Sparse Channel Estimation for OFDM-based Underwater Cooperative Systems with Amplify-and-Forward Relaying

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***Abstract*—This paper is concerned with a challenging problem of channel estimation for amplify-and-forward cooperative relay based orthogonal frequency division multiplexing (OFDM) systems in the presence of sparse underwater acoustic channels and of the correlative non-Gaussian noise. We exploit the sparse structure of the channel impulse response to improve the performance of the channel estimation algorithm, due to the reduced number of taps to be estimated. The resulting novel algorithm initially estimates the overall sparse channel taps from the source to the destination as well as their locations using the matching pursuit (MP) approach. The correlated non-Gaussian effective noise is modeled as a Gaussian mixture. Based on the Gaussian mixture model, an efficient and low complexity algorithm is developed based on the combinations of the MP and the space-alternating generalized expectation-maximization (SAGE) technique, to improve the estimates of the channel taps and their location as well as the noise distribution parameters in an iterative way. The proposed SAGE algorithm is designed in such a way that, by choosing the admissible hidden data properly on which the SAGE algorithm relies, a subset of parameters is updated for analytical tractability and the remaining parameters for faster convergence Computer simulations show that underwater acoustic (UWA) channel is estimated very effectively and the proposed algorithm has excellent symbol error rate and channel estimation performance.**

I. INTRODUCTION

Underwater wireless communication has received a growing attention and research has been active for over a decade on designing the methods for underwater applications. It has been of critical importance to provide high-speed wireless links with high link reliability in various underwater applications such as offshore oil field exploration/monitoring, oceanographic data collection, maritime archaeology, seismic observations, environmental monitoring, port and border security among many others.

An underwater acoustic channel presents a communication system designer with many challenges. The three distinguishing characteristics of this channel are frequency-dependent propagation loss, severe multipath with much longer delay spreads [1], and low speed of sound propagation. Relay-assisted cooperative diversity presents a viable solution for underwater acoustic communication to extend transmission range and mitigate the degrading effects of fading. Cooperative diversity also named as user cooperation is a transmission method which extracts spatial diversity advantages through the use of relays [2]. The concept of cooperative diversity has been recently applied to underwater acoustic (UWA) communication and the number of current studies in this area is very limited [3], [4], [5], [6]. Mainly, Decode and Forward (DF) and Amplify and Forward (AF) relays have been adopted in practice for cooperative diversity systems. As remarked in [7] that the AF operation mode puts less processing burden on the relay and that AF relay actually outperforms DF relays under certain conditions.

The orthogonal frequency division multiplexing (OFDM)-based cooperative communication systems in underwater acoustic channels assuming various cooperation protocols are promising and seem to be a primary candidate for next generation UWA systems, due to their robustness to large multipath spreads [8], [3], [9]. A reliable channel state information (CSI) is necessary at the destination, to enable high transmission speeds and high link reliability. However, almost all the existing works assume that the perfect channel knowledge is available and there are only few results exists on channel estimation for the relay networks suggested under quite nonrealistic assumptions [7], [10]. Given sufficiently wide transmission bandwidth, the impulse response of the underwater acoustic channel is often sparse as the multipath arrivals becomes resolvable [1]. Furthermore, the effective noise entering the system between the source and the destination through the relay is correlated and non Gaussian. The combination of sparse structure and correlated non-Gaussian noise type creates a challenging channel estimation problem for relay based corporation diversity UWA systems. To the best of our knowledge, the problem of channel estimation for underwater AF relay channels has not been addressed satisfactorily in the literature and this motivated our present work.

In this paper we provide a new pilot assisted channel estimation technique for relay networks that employ the AF transmission scheme. Our main contribution in this work is two folds. First, we exploit the sparse structure of the channel impulse response to improve the performance of the channel estimation algorithm, due to the reduced number of taps to be estimated. The resulting algorithm initially estimates the overall sparse channel taps from the source to the destination as well as their locations using the matching pursuit (MP) approach [11]. The correlated non-Gaussian effective noise is modeled as a Gaussian mixture. Second, based on the Gaussian mixture model we develop an efficient and low complexity novel algorithm by combining the MP and the SAGE techniques, called the MP-SAGE algorithm which relies on the concept of the admissible hidden data, to improve the estimates of the channel taps and their location as well as the noise distribution parameters in an iterative way. We demonstrate that by suitably choosing the admissible hidden data on which the SAGE algorithm relies, a subset of parameters is updated for analytical tractability and the remaining parameters for faster convergence [12].

II. SYSTEM MODEL

We consider an UWA cooperative wireless communication scenario where the source node S transmits information to the destination node D with the assistance of relay node R each of which is equipped with a single pair of transmit and receive antenna. The cooperation is based on the receive diversity (RD) protocol [13] with a single-relay amplify-and-forward (AF) relaying with half-duplex nodes. In our work, we assume that the relay node does not perform the channel estimation to keep its complexity as low as possible. As shown in Fig. 1, in the broadcasting phase, the source node transmits to the destination and the relay nodes. In the relaying phase, the relay node forwards a scaled noisy version of the signals received from the source. The channel between each node pair is assumed to be quasi-static frequency-selective Rician fading. The channel impulse responses (CIRs) for $S \to R$, $R \to D$ and $S \to D$ links are sparse and denoted by $\tilde{\mathbf{h}}^{SR}$, $\tilde{\mathbf{h}}^{RD}$ and $\tilde{\mathbf{h}}^{SD}$ having maximum discrete-valued multipath delays $\tilde{L}_{SR}$, $\tilde{L}_{RD}$ and $\tilde{L}_{SD}$, respectively. $L_{SR} \ll \tilde{L}_{SR}$, $L_{RD} \ll \tilde{L}_{RD}$ and $L_{SD} \ll \tilde{L}_{SD}$ denote the number of non-zero elements of the multipath channels. Channel coefficients (taps) on each link is a complex Gaussian random variable with independent real and imaginary parts with mean $\mu_\ell/\sqrt{2}$ and the variance $\sigma_\ell^2/2$. Let $\Omega_\ell = E\{|h_\ell|^2\} = \mu_\ell^2 + \sigma_\ell^2$

