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Blind-Phase Noise Estimation in OFDM Systems by Sequential Monte Carlo Method

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SUMMARY

One of the main drawbacks of orthogonal frequency division multiplexing (OFDM) systems is the phase noise (PN) caused by the oscillator instabilities. Unfortunately, due to the PN, the most valuable feature namely orthogonality between the carriers, is destroyed resulting in a significant degradation in the performance of OFDM systems. In this paper, based on a sequential Monte Carlo method (particle filtering), a computationally efficient algorithm is presented for estimating the residual phase noise, blindly, generated at the output of the phase tracking loop employed in OFDM systems. The basic idea is to treat the transmitted symbols as 'missing data' and draw samples sequentially of them based on the observed signal samples up to time *t*. This way, the Bayesian estimates of the phase noise is obtained through these samples, sequentially drawn, together with their importance weights. The proposed receiver structure is seen to be ideally suited for highspeed parallel implementation using VLSI technology. The performance of the proposed approaches are studied in terms of average mean square error. Through experimental results, the effects of an initialisation on the tracking performance are also explored. Copyright © 2006 AEIT.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has lately been extensively considered for use in wireless/mobile communications systems, mainly in WLAN standards such as the IEEE802.11a and its European equivalent ETSI HIPERLAN/2 due to its robustness to multipath, its high-data rates and its efficient use of bandwidth [1, 2]. The attractiveness of OFDM systems stems from the fact that these systems transform the frequency-selective channel into a set of parallel flat-fading channels. The information is thus split into different streams sent over different sub-carriers thereby reducing intersymbol interference (ISI) and allowing for high-data rates without adding complexity to the equalizers [3, 4].

One of the main drawbacks of OFDM systems is the phase noise (PN) caused by the oscillator instabilities [5]. Unfortunately, due to the PN, the most valuable feature namely orthogonality between the carriers, is destroyed resulting in a significant degradation in the performance of OFDM systems [5]. Random PN causes two effects on OFDM systems, rotating each symbol by a random phase that is referred to as the common phase error (CPE) and producing intercarrier interference (ICI) term that adds to the channel noise due to the loss of orthogonality between subcarriers [6]. Several methods have been

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proposed in the literature for the estimation and compensation of the PN in OFDM systems [7, 8]. Most of the approaches however only addresses the estimation of the CPE by assuming ICI terms are approximated by a Gaussian distribution and these techniques are implemented after the DFT process at the receiver [8]. The main drawback of these approaches is the data-dependent ICI which introduces an additional random noise on top of the additive Gaussian channel noise causes a significant degradation in the CPE estimator performance. In contrast to these approaches, we try to solve PN estimation problem in the time domain before the DFT process at the OFDM receiver. As it will be seen shortly this approach will not be faced with ICI effect during the estimation procedure resulting in more accurate random phase estimation.

A considerable amount of research has been carried out for online estimation of the timevarying as well as the fixed phase offset at digital receivers in the presence of data [6]. Estimating the phase offset by maximum likelihood (ML) technique does not seem to be analytically tractable. Even if the likelihood function can be evaluated offline; however, it is invariably a nonlinear function of the parameter to be estimated, which makes the maximisation step computationally infeasible. Phase synchronisation is typically implemented by a decision directed (or data-aided) or nondecision directed (or nondata aided). Decision-directed schemes depend on availability of reliably detected symbol for obtaining the phase estimate, and therefore, they usually require transmission of pilot or training data. However, in applications where bandwidth is the most precious resource, training data can significantly reduce the overall system capacity. Thus blind or nondata-aided techniques become an attractive alternative [9, 10]. Unlike data-aided techniques, nondata-aided methods do not require knowledge of the transmitted data, and instead, they exploit statistics of digital-transmitted signal. ML estimation techniques can also be used in nondecision-directed methods if the symbols transmitted are treated as random variables with known statistics so that the likelihood function can be averaged over the data sequence received. Unfortunately, except for few simple cases, this averaging process is mathematically impracticable and it can be obtained only by some approximations which are valid only either at high- or low-SNR values [11].

