The Effect of Primary User Bandwidth on Bayesian Compressive Sensing Based Spectrum Sensing

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***Abstract*—The application of compressive sensing (CS) theory has found great interest in wideband spectrum sensing. Although most studies have considered perfect reconstruction of the primary user signal, it is actually more important to assess the presence or absence of the signal. Among CS based methods, Bayesian CS (BCS) takes into consideration the prior information of signal coefficients to be estimated, which improves signal reconstruction performance. On the other hand, the sparsity level of the signal to be estimated has a direct impact on signal reconstruction and detection performances. Considering all of the above, the effect of sparsity level on BCS based spectrum sensing is studied in this paper. More specifically, a BCS based spectrum sensing scheme is considered and its mean-square error (MSE) performance is compared with the Bayesian Cramer-Rao bound for various user bandwidths. BCS MSE is also compared with the deterministic lower MSE (DL-MSE), which is a tight lower bound of the conventional basis pursuit approach. Furthermore, complementary receiver operating characteristic (ROC) curves are obtained to examine the trade-off between probabilities of false alarm and detection, depending on the user signal bandwidth.**

***Index Terms*—Cognitive radios, energy efficiency, Bayesian compressive sensing, spectrum sensing, probability of detection, probability of false alarm**

I. Introduction

In the last decade, limited frequency resources and energy efficiency have become two leading major issues in wireless communications. One way of efficiently utilizing the frequency resources is through cognitive radios [1], which sense the spectrum and opportunistically use the unused frequency bands. In order to assess the presence or absence of primary systems in a wider frequency range, wideband spectrum sensing studies have been conducted [2], [3]. In these studies, the increase in bandwidth corresponds to higher sampling rates at the receiver side, hence, spectrum sensing becomes less energy efficient in terms of sampling operation.

In the case a signal exhibits a sparse structure, the signal can be estimated by using compressive sensing (CS) theory proposed in [4] and [5]. According to the CS theory, an M-sample long sparse signal, which contains K nonzero coefficients, can be recovered with high probability by projecting it on an $N \times M$ random measurement matrix, where $K << N < M$. In wideband spectrum sensing, the received signal may be viewed as $sparse\ in\ frequency\ domain$, if there are only few orthogonal users active in a wide frequency range. In that case, CS based approaches can be applied for spectrum sensing [6].

In CS based spectrum sensing studies, there are different implementations to assess the primary users. In [7], spectrum identification, which is robust to interference, is evaluated by determining user locations and providing transmission powers of involved signal without reconstruction. In [8], the total iteration number in Bayesian CS (BCS) is reduced by setting a threshold for significant spikes when the wideband signal is block sparse. The probability of detection is calculated only with changing signal-to-noise-ratio (SNR). However, reconstruction performance is not particularly studied. In [9], wideband cooperative CS based spectrum sensing has been evaluated by using distributed sensing matrix. In [10], block sparse signal has been estimated based on sparse Bayesian learning. In [11], basis pursuit, Bayesian CS and multi-resolution BCS performances are compared in terms of computation time and reconstruction error. Considering [6] – [11], these studies (i) do not consider a lower bound on the estimation/reconstruction performance, and (ii) do not assess the received signal in the absence of a primary user signal. However, in conventional spectrum sensing studies [12], [13], receiver operating characteristic (ROC) curves, which show the trade-off between probabilities of false alarm and detection, are necessary to assess the actual spectrum sensing performance.

In this paper, we consider the implementation of Bayesian CS [14] for spectrum sensing and investigate the effect of primary user bandwidth on signal reconstruction and detection performances. Accordingly, we provide a lower bound on the estimation performance and present complementary ROC curves unlike [6] – [11]. Different from our earlier work in [15], the effect of sparsity (i.e., user bandwidth) on the signal reconstruction performance is studied and compared to the achievable lower bounds; deterministic lower mean square error (DL-MSE) and Bayesian Cramer-Rao bound (BCRB). Furthermore, probabilities of false alarm and misdetection are obtained for various scenarios, where [15] did not consider the possible absence of a primary user signal. This consideration is important as probability of detection alone is not a sufficient measure to understand the signal detection performance.

