Reliable Two-Path Successive Relaying

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***Abstract*—Emerging two-path successive relaying protocols generally rely on error-free source-relay channels and/or interference-free inter-relay channels to achieve high-rate and full-diversity. In this paper, by removing these optimistic assumptions, a novel two-path successive relaying scheme that benefits from relay selection and distributed space-time block coding (STBC), and transfers the data from the source to the destination via relays in a reliable fashion is proposed. The proposed scheme can achieve full diversity without the requirement of perfect decoding at relays since not only the destination but also the relays benefit from distributed STBC and relay selection. As the target STBC, coordinate interleaved orthogonal design (CIOD) for two transmit antennas is considered. The average symbol error probability of the proposed scheme is derived and its error performance is compared with reference systems.**

I. Introduction

Two-path successive relaying has emerged recently as a promising cooperative transmission strategy since it provides significant bandwidth efficiency improvements over the classical relaying methods [1]. Two-path successive relaying has been realized by exploiting distributed space-time block codes (STBC) [2], which create a virtual multi antenna scheme for STBC transmission. An effective distributed STBC for two-path relaying has been proposed in [3] which provides full-rate and full-diversity; however, this code does not permit single symbol detection, which makes its implementation difficult and costly. More recently, a new distributed STBC, which uses the coordinate interleaved orthogonal design (CIOD) [4], has been proposed for two-path relaying [5]. This scheme provides single symbol decoding and achieves better error performance than the scheme of [3] and provides full-diversity when *i)* the relays perfectly decode and forward the signals transmitted from the source, and *ii)* the channel between relays is not affected by fading; however, these are extremely optimistic assumptions which cannot hold for practical wireless networks.

In this paper, by removing these two assumptions, we propose a novel two-path successive relaying scheme which can reliably transfer the data from the source to the destination via relays under realistic network conditions. The main contributions of this work are as follows: 1) It is shown that full-diversity can be achieved with the proposed protocol at both relay nodes and the destination without any assumptions. This is accomplished by implementing distributed CIOD signaling at the destination and at one of the relays, while the other relay benefits from relay selection; 2) The average symbol error probability (ASEP) of the proposed scheme is evaluated analytically for M-ary quadrature amplitude modulation (M-QAM). Taking into account the error propagation, it is shown that analytical and computer simulation results match very well; 3) It is shown that for realistic network conditions, the proposed scheme achieves significantly better bit error rate (BER) performance than its counterparts given in the literature.*

II. System Model and The New Scheme

We consider a relay network with a single source node S, two relay nodes R_1 and R_2 and a destination node D. Each node has a single antenna and operates in half-duplex mode. We assume that there is no direct transmission from S to D. h_{SR_i} and $h_{R_iD}, i=1,2$, represent the wireless channel fading coefficients between S and relays, and relays and D, respectively, while the inter-relay channel h_R is reciprocal. We assume that the real and imaginary parts of h_{SR_i}, h_{R_iD} and h_R follow the $\mathcal{N}(0,1/2)$ distribution, and $h_{R_iD}, i=1,2$, are known at D, while h_{SR_i} and h_R are known at the relays. The variance of the zero-mean complex Gaussian noise samples $n_{R_1}(t), n_{R_2}(t)$ and $n_D(t)$ at relays and D is assumed to be N_0, where $t\,(t=1,2,3)$ represents time slots.

The proposed scheme is based on CIOD transmission with two transmit antennas which may be presented by either of the following 2×2 transmission matrices [4]:

$$\begin{bmatrix} s_1^R + js_2^I & 0 \\ 0 & s_2^R + js_1^I \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} s_2^R + js_1^I & 0 \\ 0 & s_1^R + js_2^I \end{bmatrix} \tag{2}$$

where columns and rows correspond to time slots and transmit antennas, respectively, and s_1 and s_2 are two complex information symbols drawn from a rotated M-QAM constellation. Assuming that a square M-QAM constellation with signal points $s = s^R + js^I$ where $s^R, s^I \in \left\{\pm 1, \pm 3, \ldots, \pm\sqrt{M}-1\right\}$ is rotated by an angle θ, the rotated signal constellation symbols are denoted by $s_\theta = se^{j\theta} = s_\theta^R + js_\theta^I$ whose real and imaginary components s_θ^R and s_θ^I take M distinct values from the set $\left\{s^R\cos\theta - s^I\sin\theta\right\}$, where $\theta = 31.7^\circ$ is the optimal rotation angle for square M-QAM [4]. Consequently, for a given $s_\theta^R\ \left(s_\theta^I\right)$, $s_\theta^I\ \left(s_\theta^R\right)$ can be determined uniquely. As an example, for 4-QAM, the rotated symbols with distinct real and imaginary parts are $s_\theta \in \{(-1.376 + j0.325), (-0.325 - j1.376), (0.325 + j1.376), (1.376 - j0.325)\}$, and if $s_\theta^R = -1.376$ is given for this constellation, we know that $s_\theta^I =$

***Notation:* For a complex variable $s = s^R + js^I$, s^R and s^I denote the real and imaginary parts of s, where $j=\sqrt{-1}$. $X \sim \mathcal{N}\left(\mu_X, \sigma_X^2\right)$ denotes the Gaussian distribution of a real r.v. X with mean μ_X and variance σ_X^2.

