Electricity Market Equilibrium Models with Dynamic Demand Functions

Emre Çelebi
Department of Industrial Engineering
Kadir Has University
Istanbul, Turkey
ecelebi@khas.edu.tr

***Abstract*—Many electricity market models have either mostly ignored the demand response to changing prices (e.g., day-ahead models with mostly fixed demand), or, at the other extreme, they assumed that the full demand response occurred within one hour. Moreover, the capital stock adjustment and the forward-looking nature of consumers are usually omitted. In this paper, we propose variational inequality models for electricity markets with dynamic demand models where the intertemporal nature of consumption (i.e., the current consumption decision affects capital stock/habits and thus the future preferences and demand) is recognized. It is intended that the proposed models would develop a framework for electricity market equilibrium models that incorporate the dynamics of the demand side.**

***Index Terms*--Power system economics, load modeling, demand response, variational inequality problem, habit formation, capital stock adjustment.**

I. INTRODUCTION

The deregulation in many electricity markets has allowed for competition in only wholesale markets and often, regulatory bodies have controlled the consumer prices (e.g., flat rates) in retail markets against the price volatility and spikes in the wholesale markets. Except for large industrial customers equipped with real time meters, participation of the demand side in the wholesale market is very small. Consequently, many consumers are indifferent to fluctuations in electricity prices or uninterested in curtailing power usage during price spikes in the wholesale markets. Moreover, this does not encourage consumers to reduce peak demand, thereby causing supply costs to increase due to extra peak generation capacity[1].

Due to the unique physical and operational characteristics of electricity production and transmission processes, electricity price exhibits different behaviors than other financial prices. Since the electricity prices are vital part of strategic (e.g., investment planning), tactical (e.g., offers/bids in wholesale markets) and operational (e.g., system security) decisions in electricity markets, modeling electricity price is one of the most critical component in electricity markets. There has been a growing literature addressing mainly two competing approaches to the problem of modeling electricity price processes [1]:

a) “Fundamental approach” that relies on simulation of system and market operation to arrive at market prices; and

b) “Technical approach” that attempts to model directly the stochastic behavior of market prices from historical data and statistical analysis.

The first approach provides more realistic system and transmission network modeling under specific scenarios and become computationally permissible with large number of scenarios that must be considered (for examples see [2, 3]). Using the second approach to characterize market prices is, therefore, not the focus of this paper.

In electricity market models using the first approach, typically, it is assumed that either hourly electric load is fixed, based on short term forecasts, or, at the other extreme, full demand response occurs within one hour [4-7]. Moreover, the behavioral aspects and dynamics (e.g., habit formation, capital stock adjustment) of the demand side are usually omitted (except in Çelebi and Fuller [8] where a habit formation model with separable monthly demand functions are used). However, the ongoing roll-out of smart meters creates opportunities for greater demand-side participation in electricity markets. The effective use of this communication between suppliers and consumers is expected to deliver energy savings, cost reductions, and increased reliability and security. Therefore, a target for “smart grids” is that electricity market models should be enhanced to account for adjustments of electricity consumption levels in response to frequently communicated electricity prices. Incorporating demand response into the management of the power system is one of the important challenges to achieve this target.

Federal Energy Regularity Commission (FERC) defines

[1] Demand response is not only about price spikes but also about increasing demand in case of negative prices. It also influences the base load capacity (as it offers flexibility) and the transmission system (e.g., congestion).

the demand response as [9]:

"Changes in electric use by demand-side resources from their normal consumption patterns in response to changes in the price of electricity, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized."

Demand-side participation through demand response can increase market efficiency by reducing loads when marginal benefits of consumption are less than marginal costs, and by increasing consumption when the reverse is the case. One way consumers may respond to changes in electricity prices is through automation built into smart appliances. Appliance manufacturers such as GE, Whirlpool, Samsung, Electrolux and LG have been intensely working on developing smart appliances (e.g., LG's new line of price responsive appliances called Smart THINQ [10] and Nest's recently announced smart thermostat [11] which learns about the preferences of the occupants and tracks their status whether they are at home or away). According to a report by Zpryme Research and Consulting, the smart appliance market is growing 49% annually [12].

In this paper, we simulate consumer prices using monthly demand functions rather than hourly in order to bridge the "speed of response gap" between suppliers and consumers[2]. Moreover, the demand response is dynamic in the model through a dependence of this month's demand on the previous month's demand. Note that these models are medium-term (six months to several years) and the "lag" dependence on the previous month mostly represents the consumers' habits as well as adjustment in capital stock.

The models proposed in this paper address the problem of demand response with the inclusion of dynamic demand functions in the context of bilateral markets, but the models also apply to POOLCO system with a uniform pricing mechanism (i.e., no locational marginal prices)[3]. Within this context, the proposed model can be used as a forecasting and policy analysis tool (e.g., to assess demand response effects and potential market power of large suppliers) by regulatory bodies.

