Post Outage Bus Voltage Calculations for Double Branch Outages

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***Abstract*—Secure operation of electrical power systems is vital, hence fast and accurate post-outage state calculations are important for contingency analysis. Contingency analysis includes simulations of both the single and double branch outages. This paper presents constrained optimization problem of a recently developed double branch outage model. Harmony search algorithm is used as an optimization tool. IEEE 30 Bus Test system simulation results are given, and compared with those of the AC load flow in terms of computational accuracy. Speed test results of IEEE 14, 30, 57, 118 and 300 Bus Test Systems are illustrated and compared with those of the AC load flow calculations.**

***Index Terms*—double branch outage, harmony search algorithm, heuristic methods, modeling, optimization**

I. Introduction

With the advent of technology, power systems faced up with new mobile customers, distributed generations, and storage elements. Outage of any components in the system can cause significant problems; hence remedial actions should be taken on time. Contingency analysis deal with the solutions of the single and multiple branch outages. Single branch outages have been widely studied and the results have been published in the past [1]–[6]. A recent work simulated branch outage problem by a pair of fictitious power sources and formulated it as a local constrained optimization problem [6]. The model used the information of only the outaged buses and their first order neighbours and gave good results in terms of accuracy and speed. Up to now, optimization problem of this model has been solved by differential evolution method, harmony search [7], particle swarm optimization [8], and gravitational search [9]. Based on this model, a model to simulate double branch outages was formulated as a local constrained optimization problem [10]. The number of single branch outages is proportional with the number of branches whereas the number of double branch outages is proportional with the square of the number of branches in a power system [11]. This means that time required for a complete double branch outage contingency analysis will be proportional with the square of the size of the power system under test. Hence the developed model must be fast to provide acceptable accuracy. There have been some double branch outage model applications [4], [12] in the past, but they only gave the results of very limited non problematic cases.

In this paper we have used the model in [10], and we solved the local constrained optimization problem of this model by using harmony search algorithm [13]. Being a fast alternative to the other intelligent methods such as particle swarm optimization, differential evolution, genetic algorithms, etc. this method is promising and gaining more importance day by day. It is a newly developed method that mimics of the jazz musicians. improvisation process and consists of mainly four basic steps similar to the other heuristic methods. These are; initialization of solution candidates, improvisation, update process, control of stopping criterion. Up to now, harmony search method has been applied to several areas and economic dispatch problem [14] in electrical power systems.

The proposed solution by harmony search method aims to speed up on electrical power systems. in electrical power systems. sed solution by harmony search method aims to speed up the solution without sacrificing from the accuracy. We used IEEE 30 Bus Test system for accuracy tests of the proposed double branch outage simulation method. For computational speed tests, we used IEEE 14, 30, 57, 118 and 300 Test systems. For power system related studies Matpower [16] was used, which is a Matlab based power system package.

The rest of the paper is organized as follows: In the second part, a double branch outage model and a local optimization method for solving double branch outages are defined. In the third part, algorithm for the application of the harmony search algorithm for the double branch outage problem is given. In the next part, test results of IEEE 30 bus test systems are compared to those of AC load flow results in terms of accuracy, and speed test results of IEEE 14, 30, 57, 118 and 300 Bus Test Sytems are given and compared to those of AC load flow results. Finally, conclusion part concludes the paper.

II. Double Branch Outage Model

We use a recently developed double branch outage model [10] which is based on a single branch outage model [6]. Outages can be modeled by inserting fictitious sources and minimizing the reactive power flows so that there will be no reactive flows through the branches. This idea can also be employed to double branch outages. Assume that the branches between buses i and j, and the branches between buses k and l are simultaneously outaged. The first branch outage is simulated by using fictitious source pairs O_{si} and Q_{sj} and the second branch outage is simulated by using fictitious source pairs Q_{sk} and Q_{sl}. In Figure 1 double branch outage model and corresponding reactive power flows are shown.