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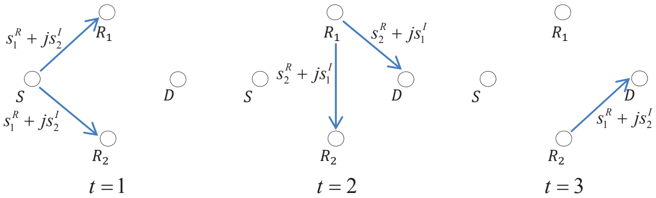

Fig. 1. Three Phase Successive Relaying with Stronger $S-R_1$ Channel

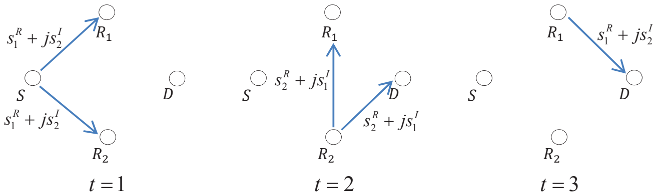

Fig. 2. Three Phase Successive Relaying with Stronger $S-R_2$ Channel

0.325. Constellation rotation is required for CIODs to achieve full-diversity, while in our scheme it is also required to identify symbols from their real or imaginary parts.

In the proposed protocol given in Figs. 1-2, within every consecutive three time slots, two information symbols s_1 and s_2 drawn from a rotated M-QAM constellation are transmitted from S as follows: In the first time slot, S processes s_1 and s_2, and transmits the coordinate interleaved symbol $s_1^R + js_2^I$ to R_1 and R_2. If the $S-R_1$ channel is stronger than the $S-R_2$ channel, R_1 decodes s_1^R and s_2^I first, i.e., it obtains s_1 and s_2 since each symbol can be identified from its real or imaginary part only, which take distinct values after the constellation rotation. Then R_1 forms and transmits the coordinate interleaved symbol $s_2^R + js_1^I$ to R_2 and D in the second time slot. As seen from (1), distributed CIOD signaling is achieved for R_2 after two time slots. In the third time slot, after detecting s_1 and s_2, R_2 transmits $s_1^R + js_2^I$ to D to create the virtual multiple-input single-output (MISO) system using the CIOD matrix given in (2) for D. As seen from Fig. 2, similar procedures can be applied when the $S-R_2$ channel is stronger than the $S-R_1$ channel. Note that a virtual MISO system is created for both D and one of the relays, while the other relay benefits from relay selection. Therefore, the overall diversity order of the system becomes two since not only D, but also R_1 and R_2 achieve a diversity order of two. On the other hand, the transmission rate of the proposed scheme is $2/3$ symbols per channel use since only two information symbols are transmitted in three time slots.

III. Symbol Error Probability (SEP) Analysis of the Proposed Scheme

In this section, we evaluate the ASEP of the proposed scheme for general M-QAM. Without loss of generality, we can analyze the error performance of the signaling scheme given in Fig. 1 where the $S-R_1$ channel is stronger than the $S-R_2$ channel since the ASEP is the same for both cases. The average destination SEP of the scheme given in Fig. 1 can be expressed as $P_D = \frac{1}{M}\sum_s \sum_{\hat{s}} P_D(s \to \hat{s})$ for $s \neq \hat{s}$ where $P_D(s \to \hat{s})$ stands for the pairwise error probability (PEP) at the destination associated with detection of symbol $\hat{s}$ given that symbol s is transmitted.

Destination PEP $P_D(s \to \hat{s})$ can be expressed as the sum of four probabilities related to the error events at the relays as $P_D(s \to \hat{s}) = P_1 + P_2 + P_3 + P_4$ where

$$P_1 = P_{R_1}^c(s)\, P_{R_2}^c(s \mid R_1^c)\, P_D(s \to \hat{s} \mid R_1^c, R_2^c)$$
$$P_2 = \sum\nolimits_{\bar{s},\bar{s}\neq s} P_{R_1}^c(s)\, P_{R_2}^e(s \to \bar{s} \mid R_1^c)\, P_D(s \to \hat{s} \mid R_1^c, R_2^e)$$
$$P_3 = \sum\nolimits_{\tilde{s},\tilde{s}\neq s} P_{R_1}^e(s \to \tilde{s})\, P_{R_2}^c(s \mid R_1^e)\, P_D(s \to \hat{s} \mid R_1^e, R_2^c)$$
$$P_4 = \sum_{\tilde{s},\tilde{s}\neq s} \sum_{\bar{s},\bar{s}\neq s} P_{R_1}^e(s \to \tilde{s})\, P_{R_2}^e(s \to \bar{s} \mid R_1^e)\, P_D(s \to \hat{s} \mid R_1^e, R_2^e)$$