Several studies have estimated the size of potential efficiency gains from adoption of time-varying pricing (e.g., real-time pricing –RTP and time-of-use –TOU pricing) and have performed reasonable sensitivity analysis for their simulations. In particular, Çelebi and Fuller [13] study the electricity pricing problem from a more operational perspective and propose a model for time-of-use (TOU) prices in electricity markets. Consumers make their consumption decision by using a predefined demand function, i.e., they do not consider the possibility that the electricity price may drop in future periods. Borenstein and Holland [14] have calculated the long-run efficiency gains of 3% to 11% of the energy bill with RTP adoption and Borenstein [15] has simulated about a quarter of these gains for seasonal TOU pricing. Holland and Mansur [16] have found that the short-run efficiency gains are modest (0.24% and 2.5% of the total energy bill, respectively) if all customers adopt RTP. Moreover, they have analyzed the environmental effects of RTP adoption.

Our approach in this paper is different in several ways. Aforementioned studies specify constant elasticity demand functions for each hour, whereas we propose monthly linear demand functions. They do not explicitly model the behavioral aspects and dynamics (e.g., habit formation, capital stock adjustment) of the demand side. Furthermore, their analyses are in the context of perfect competition only, but they have stated that market power would increase the efficiency gains.

We represent our models using the variational inequality framework, which is an effective and convenient way to create and manage our models. The variational inequality formulations are developed to estimate monthly prices (as well as hourly prices can be derived) in different market structures, namely, perfect competition, oligopoly (i.e., all firms or several large firms compete à la Cournot[4]) and monopoly. The aim is to see the range of price manipulations for different structures. The supply side of the model is deliberately simplified here, and hence transmission network representation is not included. But a network representation must be added for nodal/zonal pricing systems (e.g., PJM, New York, New England, etc.)

The paper is organized as follows. Section II introduces the model components and the dynamic demand functions with their underlying assumptions. It also gives a brief review about variational inequality problems. The paper concludes with Section III, in which directions for future research are suggested.

II. ELECTRICITY MARKET MODELS

This section presents a multi-firm, multi-period equilibrium model in electricity markets with dynamic demand functions. Çelebi and Fuller [8] provide the framework to analyze supply and demand sides with different time scales of response (e.g., hourly versus monthly), and part of this section is mostly following their notation. Section II-D is mainly different where demand side with dynamic demand

[2] The speed of response gap is the difference between the consumers' response to changes in price, which is no more frequent than the billing cycle allows (e.g., monthly), and the change in marginal costs of production, which occurs much more rapidly (e.g., hourly) [8].

[3] In the presence of arbitrage (that erase any non-cost based differences in prices) and a network representation, Cournot competition in a bilateral market is equivalent to Cournot competition among generators in a POOLCO (i.e., generators sell to a central auction) [6]. Without network representation, POOLCO and bilateral market models would be equivalent, too.

[4] Cournot oligopoly is the most common framework to model interaction among participants in electricity markets. In this framework, a supplier takes its rivals' sales and/or production quantities as fixed within its profit maximization problem. Other oligopolistic models (e.g., supply function, Bertrand, Stackelberg, tacit collusion) can be examined but the Cournot model is the most practical [5]. Furthermore, it may be sufficient to simulate market prices [17].

functions and underlying assumptions are explained in details.

The model consists of three parts: the independent system operator's (ISO's) problem, supply side (e.g., firm f's problem) and the demand side. Symbols for the ISO and supply side problems are defined in the following list. Symbols for the demand side are defined in section II-D.

Sets

set of generation facilities: $i=1,\dots,I$

set of periods (months): $t=1,\dots,T$

set of hours in period t: $h=1,\dots,H^{(t)}$ (defined by the market regulator)

set of firms: $f=1,\dots,F$

Parameters

$c_{fi}^{(t)}$ = operating cost per unit of energy for firm f's facility i in period t ($/MWh)

$\kappa_{fi}^{(t)}$ = capacity of firm f's facility i in period t (MW)

$\delta_h^{(t)}$ = fraction of total energy demand during month t that occurs during hour h (see section II-B for explanation of this load shape parameter).