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denotes the power profile of the relevant Rician multipath channel and $\sum_{\ell=1}^{L}\Omega_\ell = 1$, $L \in \{L_{SR}, L_{RD}, L_{SD}\}$. Moreover, Rician κ-factor for ℓth tap is the ratio of the power in the mean component to the power in the diffuse component, i.e. $\kappa_\ell = \mu_\ell^2/\sigma_\ell^2$. Therefore, each channel tap is given by

$$h_\ell = \sqrt{\frac{\kappa_\ell \Omega_\ell}{\kappa_\ell + 1}}\left(\frac{1+j}{\sqrt{2}}\right) + \sqrt{\frac{\Omega_\ell}{\kappa_\ell + 1}}\,\breve{h}_\ell \ , \quad \ell = 1, 2, \cdots, \acute{L} \quad \text{and} \quad \acute{L} \in \{L_{SR}, L_{RD}, L_{SD}\}, \tag{1}$$

where $\breve{h}_\ell$ is a complex Gaussian random variable with zero mean and unit variance.

The additive ambient noise, generated by underwater acoustic channels has several distinct physical origins each corresponding to particular frequency range [14]. In this paper, we assume that power spectral density of the ambient noise is modeled in 10 - 100 KHz band as a function of frequency in Hz as

$$N(f) = \frac{f_0 \sigma_v^2}{\pi(f^2 + f_0^2)} \ , \tag{2}$$

where σ_v^2 is the noise variance, and f_0 is chosen as a model parameter of the colored noise autocorrelation function ($f_0 T_s = 0.01, 0.05, 0.1, etc.$). Note that the autocorrelation function of the ambient noise can be obtained from (2) as

$$\rho(n - n') = \sigma_v^2\, e^{-2\pi|n-n'|f_0 T_s} \ , \tag{3}$$

where T_s is the sampling period. Consequently, the complex- valued additive Gaussian ambient noises on the links $S \rightarrow R$, $R \rightarrow D$ and $S \rightarrow D$ are denoted by $\mathbf{v}^{SR} = [v_0^{SR}, v_1^{SR}, \cdots, v_{N-1}^{SR}]^T$, $\mathbf{v}^{RD} = [v_0^{RD}, v_1^{RD}, \cdots, v_{N-1}^{RD}]^T$ and $\mathbf{v}^{SD} = [v_0^{SD}, v_1^{SD}, \cdots, v_{N-1}^{SD}]^T$ respectively. We assume that CIRs remain constant over a period of one block transmission and vary independently from block to block.

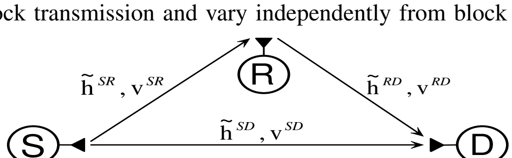

Fig. 1. Single-relay transmission model

We now now consider an OFDM based UWA relay system with N subcarriers. At the transmitter, K out of N subcarriers are actively employed to transmit data symbols and nothing is transmitted from the remaining $N - K$ carriers. The time-domain transmitted signal is denoted as

$$s(n) = \frac{1}{N}\sum_{k=0}^{K-1} d(k)\, e^{j2\pi nk/N} \ , \tag{4}$$

where n and k are the discrete-time and the discrete-frequency indices during the mth OFDM symbol, respectively. $d(k)$ stands for the frequency domain data symbol transmitted at discrete time m over the kth OFDM subchannel. To avoid inter-symbol interference (ISI) a cyclic prefix is added between adjacent OFDM blocks. After FFT and removing the cyclic prefix, time domain received data block in the broadcasting phase (1st time slot) at the relay and the destination nodes are given as

$$\overset{R}{y}_{1,n} = \sum_{l=1}^{\tilde{L}_{SR}} \tilde{h}_l^{SR} s_{n-l} + \overset{SR}{v_n} \quad \text{and} \quad \overset{D}{y}_{1,n} = \sum_{l=1}^{\tilde{L}_{SD}} \tilde{h}_l^{SD} s_{n-l} + \overset{SD}{v_n} \ , \tag{5}$$

respectively, where $s_n = \frac{1}{N}\sum_{k=1}^{N} d_k\, e^{j2\pi nk/N}$ is the time-domain signal sample transmitted from the S node at nth discrete time and d_k is the data symbol transmitted over the kth subchannel. In the relaying phase (2nd time slot), the time domain received signal at the destination node is

$$\overset{D}{y}_{2,n} = \frac{1}{\gamma}\sum_{l=1}^{\tilde{L}_{RD}} \tilde{h}_l^{RD}\, \overset{R}{y}_{1,n-l} + \overset{RD}{v_n} \tag{6}$$

where $\gamma = \sqrt{\sum_{n=1}^{N} E\{|y_{1,n}^{R}|^2\}}$ is the normalization factor. To ensure that the power budget is not violated, the relay node normalizes the receive signal $\overset{R}{y}_{1,n}$, $n = 1, 2, \cdots, N$ by γ. Inserting (5) into (6), the vector form of (6) can be expressed as