in which $P_{R_1}^c(s)$ is the probability of correct detection of s at R_1, $P_{R_2}^c(s \mid R_1^c)$ is the correct detection probability of s at R_2 conditioned on correct detection at R_1, $P_D(s \to \hat{s} \mid R_1^c, R_2^c)$ is the PEP at the destination conditioned on the correct detection of s at *both* relays, $P_{R_2}^e(s \to \bar{s} \mid R_1^c)$ is the PEP at R_2 conditioned on the correct detection of s at R_1, $P_D(s \to \hat{s} \mid R_1^c, R_2^e)$ is the PEP at the destination conditioned on correct detection of s at R_1 *and* erroneous detection of s to $\bar{s}$ at R_2, $P_{R_1}^e(s \to \tilde{s})$ is the PEP at R_1, $P_{R_2}^c(s \mid R_1^e)$ is the probability of correct detection of s at R_2 conditioned on the erroneous detection of s to $\tilde{s}$ at R_1, $P_D(s \to \hat{s} \mid R_1^e, R_2^c)$ is the PEP at the destination conditioned on correct detection of s at R_2 *and* erroneous detection of s to $\tilde{s}$ at R_1, $P_{R_2}^e(s \to \bar{s} \mid R_1^e)$ is the PEP at R_2 conditioned on the erroneous detection of s to $\tilde{s}$ at R_1, $P_D(s \to \hat{s} \mid R_1^e, R_2^e)$ is the PEP at the destination conditioned on the erroneous detection of s at *both* relays. Our analyses show that the ASEP at the destination is dominated by the case in which $\tilde{s} = \bar{s} = \hat{s}$, i.e., for the case where successive identical erroneous detections occur in the relaying scheme. Therefore we obtain the following:

$$P_2 \cong P_{R_1}^c(s)\, P_{R_2}^e(s \to \hat{s} \mid R_1^c)\, P_D(s \to \hat{s} \mid R_1^c, R_2^e)$$
$$P_3 \cong P_{R_1}^e(s \to \hat{s})\, P_{R_2}^c(s \mid R_1^e)\, P_D(s \to \hat{s} \mid R_1^e, R_2^c)$$
$$P_4 \cong P_{R_1}^e(s \to \hat{s})\, P_{R_2}^e(s \to \hat{s} \mid R_1^e)\, P_D(s \to \hat{s} \mid R_1^e, R_2^e)$$

Our analyses also show that the ASEP at the destination is mainly dominated by P_1, P_2 and P_4, and the effect of P_3 can be ignored. This can be explained by the fact that $P_{R_2}^c(s \mid R_1^e) \ll P_{R_2}^e(s \to \hat{s} \mid R_1^e)$ while $P_D(s \to \hat{s} \mid R_1^e, R_2^c) \sim P_D(s \to \hat{s} \mid R_1^e, R_2^e)$. As we will show in the sequel, independent of the SNR, the probability of symbol error increases dramatically at R_2 when R_1 makes a decision error. Therefore, the probability $P_{R_2}^c(s \mid R_1^e)$ can be neglected when compared to $P_{R_2}^e(s \to \hat{s} \mid R_1^e)$, and we can assume $P_3 \ll P_4$. Therefore the ASEP can be rewritten as the sum of the three terms as

$$\begin{aligned} P_D(s \to \hat{s}) \cong\ & P_{R_1}^c(s)\, P_{R_2}^c(s \mid R_1^c)\, P_D(s \to \hat{s} \mid R_1^c, R_2^c) \\ &+ P_{R_1}^c(s)\, P_{R_2}^e(s \to \hat{s} \mid R_1^c)\, P_D(s \to \hat{s} \mid R_1^c, R_2^e) \\ &+ P_{R_1}^e(s \to \hat{s})\, P_{R_2}^e(s \to \hat{s} \mid R_1^e)\, P_D(s \to \hat{s} \mid R_1^e, R_2^e). \quad (3) \end{aligned}$$

In order to obtain the ASEP, we calculated (3) for $s = s_1$. The derivation of each term in (3) is quite lengthy. In Appendices A, B and C, the details of the derivations are given for the terms related with R_1 $\left(P_{R_1}^c(s), P_{R_1}^e(s \to \hat{s})\right)$, R_2 $\left(P_{R_2}^c(s \mid R_1^c), P_{R_2}^e(s \to \hat{s} \mid R_1^c), P_{R_2}^e(s \to \hat{s} \mid R_1^e)\right)$ and D $\left(P_D(s \to \hat{s} \mid R_1^c, R_2^c), P_D(s \to \hat{s} \mid R_1^c, R_2^e), P_D(s \to \hat{s} \mid R_1^e, R_2^e)\right)$, respectively.