Decision variables

$z_{fih}^{(t)}$ = the energy flowing from firm f's facility i to demand for hour h in period t (MWh)

$d_f^{(t)}$ = sales by firm f in period t (MWh)

$p^{(t)}$ = Prices (e.g., flat rate) in period t ($/MWh) (a function of $d_f^{(t)}$ variables, but treated as a parameter by price-taking firms)

A. ISO's Problem

In (1), for each period t, the ISO chooses $z_{fih}^{(t)}$ to minimize total operating costs of all generation facilities of firms that inject amounts $d_f^{(t)}$, while respecting the fluctuations in demand over the hours of periods. The ISO's role in this setting is to provide incentives for generating firms to follow the historical shape of the load duration curve within the hours of demand period t.

$$\max_{z} -\sum_{f=1}^{F}\sum_{i=1}^{I}\sum_{h=1}^{H^{(t)}} c_{fi}^{(t)} z_{fih}^{(t)}$$

$$\begin{aligned}
&\textit{subject to} && && [dual] \\
&d_f^{*(t)} - \sum_{i=1}^{I}\sum_{h=1}^{H^{(t)}} z_{fih}^{(t)} \le 0 && \forall f && \left[\rho_f^{(t)}\right] \\
&z_{fih}^{(t)} \le \kappa_{fi}^{(t)} && \forall f,i,h && \left[\mu_{fih}^{(t)}\right] \\
&\delta_h^{(t)}\sum_{f=1}^{F} d_f^{*(t)} - \sum_{f=1}^{F}\sum_{i=1}^{I} z_{fih}^{(t)} = 0 && \forall h && \left[\lambda_h^{(t)}\right] \\
&z_{fijh}^{(t)} \ge 0 && \forall f,i,h &&
\end{aligned} \tag{1}$$

Here, we assume that each generator firm f reports its bilateral contract amount $d_f^{(t)}$ (but not the prices $p^{(t)}$) to the ISO before each time period t. Furthermore, it is assumed that the ISO is aware of the marginal cost (e.g., $c_{fi}^{(t)}$) and capacity of each generation facility (e.g., $\kappa_{fi}^{(t)}$) at each time period t. In the ISO's problem, the amount of each firm's sales, $d_f^{(t)}$, is treated as fixed (denoted by superscript *). The first set of constraints in (1) ensures that electricity supply of firm f is sufficient to meet its sales. The second set of constraints contains the capacity constraint for each generation facility owned by firm f. The next subsection explains in detail how the third set of constraints ensures that generation matches demand in every period.

Here, we also need to explain the meaning and measurement of the parameters $\delta_h^{(t)}$ which link the different time scales of the supply and demand sides of the model. When consumers pay the same price for energy at any hour within a period, it is reasonable to suppose that the demand variations over hours within the period are related to non-price causes, such as temperatures, natural lighting, daily meal schedules and habits of all kinds that affect electricity usage. Such non-price causes must necessarily be represented by parameters, not variables to be solved for. Çelebi and Fuller [13] proposed to measure the pattern of variation in demand that has been observed in the recent past, and to assume that the same pattern (but not the absolute values) will repeat in the near future, i.e., within the model's time horizon. These parameters for a specific demand period can be calculated from historical observations of a previous year (see [13] for calculation of these parameters), as the fraction, $\delta_h^{(t)}$, of total energy demand in period t, that occurs during hour h. Note that $\sum_{h=1}^{H^{(t)}} \delta_h^{(t)} = 1$.

The third set of constraints in (1) states that the hourly generation for all different facilities and all firms should meet the total sales of all firms at hour h. With this condition, the ISO imposes the historical shape of the load duration curve within the hours of demand period t. But if prices differ from historical ones, then the entire month's load duration curve of the solution can have a shape that is different from the historical shape. The dual variable $\lambda_h^{(t)}$ (unconstrained in sign) for this condition is the penalty/payment for firm f's hourly deviations of its sales from the average hourly demand in demand period t and hourly deviations of its output $\sum_{i=1}^{I} z_{fih}^{(t)}$ from the average output over all hours in the period t. (i.e., deviations from the historical shape of the load duration curve).

B. Supply Side: Firm f's Problem

The supply side of the model, formulated in (2), maximizes firm f's profit π_f, i.e., the total revenues of firm f minus the total operating cost of firm f's hourly generation by different technologies of production (e.g., nuclear, hydro, coal, gas/oil, indexed by i) to meet its sales in different demand periods (e.g., months indexed by t) plus the ISO's penalty/payment due to variations from hourly average demand and hourly average output in demand period t. Various market structures are modeled within this framework. The perfect competition structure –with firms treating $p^{(t)}$ as a parameter beyond their control– serves as a reference case, as it would lead to the most efficient market performance. On the other hand, the monopoly structure represents the worst outcome of exercising market power. In between is the Nash-Cournot structure where either all firms or some large firms act à la Cournot. In the monopoly and Nash-Cournot structures, firms see their knowledge of the dependence of $p^{(t)}$ on total market demand, as detailed in section II-E.