$$\overset{D}{\mathbf{y}_2} = \mathbf{\Gamma}\,\tilde{\mathbf{h}} + \mathbf{v} \ , \tag{7}$$

where $\overset{D}{\mathbf{y}_2} = [\overset{D}{y}_{2,0}, \overset{D}{y}_{2,1}, \cdots, \overset{D}{y}_{2,N-1}]^T$ is the time-domain received vector on the destination node in the relaying phase, $\mathbf{\Gamma} = \frac{1}{\gamma}\,\mathbf{F}^{-1}\mathbf{D}\,\mathbf{F}$, with $\mathbf{F}$ being the FFT matrix whose kth row and nth column entry $[\mathbf{F}]_{k,n} = e^{-j2\pi nk/N}$ and $\mathbf{D}$ is a diagonal matrix having the data symbols $\{d_k\}_{k=0}^{N-1}$ on its main diagonal. $\tilde{\mathbf{h}} = \tilde{\mathbf{h}}^{SR} \circledast \tilde{\mathbf{h}}^{RD}$ and

$$\begin{aligned} \mathbf{v} &= \tilde{\mathbf{h}}^{RD} \circledast \mathbf{v}^{SR} + \mathbf{v}^{RD} \\ &= \frac{1}{\gamma}\mathbf{F}^{-1}\mathbf{D}_{\tilde{\mathbf{H}}^{RD}}\,\mathbf{F}\,\mathbf{v}^{SR} + \mathbf{v}^{RD} \end{aligned} \tag{8}$$

denote the cascaded sparse multipath channel and additive noise on $S \rightarrow R \rightarrow D$ link, respectively, where $\circledast$ is the N-sample circular convolution operator and $\mathbf{D}_{\tilde{\mathbf{H}}^{RD}}$ represents a diagonal matrix whose main diagonal vector is $\tilde{\mathbf{H}}^{RD} = \mathbf{F}\,\tilde{\mathbf{h}}^{RD}$.

It is obvious from (8) that the ambient noise $\mathbf{v}$ is non-Gaussian and colored. Thus, without going further toward the channel estimation step, the observation model in (7) can be reduced to the one with additive white non-Gaussian noise by the use of a noise-whitening filter, based on the singular value decomposition (SVD) of the co-variance matrix of $\mathbf{v}$, $\mathbf{\Sigma_v} = \mathbf{U}\mathbf{\Upsilon}\mathbf{U}^\dagger$, where $\mathbf{U}$ is an $N \times N$ complex valued unitary transformation matrix, $\mathbf{\Upsilon}$ is an $N \times N$ diagonal matrix with positive real entries and $(\cdot)^\dagger$ denotes the conjugate transpose operator. Consequently, the colored noise can be transformed into a white noise through the linear transformation $\mathbf{\Psi}\mathbf{v} = \mathbf{w}$, where $\mathbf{w} = [w_1, w_2, \cdots, w_N]^T$ is a non-Gaussian white noise vector with identity covariance matrix and $\mathbf{\Psi} = \mathbf{\Upsilon}^{-1/2}\mathbf{U}^\dagger$ is termed as whitening matrix. Multiplying (7) by $\mathbf{\Psi}$ from the left we obtain the following observation model

$$\mathbf{y} = \mathbf{A}\tilde{\mathbf{h}} + \mathbf{w} \quad \in \mathcal{C}^{N\times 1} \ , \tag{9}$$

where $\mathbf{y} = \mathbf{\Psi}\,\overset{D}{\mathbf{y}_2}$ and $\mathbf{A} = \mathbf{\Psi}\,\mathbf{\Gamma} \in \mathcal{C}^{N\times N}$ is the convolution matrix generated from data symbols. Note that even though the components of $\mathbf{w}$ in (9) are uncorrelated due to the whitening process, they are still dependent non-Gaussian random variables. However, as we can observe from (8), the distribution of the ambient noise $\mathbf{v}$ is closed to Gaussian. Consequently, $\{w_k\}$'s may be assumed to be independent and identically distributed (i.i.d.) non-Gaussian samples of the additive noise $\mathbf{w}$.

In this work, we are mainly interested in estimation of $\tilde{\mathbf{h}}$ in (9) where $\tilde{\mathbf{h}} \in \mathcal{C}^N$ is a complex valued, sparse multipath channel vector with non-zero entries, $h_1, h_2, \cdots, h_L$ ($L \ll N$) and the associated random channel tap positions, $\eta_1, \eta_1, \cdots, \eta_L$. The received signal in (9) can be rewritten as

$$\mathbf{y} = \sum_{\ell=1}^{L} \mathbf{a}_{\eta_\ell} h_\ell + \mathbf{w} \ , \tag{10}$$

where, $\mathbf{a}_{\eta_\ell}$ is the η_ℓ th column vector of the matrix $\mathbf{A}$ corresponding to the ℓth multipath channel tap position. Note that the matrix $\mathbf{A}$ is known by the receiver completely since it contains only pilot symbols during the training phase in a given frame.

We model the white, non-Gaussian noise samples $w_n, n = 1, 2, \cdots, N$ in (9) as an i.i.d. M-term Gaussian mixture as follows

$$p(w_n) \quad = \quad \sum_{m=1}^{M} \frac{\lambda_m}{\pi\sigma_m^2}\, e^{-|w_n|^2/\sigma_m^2} \ , \tag{11}$$

where $p(w_n|\nu_n = m) \triangleq \frac{1}{\pi\sigma_m^2}\, e^{-|w_n|^2/\sigma_m^2}$, $\nu_n \in \{1, 2, \cdots, M\}$ is the nth random mixture index that identifies which term in Gaussian mixture pdf in (11) produced the additive noise sample w_n and

$p(\nu_n = m) = \lambda_m$ is the probability that w_n is chosen from the mth term in the mixture pdf, with $\sum_{m=1}^{M} \lambda_m = 1$. In (11), σ_m^2 denotes the variance of the mth Gaussian mixture.