The rest of paper is organized as follows. Bayesian CS for parameter estimation is explained in Section II. In Section III, primary user signal model is presented. System performance is evaluated in terms of reconstruction and detection performances in Section IV. In Section V, simulation results are interpreted. Concluding remarks are given in Section VI.

II. Bayesian CS for Parameter Estimation

Primary user signal is transmitted through an additive white Gaussian noise (AWGN) channel in time domain. The received signal, which is corrupted by noise, sampled below Nyquist rate can be modeled as

$$\mathbf{r} = \mathbf{A}\big(\mathbf{w}_t + \mathbf{n}_t\big) \tag{1}$$

where $\mathbf{A} \in \Re^{N\times M}$, $\mathbf{w}_t \in \Re^{M\times 1}$, and $\mathbf{n}_t \in \Re^{M\times 1}$ represent random projection matrix, primary user signal in time domain, and AWGN, respectively. The above equation can be rewritten via inverse discrete Fourier transform (IDFT) matrix, $\mathbf{F}^{-1}$, as

$$\mathbf{r} = \mathbf{A}\mathbf{F}^{-1}\mathbf{w}_f + \mathbf{A}\mathbf{n}_t = \mathbf{\Phi}\mathbf{w}_f + \mathbf{n}_f \tag{2}$$

where $\mathbf{w}_f$ corresponds to frequency spectrum occupied by primary users, $\mathbf{\Phi} = \mathbf{A}\mathbf{F}^{-1}$ can be viewed as a transition matrix representing a conversion from frequency domain to time domain, and $\mathbf{n}_f$ denotes Gaussian distributed noise samples with mean zero and variance σ^2.

In Bayesian learning technique [14], a prior information is introduced on signal coefficients, where the coefficients are assumed to be Gaussian distributed. The main objective of BCS is to estimate the sparse parameter vector $\mathbf{w}_f$ by exploiting a sparsity-promoting prior through the estimation of hyperparameters α and $\boldsymbol{\beta}$ that represent, respectively, inverse noise variance and inverse variance of signal coefficients. The full posterior probability of unknowns can be defined as

$$p(\mathbf{w}_f, \boldsymbol{\beta}, \alpha|\mathbf{r}) = \frac{p(\mathbf{r}|\mathbf{w}_f, \boldsymbol{\beta}, \alpha)p(\mathbf{w}_f, \boldsymbol{\beta}, \alpha)}{p(\mathbf{r})}. \tag{3}$$

However, it is theoretically not possible to calculate the received signal probability, which is defined by following equation, as it requires triple integration over unknowns:

$$p(\mathbf{r}) = \int\int\int p(\mathbf{r}|\mathbf{w}_f, \boldsymbol{\beta}, \alpha)p(\mathbf{w}_f, \boldsymbol{\beta}, \alpha)\,d\mathbf{w}_f d\boldsymbol{\beta} d\alpha. \tag{4}$$

Therefore, the full posterior probability can be rearranged as

$$p(\mathbf{w}_f, \boldsymbol{\beta}, \alpha|\mathbf{r}) = p(\mathbf{w}_f|\mathbf{r}, \boldsymbol{\beta}, \alpha)p(\boldsymbol{\beta}, \alpha|\mathbf{r}). \tag{5}$$

