Channel Estimation for MIMO-OFDM Systems in Fixed Broadband Wireless Applications

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Abstract

Systems employing multiple transmit and receive antennas, known as multiple input multiple output (MIMO) systems can be used with OFDM to improve the resistance to channel impairments. Thus the technologies of OFDM and MIMO are equipped in fixed wireless applications with attractive features, including high data rates and robust performance. However, since different signals are transmitted from different antennas simultaneously, the received signal is the superposition of these signals, which implies new challenges for channel estimation. In this paper we propose a time domain MMSE based channel estimation approach for MIMO-OFDM systems. The proposed approach employs a convenient representation of the discrete multipath fading channel based on the Karhunen-Loeve (KL) orthogonal expansion and finds MMSE estimates of the uncorrelated KL series expansion coefficients. Based on such an expansion, no matrix inversion is required in the proposed MMSE estimator. Also the performance of the proposed approach is studied through the evaluation of minimum Bayesian MSE.

1. Introduction

In order to accomplish the high-data rate goal and efficiently support high quality wireless services, several physical layer-related techniques have to be developed for future wireless systems. One approach that shows real promise for substantial capacity enhancement is to employ multicarrier transmission and, in particular, orthogonal frequency division multiplexing (OFDM) [1] . OFDM has lately been extensively considered for use in wireless/mobile communications systems, mainly in WLAN standards such as the IEEE802.11a and its European equivalent ETSI HIPERLAN/2 due to its robustness to multipath, its high-data rates, and its efficient use of bandwidth [1] . The attractiveness of OFDM systems stem from the fact that these systems transform the frequency-selective channel into a set of parallel flat-fading channels. The information is thus split into different streams sent over different sub-carriers thereby reducing intersymbol interference (ISI) and allowing for high data rates without adding complexity to the equalizers [2].

Systems employing multiple transmit and receive antennas, known as multiple input multiple output (MIMO) systems can be used with OFDM to improve the resistance to channel impairments. Thus the technologies of OFDM and MIMO equip fixed wireless applications with attractive features, including high data rates and robust performance. However, since different signals are transmitted from different antennas simultaneously, the received signal is the superposition of these signals, which implies new challenges for channel estimation.

Multipath fading channels have been studied extensively, and several models have been developed to describe their variations. In many cases, the channel taps are modeled as general lowpass stochastic processes, the statistics depend on mobility parameters. A different approach explicitly models the multipath channel taps by the Karhunen-Loeve (KL) series representation. In the case of KL series representation of stochastic process, a convenient choice of orthogonal basis set is one that makes the expansion coefficient random variables uncorrelated. When these orthogonal bases are employed to expand the channel taps of the multipath channel, uncorrelated coefficients are indeed represent the multipath channel. Therefore, KL representation allows one to tackle the estimation of correlated multipath parameters as a parameter estimation problem of the uncorrelated coefficients [3]. Exploiting KL expansion, the main contribution of this paper is to propose a computationally efficient, pilot-aided MMSE channel estimation algorithm. Based on such representation, no matrix inversion is required in the proposed approach. The performance of the proposed approach is explored based on the evaluation of the minimum Bayesian MSE for the random KL coefficients.

Notation used in this paper are standart. Upper- and lower-case bold letters denote matrices and vectors, respectively. $(.)^T$ denotes the transpose and $(.)^H$ denotes the Hermitian transpose. The Kronecker product of matrix $\mathbf{A}$ and $\mathbf{B}$ is denoted as $\mathbf{A}\otimes\mathbf{B}$. Matrix $\mathbf{I}_n$ and matrix $\mathbf{0}_n$ stand for identity matrix and zero matrix of order n, respectively. $diag(\mathbf{a})$ denotes the diagonal matrix whose diagonal is composed of vector $\mathbf{a}$, σ_x^2 stands for the variance of the random variable x. $tr(\mathbf{A})$ stands for the trace operator which is equals to the sum of the matrix $\mathbf{A}$'s diagonal elements.