On the other hand, in order to provide an implementable solution, recently there have been a substantial amount of work on iterative formulation of the parameter estimation problem based on the expectation-maximization (EM) technique [12]. It is known that the EM algorithm derives iteratively and converges to the true ML estimation of these unknown parameters. The main drawbacks of this approach are that the algorithm is sensitive to the initial starting values chosen for the parameters, it does not necessarily converge to the global extremum and the convergence can be slow. Furthermore, in situation where the posterior distribution must be constantly updated with arrival of the new data with missing parts, EM algorithm can be highly inefficient, because the whole iteration process must be redone with additional data. The sequential Monte Carlo (SMC) methodology, also called *particle filtering*, [14] that has emerged in the field of statistics and engineering has shown great promise to solve such problems. This technique can approximate the optimal solution directly without compromising the system model. Additionally, the decision made at time t does not depend on any decisions made previously, and thus, no error is propagated in their implementation. More importantly, the SMC yields a fully blind algorithm and allows for both Gaussian and non-Gaussian ambient noise as well as high-speed parallel implementations.

The main objective of this paper is to solve the PN estimation problem by means of the SMC technique. The basic idea is to treat the transmitted data as 'missing data' and to sequentially draw samples of them based on the current observation and computing appropriate importance sampling weights. Based on sequentially drawn samples, the Kalman filter is used to estimate the unknown phase from an extended Kalman state-space model of the underlying system. Furthermore, the tracking of the timevarying PN and the data detection are naturally integrated. The algorithm is self-adaptive and no training/pilot symbols or decision feedback are needed [13].

In the following, the system and the main model for the PN are described in Section 2, the solution of the blind-phase noise estimation problem by means of the SMC method is presented in Section 3. Resampling method is detailed in Section 4. Finally, the simulation results and the main conclusions of the paper are given in Sections 5 and 6 respectively.

2. SYSTEM DESCRIPTION

We consider an OFDM system with N subcarriers operating over a frequency-selective Rayleigh fading channel. In this paper, we assume that the multipath intensity profile has exponential distribution and the delay spread T_{d} is less than or equal to the guard interval L. With the aid of the discrete time channel model [3], the output of the frequency-selective channel can be written as

$$y_t = \sum_{k=0}^{L} h_k s_{t-k} \quad (1)$$

where the $h_k, k = 0, 1, \ldots, L$ denotes the kth tap gain and we assume to have ideal knowledge of these channel tap gains. Moreover, $s_t = \sum_{n=0}^{N-1} d_n e^{-j\frac{2\pi tn}{N}}$ where $\{d_n\}$ denotes the independent data symbols transmitted on the nth subcarrier of an OFDM symbol. Hence, s_t is a linear combination of independent, identically distributed random variables. If the number of subcarriers is sufficiently large, s_t can be modelled as a complex Gaussian process whose real and imaginary parts are independent. It has zero mean and variance $\sigma_s^2 = E\{|s_t|^2\} = E_s$, where E_s is the symbol energy per subcarrier.

Also, assuming perfect frequency and timing synchronisation, the received signal, r_t, corrupted by the additive Gaussian noise n_t and distorted by the timevarying phase noise θ_t can be expressed as

$$r_t = y_t e^{j\theta_t} + n_t, \quad t = 1, \ldots, T_0 \quad (2)$$

where n_t is the complex envelope of the additive white Gaussian noise with variance $\sigma_n^2 = E\{|n_t(k)|^2\}$. θ_t is the sample of the PN process at the output of the free-running local oscillator representing the phase noise. It is shown in Reference [16] that in the case of free-running oscillators, PN can be modelled as a Wiener process defined as

$$\theta_t = \theta_{t-1} + u_t$$

$$\theta_0 \sim \text{uniform}(-\pi, \pi) \quad (3)$$

where u_t is zero-mean Gaussian random variable with variance $\sigma_u^2 = 2\pi BT_s$ where T_s is the sampling period of the OFDM receiver A/D converter and BT refers to the PN rate, where $T = T_s(N + L)$. It is assumed that u_t and n_t are independent of each other. Defining the vectors $\boldsymbol{R}_t = [r_0, r_1, \ldots, r_t]^T$, $\boldsymbol{S}_t = [s_0, s_1, \ldots, s_t]^T$, $\boldsymbol{s}_t = [s_t, s_{t-1}, \ldots, s_{t-L}]^T$, and $\boldsymbol{h}_t = [h_0, h_1, \ldots, h_L]^T$ combining Equations (2) and (3), and taking into account the structure of $\boldsymbol{s}_t$, we obtain the following dynamic state-space representation of the communication system,

$$\theta_t = \theta_{t-1} + u_t$$

$$\boldsymbol{s}_t = \mathbf{F}\boldsymbol{s}_{t-1} + \boldsymbol{v}_t$$

$$r_t = \boldsymbol{h}_t^T \boldsymbol{s}_t e^{j\theta_t} + n_t \quad (4)$$

where

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 \\ 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & . \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix} \quad (5)$$

is a $(L+1) \times (L+1)$ shifting matrix, and $\boldsymbol{v}_t = [s_t, 0, \ldots, 0]$ is a $(L+1) \times 1$ perturbation vector that contains the new symbol s_t.