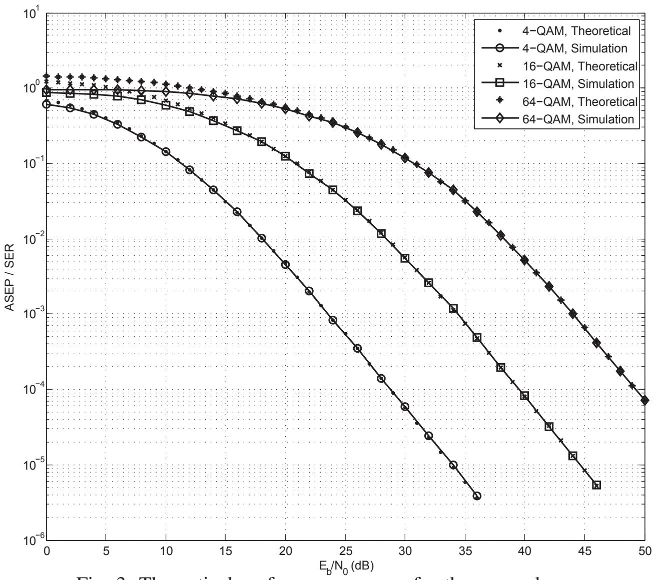

Fig. 3. Theoretical performance curves for the new scheme

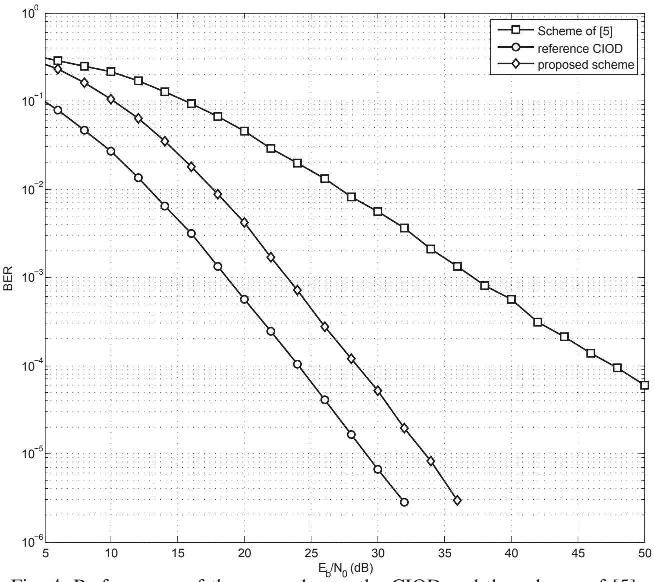

Fig. 4. Performance of the new scheme, the CIOD and the scheme of [5]

IV. Numerical Results

In this section, we present numerical results based on the above analytical expressions as well as Monte Carlo simulation results for quasi-static uncorrelated Rayleigh fading channels.

In Fig. 3, we show the theoretical ASEP curves and computer simulation results for the proposed scheme using 4-QAM, 16-QAM and 64-QAM modulation schemes. As seen from Fig. 3, with increasing SNR, the theoretical curves become extremely tight with the computer simulation curves, and therefore support our analytical results.

In Fig. 4, we compare the bit error rate performance of the proposed scheme with the scheme of [5] and the classical CIOD (i.e., the scheme of [5] with error-free relays) which provides a performance benchmark. We employ 4-QAM modulation for all systems. For the new scheme and the scheme of [5], we consider realistic network conditions in which relays can make erroneous detections. As seen from Fig. 4, without perfect decoding at relays, the scheme of [5] cannot achieve full diversity while the proposed scheme does. We also observe from Fig. 4 that the reference distributed CIOD with error-free relays and the proposed scheme achieve the same diversity order; however, the difference in the error performance can be explained by the additional errors at the relays which increase the overall ASEP at the destination of the proposed scheme.

V. Conclusion

A novel reliable two-path successive relaying protocol has been proposed and its error performance has been investigated comprehensively. It has been shown that by the achievement of full-diversity at all of the nodes of the network, it is possible to transfer data from the source to the destination via relays in a reliable manner.

Appendix A
Calculation of Probabilities Related to R_1 $P^c_{R_1}(s), P^e_{R_1}(s \to \hat{s})$

As we mentioned earlier, we consider the case in which the $S-R_1$ channel is stronger than the $S-R_2$ channel, i.e., $h_1 > h_2$, where $h_1 = |h_{SR_1}|^2$ and $h_2 = |h_{SR_2}|^2$. On analyzing the order statistics we obtain $f_{h_1}(h_1) = 2\left(1 - e^{-h_1}\right)e^{-h_1}, h_1 > 0$ and $f_{h_2}(h_2) = 2e^{-2h_2}, h_2 > 0$ [6].

i) $P^c_{R_1}(s_1)$: The received signal at R_1 for $t = 1$ is given as $r_{R_1}(1) = h_{SR_1}\left(s^R_1 + js^I_2\right) + n_{R_1}(1)$. Thanks to coordinate interleaving, for the independent detection of s^R_1, R_1 obtains

$$y_{R_1} = h^R_{SR_1} r^R_{R_1}(1) + h^I_{SR_1} r^I_{R_1}(1) = h_1 s^R_1 + w_{R_1} \qquad (4)$$

where $w_{R_1} \triangleq h^R_{SR_1} n^R_{R_1}(1) + h^I_{SR_1} n^I_{R_1}(1)$. Therefore, the detection problem of s^R_1 becomes the detection of a modified M-PAM signal subject to fading. Let us denote the possible values of s^R_1 in ascending order as $a_1, a_2, \ldots, a_M$. Considering the transmission model given in (4), we have M decision intervals separated by the threshold values (normalized by h_1) $\lambda_1, \lambda_2, \ldots, \lambda_{M-1}$. As an example for 4-QAM, we have $\lambda_1 = -\cos\theta, \lambda_2 = 0$ and $\lambda_3 = \cos\theta$. Considering that w_{R_1} in (4) is distributed as $\mathcal{N}(0, \psi^2)$, where $\psi^2 = \sigma^2 h_1$ and $\sigma^2 = N_0/2$, the correct detection probability for $a_i, i = 1, \ldots, M$, conditioned on h_1 can be written as

$$P^c_{R_1}(a_i)\Big|_{h_1} = \int_a^b f_{y_{R_1}}(y_{R_1} \mid a_i, h_1)\, dy_{R_1}$$

where $a = -\infty, b = h_1\lambda_1$ and $a = h_1\lambda_{i-1}, b = h_1\lambda_i$ for $i = 1, M$ and $2 \le i \le M-1$, respectively, and conditioned on a_i and h_1, y_{R_1} follows the $\mathcal{N}\left(h_1 a_i, \psi^2\right)$ distribution for all i. Simple manipulation gives

$$P^c_{R_1}(a_i)\Big|_{h_1} = \begin{cases} 1 - Q\left(\frac{h_1(\lambda_1 - a_1)}{\psi}\right), & i = 1, i = M \\ 1 - Q\left(\frac{h_1(a_i - \lambda_{i-1})}{\psi}\right) - Q\left(\frac{h_1(\lambda_i - a_i)}{\psi}\right), & \text{o.w.} \end{cases}$$