2. System Model

2.1. Channel Model

The system under consideration is depicted in Figure 1. Consider a system with N_T transmit antennas and N_R receive antennas. At each transmit antenna the data is first modulated

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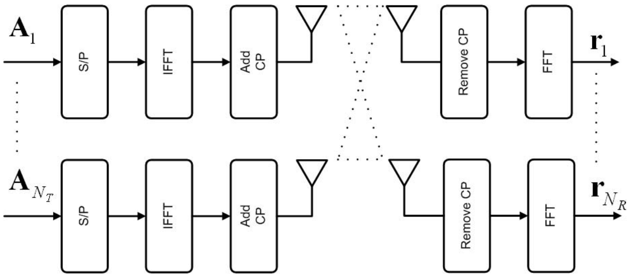

Figure 1: MIMO-OFDM System

by an IFFT, and a cyclic prefix of length v is added. Here $v \geq L-1$, where L is the maximum length of all channels. The channel impulse response vector between i^{th} transmit antenna and j^{th} receive antenna is

$$\mathbf{h}_{ji} = [h_{ji}[0], h_{ji}[1], \cdots, h_{ji}[L-1]]^T \ . \tag{1}$$

The channel frequency response vector between i^{th} transmit antenna and j^{th} receive antenna is

$$\mathbf{H}_{ji} = diag\{\mathbf{F}_K^L \mathbf{h}_{ji}\}. \tag{2}$$

where $\mathbf{F}_K^L \in \mathbb{C}^{K\times L}$ is composed of the first left L columns of K-point FFT matrix, and K is the number of the subcarriers in one OFDM symbol.

$$\mathbf{F} \triangleq \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{j2\pi/K} & \cdots & e^{j2\pi(K-1)/K} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j2\pi(K-1)/K} & \cdots & e^{j2\pi(K-1)(K-1)/K} \end{bmatrix}. \tag{3}$$

Vector $\mathbf{h}_j \in \mathbb{C}^{N_T L}$ is denoted as the combined channel impulse response vector from all the transmit antennas to the j^{th} receive antenna:

$$\mathbf{h}_j = \left[\mathbf{h}_{j1}^T, \mathbf{h}_{j2}^T, ..., \mathbf{h}_{jN_T}^T\right]^T \ . \tag{4}$$

According to (4) all of the channel impulse responses vector $\mathbf{h} \in \mathbb{C}^M$ between entire transmit and receive antennas where $M \triangleq N_R N_T L$ can be shown

$$\mathbf{h} = \left[\mathbf{h}_1^T, \mathbf{h}_2^T, \ldots, \mathbf{h}_{N_R}^T\right]^T \ . \tag{5}$$

The channel considered in this paper is SUI (Stanford University Interim) model which can be used for simulations, design, development and testing of technologies suitable for fixed broadband wireless applications. The parameters of these models were selected based upon statistical model described in [5].

2.2. MIMO-OFDM System Model

We assume that the transmitted OFDM symbol is $\mathbf{a}_i = [a_i[1], a_i[2], \cdots, a_i[K]]^T, \ \in \Xi^K$ where Ξ modulation symbol alphabet.Thus, the received superposition signal block vector $\mathbf{r}_j = [r_j[1], r_j[2], \cdots, r_j[K]]^T \in \mathbb{C}^K$ of the j^{th} receiver after FFT can be expressed as

$$\mathbf{r}_j = \mathbf{A}\left(\mathbf{I}_{N_T} \otimes \mathbf{F}_K^L\right)\mathbf{h}_j + \boldsymbol{\eta}_j. \tag{6}$$

where $\mathbf{A} = [\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_{N_T}] \in \mathbb{C}^{K\times N_T K}$ is data super matrix that transmitted from all transmit antennas, $\mathbf{A}_i = diag(\mathbf{a}_i)$ is data matrix that transmitted from i^{th} transmit antenna. $\boldsymbol{\eta}_j = [\eta_j[1], \eta_j[1], \cdots, \eta_j[K]]^T \in \mathbb{C}^K$ is the noise vector at the j^{th} receiver. We assume that the noise is white and the noise autocorrelation matrix is $\mathbf{C}\boldsymbol{\eta}_j = E\left[\boldsymbol{\eta}_j \boldsymbol{\eta}_j^H\right] = \sigma^2 \mathbf{I}_K$. The system model given in equation (6) is similar to a multi-input single-output system model.The system model which includes all received signals that will used as a basis for the channel estimation is:

$$\mathbf{r} = \left(\mathbf{I}_{N_R} \otimes \mathbf{A}\left(\mathbf{I}_{N_T} \otimes \mathbf{F}_K^L\right)\right)\mathbf{h} + \boldsymbol{\eta}. \tag{7}$$

where $\mathbf{r} = \left[\mathbf{r}_1{}^T, \mathbf{r}_2{}^T, \cdots, \mathbf{r}_{N_R}{}^T\right]^T \in \mathbb{C}^{N_R K}$ is received signal block vector of all receiver after FFT, $\boldsymbol{\eta} \in \mathbb{C}^{N_R K}$ is white noise and its autocorrelation matrix is $\mathbf{C}\boldsymbol{\eta} = \sigma^2 \mathbf{I}_{N_R K}$. An approach adapted herein explicitly models the channel parameters by the KL series representation and estimates the uncorrelated expansion coefficients.

3. MMSE Estimation of KL Coefficients

3.1. Pilot aided estimation

Pilot symbol assisted techniques can provide information about an undersampled version of the channel that may be easier to identify. In this paper, we therefore address the problem of estimating multipath channel parameters by exploiting the distributed training symbols. Considering (7), and in order that the pilot symbols are included in the output vector for our estimation purposes, we focus on a under-sampled signal model. Assuming K_p pilot symbols are uniformly inserted at known locations of the OFDM block at i^{th} transmit antenna, the $N_R K_p \times 1$ vector corresponding the FFT output at the pilot locations becomes

$$\mathbf{r}^p = \left(\mathbf{I}_{N_R} \otimes \mathbf{A}^p\left(\mathbf{I}_{N_T} \otimes \mathbf{F}_{K_p}^L\right)\right)\mathbf{h} + \boldsymbol{\eta}^p \in \mathbb{C}^{N_R K_p}. \tag{8}$$

where

$$\begin{aligned} \mathbf{A}^p &= \left[\mathbf{A}_1^p, \mathbf{A}_2^p, \cdots, \mathbf{A}_{N_T}^p\right] \in \mathbb{C}^{K_p \times N_T K_p} \\ \mathbf{A}_i^p &= diag\left(\mathbf{a}_{\triangle}, \mathbf{a}_{2\triangle}, \cdots, \mathbf{a}_{K_p\triangle}\right) \end{aligned}$$

and $\triangle$ is pilot spacing interval, $\mathbf{F}_{K_p}^L$ is a FFT matrix generated based on pilot indices, and similarly $\boldsymbol{\eta}^p$ is the under-sampled noise vector that is statistically equivalent to $\boldsymbol{\eta}$.

For the convenience of description in the time domain MMSE channel estimation, the received signal vector in equation (8) can be rewritten as:

$$\mathbf{r}^p = \mathcal{A}\,\mathbf{h} + \boldsymbol{\eta}^p \ . \tag{9}$$

where $\tilde{\mathbf{A}} \triangleq \mathbf{A}^p\left(\mathbf{I}_{N_T} \otimes \mathbf{F}_{K_p}^L\right)$ and $\mathcal{A} \triangleq \left(\mathbf{I}_{N_R} \otimes \tilde{\mathbf{A}}\right)$. Equation (9) offers a Bayesian linear model representation. Based on this representation, the minimum variance estimator for the time domain channel vector $\mathbf{h}$, conditional mean of $\mathbf{h}$ given $\mathbf{r}$, can be obtained using MMSE estimator. We should clearly make the assumptions that $\mathbf{h} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_\mathbf{h})$, $\boldsymbol{\eta}^p \sim \mathcal{N}\left(\mathbf{0}, \mathbf{C}\boldsymbol{\eta}^\mathbf{p}\right)$ and $\mathbf{h}$ is uncorrelated with $\boldsymbol{\eta}^p$. Therefore, MMSE estimate of $\mathbf{h}$ is given by [6]:

$$\begin{aligned}\hat{\mathbf{h}} &= \left(\mathcal{A}^H \mathbf{C}_{\boldsymbol{\eta}^p}^{-1} \mathcal{A} + \mathbf{C}_{\mathbf{h}}^{-1}\right)^{-1} \mathcal{A}^H \mathbf{C}_{\boldsymbol{\eta}^p}^{-1} \mathbf{r}^p \\ &= \left(\frac{1}{\sigma^2} \mathcal{A}^H \mathbf{I}_{N_R K_p} \mathcal{A} + \mathbf{C}_{\mathbf{h}}^{-1}\right)^{-1} \frac{1}{\sigma^2} \mathcal{A}^H \mathbf{I}_{N_R K_p}\, \mathbf{r}^p \\ &= \left(\mathcal{A}^H \mathcal{A} + \sigma^2 \mathbf{C}_{\mathbf{h}}^{-1}\right)^{-1} \mathcal{A}^H \mathbf{r}^p \,. \end{aligned} \tag{10}$$