3. SMC FOR BLIND-PHASE NOISE ESTIMATION

Since we are interested in estimating the phase noise θ_t blindly at time t based on the observation $\boldsymbol{R}_t$, the Bayes solution requires the posterior distribution

$$p(\theta_t|\boldsymbol{R}_t) = \int p(\theta_t|\boldsymbol{R}_t, \boldsymbol{S}_t) p(\boldsymbol{S}_t|\boldsymbol{R}_t) d\boldsymbol{S}_t \quad (6)$$

Note that with a given $\boldsymbol{S}_t$, the nonlinear (Kalman filter) model (4) can be converted into a linear model by linearising the observation Equation (2) as follows [15]:

$$\theta_t = \theta_{t-1} + u_t$$

$$r_t = \boldsymbol{h}_t^T \boldsymbol{s}_t (V_t \theta_t + Q_t) + n_t \quad (7)$$

where

$$V_t = j e^{j\hat{\theta}_{t|t-1}}$$

$$Q_t = (1 - j\hat{\theta}_{t|t-1}) e^{j\hat{\theta}_{t|t-1}} \quad (8)$$

$\hat{\theta}_{t|t-1}$ denotes the estimator of θ_t based on the observations $\boldsymbol{R}_{t-1} = (r_0, r_1, \ldots, r_{t-1})$. Then the state-space model (4) becomes a linear Gaussian system. Hence, $p(\theta_t|\boldsymbol{S}_t, \boldsymbol{R}_t) \sim N(\mu_{\theta_t}(\boldsymbol{S}_t), \sigma^2_{\theta_t}(\boldsymbol{S}_t))$, where the mean $\mu_{\theta_t}(\boldsymbol{S}_t)$ and the variance $\sigma^2_{\theta_t}(\boldsymbol{S}_t)$ can be obtained as follows. Denoting $\mu_{\theta_t}(\boldsymbol{S}_t) \triangleq {}_{t|t}\theta_t$, and $\sigma^2_{\theta_t}(\boldsymbol{S}_t) \triangleq M_{t|t}$.

$\hat{\theta}_{t|t}$ and $\hat{M}_{t|t}$ can be calculated recursively by using the Extended Kalman Technique [[15], pages 449–452] with the given $\boldsymbol{S}_t$ as:

$$\hat{\theta}_{t|t} = \hat{\theta}_{t|t-1} + K_t(r_t - \boldsymbol{h}^T \boldsymbol{s}_t e^{j\hat{\theta}_{t|t-1}}) \quad (9)$$

$$M_{t|t} = (1 - K_t \boldsymbol{h}_t^T \boldsymbol{s}_t V_t) M_{t|t-1}$$

where

$$K_t = \frac{M_{t|t-1}(\boldsymbol{h}_t^T \boldsymbol{s}_t V_t)^*}{|\boldsymbol{h}_t^T \boldsymbol{s}_t|^2 M_{t|t-1} + \sigma_n^2}$$

$$\hat{\theta}_{t|t-1} = \hat{\theta}_{t-1|t-1}$$

$$M_{t|t-1} = M_{t-1|t-1} + \sigma_u^2 \tag{10}$$

We can now make timely estimates of θ_t based on the currently available observation $\boldsymbol{R}_t$, up to time t, blindly, as follows. With the Bayes theorem, we realise that the optimal solution to this problem is

$$\hat{\theta}_t = E\{\theta_t|\boldsymbol{R}_t\}$$
$$= \int_{\boldsymbol{S}_t} \underbrace{\left[\int_{\theta_t} \theta_t p(\theta_t|\boldsymbol{S}_t, \boldsymbol{R}_t) \mathrm{d}\theta_t\right]}_{\mu\theta_t(\boldsymbol{S}_t)} p(\boldsymbol{S}_t|\boldsymbol{R}_t), \mathrm{d}\boldsymbol{S}_t \tag{11}$$

In most cases, an exact evaluation of the expectation (11) is analytically intractable. Sequential Monte Carlo technique can provide us an alternative way for the required computation. Specifically, following the notation adopted in Reference [4], if we can draw m independent random samples $\{\boldsymbol{S}_t^{(j)}\}_{j=1}^m$ from the distribution $p(\boldsymbol{S}_t|\boldsymbol{R}_t)$, then we can approximate the quantity of interest $E\{\theta_t|\boldsymbol{R}_t\}$ in Equation (11) by

$$E\{\theta_t|\boldsymbol{R}_t\} \cong \frac{1}{m}\sum_{j=1}^{m} \mu_{\theta_t}(\boldsymbol{S}_t^{(j)}) \tag{12}$$