Before presenting received signal's probability density function, independent identically distributed zero-mean Gaussian noise process can be defined as

$$p(\mathbf{n}_f) = \prod_{i=1}^{N} \mathcal{N}(n_{f_i}|0, \sigma^2). \tag{6}$$

Accordingly, received compressed signal distribution will be

$$p(\mathbf{r}|\mathbf{w}_f, \alpha) = (2\pi\sigma^2)^{-N/2} \exp\left(-\frac{1}{2\sigma^2}||\mathbf{r} - \mathbf{\Phi}\mathbf{w}_f||_2^2\right) \tag{7}$$

where $||\cdot||_2$ represents ℓ_2-norm. In Bayesian approach, prior information on spectrum coefficients is included unlike basis pursuit approach. The distribution of prior information can be expressed as [14]

$$p(\mathbf{w}_f|\boldsymbol{\beta}) = \prod_{i=1}^{M}(2\pi\beta_i^{-1})^{-1/2} \exp\left(-\frac{\beta_i w_{f,i}^2}{2}\right) \tag{8}$$

where $\mathbf{w}_f = [w_{f,1}, w_{f,2}, \ldots, w_{f,M}]^T$.

The first multiplier of posterior probability given in (5) can be expanded via Bayes' rule as

$$p(\mathbf{w}_f|\mathbf{r}, \boldsymbol{\beta}, \alpha) = \frac{p(\mathbf{r}|\mathbf{w}_f, \alpha)p(\mathbf{w}_f|\boldsymbol{\beta})}{p(\mathbf{r}|\boldsymbol{\beta}, \alpha)}. \tag{9}$$

The posterior probability given in (9) results with the distribution $\mathcal{N}(\boldsymbol{\mu}, \mathbf{\Sigma})$, where [14]

$$\begin{aligned} \boldsymbol{\mu} &= \alpha\mathbf{\Sigma}\mathbf{\Phi}^T\mathbf{r} \\ \mathbf{\Sigma} &= \big(\mathrm{diag}(\boldsymbol{\beta}) + \alpha\mathbf{\Phi}^T\mathbf{\Phi}\big)^{-1}. \end{aligned} \tag{10}$$

The distribution of unknown parameters, $p(\boldsymbol{\beta}, \alpha|\mathbf{r})$, which is the second multiplier of (5), is obtained via type-II maximum likelihood technique by operating relevance vector machines (RVM) [16]. The maximization process can also be applied to $p(\mathbf{r}|\boldsymbol{\beta}, \alpha)$, which is proportional to $p(\boldsymbol{\beta}, \alpha|\mathbf{r})$ [14]. The marginal likelihood function can be described as

$$p(\mathbf{r}|\boldsymbol{\beta}, \alpha) = \int_{-\infty}^{+\infty} p(\mathbf{r}|\mathbf{w}_f, \alpha)p(\mathbf{w}_f|\boldsymbol{\beta})d\mathbf{w}_f. \tag{11}$$

It is more appropriate to use log-marginal likelihood function in maximization process and it can be given as [17]

$$\begin{aligned} \log p(\mathbf{r}|\boldsymbol{\beta}, \alpha) &= \log \int_{-\infty}^{+\infty} p(\mathbf{r}|\mathbf{w}_f, \alpha)p(\mathbf{w}_f|\boldsymbol{\beta})d\mathbf{w}_f \quad (12) \\ &= \frac{N}{2}\log\alpha - \frac{1}{2}(\alpha\mathbf{r}^T\mathbf{r} - \boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}) \\ &- \frac{1}{2}\log|\mathbf{\Sigma}| - \frac{N}{2}\log(2\pi) + \frac{1}{2}\sum_{i=1}^{M}\log\beta_i. \end{aligned}$$