$\mathcal{A}^H \mathcal{A}$ term in equation (10) is easily simplified as follows by Kronecker product properties [7]:

$$\begin{aligned}\mathcal{A}^H \mathcal{A} &= \left[\left(\mathbf{I}_{N_R} \otimes \tilde{\mathbf{A}}\right)^H \left(\mathbf{I}_{N_R} \otimes \tilde{\mathbf{A}}\right)\right] \\ &= \left(\mathbf{I}_{N_R} \otimes\, \tilde{\mathbf{A}}^H \tilde{\mathbf{A}}\right) \,. \end{aligned} \tag{11}$$

$$\tilde{\mathbf{A}}^H \tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{B}_{11} & \cdots & \mathbf{B}_{1N_T} \\ \vdots & \ddots & \vdots \\ \mathbf{B}_{N_T 1} & \cdots & \mathbf{B}_{N_T N_T} \end{bmatrix} \tag{12}$$

where

$$\mathbf{B}_{ij} \triangleq (\mathbf{F}_{K_p}^L)^H (\mathbf{A}_i^p)^H \mathbf{A}_j^p\, \mathbf{F}_{K_p}^L \,. \tag{13}$$

For MMSE channel estimation we require $\tilde{\mathbf{A}}^H \tilde{\mathbf{A}}$ to be diagonal. To do this the training sequence on different transmit antennas must not only be orthogonal but also phase shift orthogonal for phase shifts in the range $\{-L+1, \cdots, L+1\}$ [8]. In this case (12) will be

$$\mathbf{B}_{ij} = \begin{cases} K_p\, \mathbf{I}_L & i = j \\ \mathbf{0}_L & i \neq j \end{cases} \tag{14}$$

$$\tilde{\mathbf{A}}^H \tilde{\mathbf{A}} = K_p\, \mathbf{I}_{N_T L} \tag{15}$$

Substituting (15) in (11)

$$\mathcal{A}^H \mathcal{A} = \left(\mathbf{I}_{N_R} \otimes\, \tilde{\mathbf{A}}^H \tilde{\mathbf{A}}\right) = K_p\, \mathbf{I}_M \tag{16}$$

Then according to (16) and (10), we arrive at the expression

$$\hat{\mathbf{h}} = \left(K_p\, \mathbf{I}_M + \sigma^2 \mathbf{C}_{\mathbf{h}}^{-1}\right)^{-1} \mathcal{A}^H \mathbf{r}^p \,. \tag{17}$$

Since as obtained in (17) MMSE estimation still requires the inversion of $\mathbf{C}_{\mathbf{h}}^{-1}$, it therefore suffers from a high computational complexity. However, it is possible to reduce complexity of the MMSE algorithm by diagonalizing channel covariance matrix with an KL expansion.

3.2. KL Expansion

Channel impulse response $\mathbf{h}$ is a zero-mean Gaussian process with covariance matrix $\mathbf{C}_{\mathbf{h}}$. The KL transformation is therefore employed here to rotate the vector $\mathbf{h}$ so that all its components are uncorrelated.

The vector $\mathbf{h}$, can be expressed as a linear combination of the orthonormal basis vectors as follows:

$$\mathbf{h} = \sum_{l=1}^{M} g_l \psi_l = \mathbf{\Psi} \mathbf{g} \,. \tag{18}$$

where $\mathbf{\Psi} = [\psi_1, \psi_2, \cdots, \psi_M]$, ψ_i's are the orthonormal basis vectors, $\mathbf{g} = [g_1, g_2, \cdots, g_M]^T$, and g_l is the weights of the expansion. If we form the covariance matrix $\mathbf{C}_{\mathbf{h}}$ as

$$\mathbf{C}_{\mathbf{h}} = \mathbf{\Psi} \mathbf{\Lambda}_{\mathbf{g}} \mathbf{\Psi}^H \,. \tag{19}$$