As explained in [13], the supply model can be extended to be more realistic as long as each firm's model remains as a linear program (e.g., a linearized DC power network, line limits and ramping constraints can be included at the expense of problem size) or more generally as a convex program.

$$\max_{d,z} \pi_f = \sum_{t=1}^{T}\left[p^{(t)} - \sum_{h=1}^{H^{(t)}} \delta_h^{(t)} \lambda_h^{*(t)}\right] d_f^{(t)} - \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{h=1}^{H^{(t)}} \left[c_{fi}^{(t)} - \lambda_h^{*(t)}\right] z_{fih}^{(t)}$$

$$\begin{aligned} &\textit{subject to} && && [dual] \\ &d_f^{(t)} - \sum_{i=1}^{I}\sum_{h=1}^{H^{(t)}} z_{fih}^{(t)} \leq 0 && \forall\, t && \left[\rho_f^{(t)}\right] \\ &z_{fih}^{(t)} \leq \kappa_{fi}^{(t)} && \forall\, i,h,t && \left[\mu_{fih}^{(t)}\right] \\ &z_{fih}^{(t)} \geq 0 && \forall\, i,h,t && \end{aligned} \tag{2}$$

In firm *f*'s problem, it is presumed that the ISO's penalty/payment term, $\lambda_h^{(t)}$, is fixed (denoted by superscript *). There is no discounting but it could be included for longer time horizons. The first set of constraints ensures that electricity supply of firm *f* is sufficient to meet its sales to demand period *t*; at an optimal solution, these constraints are binding equalities. The second set of constraints contains the capacity constraint for each generation facility owned by firm *f*. It should be noted that the ISO's and firm *f*'s problem have the common variable, $z_{fih}^{(t)}$, which, in equilibrium, are equivalent as shown in section II-E.

C. *Demand Side*

The dynamics of demand side can be modeled in many ways but the default assumption in both empirical and theoretical demand analysis is constant tastes and, more generally, constant demand parameters. However, habit formation models allow demand parameters to depend in a specified way on previous levels of consumption [18].

In analyzing the dynamics of demand, Taylor and Houthakker [19] have used both complete demand system and single equation analysis. They use unobserved "state variables" that depend on lagged values of consumption. In their single equation analysis, they incorporate state variables directly into the demand equations without derivation from an underlying utility function or preference ordering. By contrast, they incorporate state variables into the utility function in their complete system analysis and derive the implied demand functions from this system. In both cases, they interpret the state variables as representing either a "stock of habits" or a "stock of consumer durables" depending on whether the coefficient of the state variable in the demand equation is positive or negative. Thus, Houthakker and Taylor use state variables to generate dynamic demand equations corresponding to both habit formation and consumer durables for both single equation and complete demand system analyses [20]. Nevertheless, compared to the complete demand system approach, single equation demand analysis may seem ad hoc and old-fashioned. But, the state variable and lagged consumption specifications provide empirically tractable dynamic demand models [19]. Despite the empirical success of dynamic demand models, however, the static specification remains the default assumption in both theoretical and empirical demand analysis.

For capital stock adjustment models, Taylor and Houthakker's dynamic demand model ([19] and same titled 2nd edition of their book in 1970) may be used, although, estimation of the parameters and nonlinear nature of their demand equations are drawbacks for using their model. But, an advantage of their demand system is that it is derived from an additive quadratic utility function.

Phlips [21] presents the results from estimating a complete system of demand equations (using Taylor and Houthakker's additive quadratic model and linear expenditure system), for eleven United States expenditure categories, allowing for habit formation, capital stock adjustment, and depreciation, taking into account the necessary or unnecessary character of commodities. He concludes that the linear expenditure system is a better vehicle of empirical analysis than the additive quadratic model.

The theoretical difficulty with habit formation arises in models of "naive" as opposed to "rational" habit formation [22]. With naive habit formation, in each period the consumer chooses a one-period consumption pattern to maximize a one-period utility function, thus, ignores intertemporal allocation. The advantage of "naive" habit formation models is their tractability. Çelebi and Fuller [8] employ this naive habit formation model in TOU pricing models under different market structures. In their demand models, it is assumed that maximizing successive one-period utility functions is rational only if preferences are separable over time (i.e., if the marginal rates of substitution involving consumption within each period are independent of consumption in other periods). The problem is that naive habit model assumes that individuals fail to recognize the influence of current consumption on their future) [20]. However, models of rational habit formation avoid this difficulty, but encounter others. With rational habit formation, individuals correctly recognize that their current consumption affects future behavior. As Pollak [18] discussed:

> *"A further complication in rational habit formation models arises from the need to make explicit assumptions about the individual's ability to "commit" or "pre-commit" to a lifetime consumption plan. At one extreme, if an individual can pre-commit without cost, and if the individual accepts the primacy of current preferences, then the optimal lifetime*

consumption plan is one which maximizes the intertemporal utility function reflecting current preferences. Under these assumptions, rational habit formation is equivalent to maximizing a non-separable intertemporal utility function. At the other extreme, if pre-commitment is impossible, then the optimal plan is one that takes full account of the effect of current decisions on future preferences. Under this assumption, the optimal plan is the optimal feasible plan, where feasibility takes full account of future decisions that are nonoptimal from the perspective of current tastes. Intermediate cases in which pre-commitment is costly are even more complex."