But, usually drawing samples from $p(\boldsymbol{S}_t|\boldsymbol{R}_t)$ directly is difficult. Instead, sample generation from some *trial distribution* may be easier. In this case, the idea of *importance sampling* can be used [4]. By associating the weight $w_t^{(j)} = \frac{p(\boldsymbol{S}_t^{(j)}|\boldsymbol{R}_t)}{q(\boldsymbol{S}_t^{(j)}|\boldsymbol{R}_t)}$ to the samples, the quantity of interest, $E\{\theta_t|\boldsymbol{S}_t\}$ can be approximated as follows:

$$E\{\theta_t|\boldsymbol{R}_t\} \cong \frac{1}{W_t}\sum_{j=1}^{m} \mu_{\theta_t}(\boldsymbol{S}_t^{(j)}) w_t^{(j)} \tag{13}$$

with $W_t = \sum w_t^{(j)}$. The pair $(\boldsymbol{S}_t^{(j)}, w_t^{(j)}), j = 1, 2, \ldots, m$ is called a properly weighted sample with respect to distribution $p(\boldsymbol{S}_t|\boldsymbol{R}_t)$.

Specifically, it was shown in Reference [4] that a suitable choice for the trial distribution is of the form $q(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)}) = p(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)})$. For this trial sampling distribution, it is shown in Reference [4] that the importance weight is updated according to

$$w_t^{(j)} = w_{t-1}^{(j)} p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}), \quad t = 1, 2, \ldots \tag{14}$$

The optimal trial distribution in $p(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)})$ can be computed as follows:

$$p(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)}) = p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t) P(s_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) \tag{15}$$

Furthermore, it can be shown from the state and observation equations in (4) that $p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t) \sim \mathcal{N}(\mu_{r_t}^{(j)}, \sigma_{r_t}^{2(j)})$ with mean and variance given by

$$\begin{aligned}\mu_{r_t}^{(j)} &= E\{r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t\} \\ &= \boldsymbol{h}_t^{\mathrm{T}} \boldsymbol{s}_t (V_t \hat{\theta}_{t|t-1}^{(j)} + Q_t)\end{aligned} \tag{16}$$

$$\begin{aligned}\sigma_{r_t}^{2(j)} &= \mathrm{Var}\{r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t\} \\ &= |\boldsymbol{h}_t^T \boldsymbol{s}_t|^2 M_{t|t-1}^{(j)} + \sigma_n^2\end{aligned} \tag{17}$$

where the quantities $\hat{\theta}_{t|t-1}^{(j)}$ and $M_{t|t-1}^{(j)}$ in Equations (16) and (17) respectively can be computed recursively for the Extended Kalman equations given in Equations (9) and (10). Also since s_t is independent of $\boldsymbol{S}_{t-1}$ and $\boldsymbol{R}_{t-1}$, the second term in Equation (15), it can be written as $p(s_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) = p(s_t)$ where it was pointed out earlier that $p(s_t) \sim \mathcal{N}(0, \sigma_s^2)$.

Note that dependency of the $\sigma_{r_t}^{2(j)}$ in (16) to s_t precludes combining the product of Gaussian densities in Equation (15) into a single Gaussian, hence obtaining a tractable sampling distribution. This problem can be circumvented by approximating the $\sigma_{r_t}^{2(j)}$ as follows. From Equation (4), we can use the approximation $\boldsymbol{s}_t \approx \mathbf{F}\boldsymbol{s}_{t-1}$ in Equation (16) to obtain

$$\sigma_{r_t}^{2(j)} \cong |\boldsymbol{h}_t^T \mathbf{F} \boldsymbol{s}_{t-1}^{(j)}|^2 M_{t|t-1}^{(j)} + \sigma_n^2 \tag{18}$$

Similarly using Equations (11) in (16), the mean $\mu_{r_t}^{(j)}$ can be expressed as

$$\mu_{r_t}^{(j)} = (\boldsymbol{h}_t^{\mathrm{T}} \mathbf{F} \boldsymbol{s}_{t-1}^{(j)} + h_0 s_t) G_t^{(j)} \quad \text{where} \quad G_t^{(j)} \triangleq V_t \hat{\theta}_{t|t-1}^{(j)} + Q_t \tag{19}$$