III. Sparse Multipath Channel Estimation with MP-SAGE Algorithm

We now propose a new iterative algorithm, called the MP-SAGE algorithm, based on the SAGE and the MP techniques for channel estimation employing the signal model given by (9). The SAGE algorithm, proposed by Fessler et al. [15] provides updated estimates for an unknown parameter set $\mathbf{\Theta}$ iteratively as follows. At iteration (i) only a subset of $\mathbf{\Theta}_S$ indexed by $S = S[i]$ is updated while keeping the parameters in the complement set $\mathbf{\Theta}_{\overline{S}}$ fixed.

The MP algorithm is an iterative procedure which can sequentially identify the dominant channel taps and estimate the associated tap coefficients by choosing the the column $\mathbf{a}_{\eta_\ell}$ of $\mathbf{A}$ in (9) which best aligned with the residual vector until all the taps are identified. The detail description of the MP algorithm is given in Sec. III-C. Finally our proposed MP-SAGE algorithm implements the MP algorithm at each SAGE iteration step by updating, all the dominant channel taps and the associated tap coefficients sequentially. The details of the MP-SAGE algorithm is presented below:

The unknown parameter set to be estimated in our problem is

$$\mathbf{\Phi} = \{\mathbf{h}, \boldsymbol{\eta}, \boldsymbol{\alpha}\} \ , \tag{12}$$

where $\mathbf{h} = [h_1, h_2, \cdots, h_L]^T$, $\boldsymbol{\eta} = [\eta_1, \eta_2, \cdots, \eta_L]^T$ and $\boldsymbol{\alpha} = \{\lambda_1, \cdots, \lambda_M, \sigma_1^2, \cdots, \sigma_M^2\}$.

The first step in deriving the MP-SAGE algorithm for estimating $\mathbf{\Phi}$ based on the received vector $\mathbf{y}$ is the specifications of "complete data" and "admissible hidden data" sets whose pdfs are characterized by the common parameters set $\mathbf{\Phi}$. To obtain a receiver architecture that iterates between soft-data and channel estimation in the MP-SAGE algorithm, we decompose $\mathbf{\Phi}$ into $L+1$ subsets, representing the parameters, $\mathbf{h}$, $\boldsymbol{\eta}$ and $\boldsymbol{\alpha}$, as follows.

- The first L subsets of $\mathbf{\Phi}$ are chosen as $\mathbf{\Phi}_\ell = \{h_\ell, \eta_\ell\}$, $\ell = 1, 2, \cdots, L$. For each subset we define $\bar{\mathbf{\Phi}}_\ell = \mathbf{\Phi} \setminus \mathbf{\Phi}_\ell = \{\bar{\mathbf{h}}_\ell, \bar{\boldsymbol{\eta}}_\ell, \boldsymbol{\alpha}\}$, $\bar{\mathbf{h}}_\ell = \mathbf{h} \setminus h_\ell$ and $\bar{\boldsymbol{\eta}}_\ell = \boldsymbol{\eta} \setminus \eta_\ell$, where $\setminus$ denotes the exclusion operator.
- The $(L+1)$st subset of $\mathbf{\Phi}$ is chosen as by $\mathbf{\Phi}_{L+1} = \boldsymbol{\alpha}$ and $\bar{\mathbf{\Phi}}_{L+1} \triangleq \mathbf{\Phi} \setminus \mathbf{\Phi}_{L+1} = \mathbf{\Phi} \setminus \boldsymbol{\alpha} = \{\mathbf{h}, \boldsymbol{\eta}\}$

At the SAGE iteration (i), only the parameters in one set are updated, whereas the other parameters are kept fixed, and this process is repeated until all parameters are updated. According to the above parameter subset definitions, each iteration of the SAGE algorithm for our problem has two steps:

1) $\mathbf{\Phi}_\ell, \ell = 1, 2, \cdots, L$ is updated with the MP-SAGE algorithm while $\mathbf{\Phi}_{L+1}$ is fixed.

2) $\mathbf{\Phi}_{L+1}$ is updated with the SAGE algorithm while $\mathbf{\Phi}_\ell, \ell = 1, 2, \cdots, L$ is fixed.

We now derive the MP-SAGE algorithm below by also specifying the corresponding admissible hidden data and complete data sets.

A. Estimation of $\mathbf{\Phi}_\ell = \{h_\ell, \eta_\ell\}, \ell = 1, 2, \cdots, L$

A suitable approach for applying the MP-SAGE algorithm for estimation of $\mathbf{\Phi}_\ell$ is to decompose the nth sample of the receive signal in (10) into the sum

$$y_n = x_n^{(\ell)} + \bar{x}_n^{(\ell)} \ , \tag{13}$$

where

$$x_n^{(\ell)} = a_{n,\eta_\ell} h_\ell + w_n \quad \text{and} \quad \bar{x}_n^{(\ell)} = \sum_{\ell'=1,\,\ell'\neq\ell}^{L} a_{n,\eta_{\ell'}} h_{\ell'} \tag{14}$$

and a_{n,η_ℓ} denotes nth element of the $\mathbf{a}_{\eta_\ell}$. We define the admissible hidden data as $\chi_\ell = \{\mathbf{x}^{(\ell)}, \boldsymbol{\nu}\}$, where $\mathbf{x}^{(\ell)} = [x_1^{(\ell)}, x_2^{(\ell)}, \cdots, x_N^{(\ell)}]^T$ and $\boldsymbol{\nu} = [\nu_1, \nu_2, \cdots, \nu_N]^T$.