Using the alternative form of the Q-function

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta \qquad (5)$$

and considering the moment generating function (m.g.f.) of h_1 given as $M_{h_1}(s) = 2/\left(2 - 3s + s^2\right)$, the unconditional correct detection probability for a_i is obtained as follows:

$$P_{R_1}^c(a_i)=\begin{cases}1-q\left(\frac{(\lambda_1-a_1)^2}{N_0}\right), & i=1,\, i=M\\ 1-q\left(\frac{(a_i-\lambda_{i-1})^2}{N_0}\right)-q\left(\frac{(\lambda_i-a_i)^2}{N_0}\right), & 2\le i\le M-1\end{cases}$$

where

$$q(x)\triangleq\frac{1}{\pi}\int_0^{\pi/2}M_{h_1}\left(\frac{-x}{\sin^2\theta}\right)d\theta=\frac{1}{2}\left(1-\frac{2}{\sqrt{1+\frac{1}{x}}}+\frac{1}{\sqrt{1+\frac{2}{x}}}\right). \tag{6}$$

ii) $P_{R_1}^e(s_1\to\hat{s}_1)$: For the signaling scheme of (4), by the integration of the conditional p.d.f. of y_{R_1} over the decision intervals mentioned above, the exact conditional PEP (CPEP) can be written in general form as

$$P_{R_1}^e(a_i\to a_j)\Big|_{h_1}=\int_a^b f_{y_{R_1}}(y_{R_1}\mid a_i,h_1)\,dy_{R_1} \tag{7}$$

where (a,b) pairs are given by $(-\infty,h_1\lambda_1)$, $(h_1\lambda_{j-1},h_1\lambda_j)$ and $(h_1\lambda_{M-1},\infty)$ for $j=1$, $2\le j\le M-1$ and $j=M$, respectively for $1\le i\le M$. Using (5), $M_{h_1}(s)$ and (6), the unconditional PEP (UPEP) is derived as follows (for $i=1,\ldots,M$, $j\ne i$):

$$P_{R_1}^e(a_i\to a_j)=\begin{cases}q\left(\frac{(a_i-\lambda_1)^2}{N_0}\right), & j=1\\ \left|q\left(\frac{(\lambda_{j-1}-a_i)^2}{N_0}\right)-q\left(\frac{(\lambda_j-a_i)^2}{N_0}\right)\right|, & \text{o.w.}\\ q\left(\frac{(\lambda_{M-1}-a_i)^2}{N_0}\right), & j=M.\end{cases}$$

Since the coordinate interleaving technique allows us to distinguish symbols from only their real (or imaginary) parts, $P_{R_1}^c(a_i)=P_{R_1}^c(s_1)$ and $P_{R_1}^e(a_i\to a_j)=P_{R_1}^e(s_1\to\hat{s}_1)$, for $s_1\ne\hat{s}_1$.

Appendix B

Calculation of Probabilities Related to R_2 $P_{R_2}^c(s\mid R_1^c),P_{R_2}^e(s\to\hat{s}\mid R_1^c),P_{R_2}^e(s\to\hat{s}\mid R_1^e)$

i) $P_{R_2}^e(s_1\to\hat{s}_1\mid R_1^e)$: Assuming that $s_1=s_1^R+js_1^I$ has been erroneously detected at R_1 as $\tilde{s}_1=\tilde{s}_1^R+j\tilde{s}_1^I$, the received signals at R_2 at the first two time slots can be written as

$$\begin{bmatrix}r_{R_2}(1)\\ r_{R_2}(2)\end{bmatrix}=\begin{bmatrix}s_1^R+js_2^I & 0\\ 0 & s_2^R+j\tilde{s}_1^I\end{bmatrix}\begin{bmatrix}h_{SR_2}\\ h_R\end{bmatrix}+\begin{bmatrix}n_{R_2}(1)\\ n_{R_2}(2)\end{bmatrix}.$$

R_2 applies CIOD detection procedures to decode s_1 and s_2 by combining the received signals as $\gamma_{R_2}=\alpha_{R_2}^R+j\beta_{R_2}^I$ and $\delta_{R_2}=\beta_{R_2}^R+j\alpha_{R_2}^I$ where $\alpha_{R_2}=h_{SR_2}^*r_{R_2}(1)$ and $\beta_{R_2}=h_R^*r_{R_2}(2)$. Then, the maximum likelihood (ML) decision rules for s_1 and s_2 are given as follows [4]: $\hat{s}_i=\arg\min_{s_i}\{m_{R_2}(s_i)\}$, $i=1,2$, where

$$m_{R_2}(s_1)=h_3\left(\gamma_{R_2}^R-h_2s_1^R\right)^2+h_2\left(\gamma_{R_2}^I-h_3s_1^I\right)^2$$
$$m_{R_2}(s_2)=h_2\left(\delta_{R_2}^R-h_3s_2^R\right)^2+h_3\left(\delta_{R_2}^I-h_2s_2^I\right)^2$$

with $h_2\triangleq|h_{SR_2}|^2$ and $h_3\triangleq|h_R|^2$. The CPEP of detecting $\hat{s}_1$ when s_1 is transmitted can be given as

$$P_{R_2}^e(s_1\to\hat{s}_1\mid R_1^e)\Big|_{h_2,h_3}=P\left(m_{R_2}(\hat{s}_1)<m_{R_2}(s_1)\right). \tag{8}$$