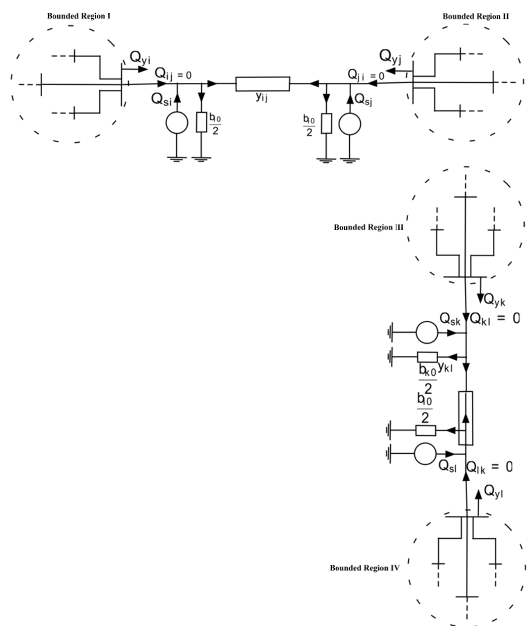

Fig. 1. Double branch outage model and corresponding reactive power flows

The steps of double branch outage modeling can be given as follows [10],

- Select two branches to be outaged and number them as: ij, and kl.
- Compute the bus voltage angles by using the linearized active power equations as shown below.

$$\delta_m = \delta_m + (X_{mi} - X_{mj}) \triangle P_n + (X_{mk} - X_{ml}) \triangle P_r \quad l = 2, 3, \cdots, \text{NB}$$

$$\triangle P_n = \frac{P_{ij}}{[1 - (X_{ii} + X_{jj} - 2X_{ij})/x_n]} \qquad (1)$$
$$\triangle P_r = \frac{P_{kl}}{[1 - (X_{kk} + X_{ll} - 2X_{kl})/x_r]}$$

  where, X_{ij} represents the ith row, jth column element of the bus susceptance matrix, X_{kl} represents the kth row, lth column element of the bus susceptance matrix, P_{ij} and P_{kl} are the pre-outage active power flows through the outaged branches, and x_n and x_r represent the reactance of the branches at hand.
- Calculate the loss reactive powers, represented as $\tilde{Q}_{Li} \cong \tilde{Q}_{Lj}$, $\tilde{Q}_{Lk} \cong \tilde{Q}_{Ll}$, for the optimization cycle.
- Minimize reactive power mismatches at buses i, j, k and l. This process is mathematically equivalent to the following constrained optimization problem.

$$\begin{aligned} \min_{wrt\ Q_{si}, Q_{sk}} \quad & \| Q_i - (\overline{Q}_{ij} + \overline{Q}_{Li}) + Q_{Di} \\ & Q_j - (-\overline{Q}_{ij} + \overline{Q}_{Li}) + Q_{Dj} \\ & Q_k - (\overline{Q}_{kl} + \overline{Q}_{Lk}) + Q_{Dk} \\ & Q_l - (-\overline{Q}_{kl} + \overline{Q}_{Li}) + Q_{Dl} \| \\ \text{subject to} \quad & g_q(V_b) = \triangle Q_b - B_b \triangle V_b = 0 \end{aligned} \qquad (2)$$

where, $\| \cdot \|$ is the Euclidean norm of a vector. The constraint part of (2) is linearized reactive power equation for load buses, $\triangle Q_b$ is the reactive power mismatch vector in the bounded region, V_b is the load bus voltage magnitude vector in the bounded region and B_b is the bus susceptance matrix for the bounded region.

III. Harmony Search Algortihm for Double Branch Outage Model

Harmony search algorithm for double branch outage problem can be summarized as follows:

- Decide on harmony search algorithm parameters, harmony memory size (HMS), haamony memeory consideration rate (HMCR), pitch adjusting rate (PAR).
- Run power flow to obtain pre-outage bus voltage magnitudes.
- Create matrix **A** whose size is HMS by two. The elements of the first column of **A** are between $Q_{ij} + \omega$ and $Q_{ij} - \omega$ and the elements of second column of **A** are between $Q_{ij} + \omega$ and $Q_{ij} - \omega$
- Let buses in the bounded region obtained for the first bounded region be named as Bound1 and for the second bounded region be named as Bound2. Find the union of these two sets as follows:

$$\text{Union} = \text{Bound1} \cup \text{Bound2} \qquad (3)$$

- For every element of the first column of **A** matrix, perform the computations given below.