On the other hand, the complete demand system analysis has the theoretical advantage of its consistency with preference maximization. However, the theory implies consistency with preference maximization for individual level data (and household level data with some additional assumptions) [18]. Hence, using preference maximization to structure the analysis of aggregate demand data requires postulating the existence of a "representative consumer" or making some other very special assumption to exercise the aggregation problem and ensure that market demand functions behave as individual demand functions. Another major disadvantage of the complete demand system analysis is that, when the number of "goods" is large, it may require estimating too many parameters. But, parametric size is manageable for demand systems corresponding to additive utility functions[5]; for example, the linear expenditure system or the additive quadratic model used by Taylor and Houthakker contains *2n - 1* parameters, where *n* is the number of goods.

Pollak and Wales's [20] "habit formation" assumptions imply that consumption in the previous period influences current preference and demand, but that consumption in the more distant past does not. This assumption may be generalized by allowing the necessary quantity of each good to depend on a geometrically weighted average of all past consumption of that good (e.g., distributed lag). A fundamental assumption of the habit formation model is that the individual does not take account of the effect of his current purchase on his future preferences and future consumption. On the contrary, capital stock adjustment models must explicitly recognize the intertemporal nature of consumption (i.e., current consumption also affects the future preferences and demand).

An important feature of the dynamic (e.g., lagged) demand formulation is that demand in a period is also dependent of future prices. This strongly suggests that, to be consistent with lagged demand, we must assume that consumers have "foresight" as in the stock adjustment model. In this paper, we have employed the habit formation assumptions in the demand models with a "foresight" nature of consumers. We emphasize both the habit formation model and the capital stock adjustment. We assume that consumers are making their decisions for each period dependent on past and the future consumption levels.

The demand side is represented by demand equations that use the prices and lagged demand as independent variables. A distributed lag model can represent the dynamics of demand side in time. One form of a one commodity model is the linear distributed lag model:

$$d^{(t)} = a^{(t)} + b^{(t)}p^{(t)} + e^{(t)}d^{(t-1)} \tag{3}$$

where

$a^{(t)}$ = factors representing non-price effects at period t (e.g., weather conditions, socio-demographic factors)

$d^{(t)}$ = total demand for electricity in period t where $d^{(t)} = \sum_{f=1}^{F} d_f^{(t)}$ and $d_f^{(t)}$ is the sales of firm f in period t.

$d^{(t-1)}$ = lagged demand where $d^{(t-1)} = \sum_{f=1}^{F} d_f^{(t-1)}$

$p^{(t)}$ = electricity price in period t (i.e., flat rate)

$b^{(t)}$ = price coefficients (i.e., own-price only) for period t, where $b^{(t)} < 0$.

$e^{(t)}$ = lag coefficients for period t, where $e^{(t)} > 0$.

In a real world application, a careful econometric study would be needed, to establish the best functional form, and its parameters. Equation (3) can also be extended to a multi-commodity case where each commodity is the electricity demand in different times of day (e.g., demand blocks in TOU pricing: on-peak, mid-peak, off-peak). Note that an alternative but very similar procedure for incorporating dynamics into demand analysis is to allow demand function parameters to depend directly on lagged consumption, i.e., $a'^{(t)} = a^{(t)} + e^{(t)}d^{(t-1)}$, and the general form of demand functions are in the linear form (see [22] for further discussion):

$$d^{(t)} = a'^{(t)} + b^{(t)}p^{(t)}. \tag{4}$$

D. *Variational Inequality (VI) Approach*

The models proposed in this paper are represented and solved by the variational inequality (VI) problem approach [23]. In general, a finite dimensional VI problem is defined as follows:

$$\begin{aligned} &VI(\boldsymbol{F},K)\text{: Find a vector } \boldsymbol{x}^* \in K \subset R^n \text{, such that:} \\ &\boldsymbol{F}(\boldsymbol{x}^*)^T.(\boldsymbol{x}-\boldsymbol{x}^*) \geq 0, \quad \forall \boldsymbol{x} \in K \end{aligned} \tag{5}$$

where $\boldsymbol{F}$ is a given continuous function from K to R^n and K is a nonempty, closed and convex set [23].

To aid readers who are familiar with MCP models, but not VI problems, we first formulate the perfect competition model as a MCP, followed by the VI form and a justification for the equivalence of the two forms. The oligopoly and monopoly models are presented only in the VI form.

In (6), we formulate the perfect competition model as a MCP, by writing out the necessary Karush-Kuhn-Tucker (KKT) conditions for the ISO's and firm *f*'s problems along with the demand equation.