Considering $\gamma_{R_2}=h_2s_1^R+jh_3\tilde{s}_1^I+w_{R_2}^R(1)+jw_{R_2}^I(2)$, where $w_{R_2}(1)=h_{SR_2}^*n_{R_2}(1)$ and $w_{R_2}(2)=h_R^*n_{R_2}(2)$, we obtain

$$P_{R_2}^e(s_1\to\hat{s}_1\mid R_1^e)\Big|_{h_2,h_3}= P\Big(h_3\left[h_2\Delta_1+w_{R_2}^R(1)\right]^2+h_2\left[h_3\Delta_2+w_{R_2}^I(2)\right]^2 < h_3\left(w_{R_2}^R(1)\right)^2+h_2\left[h_3\Delta_3+w_{R_2}^I(2)\right]^2\Big) \tag{9}$$

where $\Delta_1\triangleq s_1^R-\hat{s}_1^R$, $\Delta_2\triangleq\tilde{s}_1^I-\hat{s}_1^I$ and $\Delta_3\triangleq\tilde{s}_1^I-s_1^I$. After manipulation, we arrive at

$$P_{R_2}^e(s_1\to\hat{s}_1\mid R_1^e)\Big|_{h_2,h_3}=P(A<0) \tag{10}$$

where $A\sim\mathcal{N}(\mu_A,\sigma_A^2)$, $\mu_A=h_2h_3\left(h_2\Delta_1^2+h_3\left(\Delta_2^2-\Delta_3^2\right)\right)$ and $\sigma_A^2=4\sigma^2h_2^2h_3^2\left(h_2\Delta_1^2+h_3\left(\Delta_2-\Delta_3\right)^2\right)$. From (10), the desired CPEP expression can be obtained as

$$P_{R_2}^e(s_1\to\hat{s}_1\mid R_1^e)\Big|_{h_2,h_3}=Q\left(\frac{h_2\Delta_1^2+h_3\left(\Delta_2^2-\Delta_3^2\right)}{2\sigma\sqrt{h_2\Delta_1^2+h_3\left(\Delta_2-\Delta_3\right)^2}}\right). \tag{11}$$

Let us define $\Phi_1=s_1^R-\hat{s}_1^R$ and $\Phi_2=s_1^I-\hat{s}_1^I$. As we mentioned earlier, the average SEP at the destination is dominated by the case where $\tilde{s}_1=\hat{s}_1$. For this case we have $\Delta_1^2=\Phi_1^2$, $\Delta_2=0$ and $\Delta_3=\Phi_2^2$ in (11), which gives

$$P_{R_2}^e(s_1\to\hat{s}_1\mid R_1^e)\Big|_{h_2,h_3}=Q\left(\frac{h_2\Phi_1^2-h_3\Phi_2^2}{2\sigma\sqrt{h_2\Phi_1^2+h_3\Phi_2^2}}\right). \tag{12}$$

Please note that (12) can take very large values when $h_3\Phi_2^2>h_2\Phi_1^2$, which supports our assumption that $P_{R_2}^c(s\mid R_1^e)\ll P_{R_2}^e(s\to\hat{s}\mid R_1^e)$. Thus, we can approximate the UPEP for high SNR after tedious calculations as

$$P_{R_2}^e(s_1\to\hat{s}_1\mid R_1^e)\approx P\left(h_2\Phi_1^2<h_3\Phi_2^2\right)=\frac{2\Phi_2^2}{\Phi_1^2+2\Phi_2^2} \tag{13}$$

which proves that the probability of error becomes very high at R_2 if R_1 forwards an erroneously detected signal.

ii) $P_{R_2}^e(s_1\to\hat{s}_1\mid R_1^c)$: With correct detection at R_1, the CPEP of detection $\hat{s}_1$ when s_1 is transmitted can be obtained by setting $\tilde{s}_1^I=s_1^I$ in (11) $\left(\Delta_2^2=\Phi_2^2,\Delta_3=0\right)$ as

$$P_{R_2}^e(s_1\to\hat{s}_1\mid R_1^c)\Big|_{h_2,h_3}=Q\left(\sqrt{\frac{h_2\Phi_1^2+h_3\Phi_2^2}{2N_0}}\right) \tag{14}$$

which is the CPEP of the classical CIOD. Let us define $u=u_1+u_2$ where $u_1\triangleq h_2\Phi_1^2$ and $u_2\triangleq h_3\Phi_2^2$. Using (5) and defining $M_u(s)=E\{e^{su}\}$, the corresponding UPEP can be evaluated as follows:

$$P_{R_2}^e(s_1\to\hat{s}_1\mid R_1^c)=\frac{1}{\pi}\int_0^{\pi/2}M_u\left(\frac{-1}{4N_0\sin^2\theta}\right)d\theta. \tag{15}$$