To perform the *E-Step* of the MP-SAGE algorithm, the conditional expectation is taken over χ_ℓ given the observation $\mathbf{y}$ and given that $\mathbf{\Phi}$ equals its estimate calculated at ith iteration:

$$\begin{aligned} Q_\ell(\mathbf{\Phi}_\ell|\mathbf{\Phi}^{(i)}) &= E\big\{\log p(\chi_\ell|\mathbf{\Phi}_\ell, \bar{\mathbf{\Phi}}_\ell^{(i)})\big|\mathbf{y}, \mathbf{\Phi}^{(i)}\big\} \\ &\sim \sum_{n=1}^{N} \delta_n^{(i)} \Big(2\Re\big\{\widehat{x_n^{(\ell)}}^{(i)} a_{n,\eta_\ell}^* h_\ell^*\big\} - |a_{n,\eta_\ell} h_\ell|^2\Big), \end{aligned} \tag{15}$$

where $\Re(\cdot)$ and $(\cdot)^*$ denote the real part and the conjugate operators, respectively, and $\widehat{x_n^{(\ell)}}^{(i)}$ is defined as

$$\widehat{x_n^{(\ell)}}^{(i)} \triangleq E\big\{x_n^{(\ell)}|\nu_n, y_n, \mathbf{h}^{(i)}, \boldsymbol{\eta}^{(i)}, \boldsymbol{\alpha}^{(i)}\big\} \ .$$

Recalling (13) it follows that

$$\widehat{x_n^{(\ell)}}^{(i)} = y_n - \sum_{\ell'=1,\ell'\neq\ell}^{L} a_{n,\eta_{\ell'}^{(i)}} h_{\ell'}^{(i)} \ , \tag{16}$$

and $\delta_n^{(i)}$ in (15) is defined as

$$\begin{aligned} \delta_n^{(i)} &\triangleq E_{\{\nu_n|y_n,\mathbf{h}^{(i)},\boldsymbol{\eta}^{(i)},\boldsymbol{\alpha}^{(i)}\}}\Big\{\frac{1}{(\sigma_{\nu_n}^2)^{(i)}}\Big\} \\ &= \sum_{m=1}^{M} \frac{1}{(\sigma_m^2)^{(i)}} p_{\nu_n}^{(i)}(m) \quad , \quad n = 1, 2, \cdots, N. \end{aligned} \tag{17}$$

Keeping in mind $p(\nu_n = m|\boldsymbol{\alpha}^{(i)}) = \lambda_m^{(i)}$, the posterior probability density function of the random mixture index ν_n at ith iteration, $p_{\nu_n}^{(i)}(m)$, is evaluated as follows

$$\begin{aligned} p_{\nu_n}^{(i)}(m) &\triangleq p(\nu_n{=}m\,|\,y_n, \mathbf{h}^{(i)}, \boldsymbol{\eta}^{(i)}, \boldsymbol{\alpha}^{(i)}) \\ &= \frac{\lambda_m^{(i)}\, e^{-\left|y_n - \sum_{\ell=1}^{L} a_{n,\eta_\ell^{(i)}} h_\ell^{(i)}\right|^2/(\sigma_m^2)^{(i)}} \Big/ \big(\pi(\sigma_m^2)^{(i)}\big)}{\sum_{m'=1}^{M} \lambda_{m'}^{(i)}\, e^{-\left|y_n - \sum_{\ell=1}^{L} a_{n,\eta_\ell^{(i)}} h_\ell^{(i)}\right|^2/(\sigma_{m'}^2)^{(i)}} \Big/ \big(\pi(\sigma_{m'}^2)^{(i)}\big)}. \end{aligned} \tag{18}$$

The vector form of (15) can be written as follows

$$Q_\ell(\mathbf{\Phi}_\ell|\mathbf{\Phi}^{(i)}) = 2\Re\big\{\mathbf{a}_{\eta_\ell}^\dagger \mathbf{D}_\delta^{(i)} \widehat{\mathbf{x}^{(\ell)}}^{(i)} h_\ell^*\big\} - \mathbf{a}_{\eta_\ell}^\dagger \mathbf{D}_\delta^{(i)} \mathbf{a}_{\eta_\ell} |h_\ell|^2 \ , \tag{19}$$

where from (16) $\widehat{\mathbf{x}^{(\ell)}}^{(i)} = [\widehat{x_1^{(\ell)}}^{(i)}, \cdots, \widehat{x_N^{(\ell)}}^{(i)}]^T = \mathbf{y} - \sum_{p=1,p\neq\ell}^{L} \mathbf{a}_{\eta_p^{(i)}} h_p^{(i)}$ and $\mathbf{D}_\delta^{(i)}$ is a diagonal matrix with entries $\delta_1^{(i)}, \delta_2^{(i)}, \cdots, \delta_N^{(i)}$ which are calculated from (17).