[5] As Pollak pointed out [18]: *"When the number of goods (that is, consumption categories) is large, additivity is implausible. For example, additivity of the direct utility function holds if and only if the marginal rate of substitution involving each pair of goods depends only on the quantities of those two goods and is independent of the quantities of all other goods. For narrowly defined consumption categories, this condition is unlikely to hold: the marginal rate of substitution of bread for butter is unlikely to be independent of the quantity of jam."*

MCP: Find $d_f^{(t)}, z_{fih}^{(t)}, p^{(t)}, \rho_f^{(t)}, \mu_{fih}^{(t)}, \lambda_h^{(t)}$ that satisfy

$$
\begin{array}{lll}
d_f^{(t)} \geq 0 \perp & -p^{(t)} + \sum_{h=1}^{H^{(t)}} \delta_h^{(t)} \lambda_h^{(t)} + \rho_j^{(t)} \geq 0 & \forall f, t \\
z_{fih}^{(t)} \geq 0 \perp & c_{fi}^{(t)} - \rho_f^{(t)} + \mu_{fih}^{(t)} - \lambda_h^{(t)} \geq 0 & \forall f, i, h, t \\
\rho_f^{(t)} \geq 0 \perp & d_f^{(t)} - \sum_{i=1}^{I} \sum_{h=1}^{H^{(t)}} z_{fih}^{(t)} \leq 0 & \forall f, t \\
\mu_{fih}^{(t)} \geq 0 \perp & z_{fih}^{(t)} \leq \kappa_{fi}^{(t)} & \forall f, i, h, t \\
\lambda_h^{(t)} free \perp & \delta_h^{(t)} \sum_{f=1}^{F} d_f^{(t)} - \sum_{f=1}^{F} \sum_{i=1}^{I} z_{fih}^{(t)} = 0 & \forall h, t \\
\multicolumn{2}{l}{\sum_{f=1}^{F} d_f^{(t)} = a^{(t)} + b^{(t)} p^{(t)} + e^{(t)} \sum_{f=1}^{F} d_f^{(t-1)}} & \forall t
\end{array}
\tag{6}
$$

The first four conditions in (6) are the necessary KKT conditions for firm *f*'s problem. The second to fifth conditions in (6) are the necessary KKT conditions for the ISO's problem (i.e., second to fourth conditions are common for the ISO's and firm *f*'s problem). The last equation is the linear distributed lagged demand equation. Note that the first condition does not include the extra term for the marginal revenue of firm *f* (i.e., recognizing that $p^{(t)}$ is a function of $\sum_{f=1}^{F} d_{fj}^{(t)}$) that appears in monopoly and Cournot models. Because all firms are price takers in a perfect competition structure, this condition only has the $p^{(t)}$ term as the marginal revenue term. Also note that the third and fifth conditions in (6) are linearly dependent at a solution, where the third constraints are binding (i.e., summing the third condition over all firms *f* equals the sum of the fifth condition over all hours *h*) and one combination of *t*, *f* in the third set of constraints can be dropped from (6).

When $d_f^{(t)} > 0$ (implying $z_{fih}^{(t)} > 0$ from the third condition), the first two conditions in (6) become equalities and we can derive the following condition by summing them:

$$
p^{(t)} - \sum_{h=1}^{H^{(t)}} \delta_h^{(t)} \lambda_h^{(t)} + \lambda_h^{(t)} = c_{fi}^{(t)} + \mu_{fih}^{(t)} \tag{7}
$$

The left hand side can be understood as the hourly price; let it be denoted by $p_h^{(t)}$. All firms receive this hourly price whereas consumers are paying the period *t*'s price ($p^{(t)}$) in the models. Note that firms have adequate revenue when $z_{fih}^{(t)} > 0$, because $p_h^{(t)} - c_{fi}^{(t)} = \mu_{fih}^{(t)} \geq 0$. If we multiply the left hand side of (7) by $\delta_h^{(t)}$ and sum over all hours *h*, we derive the condition, $p^{(t)} = \sum_{h=1}^{H_j^{(t)}} \delta_h^{(t)} p_h^{(t)}$. This is the weighted average condition imposed in [13] (without the discount factor), which relates the hourly marginal cost $p_h^{(t)}$ to consumer TOU price $p^{(t)}$ in the perfect competition case. This also ensures that the revenue requirement of all firms for demand in period *t* is met by revenue collected from consumers.

$$
p^{(t)} \sum_{f=1}^{F} d_j^{(t)} = \sum_{h=1}^{H^{(t)}} \delta_h^{(t)} p_h^{(t)} \sum_{f=1}^{F} d_f^{(t)} = \sum_{h=1}^{H^{(t)}} \sum_{f=1}^{F} \sum_{i=1}^{I} p_h^{(t)} z_{fih}^{(t)}
$$

Related to this is the fact that the penalties/payments imposed by the ISO sum to zero, over all firms, within every period *t*:

$$
\begin{aligned}
&- \sum_{f=1}^{F} \sum_{h=1}^{H^{(t)}} \delta_h^{(t)} \lambda_h^{(t)} d_f^{(t)} + \sum_{f=1}^{F} \sum_{i=1}^{I} \sum_{h=1}^{H^{(t)}} \lambda_h^{(t)} z_{fih}^{(t)} \\
&= - \sum_{h=1}^{H^{(t)}} \sum_{f=1}^{F} \sum_{i=1}^{I} \lambda_h^{(t)} z_{fih}^{(t)} + \sum_{f=1}^{F} \sum_{i=1}^{I} \sum_{h=1}^{H^{(t)}} \lambda_h^{(t)} z_{fih}^{(t)} = 0
\end{aligned}
$$

Thus the ISO's penalty/payment scheme shifts money around among firms, but does not directly involve consumers.