Considering $f_{u_1}(u_1)=\left(2/\Phi_1^2\right)e^{-2u_1/\Phi_1^2}$ and $f_{u_2}(u_2)=\left(1/\Phi_2^2\right)e^{-u_2/\Phi_2^2}$ the p.d.f. of u, which is the sum of two exponential r.v.'s, can be calculated by [6]

$$f_u(u)=\int_0^u f_{u_2}(u_2)f_{u_1}(u-u_2)\,du_2=\frac{2\left(e^{-2u/\Phi_1^2}-e^{-u/\Phi_2^2}\right)}{\left(\Phi_1^2-2\Phi_2^2\right)}$$

Then, the corresponding m.g.f. is evaluated as

$$M_u(s)=\frac{2}{\left(\Phi_1^2s-2\right)\left(\Phi_2^2s-1\right)}. \tag{16}$$

Combining (15) and (16), we obtain

$$P^e_{R_2}\left(s_1 \to \hat{s}_1 \mid R^c_1\right) = \frac{\Phi_1^2\left(1-\rho_1\right)+2\Phi_2^2\left(-1+\rho_2\right)}{2\left(\Phi_1^2-2\Phi_2^2\right)} \quad (17)$$

where $\rho_1 \triangleq 1/\sqrt{1+8N_0/\Phi_1^2}$ and $\rho_2 \triangleq 1/\sqrt{1+4N_0/\Phi_2^2}$.

iii) $P^c_{R_2}\left(s_1 \mid R^c_1\right)$: The correct detection probability of s_1 at R_2 given that R_1 forwarded the correct s_1 component can be easily obtained by using (17) in the following union bound:

$$P^c_{R_2}\left(s_1 \mid R^c_1\right) \le 1 - \frac{1}{M}\sum_{\hat{s}_1, \hat{s}_1 \ne s_1} P^e_{R_2}\left(s_1 \to \hat{s}_1 \mid R^c_1\right). \quad (18)$$

APPENDIX C
CALCULATION OF PROBABILITIES RELATED TO D
$P_D\left(s \to \hat{s} \mid R^c_1, R^e_2\right), P_D\left(s \to \hat{s} \mid R^c_1, R^e_2\right), P_D\left(s \to \hat{s} \mid R^e_1, R^e_2\right)$

i) $P_D\left(s_1 \to \hat{s}_1 \mid R^e_1, R^e_2\right)$: Assuming that $s_1 = s_1^R + js_1^I$ has been erroneously detected as $\tilde{s}_1 = \tilde{s}_1^R + j\tilde{s}_1^I$ and $\bar{s}_1 = \bar{s}_1^R + j\bar{s}_1^I$ at R_1 and R_2, respectively, the received signals at the destination for $t = 2$ and 3 can be expressed as

$$\begin{bmatrix} r_D(3) \\ r_D(2) \end{bmatrix} = \begin{bmatrix} \bar{s}_1^R + js_2^I & 0 \\ 0 & s_2^R + j\tilde{s}_1^I \end{bmatrix} \begin{bmatrix} h_{R_2D} \\ h_{R_1D} \end{bmatrix} + \begin{bmatrix} n_D(3) \\ n_D(2) \end{bmatrix}. \quad (19)$$

According to the CIOD detection procedures, after processing and interleaving these signals as $\alpha_D = h^*_{R_2D} r_D(3)$, $\beta_D = h^*_{R_1D} r_D(2)$ and $\gamma_D = \alpha_D^R + j\beta_D^I$, $\delta_D = \beta_D^R + j\alpha_D^I$, the receiver calculates the ML decision metrics as

$$\hat{s}_1 = \arg\min_{s_1}\left\{h_4\left(\gamma_D^R - h_5 s_1^R\right)^2 + h_5\left(\gamma_D^I - h_4 s_1^I\right)^2\right\}$$
$$\hat{s}_2 = \arg\min_{s_2}\left\{h_5\left(\delta_D^R - h_4 s_2^R\right)^2 + h_4\left(\delta_D^I - h_5 s_2^I\right)^2\right\}$$

where $h_4 \triangleq \left|h_{R_1D}\right|^2$ and $h_5 \triangleq \left|h_{R_2D}\right|^2$. Considering $\gamma_D = h_5\bar{s}_1^R + jh_4\tilde{s}_1^I + w_D^R(3) + jw_D^I(2)$ where $w_D(3) = h^*_{R_2D}n_D(3)$ and $w_D(2) = h^*_{R_1D}n_D(2)$, similar to (8), the CPEP can be calculated as

$$\begin{aligned} &P_D\left(s_1 \to \hat{s}_1 \mid R^e_1, R^e_2\right)\Big|_{h_4,h_5} = \\ &P\Big(h_4\left[h_5\Delta_4 + w_D^R(3)\right]^2 + h_5\left[h_4\Delta_2 + w_D^I(2)\right]^2 \\ &\quad < h_4\left[h_5\Delta_5 + w_D^R(3)\right]^2 + h_5\left[h_4\Delta_3 + w_D^I(2)\right]^2\Big) \end{aligned} \quad (20)$$

where Δ_2 and Δ_3 are defined in (9), and $\Delta_4 \triangleq \bar{s}_1^R - \hat{s}_1^R$, $\Delta_5 \triangleq \bar{s}_1^R - s_1^R$. After simple calculations, we obtain

$$\begin{aligned} &P_D\left(s_1 \to \hat{s}_1 \mid R^e_1, R^e_2\right)\Big|_{h_4,h_5} \\ &= Q\left(\frac{h_5\left(\Delta_4^2 - \Delta_5^2\right) + h_4\left(\Delta_2^2 - \Delta_3^2\right)}{2\sigma\sqrt{h_5\left(\Delta_4 - \Delta_5\right)^2 + h_4\left(\Delta_2 - \Delta_3\right)^2}}\right). \end{aligned} \quad (21)$$