$$\begin{aligned} \triangle \boldsymbol{Q}_1 &= [0, 0, \cdots, A_{(1,i)}, \cdots, A_{(1,j)}, \cdots, 0]^T \\ A_{(1,j)} &= -A_{(1,i)} + 2Q_{L1i} \end{aligned} \qquad (4)$$

$$\begin{aligned} \triangle \mathbf{V}_{\text{union}} &= \mathbf{B}_{\text{union}}^{-1} \triangle \mathbf{Q_1} \\ \mathbf{V}_{\text{union}_{\text{one}}} &= \mathbf{V}_{\text{union}} + \triangle \mathbf{V}_{\text{union}} \end{aligned} \qquad (5)$$

Similarly, using the second column elements of matrix **A**, perform the following computations:

$$\begin{aligned} \triangle \boldsymbol{Q}_2 &= [0, 0, \cdots, A_{(2,k)}, \cdots, A_{(2,l)}, \cdots, 0]^T \\ A_{(2,l)} &= -A_{(2,k)} + 2Q_{L2k} \end{aligned} \qquad (6)$$

$$\begin{aligned} \triangle \mathbf{V}_{\text{union}} &= \mathbf{B}_{\text{union}}^{-1} \triangle \mathbf{Q_2} \\ \mathbf{V}_{\text{union}_{\text{two}}} &= \mathbf{V}_{\text{union}_{\text{one}}} + \triangle \mathbf{V}_{\text{union}} \end{aligned} \qquad (7)$$

- Compute the objective functions (mismatch vector) for the revised load bus voltage magnitudes.
- Perform the following steps until a stopping criterion is reached
  - Create a random number in $[0, 1]$ interval. If this number is smaller than HMCR, select a random element from $\mathbf{A}$ matrix, otherwise create another value in the range that has been specified by the user. Let this newly selected variable or newly created value be named as new variable.
  - Create two different numbers in $[0.1]$ interval. If the first one these two numbers is smaller than PAR don't perform any operation otherwise depending on the comparison of the second number to 0.5, add brange multiplied by a random number in $[0, 1]$ interval to new variable or not.
  - Perform the following computations.

$$\triangle \boldsymbol{Q}_1 \quad = \quad [0, 0, \cdots, \text{n. var.}, \cdots, \text{-n var.}, \cdots, 0]^T \tag{8}$$

$$\begin{aligned} \triangle \mathbf{V_{union}} &= \mathbf{B_{union}^{-1}} \triangle \mathbf{Q_1} \\ \mathbf{V_{union_{one}}} &= \mathbf{V_{union}} + \triangle \mathbf{V_{union}} \end{aligned} \tag{9}$$

where, n. var. represents new variable. Similarly, using the second column elements of matrix $\mathbf{A}$, perform the following computations:

$$\triangle \boldsymbol{Q}_2 \quad = \quad [0, 0, \cdots, \text{n. var.}, \cdots, \text{-n.var}, \cdots, 0]^T \tag{10}$$

$$\begin{aligned} \triangle \mathbf{V_{union}} &= \mathbf{B_{union}^{-1}} \triangle \mathbf{Q_2} \\ \mathbf{V_{union_{two}}} &= \mathbf{V_{union_{one}}} + \triangle \mathbf{V_{union}} \end{aligned} \tag{11}$$

  - By using the new computed voltage magnitudes, compute the objective function values. Sort the elements of $\mathbf{A}$matrix according to these values.

IV. Test Results

Double branch outage simulations are tested on IEEE 30 bus test systems. Matlab based open-source software Matpower [16] is used as a tool. Programs for simulating the outages are written in Matlab.

One can easily see that there are two formations possible in terms of system topology. The first topology, includes no buses in the set of intersections of the bounded regions, as shown in Figure 1, and the second topology includes at least one element in the set of intersections of the bounded regions [10].

On the other hand, there are 3 different outage cases for each of the two topologies, namely [10]: (a)

1) Outaged branches are either power transmission lines or underground cables. None of the outaged branches include a tap changing transformer between the outaged bus pairs.
2) One of the outaged branch is a transmission line/underground cable and the other one includes a transformer between the outaged bus pairs.
3) Both of the outaged branches include a transformer between the outaged bus pairs.

