(X_k)}} = \frac{1}{1+e^{L(X_k)}}. \tag{45}$$

For BPSK signalling, the mean value of X_k is

$$\begin{aligned} \bar{X}_k &= \sum_{x_b\in\{+a,-a\}} x_b.P(X_k = x_b) \\ &= a(P(X_k=+a) - P(X_k=-a)). \end{aligned} \tag{46}$$

Finally, substituting $P(X_k = +a)$ and $P(X_k = -a)$ into (46), the mean value of X_k could be written as

$$\begin{aligned} \bar{X}_k &= a\left(\frac{e^{L(X_k)}}{1+e^{L(X_k)}} - \frac{1}{1+e^{L(X_k)}}\right) \\ &= \frac{e^{L(X_k)}-1}{e^{L(X_k)}+1} = a\tanh(L(X_k)/2) \end{aligned} \tag{47}$$

APPENDIX III
CALCULATION OF LLRS FOR ENCODED SYMBOLS

A. LLRs of Encoded Symbols $(C_{e,k}, C_{o,k})$ *for Generator Matrix (1,5/7)*

$$\begin{aligned} L(C_{e,k}) &= \underset{\substack{m,m' \\ C_{e,k}=-a}}{\mathbb{E}}\left[\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)\right] \\ &- \underset{\substack{m,m' \\ C_{e,k}=+a}}{E}\left[\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)\right] \end{aligned}$$

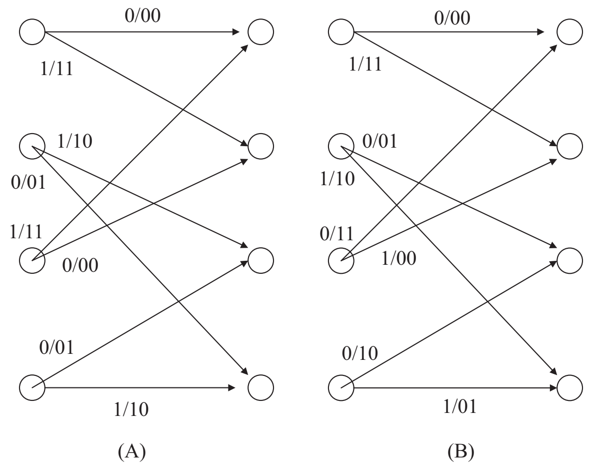

Fig. 11. (A) State transition diagram for the generator matrix (1,5/7); (B) shows state transition diagram for the generator matrix (5,7).

$$\begin{aligned}
&= \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,4)+\mathrm{B}_k(4)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,3)+\mathrm{B}_k(3)]) \\
&\quad - \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,3)+\mathrm{B}_k(3)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,4)+\mathrm{B}_k(4)])
\end{aligned} \tag{48}$$

$$\begin{aligned}
L(C_{o,k}) &= \mathop{\mathbb{E}}_{\substack{m,m' \\ C_{o,k}=-a}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&\quad - \mathop{\mathbb{E}}_{\substack{m,m' \\ C_{o,k}=+a}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&= \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,3)+\mathrm{B}_k(3)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,4)+\mathrm{B}_k(4)]) \\
&\quad - \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,4)+\mathrm{B}_k(4)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,3)+\mathrm{B}_k(3)])
\end{aligned} \tag{49}$$

B. LLRs of Encoded Symbols $(C_{e,k}, C_{o,k})$ for Generator Matrix (5,7)

$$\begin{aligned}
L(C_{e,k}) &= \mathop{\mathbb{E}}_{\substack{m,m' \\ C_{e,k}=-a}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&\quad - \mathop{\mathbb{E}}_{\substack{m,m' \\ C_{e,k}=+a}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&= \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,3)+\mathrm{B}_k(3)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,2) + \mathrm{B}_k(2)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,4)+\mathrm{B}_k(4)]) \\
&\quad - \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,4)+\mathrm{B}_k(4)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,3)+\mathrm{B}_k(3)])
\end{aligned} \tag{50}$$

$$\begin{aligned}
L(C_{o,k}) &= \mathop{\mathbb{E}}_{\substack{m,m' \\ C_{o,k}=-a}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&\quad - \mathop{\mathbb{E}}_{\substack{m,m' \\ C_{o,k}=+a}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&= \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,4)+\mathrm{B}_k(4)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,3)+\mathrm{B}_k(3)]) \\
&\quad - \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,3)+\mathrm{B}_k(3)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,4)+\mathrm{B}_k(4)])
\end{aligned} \tag{51}$$