Then, the true trial sampling distribution $p(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)})$ in Equation (15) can be expressed as follows:

$$p(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)}) \sim \mathcal{N}(\mu_{s_t}^{(j)}, \sigma_{s_t}^{2(j)}) \tag{20}$$

where

$$\mu_{s_t}^{(j)} = \frac{(r_t - \boldsymbol{h}_t^{\mathrm{T}} \mathbf{F} \boldsymbol{s}_{t-1}^{(j)} G_t^{(j)})}{h_0 G_t^{(j)}} \left(\frac{\sigma_{r_t}^{2(j)}}{|h_0 G_t^{(j)}|^2 \sigma_{\mathrm{s}}^2} + 1\right)^{-1}$$

$$\sigma_{s_t}^{2(j)} = \frac{\sigma_{r_t}^{2(j)} \sigma_{\mathrm{s}}^2}{\sigma_{r_t}^{2(j)} + |h_0 G_t^{(j)}|^2 \sigma_{\mathrm{s}}^2}$$

and $\sigma_{r_t}^{2(j)}$ is defined as Equation (16).

In order to obtain the recursion for the weighting factor $w_t^{(j)}$, the predictive distribution $p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)})$ in Equation (15) should be evaluated. It is given by

$$p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) = \int_{s_t} p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t)P(s_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)})\mathrm{d}s_t$$
$$= \int_{s_t} p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t)p(s_t)\mathrm{d}s_t \tag{21}$$

where Equation (21) holds because s_t is independent of $\boldsymbol{S}_{t-1}$ and $\boldsymbol{R}_{t-1}$. Since the both terms in the integrand of Equation (21), are Gaussian densities, the product of the Gaussian densities are integrated with respect to s_t is also Gaussian. Therefore the predictive distribution is found to be

$$p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) \sim \mathcal{N}(M_{r_t}^{(j)}, \Sigma_{r_t}^{2(j)}) \tag{22}$$

where

$$M_{r_t}^{(j)} = \boldsymbol{h}^{\mathrm{T}}\mathbf{F}\boldsymbol{s}_{t-1}^{(j)}G_t^{(j)}$$

$$\Sigma_{r_t}^{2(j)} = |h_0 G_t^{(j)}|^2\sigma_s^2 + |\boldsymbol{h}^{\mathrm{T}}\mathbf{F}\boldsymbol{s}_{t-1}^{(j)}|^2 M_{t|t-1}^{(j)} + \sigma_n^2 \tag{23}$$

We now summarise the SMC blind data phase noise estimation algorithm in Table 1:

The proposed SMC approach perform three basic operations: generation of new particles (sampling from the space of unobserved states), computation of particle weights (probability masses associated with the particles) and resampling (a process of removing particles with small weights and replacing them with particles with large weights). Particle generation and weight computation steps are computationally the most intensive ones. The particle filtering speed can be increased through both algorithmic modifications and architecture development [4]. On the algorithmic level, the main challenges for speed increase include reducing the number of operations and exploiting operational concurrency between the particle

Table1. SMC algorithm for blind-phase noise estimation.
Given $\{h_0, h_1, \ldots, h_L\}$

- *Initialisation*:

— *Initialise the extended Kalman filter*: Choose the initial mean and the variance of the estimated θ_t as

$$\mu_{\theta_0}^{(j)} = \hat{\theta}_{0|0}^{(j)} = 0, \quad \sigma_{\theta_0}^{2(j)} = M_{0|0}^{(j)} = \pi^2/12, \quad j = 1, 2, \ldots, m. \tag{24}$$

— *Initialise the importance weights*: All importance weights are initialised as $w_0^{(j)} = 1, j = 1, 2, \ldots, m$.
For $j = 1, m$
For $t = 1, T_0$

- *Compute* $\hat{\theta}_{t|t-1}, M_{t|t-1}^{(j)}$ *from Equation (8).*
- *Compute* $\mu_{r_t}^{(j)}, \sigma_{r_t}^{2(j)}$ *from Equations (16).*
- *Compute sampling distribution mean/variance* $\mu_{s_t}^{(j)}, \sigma_{s_t}^{2(j)}$ *from the Equation (20).*
- *Sample* $s_t^{(j)} \sim N(\mu_{s_t}^{(j)}, \sigma_{s_t}^{(j)})$ *and Append* $s_t^{(j)}$ to $\boldsymbol{S}_{t-1}^{(j)}$ *to obtain* $\boldsymbol{S}_t^{(j)} = (s_t^{(j)}, \boldsymbol{S}_{t-1}^{(j)})$.
- *Compute the importance weights*:

$$w_t^{(j)} = w_{t-1}^{(j)} p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}),$$

where $p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)})$ *is computed from Equation (22).*