In order to find the hyperparameter values that maximize marginal likelihood function, the derivative of this function over α and $\boldsymbol{\beta}$ should be equal to zero. The estimated hyperparameter values which are updated every iteration during iteration process are obtained as

$$\begin{aligned} \beta_m^{new} &= \frac{1 - \beta_m\Sigma_{mm}}{\mu_m^2} \\ \alpha^{new} &= \frac{N - \sum_{m=1}^{M}(1 - \beta_m\Sigma_{mm})}{||\mathbf{r} - \mathbf{\Phi}\boldsymbol{\mu}||_2^2} \end{aligned} \tag{13}$$

where Σ_{mm} is the m-th diagonal element of the covariance matrix and μ_m is the m-th posterior mean value. After calculating hyperparameter values at each iteration, a stopping criterion can be applied to finish the iterative algorithm. Therefore, a difference value, δ, can be defined as [17]

$$\delta = \sum_{i=1}^{M}\left|\beta_i^{n+1} - \beta_i^{n}\right| \tag{14}$$

where β_i^{n+1} and β_i^{n} denote inverse variance of the prior belonging to i^{th} hyperparameter at the $(n+1)^{th}$ and n^{th} iterations, respectively. When the difference value is smaller than a threshold value, δ_{thold}, (i.e., $\delta < \delta_{thold}$), the estimation process will be terminated. At the end of the iterations, the unknown primary user signal can be reconstructed as

$$\hat{\mathbf{w}}_f = \boldsymbol{\mu}. \tag{15}$$

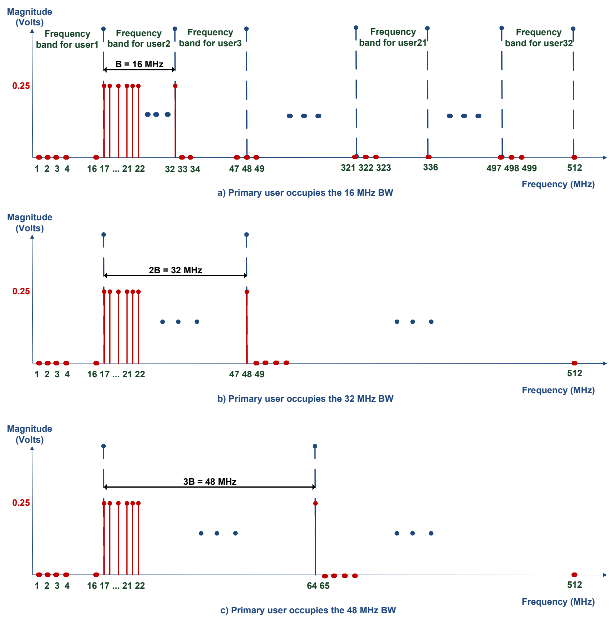

Fig. 1. Frequency domain primary user localization

III. Primary User Signal Model

Primary user signal model can be summarized as in (2) as

$$\mathbf{r} = \mathbf{\Phi}\mathbf{w}_f + \mathbf{n}_f \tag{16}$$

where $\mathbf{w}_f \in \Re^{M \times 1}$ is the primary user signal to be estimated. Frequency spectrum consists of L orthogonal bands where each frequency band has (M/L)-sample long representation. Primary user occupies the spectrum with bandwidth, B, which is defined as $B = (M/L)\Delta f$ where Δf is the frequency resolution. While SNR per primary user signal is given as

$$\gamma = \frac{\mathbf{w}_f^T \mathbf{w}_f}{\sigma^2}, \tag{17}$$

SNR per subcarrier can be defined as

$$\gamma_{SC} = \frac{w_{f_i}^2}{\sigma^2} = \gamma \frac{L}{M} \tag{18}$$

where w_{f_i} represents the i^{th} subcarrier coefficient. A primary user signal has a bandwidth of $BW = kB$, where $k \in \{1, 2, ..., L\}$. This is illustrated in Fig. 1 for $M = 512, L = 32, \Delta f = 1$ MHz and $k \in \{1, 2, 3\}$. As seen in the figure, there are three different spectrum utilization scenarios for primary users with different bandwidths of 16 MHz, 32 MHz, and 48 MHz, respectively. Also, note that SNR per subcarrier is fixed for all cases. While the above assumption of SNR is fair, reconstruction error performance degradation is expected due to bandwidth increasing (i.e., sparsity level degrading) and SNR per subcarrier decreasing when *SNR per user is fixed* for each scenario.