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Hakan Doğan (S'02) was born in Istanbul, Turkey, in 1979. He received the B.S. and M.S. degrees in electronics engineering from Istanbul University, Istanbul, Turkey, in 2001 and 2003, respectively, and is currently working toward the Ph.D. degree from the same university. Since 2001, he has been a Research Assistant at the Department of Electrical and Electronics Engineering, Istanbul University. His general research interests cover communication theory, estimation theory, statistical signal processing, and information theory. His current research areas are focused on wireless communication concepts with specific attention to equalization and channel estimation for spread-spectrum and multicarrier (orthogonal frequency-division multiplexing) systems.

Hakan A. Çırpan (M'97) received the B.S. degree from Uludag University, Bursa, Turkey, in 1989, the M.S. degree from the University of Istanbul, Istanbul, Turkey, in 1992, and the Ph.D. degree from Stevens Institute of Technology, Hoboken, NJ, in 1997, all in electrical engineering. From 1995 to 1997, he was a Research Assistant at Stevens Institute of Technology, working on signal processing algorithms for wireless communication systems. In 1997, he joined the faculty of the Department of Electrical and Electronics Engineering, University of Istanbul. His general research interests cover wireless communications, statistical signal and array processing, system identification, and estimation theory. His current research activities are focused on signal processing and communication concepts with specific attention to channel estimation and equalization algorithms for space-time coding and multicarrier (orthogonal frequency-division multiplexing) systems. Dr. Çirpan is a member of Sigma Xi. He received the Peskin Award from Stevens Institute of Technology as well as the Prof. Nazım Terzioglu Award from the Research Fund of the University of Istanbul.

Erdal Panayırcı (F'03) received the Diploma Engineering degree in electrical engineering from the Istanbul Technical University, Istanbul, Turkey, in 1964 and the Ph.D. degree in electrical engineering and system science from Michigan State University, East Lansing, in 1970. From 1970 to 2000, he was with the Faculty of Electrical and Electronics Engineering, Istanbul Technical University, where he was a Professor and the Head of the Telecommunications Chair. He has also been a part-time Consultant to several leading companies in telecommunications in Turkey. From 1979 to 1981, he was with the Department of Computer Science, Michigan State University, as a Fulbright-Hays Fellow and a NATO Senior Scientist. Between 1983 and 1986, he served as a NATO Advisory Committee Member for the Special Panel on Sensory Systems for Robotic Control. From August 1990 to December 1991, he was a Visiting Professor at the Center for Communications and Signal Processing, New Jersey Institute of Technology, Newark, and took part in the research project on interference cancellation by array processing. Between 1998 and 2000, he was a Visiting Professor at the Department of Electrical Engineering, Texas A&M University, College Station, and took part in research on developing efficient synchronization algorithms for orthogonal frequency-division multiplexing (OFDM) systems. In 2005 he was a Visiting Professor at the Department of Electrical and Electronics Engineering, Bilkent University, Ankara, Turkey. He is currently a Professor and Department Head at the Electronics Engineering Department, Kadir Has University, Istanbul, Turkey. He is engaged in research and teaching in digital communications and wireless systems, equalization and channel estimation in multicarrier (OFDM) communication systems, and efficient modulation and coding techniques (TCM and turbo coding). Prof. Panayırcı is a member of Sigma Xi. He was the Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS in the fields of synchronization and equalizations from 1995 to 1999. He is currently the Head of the Turkish Scientific Commission on Signals, Systems, and Communications of the International Union of Radio Science.