- *Update the a posteriori mean and variance of the phase noise according to Kalman equations (7–8)*
If the samples drawn up to time t is $\boldsymbol{S}_t$, *set*

$$\mu_{\theta_t}(\boldsymbol{S}_t^{(j)}) \triangleq \mu_{\theta_t}^{(j)} = \hat{\theta}_{t|t}^{(j)}$$

$$\sigma_{\theta_t}^{2(j)}(\boldsymbol{S}_t^{(j)}) \triangleq \sigma_{\theta_t}^{2(j)} = M_{t|t}^{(j)} \; j = 1, 2, \ldots, m.$$

- *Do the re-sampling as described in Equation [4].*
next j
- *Estimate phase noise* $\hat{\theta}_t = \frac{1}{m}\sum_{j=1}^{m} \mu_{\theta_t}(\boldsymbol{S}_t^{(j)})$

next t

generation and weight computation steps. Moreover, a parallel implementation with multiple processing elements can be employed to increase speed further [4].

4. RESAMPLING METHOD

A major problem in the practical implementation of the SMC method described so far is that after a few iteration most of the importance weights have negligible values that is $w_t^{(j)} \approx 0$. A relatively small weight implies that the sample is drawn far from the main body of the posterior distribution and has a small contribution in the final estimation. Such sample is said to be ineffective. The SMC algorithm becomes ineffective if there are too many ineffective samples. The common solution to this problem is *resampling*. Resampling is an algorithmic step that stochastically eliminates those samples with small weights. Basically, the resampling method takes the samples, to be generated sequentially $\Xi_t = \{\boldsymbol{S}_t^{(j)}, \mu_{\theta_t}^{(j)}, \sigma_{\theta_t}^{2(j)}\}_{j=1}^m$ with corresponding weights $\{w_t^{(j)}\}_{j=1}^m$ as an input and generates a new set of samples $\tilde{\Xi}_t = \{\tilde{\boldsymbol{S}}_t^{(j)}, \tilde{\mu}_{\theta_t}^{(j)}, \tilde{\sigma}_{\theta_t}^{2(j)}\}_{j=1}^m$ with equal weights, that is $\{w_t^{(j)} = 1/m\}_{j=1}^m$, assuming they are normalised to $\sum_{j=1}^m w_t^{(j)} = 1$. A simple procedure to achieve this goal is, for each $j = 1, 2, \ldots, m$, to choose $(\tilde{\boldsymbol{S}}_t^{(j)}, \tilde{\mu}_{\theta_t}^{(j)}, \tilde{\sigma}_{\theta_t}^{2(j)}) = (\boldsymbol{S}_t^{(j)}, \mu_{\theta_t}^{(i)}, \sigma_{\theta_t}^{2(i)})$ with probability $w_t^{(i)}$.

In this paper, a resampling technique suggested by Reference [13] is employed. This technique forms a new set of weighted samples $\tilde{\Xi}_t = \{\tilde{\boldsymbol{S}}_t^{(j)}, \tilde{\mu}_{\theta_t}^{(j)}, \tilde{\sigma}_{\theta_t}^{2(j)}\}_{j=1}^m$ according to the following algorithm. (assume that $\sum_{j=1}^m w_t^j = m$)

(1) For $j = 1, 2, \ldots, m$, retain $\ell_j = w_t^j$ copies of the samples $(\boldsymbol{S}_t^{(j)}, \mu_{\theta_t}^{(i)}, \sigma_{\theta_t}^{2(i)})$. Denote $L_r = m - \sum_{j=1}^m \ell_j$.
(2) Obtain L_r i.i.d. draws from the original sample set $\{(\boldsymbol{S}_t^{(j)}, \mu_{\theta_t}^{(i)}, \sigma_{\theta_t}^{2(i)})\}_{j=1}^m$, with probabilities proportional to $(w_t^j - \ell_j), j = 1, 2, \ldots, m$.
(3) Assign equal weights, that is set $w_t^j = 1$, for each new sample.