In the *M-step* of the MP-SAGE algorithm, the estimates of $\mathbf{\Phi}_\ell = \{h_\ell, \eta_\ell\}$ are updated at the $(i+1)$st iteration according to

$$\mathbf{\Phi}_\ell^{(i+1)} = \arg\max_{\mathbf{\Phi}_\ell} Q_\ell(\mathbf{\Phi}_\ell|\mathbf{\Phi}^{(i)}) \ , \tag{20}$$

where $Q_\ell(\mathbf{\Phi}_\ell|\mathbf{\Phi}^{(i)})$ is given by (19). So, taking the derivative of $Q_\ell(\mathbf{\Phi}_\ell|\mathbf{\Phi}^{(i)})$ with respect to h_ℓ^* and equating to zero, we find the final SAGE estimates of (η_ℓ, h_ℓ) at $(i+1)$st iteration as follows:

$$\begin{aligned} \eta_\ell^{(i+1)} &= \arg\max_{\eta_\ell} \frac{\big|\mathbf{a}_{\eta_\ell}^\dagger \mathbf{D}_\delta^{(i)} \widehat{\mathbf{x}^{(\ell)}}^{(i)}\big|^2}{\mathbf{a}_{\eta_\ell}^\dagger \mathbf{D}_\delta^{(i)} \mathbf{a}_{\eta_\ell}} \ , \ \eta_\ell \in \{1, 2, \cdots, N\}, \\ &\qquad \eta_\ell \notin \{\eta_1^{(i+1)}, \cdots, \eta_{\ell-1}^{(i+1)}\} \ , \\ h_\ell^{(i+1)} &= \frac{\mathbf{a}_{\eta_\ell^{(i+1)}}^\dagger \mathbf{D}_\delta^{(i)} \widehat{\mathbf{x}^{(\ell)}}^{(i)}}{\mathbf{a}_{\eta_\ell^{(i+1)}}^\dagger \mathbf{D}_\delta^{(i)} \mathbf{a}_{\eta_\ell^{(i+1)}}} \ . \end{aligned} \tag{21}$$

Based on the above result, $\{\eta_\ell, h_\ell\}$ can be sequentially estimated for $\ell = 1, 2, \cdots, L$, incorporating the previous estimates in the MP-SAGE mode as follows:

Step 1) For $i = 0$, determine the initial estimates $\{\eta_\ell^{(0)}, h_\ell^{(0)}\}, \ell = 1, 2, \cdots, L$, from the MP algorithm as described in Sec. III-C.

Step 2) For $i \leftarrow (i+1)$, and $\ell = 1, 2, \cdots, L$, compute $\{\eta_\ell^{(i+1)}, h_\ell^{(i+1)}\}$ from (21), replacing $\widehat{\mathbf{x}^{(\ell)}}^{(i)}$ with the residual vector $\mathbf{r}_\ell^{(i)}$ of the MP algorithm. It can be shown that, the residual vector can be computed recursively as

$$\mathbf{r}_\ell^{(i)} = \mathbf{r}_{\ell-1}^{(i)} - (\mathbf{a}_{\eta_\ell^{(i)}} h_\ell^{(i)} - \mathbf{a}_{\eta_{\ell-1}^{(i+1)}} h_{\ell-1}^{(i+1)}) \tag{22}$$

where $\mathbf{r}_0^{(i)} = \widehat{\mathbf{x}^{(1)}}^{(i)}$ and $\mathbf{a}_{\eta_0^{(i)}} = 0, h_0^{(i)} = 0$ for all (i).

Step 3) If $\ell = L$ go to the next SAGE iteration step.

Step 4) continue the SAGE iterations until convergence. END

B. Estimation of $\mathbf{\Phi}_{L+1} = \boldsymbol{\alpha} = \{\lambda_1, \cdots, \lambda_M, \sigma_1^2, \cdots, \sigma_M^2\}$

We define the complete data as $\chi_{L+1} = \{\mathbf{y}, \boldsymbol{\nu}\}$ to estimate the mixture parameters $\boldsymbol{\alpha} = \{\lambda_1, \cdots, \lambda_M, \sigma_1^2, \cdots, \sigma_M^2\}$. Now, let us derive the MP-SAGE algorithm.

To perform the *E-Step* of the MP-SAGE algorithm, the conditional expectation is taken over χ_{L+1} given the observation $\mathbf{y}$ and given that $\mathbf{\Phi}$ equals its estimate calculated at ith iteration can be expressed as follows

$$Q_{L+1}(\mathbf{\Phi}_{L+1}|\mathbf{\Phi}^{(i)}) = \sum_{n=1}^{N}\sum_{m=1}^{M} p_{\nu_n}^{(i)}(m)\Big[\log\big(\frac{\lambda_m}{\sigma_m^2}\big) - \frac{1}{\sigma_m^2}\Big|y_n - \sum_{\ell=1}^{L} a_{n,\eta_\ell^{(i)}} h_\ell^{(i)}\Big|^2\Big], \tag{23}$$

where $p_{\nu_n}^{(i)}(m)$ is given in (18).

In the *M-step* of the SAGE algorithm, the estimates $\mathbf{\Phi}_{L+1} = \boldsymbol{\alpha}$ are updated at the $(i+1)$st iteration according to the following constraint maximization problem:

$$\mathbf{\Phi}_{L+1}^{(i+1)} = \arg\max_{\mathbf{\Phi}_{L+1}} Q_{L+1}(\mathbf{\Phi}_{L+1}|\mathbf{\Phi}^{(i)}) \tag{24}$$

subject to :

$$\sum_{m=1}^{M}\lambda_m = 1 \ , \ \lambda_m \geqslant 0 \ , \ m = 1, 2, \cdots, M \ .$$

The optimization problem in (24) can be decoupled into two minimization problems. The first one is a convex minimization problem with a constraint and the other is a simple minimization problem without constraint. These problems are given as

1)

$$\min_{\lambda_1,\cdots,\lambda_M} - \sum_{n=1}^{N}\sum_{m=1}^{M} p_{\nu_n}^{(i)}(m)\log(\lambda_m) \tag{25}$$

subject to :

$$\sum_{m=1}^{M}\lambda_m = 1 \ , \ \lambda_m \geqslant 0 \ , \ m = 1, 2, \cdots, M$$