IV. System Performance

The system performance will be evaluated from both reconstruction and detection performance aspects.

A. Reconstruction Performance Evaluation

Reconstruction performance is measured by calculating MSE, and comparing it to DL-MSE which is a tight bound on basis pursuit [18] as shown in [19], and to BCRB which is a lower bound for BCS based techniques.

The signal reconstruction error is measured with MSE as

$$\text{MSE} = \text{E}\Big\{||\mathbf{w}_f - \hat{\mathbf{w}}_f||_2^2\Big\}. \tag{19}$$

A lower bound on MSE, DL-MSE, can be calculated when the locations of nonzero coefficients are known. It defines the best performance that basis pursuit can obtain. Thus, DL-MSE is defined as [20]

$$\text{DL-MSE} = \frac{K}{N\gamma_{SC}} \tag{20}$$

where N is the number of observations used for compression under the condition $N < M$.

On the other hand, in Bayesian approach, a lower bound named BCRB contains extra information compared to DL-MSE, as the estimator has prior knowledge about the distribution of signal coefficients. BCRB can be obtained following [20] as:

$$\text{BCRB} = K\left(N\gamma_{SC} + \frac{1}{\sigma_i^2}\right)^{-1} \tag{21}$$

where σ_i^2 represents the variance of the i^{th} prior. The derivation is not provided due to space constraints but can be inferred from [20].

B. Detection Performance Evaluation

In addition to reconstruction, the detection of the signal is also important because generally the knowledge of frequency band usage, whether it is in use or not, is required. Therefore, the detection performance should be used to assess the primary users in addition to reconstruction.

The detection probability of the bandwidth of interest can be expressed as

$$P_{d_{BW}} = \text{Pr}\left[\hat{\mathbf{w}}_{f_{BW}}^T \hat{\mathbf{w}}_{f_{BW}} \geq \lambda \,|\, l^{th} \text{ user active}\right] \tag{22}$$

where $\hat{\mathbf{w}}_{f_{BW}}$ denotes the spectrum coefficients estimated in the specified bandwidth, BW, which is equal to kB for different bandwidth occupation scenarios when $k \in \{1, 2, ..., L\}$ and λ is the energy threshold value of the detector. Since BCS may provide high detection probability, it is more appropriate to define probability of misdetection, which is

$$P_{md_{BW}} = 1 - P_{d_{BW}}. \tag{23}$$

Similarly, the probability of false alarm that belongs to bandwidth of interest can be defined as

$$P_{f_{BW}} = \text{Pr}\left[\hat{\mathbf{w}}_{f_{BW}}^T \hat{\mathbf{w}}_{f_{BW}} \geq \lambda \,|\, l^{th} \text{ user not active}\right]. \tag{24}$$

In order to evaluate detection performances, complementary ROC curves ($P_{md_{BW}}$ vs $P_{f_{BW}}$) will be presented via simulations in the next section to monitor the performance trade-off.