It is shown in Reference [13] that the samples drawn by the above procedure are properly weighted with respect to $p(\boldsymbol{S}_t|\boldsymbol{Y}_t)$, provided that m is sufficiently large. Note that resampling at every time step is not needed in general. In one way the resampling can be done every k_0 recursions where k_0 is a prefixed resampling interval. On the other hand, the resampling can be carried out whenever the effective sample size, approximated as

$$\hat{N}_{\mathrm{eff}} = \frac{1}{\sum_{j=1}^m (w_t^j)^2} \leq m \qquad (25)$$

goes below a certain threshold, typically a fraction of m. Intuitively, $\hat{N}_{\mathrm{eff}}$ reflects the equivalent size of i.i.d samples from the true posterior densities of interest for the set of m weighted ones. It is suggested in Reference [4] that resampling should be performed when $\hat{N}_{\mathrm{eff}} < m/10$. Alternatively, one can conduct the first approach to conduct resampling at every fixed-length time interval say every five steps.

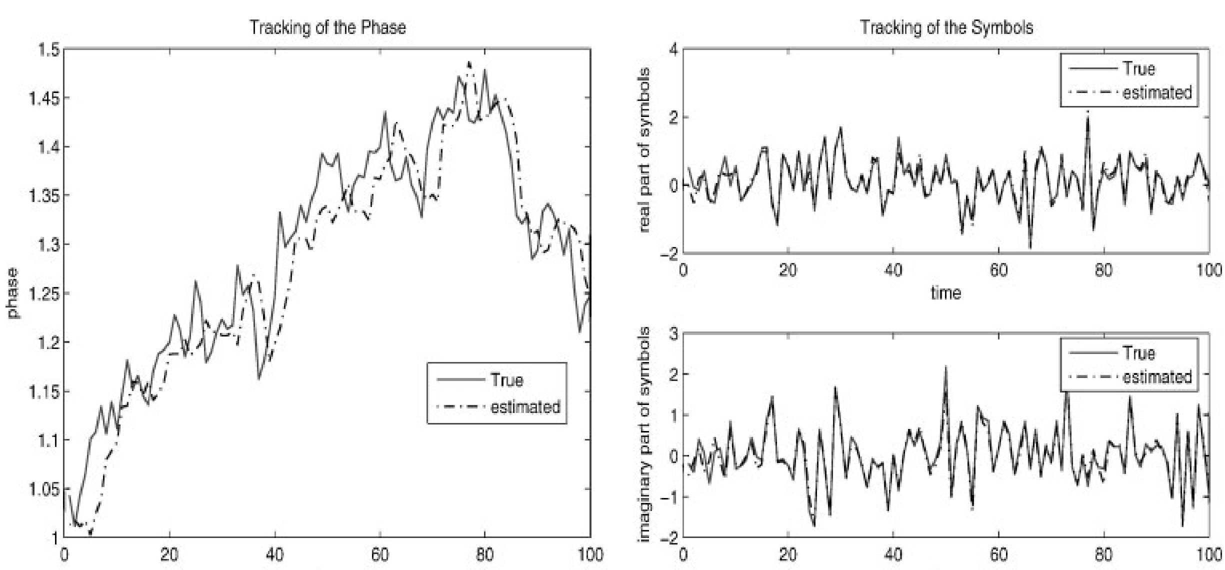

Figure 1. Tracking performance.

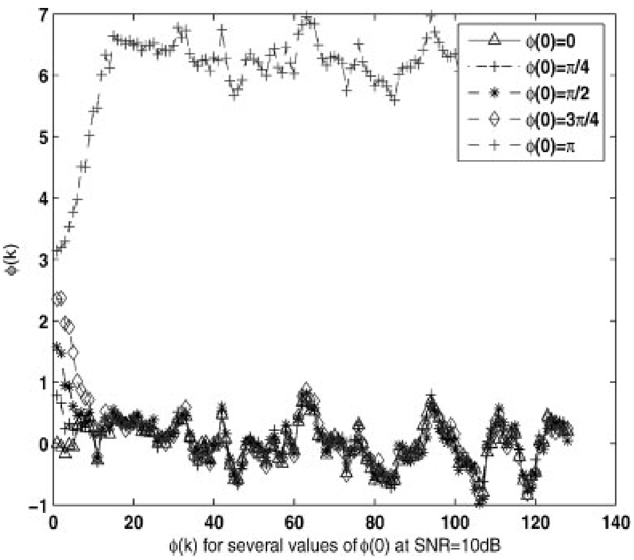

Figure 2. Tracking performance for different initialisations at SNR = 10 dB.