As mentioned earlier, we assume that for the dominant case $\bar{s}_1^R = \hat{s}_1^R$ and $\tilde{s}_1^I = \hat{s}_1^I$, which gives $\Delta_2 = \Delta_4 = 0$ and $\Delta_5^2 = \Phi_1^2$, $\Delta_3^2 = \Phi_2^2$, and (21) simplifies to

$$\begin{aligned} P_D\left(s_1 \to \hat{s}_1 \mid R^e_1, R^e_2\right)\Big|_{h_4,h_5} &= Q\left(-\sqrt{\frac{h_5\Phi_1^2 + h_4\Phi_2^2}{2N_0}}\right) \\ &= 1 - P_D\left(s_1 \to \hat{s}_1 \mid R^c_1, R^c_2\right)\Big|_{h_4,h_5} \end{aligned} \quad (22)$$

where $P_D\left(s_1 \to \hat{s}_1 \mid R^c_1, R^c_2\right)\big|_{h_4,h_5}$ is the CPEP of the classical CIOD, which will be calculated in the sequel. As seen from (22), the error probability approaches unity with increasing SNR, which is somehow expected since both relays erroneously detected s_1 and forwarded this signal to D.

ii) $P_D\left(s_1 \to \hat{s}_1 \mid R^c_1, R^e_2\right)$: Assuming that $s_1 = s_1^R + js_1^I$ has been correctly detected at R_1 ($\tilde{s_1} = s_1$) and erroneously detected as $\hat{s}_1 = \hat{s}_1^R + j\hat{s}_1^I$ at R_2, by setting $\Delta_3 = \Delta_4 = 0, \Delta_2^2 = \Phi_2^2$ and $\Delta_5^2 = \Phi_1^2$ in (21) for this case, we obtain

$$P_D\left(s_1 \to \hat{s}_1 \mid R^c_1, R^e_2\right)\Big|_{h_4,h_5} = Q\left(\frac{-h_5\Phi_1^2 + h_4\Phi_2^2}{2\sigma\sqrt{h_5\Phi_1^2 + h_4\Phi_2^2}}\right) \quad (23)$$

which has a similar structure as that of (12), and the corresponding UPEP can be calculated as

$$P_D\left(s_1 \to \hat{s}_1 \mid R^c_1, R^e_2\right) \approx P\left(h_4\Phi_2^2 < h_5\Phi_1^2\right) = \frac{\Phi_1^2}{\Phi_1^2 + \Phi_2^2}. \quad (24)$$

iii) $P_D\left(s_1 \to \hat{s}_1 \mid R^c_1, R^c_2\right)$: Assuming that both of the relays have been successfully detected s_1, i.e., $\tilde{s}_1 = s_1$ and $\bar{s}_1 = s_1$, we have $\Delta_3 = \Delta_5 = 0$, $\Delta_4^2 = \Phi_1^2$ and $\Delta_2^2 = \Phi_2^2$, and (21) simplifies to the CPEP of the classical CIOD as

$$P_D\left(s_1 \to \hat{s}_1 \mid R^c_1, R^c_2\right)\Big|_{h_4,h_5} = Q\left(\sqrt{\frac{h_5\Phi_1^2 + h_4\Phi_2^2}{2N_0}}\right) \quad (25)$$

for which the UPEP can be calculated as follows:

$$P_D\left(s_1 \to \hat{s}_1 \mid R^c_1, R^c_2\right) = \frac{1}{\pi}\int_0^{\pi/2} M_v\left(\frac{-1}{4N_0\sin^2\theta}\right)d\theta \quad (26)$$

where $v \triangleq h_5\Phi_1^2 + h_4\Phi_2^2$ with

$$\begin{aligned} f_v(v) &= \frac{e^{-v/\Phi_1^2} - e^{-v/\Phi_2^2}}{\Phi_1^2 - \Phi_2^2}, \\ M_v(s) &= \frac{1}{\left(\Phi_1^2 s - 1\right)\left(\Phi_2^2 s - 1\right)}. \end{aligned} \quad (27)$$

Substituting (27) into (26), the desired UPEP, which is also the UPEP of the classical CIOD, can be calculated as

$$P_D\left(s_1 \to \hat{s}_1 \mid R^c_1, R^c_2\right) = \frac{\Phi_1^2\left(1-\rho_1\right) + \Phi_2^2\left(-1+\rho_2\right)}{2\left(\Phi_1^2 - \Phi_2^2\right)} \quad (28)$$

where $\rho_1 \triangleq 1/\sqrt{1+4N_0/\Phi_1^2}$ and $\rho_2 \triangleq 1/\sqrt{1+4N_0/\Phi_2^2}$.

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