where $\mathbf{\Lambda}_{\mathbf{g}} = E\left[\mathbf{g}^H \mathbf{g}\right]$, the KL expansion is the one in which $\mathbf{\Lambda}_{\mathbf{g}}$ of $\mathbf{C}_{\mathbf{h}}$ is a diagonal matrix (i.e., the coefficients are uncorrelated). If $\mathbf{\Lambda}_{\mathbf{g}}$ is diagonal, then the form $\mathbf{\Psi} \mathbf{\Lambda}_{\mathbf{g}} \mathbf{\Psi}^H$ is called an eigendecomposition of $\mathbf{C}_{\mathbf{h}}$. The fact that only the eigenvectors diagonalize $\mathbf{C}_{\mathbf{h}}$ leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in Gaussian case, the uncorrelateness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem in this application equivalent to estimating the iid complex Gaussian vector $\mathbf{g}$ KL expansion coefficients.

3.3. Estimation of KL Coefficients

In contrast to (9) in which only $\mathbf{h}$ is to be estimated, we now assume the KL coefficients $\mathbf{g}$ is unknown. Thus the data model (9) is rewritten as

$$\mathbf{r}^p = \mathcal{A} \mathbf{\Psi} \mathbf{g} + \boldsymbol{\eta}^p \,. \tag{20}$$

which is also recognized as a Bayesian linear model, and recall that $\mathbf{g} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Lambda}_{\mathbf{g}})$. As a result, the MMSE estimator of $\mathbf{g}$ is

$$\hat{\mathbf{g}} = \mathbf{\Lambda}_{\mathbf{g}} \left(K_p \mathbf{\Lambda}_{\mathbf{g}} + \sigma^2 \mathbf{I}_M\right)^{-1} \mathbf{\Psi}^H \mathcal{A}^H r^p = \mathbf{\Gamma} \mathbf{\Psi}^H \mathcal{A}^H r^p \tag{21}$$

where

$$\begin{aligned}\mathbf{\Gamma} &= \mathbf{\Lambda}_{\mathbf{g}} \left(K_p \mathbf{\Lambda}_{\mathbf{g}} + \sigma^2 \mathbf{I}_M\right)^{-1} \\ &= diag\{\frac{\lambda_{g_1}}{\lambda_{g_1} K_p + \sigma^2}, \ldots, \frac{\lambda_{g_M}}{\lambda_{g_M} K_p + \sigma^2}\} \end{aligned} \tag{22}$$

and $\lambda_{g_1}, \lambda_{g_2}, \cdots, \lambda_{g_M}$ are the singular values of the $\mathbf{\Lambda}_{\mathbf{g}}$

It is clear that the complexity of the MMSE estimator in (17) is reduced by the application of KL expansion. However, the complexity of the $\mathbf{g}$ can be further reduced by exploiting the optimal truncation property of the KL expansion [9].

4. Performance Analysis: Bayesian MSE

For the MMSE estimator $\hat{\mathbf{g}}$, the error is

$$\epsilon = \mathbf{g} - \hat{\mathbf{g}}. \tag{23}$$

Since the diagonal entries of the covariance matrix of the error represent the minimum Bayesian MSE, we now derive covariance matrix $\mathbf{C}_\epsilon$ of the error vector. From the Performance of the MMSE estimator for the Bayesian Linear model Theorem [6], the error covariance matrix is obtained as

$$\begin{aligned}\mathbf{C}_\epsilon &= \left(\mathbf{\Lambda}_{\mathbf{g}}^{-1} + (\mathcal{A}\mathbf{\Psi})^H \mathbf{C}_{\boldsymbol{\eta}}^{-1} \mathcal{A} \mathbf{\Psi}\right)^{-1} \\ &= \sigma^2 \left(K_p \mathbf{I}_M + \sigma^2 \mathbf{\Lambda}_{\mathbf{g}}^{-1}\right)^{-1} = \sigma^2 \mathbf{\Gamma} \end{aligned} \tag{24}$$

and then the minimum Bayesian MSE of the full rank estimator becomes

$$\begin{aligned}\mathbf{B}_{MSE}(\hat{\mathbf{g}}) &= \frac{1}{M} tr\left(\mathbf{C}_\epsilon\right) = \frac{1}{M} tr\left(\sigma^2 \mathbf{\Gamma}\right) \\ &= \frac{1}{M} \sum_{i=1}^{M} \frac{\lambda_{g_i}}{1 + K_p \lambda_{g_i} SNR} \end{aligned} \tag{25}$$

where $SNR = 1/\sigma^2$ and tr denotes trace operator on matrices.