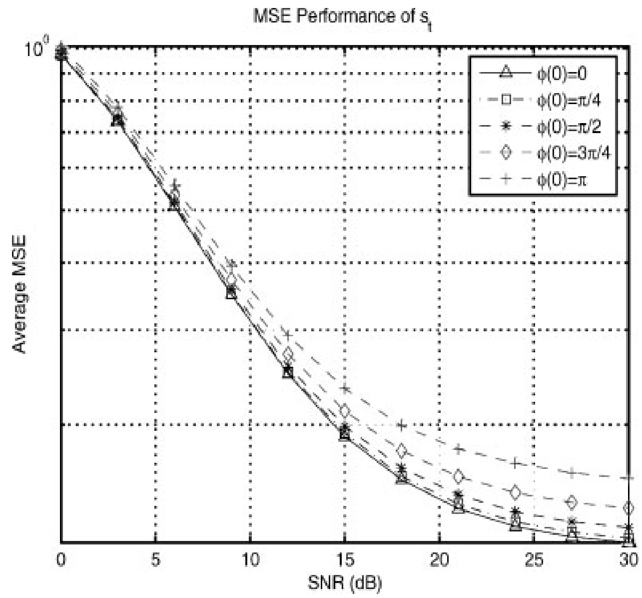

Figure 4. Average MSE performance of s_t for different initialization.

5. SIMULATION RESULTS

In this section, we provide some computer simulation examples to demonstrate the performance of the proposed SMC approach for blind-phase noise estimation and data detection in OFDM systems. The phase process is modelled by AR process driven by a white Gaussian noise with $\sigma_u^2 = 0.1$. s_t is modelled as a complex Gaussian process which has zero mean and variance $\sigma_s^2 = 1$. The impulse response of the channel has five uniformly distributed taps with spacing equal to the sampling period and with exponentially decaying profile.

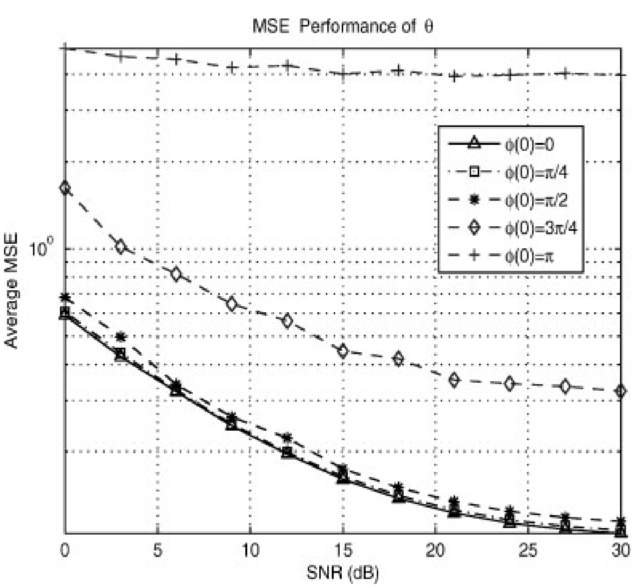

Figure 3. Average MSE performance of phase noise for different initialisations.

In order to demonstrate the performance of the adaptive SMC approach, we first present the tracking performance for both phase and symbols at SNR = 20 dB in Figure 1. It is shown through simulations that the performance of the proposed SMC algorithm can track the phase as well as transmitted symbols close to the true values.

We then consider the performance (in terms of the phase error $\phi(k) = (\theta_t - \hat{\theta}_t)$ for 1000 Monte Carlo trials for different initial phase errors $\phi(k) = 0, \pi/4, \pi/2, 3\pi/4, \pi$. The phase error for several values of $\phi(0)$ for a wide range of SNR values. The results are shown in Figure 2.

The performance of the proposed algorithm is further exploited by the evaluation of average MSE over observed subcarriers for different SNRs and different initial phase errors. The average MSE performance of this adaptive approach for both phase and symbols are plotted in Figures 3 and 4.

Our simulations indicate that as the initial phase error $\phi(0)$ approaches π, the probability that the phase error converges to the dual equilibrium point becomes very high.

Moreover, the relevant simulation results show that the proposed scheme enables to perform blind reliable phase tracking with relatively good initialisation.

6. CONCLUSIONS

We have developed a new adaptive Bayesian approach for blind-phase noise estimation and data detection for OFDM systems based on sequential Monte Carlo methodology. The optimal solutions to joint symbol detection and phase noise estimation problem is computationally prohibitive to implement by conventional methods. Thus the proposed sequential approach offers an novel and powerful approach to tackling this problem at a reasonable computational cost. The performance merits of our blind-phase noise estimation algorithm is confirmed by corroborating simulations. Sensitivity to initialisation of the proposed algorithm are investigated for OFDM systems. It is observed from simulations that as the initial phase error $\phi(0)$ approaches π, the probability that the phase error converges to the dual equilibrium point becomes very high.

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