2)

$$\min_{\sigma_1^2,\cdots,\sigma_M^2} \sum_{n=1}^{N}\sum_{m=1}^{M} p_{\nu_n}^{(i)}(m)\Big[\log(\sigma_m^2) + \frac{1}{\sigma_m^2}\Big|y_n - \sum_{\ell=1}^{L} a_{n,\eta_\ell^{(i)}} h_\ell^{(i)}\Big|^2\Big] \tag{26}$$

Solving the first problem in (25) using a convex optimization technique with lagrangian we have

$$\lambda_m^{(i+1)} = \frac{1}{N}\sum_{n=1}^{N} p_{\nu_n}^{(i)}(m) \ , \ m = 1, 2, \cdots, M. \tag{27}$$

where $p_{\nu_n}^{(i)}(m)$ is given in (18). Keeping in mind (27), it is straightforward to show that the solution of the minimization problem in (26) is

$$(\sigma_m^2)^{(i+1)} = \frac{1}{N\lambda_m^{(i+1)}}\Big(\mathbf{y} - \sum_{\ell=1}^{L}\mathbf{a}_{\eta_\ell^{(i)}} h_\ell^{(i)}\Big)^\dagger \mathbf{D}_{\mathcal{P}}^{(i)}(m)\Big(\mathbf{y} - \sum_{\ell=1}^{L}\mathbf{a}_{\eta_\ell^{(i)}} h_\ell^{(i)}\Big), \tag{28}$$

where $\mathbf{D}_{\mathcal{P}}^{(i)}(m)$ is a diagonal matrix with entries $p_{\nu_1}^{(i)}(m), p_{\nu_2}^{(i)}(m), \cdots, p_{\nu_N}^{(i)}(m)$.

Note that, as seen from (21), the positions of the dominant channel taps are identified and the associated channel tap coefficients are estimated sequentially during the estimation of $\mathbf{\Phi}_\ell = \{h_\ell, \eta_\ell\}, \ell = 1, 2, \cdots, L$, of the SAGE algorithm within the MP framework. More clearly, at $(i+1)st$ step of the SAGE algorithm, the columns of $\mathbf{A}$

C. Initialization of the Algorithm

1) Initialization of $\mathbf{\Phi}^{(0)} = \{\eta_\ell^{(0)}, h_\ell^{(0)}, \ell = 1, 2, \cdots, L\}$: We apply the matching pursuit (MP) algorithm to determine $\mathbf{\Phi}^{(0)}$ considering the observation model in (9). As a first step in the MP algorithm, the column in the matrix $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_{N-1}]$ which is best aligned with the residue vector $\mathbf{r}_0 = \mathbf{y}$ is found and denoted $\mathbf{a}_{\eta_1}$. Then the projection of $\mathbf{r}_0$ along this direction is removed from $\mathbf{r}_0$ and the residual $\mathbf{r}_1$ is obtained. The algorithm proceeds by sequentially choosing the column which is the best matches until termination criterion is met. At the ℓth iteration, the index of the vector from $\mathbf{A}$ most closely aligned with the residual vector $\mathbf{r}_{\ell-1}$ is obtained as follows

$$\eta_\ell^{(0)} = \arg\max_j \frac{|\mathbf{a}_j^\dagger \mathbf{r}_{\ell-1}|^2}{\mathbf{a}_j^\dagger \mathbf{a}_j} \ , \quad j = 1, 2, \cdots, N \ \text{ and } \ j \notin \{\eta_1^{(0)}, \eta_2^{(0)}, \cdots, \eta_{\ell-1}^{(0)}\}, \tag{29}$$

and the channel tap at position $\eta_\ell^{(0)}$ is

$$h_\ell^{(0)} = \frac{\mathbf{a}_{\eta_\ell^{(0)}}^\dagger \mathbf{r}_{\ell-1}}{\mathbf{a}_{\eta_\ell^{(0)}}^\dagger \mathbf{a}_{\eta_\ell^{(0)}}} \ . \tag{30}$$

The new residual vector is computed as $\mathbf{r}_\ell = \mathbf{r}_{\ell-1} - h_\ell^{(0)}\mathbf{a}_{\eta_\ell^{(0)}}$.

2) Initializations of the Gaussian-mixture parameters $\{\lambda_m^{(0)}, (\sigma_m^2)^{(0)}, m = 1, 2, \cdots, M\}$: The empirical pdf of the Gaussian mixture noise in (11) is obtained first by means of the samples generated from the Gaussian distribution of the random vectors $\mathbf{v}^{SR}, \mathbf{v}^{RD}, \mathbf{h}^{RD}$, representing the additive Gaussian noise and the channel impulse response on the links $S \to R \to D$, having known covariance matrices. The Gaussian-mixture parameters are then determined by solving the following constrained optimization problem numerically.

$$J(\lambda_1, \cdots, \lambda_M, \sigma_1^2, \cdots, \sigma_M^2) = \sum_{j=1}^{N_s}\Big|p_{emp}(w_j) - \sum_{m=1}^{M}\frac{\lambda_m}{\pi\sigma_m^2}e^{-|w_j|^2/\sigma_m^2}\Big|^2$$

with constraints $\sum_{m=1}^{M}\lambda_m = 1$ and $\forall m, \ \lambda_m > 0$, where N_s is the number of noise samples.