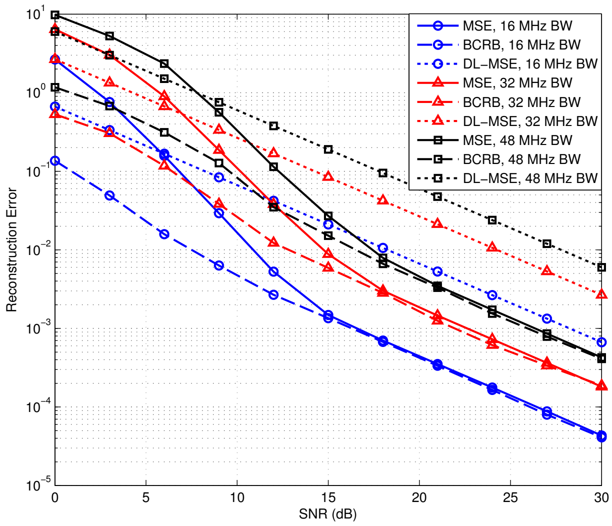

Fig. 2. Reconstruction error vs. SNR when CR=0.75

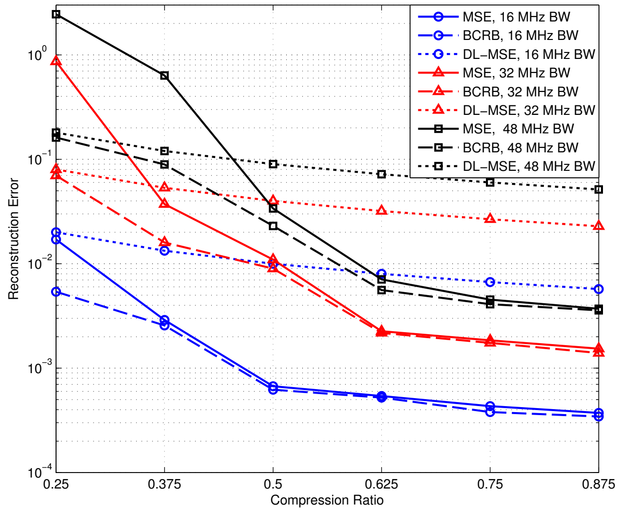

Fig. 3. Reconstruction error vs. compression ratio when SNR=20dB

V. Simulation Results

In this section, spectrum sensing performance will be assessed by providing reconstruction and detection performances. Reconstruction MSE results of BCS will be compared with the lower bounds of DL-MSE and BCRB. For the detection performance on allocated frequency bands, probabilities of misdetection and false alarm will be presented. While evaluating reconstruction and detection performances, the effect of primary user bandwidth will be examined. To do so, three different spectrum usage scenarios presented in Fig. 1 are considered. Accordingly, it is assumed that frequency spectrum can support up to $L = 32$ orthogonal users at the same time when each user's bandwidth is $B = 16$ MHz (i.e., $B = (M/L)\Delta f$), where $\Delta f = 1$ MHz and $M = 512$. On the other hand, it is also assumed that a user may have a wider bandwidth $BW = kB$, where $k \in \{1, 2, 3\}$. Sparsity level is defined as the ratio of the nonzero components to the length of the discrete spectrum. Sparsity levels (K/M) are therefore $\{16/512, 32/512, 48/512\}$ for various bandwidth considerations with corresponding signal energies being 1, 2, and 3 Joules, respectively. Compression ratios (N/M) are selected as CR=$\{0.25, 0.375, 0.5, 0.625, 0.75, 0.875\}$.

In Fig. 2, reconstruction error performances are plotted for a compression ratio fixed at 0.75. Reconstruction performance with corresponding lower bounds that belong to the minimum bandwidth scenario is the best since it has the minimum number of nonzero spectrum coefficients, as expected. In addition to that, BCRB is a tight bound for BCS MSE and lower than the DL-MSE bound since it has prior information of the probability distribution of spectrum coefficients. BCRB bounds are attained by BCS MSE at $\{15, 18, 21\}$dB SNR for $\{16, 32, 48\}$ MHz BW, respectively. Note that DL-MSE serves as the best possible performance of basis pursuit, which is inferior to BCS MSE for medium to high SNR range.

In Fig. 3, SNR is fixed at 20dB and reconstruction error performances over various compression ratios are shown. Estimation performance is improved as compression ratio increases, as expected. Furthermore, BCS MSE attains the BCRB at $\{$0.375, 0.5, 0.75$\}$ compression ratios for $\{$16, 32, 48$\}$ MHz BW, respectively. On the other hand, BCS MSE outperforms DL-MSE for all compression ratios when the BW is 16 MHz (the most sparse case), and for compression ratios greater than 0.375 when the BW is doubled or tripled. It should also be noted that the MSE performance does not improve much when the compression ratio is further increased. For example, when the BW is 16 MHz, it is better to select the compression ratio as 0.5 as opposed to 0.875, since the MSE performances are almost the same.