We can also formulate (6) as a VI problem. The feasible set for the VI problem is defined as follows:

$$
K = \left\{ d_f^{(t)}, z_{fih}^{(t)}, p^{(t)} \left| \begin{array}{ll}
d_f^{(t)} - \sum_{i=1}^{I} \sum_{h=1}^{H^{(t)}} z_{fih}^{(t)} \leq 0 & \forall f, t \\
z_{fih}^{(t)} \leq \kappa_{fi}^{(t)} & \forall f, i, h, t \\
z_{fih}^{(t)} \geq 0 & \forall f, i, h, t \\
\delta_h^{(t)} \sum_{f=1}^{F} d_f^{(t)} - \sum_{f=1}^{F} \sum_{i=1}^{I} z_{fih}^{(t)} = 0 & \forall h, t \\
\sum_{f=1}^{F} d_j^{(t)} = a^{(t)} + b^{(t)} p^{(t)} + e^{(t)} \sum_{f=1}^{F} d_f^{(t-1)} & \forall t
\end{array} \right. \right\}
$$

Note that $p^{(t)}$ variables are implicitly defined by the $d_f^{(t)}$ variables. Instead, an explicit inverse demand function can be used for a more compact formulation without $p^{(t)}$ variables, but for ease of readability of the formulation, the $p^{(t)}$ variables are used.

In the feasible set K, the first four constraints are from the ISO's and firm *f*'s problems and the last equation is the linear distributed lagged demand function.

The VI problem for the perfect competition model is as in (8). To relate (8) to the general VI form (5), the vector $\boldsymbol{x}$ contains the variables $d_f^{(t)}, z_{fih}^{(t)}$ and $p^{(t)}$ for all f, i, h and t, and the elements of the vector-valued mapping $\boldsymbol{F}(\boldsymbol{x})$ are as follows: $-p^{(t)}$ is the element of $\boldsymbol{F}$ that corresponds to $d_f^{(t)}$; $c_{fi}^{(t)}$ is the element of $\boldsymbol{F}$ that corresponds to $z_{fih}^{(t)}$; and the element of $\boldsymbol{F}$ that corresponds to $p^{(t)}$ is zero. Note that for $d_f^{(t)}$ and $z_{fih}^{(t)}$, the corresponding elements of $\boldsymbol{F}$ are the partial derivatives of the objective function of firm *f*'s problem (2) (i.e., the $\lambda_h^{(t)}$ terms are cancelled out.)

Find $\left(d_f^{(t)*}, z_{fih}^{(t)*}, p^{(t)*}\right) \in K$ such that

$$
\begin{aligned}
&- \sum_{t=1}^{T} \sum_{f=1}^{F} p^{(t)*} \left(d_f^{(t)} - d_f^{(t)*}\right) \\
&\qquad + \sum_{t=1}^{T} \sum_{f=1}^{F} \sum_{i=1}^{I} \sum_{h=1}^{H^{(t)}} c_{fi}^{(t)} \left(z_{fih}^{(t)} - z_{fih}^{(t)*}\right) \geq 0 \\
&\qquad \qquad \forall \left(d_f^{(t)}, z_{fih}^{(t)}, p^{(t)}\right) \in K
\end{aligned}
\tag{8}
$$

The VI problem (8) in primal variables has the KKT conditions listed in (6) and hence is equivalent to the MCP (6) (see [23, 24] for discussion of the KKT conditions for VI problems).

There is a minor technicality in the derivation of (6) from the KKT conditions of (8). Let $\upsilon^{(t)}$ be the dual variable of the distributed lagged demand equation in the definition of K. The KKT conditions which correspond to the $p^{(t)}$ variables are $b^{(t)}\upsilon^{(t)} = 0$. Because $b^{(t)} < 0$, it follows that $\upsilon^{(t)} = 0$ for all t. Therefore, $\upsilon^{(t)}$ and $b^{(t)}\upsilon^{(t)} = 0$ can be dropped from the list of KKT conditions of the VI problem, giving rise to the MCP (6).