IV. Simulation Results

The simulation parameters are chosen as in Table-I. The initial estimates of the multipath channel positions and taps used in MP-SAGE algorithm are determined by the reduced complexity MP algorithm. The channel multipath delays are chosen randomly within the cyclic prefix duration. The initial estimates of the multipath channel positions are employed in the best linear unbiased estimator (BLUE) as well. In our simulations, we employ the pilot pattern in which all subcarriers in a given time slot are dedicated to pilot symbols. Consequently, Figs. 2 and 3 show MSE and SER performance curves of the MP, BLUE and MP-SAGE algorithms for binary phase shift-keying (BPSK), quadrature phase shift-keying (QPSK) and 16-ary

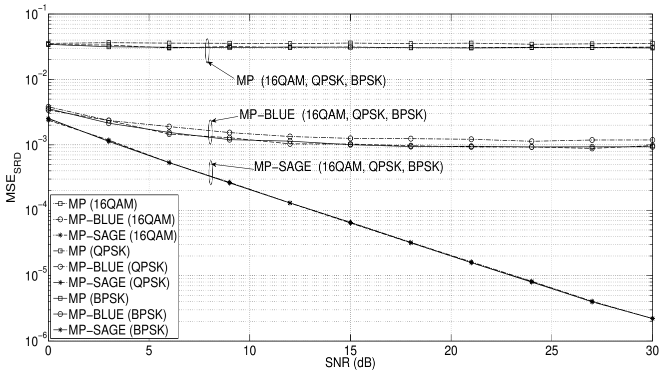

Fig. 2. MSE performance comparisons of the MP-SAGE and MP-BLUE algorithms

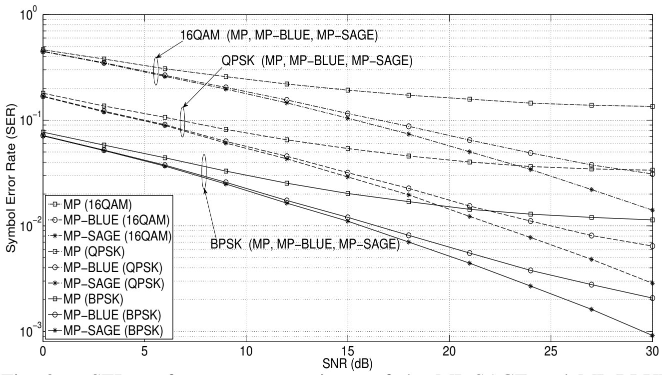

Fig. 3. SER performance comparisons of the MP-SAGE and MP-BLUE algorithms

quadrature amplitude modulation (16QAM) signaling format. As seen from these curves, the MP-SAGE algorithm, having excellent channel estimation and the symbol error rate performances outperforms the MP and BLUE estimators. We also conclude from these curves that our MP-SAGE algorithm exhibits a superior performance in the estimation of channel tap positions and the additive Gaussian mixture parameters that model the pdf of the correlative non-Gaussian ambient noise. Particularly, as seen from Fig.3 that our estimation algorithm has approximately 3 dB performance gain over the BLUE estimator at SER$= 10^{-3}$ when QPSK signaling is employed.

TABLE I
SIMULATION PARAMETERS

Number of Subcarriers (N)	256
Channel Bandwidth (BW)	3 KHz
Sampling Frequency (f_s)	BW
Sampling Frequency (f_c)	12 KHz
Number of Multipath Delays	4
Multipath Powers (Ω)	$[0.25 \quad 0.5 \quad 0.15 \quad 0.1]$
Rician Factor (κ)	3 dB
$f_o T_s$	0.01
Number of Gaussian Mixtures (M)	5
Number of OFDM Frame Length (N_f)	2
Number of iterations	5

We also investigate and compare the effect of Doppler mismatch on the system performance. Fig. 4 exhibits the robustness of the MP-SAGE algorithm to the doppler mismatch as well as the superiority to the MP-BLUE algorithm even under the effects of doppler mismatch. In Fig. 4, the performance of the proposed algorithm is examined under a doppler mismatch with the multipath doppler velocities are generated from a Thikhonov distribution having the support set $[-\Delta v, +\Delta v]$. Fig. 4 shows that the proposed MP-SAGE algorithm is quite robust to the changes of doppler velocity for BPSK, QPSK and 16QAM modulation types for velocities up to 0.2 meter/sec.

V. CONCLUSIONS

In this work we have presented a novel channel estimation algorithm for AF cooperative relay based OFDM systems in the presence

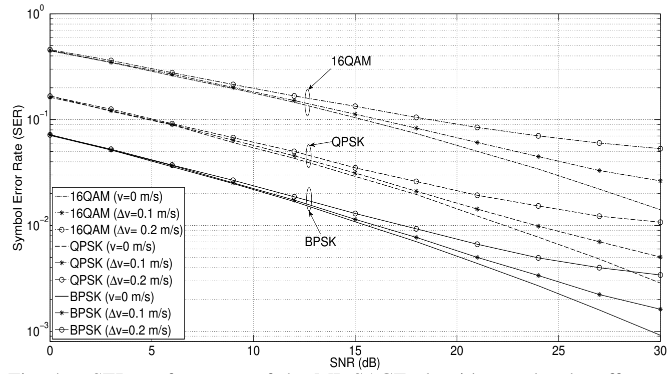

Fig. 4. SER performance of the MP-SAGE algorithm under the effects of doppler mismatch

of sparse underwater acoustic channels and of the correlative non-Gaussian noise modeled with a finite Gaussian mixture pdf. The proposed algorithm is based on the SAGE and the MP techniques. The MP algorithm was combined with the SAGE algorithm in such a way that at each SAGE iteration step, the nonzero taps and the locations of the sparse channel taps were determined and associated channel taps estimated by the MP algorithm. Finally, the computer simulations have shown that UWA channel is estimated very effectively and the proposed algorithm has excellent symbol error rate and channel estimation performance, and is robust to the effects of doppler mismatch.

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