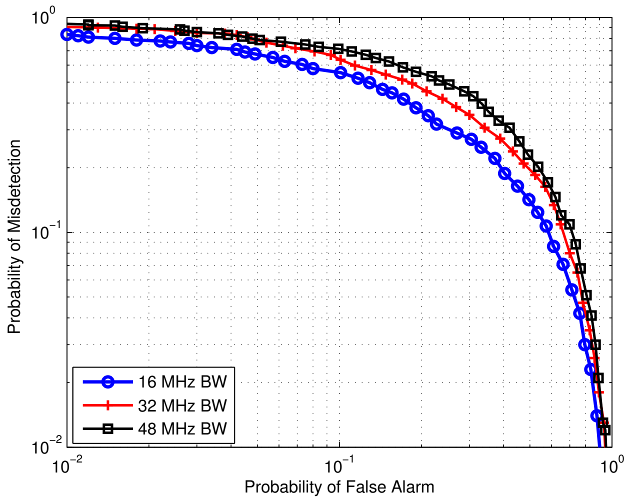

Fig. 4. Probability of misdetection vs. probability of false alarm when CR=0.375, SNR=-10dB

In Figs. 4 and 5, complementary ROC curves are plotted when compression ratios are 0.375 and 0.75, respectively, at SNR=-10dB. The SNR level selected is low so that proba-

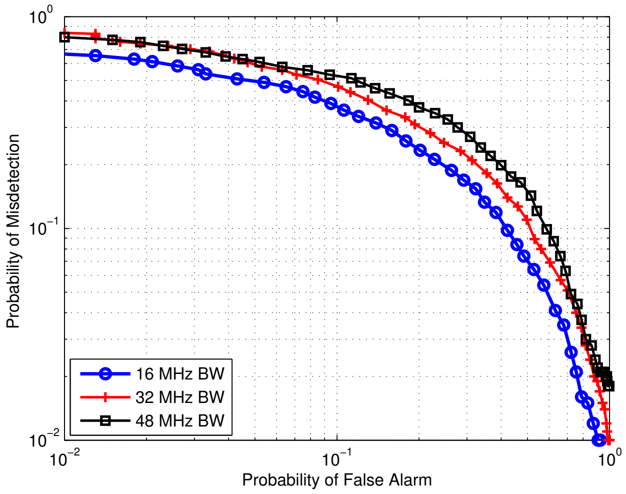

Fig. 5. Probability of misdetection vs. probability of false alarm when CR=0.75, SNR=-10dB

bility of false alarm and probability of misdetection can be observed. For a fixed false alarm rate, it can be observed that a sparser structure (i.e., narrower bandwidth) always has better misdetection performance. When the false alarm rate is set to 0.1, by increasing the number of observations from $0.375M$ to $0.75M$, probability of misdetection reduces from 0.55 to 0.37 for 16 MHz BW, from 0.64 to 0.47 for 32 MHz BW, and from 0.71 to 0.52 for 48 MHz BW.

While this study focused on the effect of the user bandwidth on the reconstruction and detection performances in an AWGN channel, future work will include the effect of fading.

VI. Conclusion

In this study, the effect of primary user bandwidth on the reconstruction and detection performances was investigated for BCS based spectrum sensing. It was observed that the BCS MSE can attain the BCRB at medium to high compression ratios and SNR values. Furthermore, the MSE performance is better for narrower bandwidths (i.e., sparser structure). More importantly, the detection performance was determined in terms of probabilities of misdetection and false alarm for the low SNR region and their trade-off is presented. It should be noted that the absence of a primary user is not considered in most of the CS based spectrum sensing studies. The results of this work are important as the BCS based spectrum sensing provides sampling reduction at the receiver and yet achieves superior performance compared to DL-MSE for a wide range of compression ratio and SNR values.

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