We only provide the VI formulations for other market structures, for ease of representation. It is straightforward to derive their equivalent MCP formulations as in the perfect competition case. The other market structures have the same feasible set K. The VI problem for the Nash-Cournot model is formulated as follows:

$$\text{Find } \left(d_f^{(t)*}, z_{fih}^{(t)*}, p^{(t)*}\right) \in K \text{ such that}$$

$$-\sum_{t=1}^{T}\sum_{f=1}^{F}\left(p^{(t)*} + \theta_f^{(t)^*}\right)\left(d_f^{(t)} - d_f^{(t)*}\right) + \sum_{t=1}^{T}\sum_{f=1}^{F}\sum_{i=1}^{I}\sum_{h=1}^{H^{(t)}} c_{fi}^{(t)}\left(z_{fih}^{(t)} - z_{fih}^{(t)*}\right) \geq 0 \quad \forall \left(d_f^{(t)}, z_{fih}^{(t)}, p^{(t)}\right) \in K \tag{9}$$

where the term $p^{(t)^*} + \theta_f^{(t)^*}$ is the marginal revenue for firm f in period t, and $\theta_{fj}^{(t)^*}$ is the "extra" marginal revenue term. This marginal revenue term is derived from the partial derivative of the objective function in (2), with respect to $d_f^{(t)}$, when the firm is aware of the price-quantity relation of the distributed lagged demand equation:

$$\frac{\partial \pi_f}{\partial d_f^{(t)}} = p^{(t)} + \frac{\partial p^{(t)}}{\partial d_f^{(t)}} d_f^{(t)} + \frac{\partial p^{(t+1)}}{\partial d_f^{(t)}} d_f^{(t+1)} = p^{(t)} + \theta_f^{(t)}$$

where $\left[\frac{\partial p^{(t)}}{\partial d_f^{(t)}}\right] = \left(b^{(t)}\right)^{-1}$ and $\left[\frac{\partial p^{(t+1)}}{\partial d_f^{(t)}}\right] = -e^{(t+1)}\left(b^{(t+1)}\right)^{-1}$.

Note that the penalties/payments imposed by the ISO are neither included in the VI formulation (9) nor in the marginal revenue term, because they sum to zero, over all firms.

Lastly we can define a VI problem for the monopoly structure:

$$\text{Find } \left(d_f^{(t)*}, z_{fih}^{(t)*}, p^{(t)*}\right) \in K \text{ such that}$$

$$-\sum_{t=1}^{T}\sum_{f=1}^{F}\left(p^{(t)*} + \sum_{f'=1}^{F}\theta_{f'j}^{(t)^*}\right)\left(d_f^{(t)} - d_f^{(t)*}\right) + \sum_{t=1}^{T}\sum_{f=1}^{F}\sum_{i=1}^{I}\sum_{h=1}^{H^{(t)}} c_{fi}^{(t)}\left(z_{fih}^{(t)} - z_{fih}^{(t)*}\right) \geq 0 \quad \forall \left(d_f^{(t)}, z_{fih}^{(t)}, p^{(t)}\right) \in K \tag{10}$$

Note that f' is an alias index for f, and that all firms are owned by the monopolist.

For each VI problem (8) to (9), the expression in the inequality is an estimate of the change in the negative of profits (summed over all firms) due to feasible deviations from equilibrium, using marginal revenues of the firms as measured at equilibrium. Therefore, at equilibrium, no firm sees any advantage in changing its variables $d_f^{(t)}$ and $z_{fih}^{(t)}$ in a feasible way. For the monopoly model (10), the monopolist firm sees no advantage in deviating from the equilibrium solution, i.e., the solution is a local maximum of profit (and a global maximum, due to convexity).

Instead of flat pricing, some consumers may prefer TOU prices, or regulatory bodies in electricity markets may choose to implement a TOU pricing scheme. In this case, consumers' prices vary by time of day, and they may or may not vary by month. We can add demand blocks as introduced in Çelebi and Fuller [13] to model TOU prices that differs by time-of-use demand blocks (e.g., off-peak, mid-peak, on-peak denoted by index j in their paper). In fact, the market models presented in this paper are equivalent to having a single demand block (e.g., j with one element only) of Çelebi and Fuller's [8] market equilibrium models with TOU pricing. For our purposes in this paper, we do not present the TOU pricing model and rather focus on the "foresight" behavior (i.e., habit formation and capital stock adjustment) of consumers.

III. CONCLUSIONS & FUTURE RESEARCH

In this paper, we propose variational inequality models for electricity markets with dynamic demand models where the intertemporal nature of consumption (i.e., the current consumption decision affects capital stock and thus the future preferences and demand) is recognized. It is intended that the proposed models would develop a framework for electricity market equilibrium models that incorporates the dynamics of the demand side.

By introducing a linearized DC network, line limits, and ramp limits, a more realistic model can be built and the impact of transmission network (e.g., the effect of location) can be examined in detail, such as the market power issues in load pockets [5, 25]. However, the problem size grows with this added realism and a need for algorithms to solve large scale equilibrium problems arises. In such a case, decomposition methods (e.g., Dantzig-Wolfe [2] or Benders decomposition [3] for VI problems) may surmount difficulties that may arise in computation of equilibrium.

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