5. Simulations

In this section, we will illustrate the merits of our channel estimator through simulations. We use avarage mean square error (MSE) and bit-error rate (BER) on a 2×2 MIMO-OFDM system as our figure of merits. We consider SUI-3 type fading multipath channel for fixed broadband wireless applications. In [5], it is shown that the number of taps is $L = 3$, the tap delays are fixed, the equivalent taps in all channels have equal power and that taps with different delays are uncorrelated within a channel as well as between channels and, the antenna correlation coefficient for this type of SUI channel is $\rho_{env} = 0.4$.Transmission bandwidth is divided into 128 tones. A QPSK MIMO-OFDM sequence passes through channel taps and is corrupted by AWGN (0dB, 5dB, 10dB, 15dB, 20dB, 25dB and 30dB respectively). We use a pilot symbol for every eight ($\Delta = 8$) and sixteen ($\Delta = 16$) symbols. The MSE at each SNR point is averaged over 500 realizations. We compare the experimental MSE performance and its theoretical Bayesian MSE of the proposed MMSE estimator with Least Squares (LS) estimator. Fig. 2 confirms that MMSE estimator performs better than LS estimator at low SNR. However, two approaches has comparable performance at high SNRs. To observe the performance, we also present the MMSE and LS estimated channel BER results together in Fig. 3. It also shows that the MMSE estimated channel BER results are better than LS estimated channel BER especially at high SNR.

6. Conclusion

We consider the design of low complexity MMSE channel estimator for MIMO-OFDM systems in Fixed Broadband Wireless channels. We first derive the MMSE estimator based on the stochastic orthogonal expansion representation of the channel via KL transform. Based on such representation, we show that no matrix inversion is needed in the MMSE algorithm. Therefore, the computational cost for implementing the proposed MMSE estimator is low and computation is numerically stable. Moreover, the performance of our proposed method was studied through the MMSE estimator performance measure Bayesian MSE.

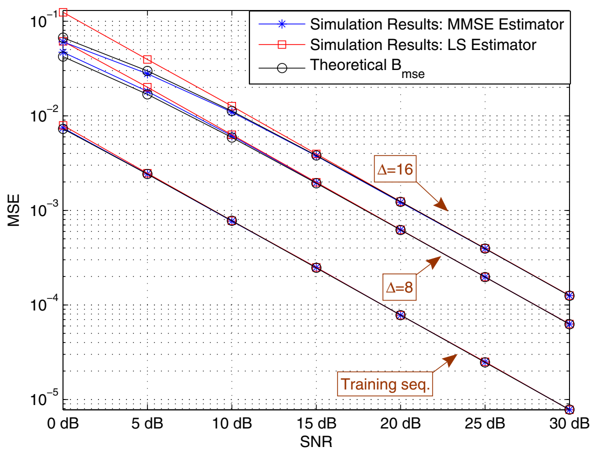

Figure 2: Mean Square Error

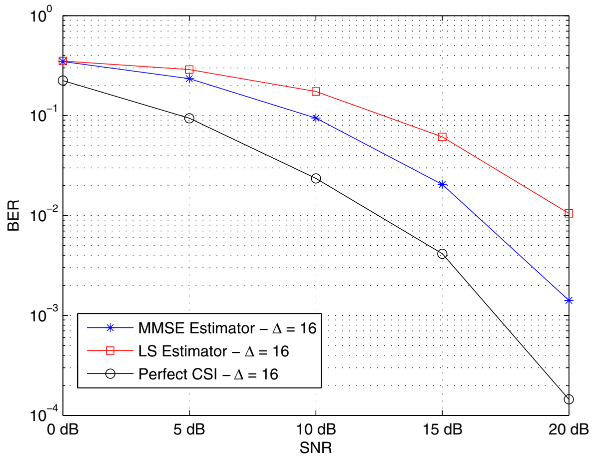

Figure 3: Bit Error Rate

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