In Figure (2) percentage errors versus number of branch outages are shown for the cases that doesn't include common elements in the bounded regions. Since there are no transformer\transformer outages this case is not shown. Note that, there are same number of double transformer/line outages more, which have the same error values so graph of this is not shown.

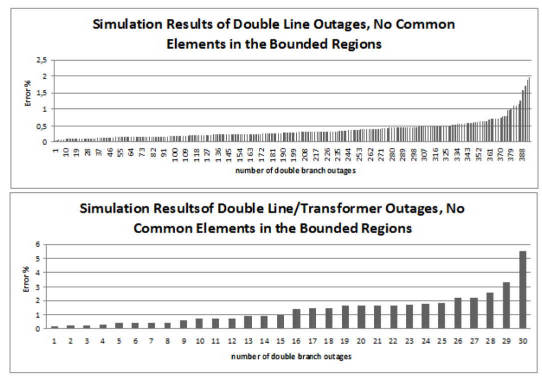

Fig. 2. Graphs of percentage error versus number of double branch outages, no-common elements in the bounded region, for IEEE 30 Bus Test System

In Figure (3) percentage errors versus number of branch outages are shown for the cases that include common elements in the bounded regions. Graphs of three different cases are also shown in the figure. As in the previous figure, there are same number of double transformer/line outages more, which have the same error values so graph of this is not shown.

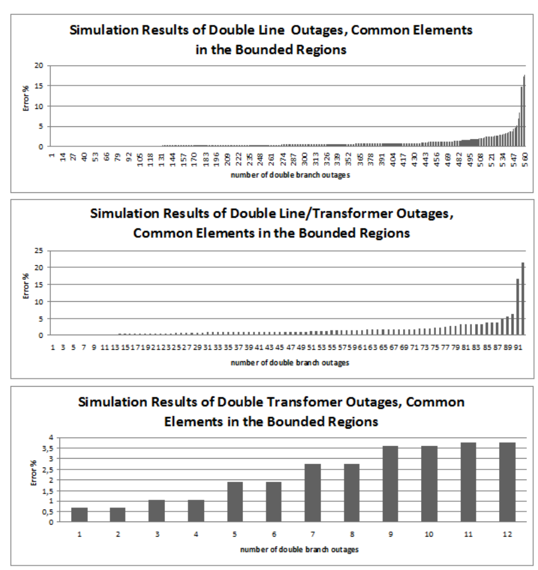

Fig. 3. Graphs of percentage error versus number of double branch outages, common elements in the bounded region, for IEEE 30 Bus Test System

Following tables give some results for the different double branch outage simulations for IEEE 30 bus test systems. In the tables V_{AC} repesents the calculated post-outage voltage magnitude of a specific bus by using AC load flow, on the other hand V_{HS} symbolizes the calculated post-outage voltage magnitude of a specific bus by using harmony search method. $Err\%$ represents the percentage error of the specific bus voltage magnitude, and is compued as follows.

$$\text{Err\%} = 100 \times \frac{abs(V_{AC} - V_{GSA})}{V_{AC}} \tag{12}$$

The outages, which do not create convergence problems, do not cause islanding problem and do result in load bus voltages less than 0.8 p.u were tested. Total of 1214 double branch outage simulations were performed for IEEE 30 bus test system.

We present sample results of double branch outage simulations in the following tables. The first one, simultanesous outage of lines $3-4$ and $21-22$ is an example that does not have a common element in the bounded regions, and is composed of only transmission lines or cables. Table (I) illustrates simulation results for this line-line outage. In the table, only voltage magnitudes that show a percentage error higher than 0.04 are presented, due to limited space. In the following tables, Bus No. represents the bus number, $V_{(AC)}$ represents the bus voltage magnitudes computed by using full AC load flow in per unit, $V_{(HS)}$ represents the bus voltage magnitudes computed by the proposed HS based solution of local constrained optimization problem in per unit, and Error % represents the percentage difference between $V_{(AC)}$ and $V_{(HS)}$. Note that maximum percentage error for this representative line-line outage is less than 0.08%.

TABLE I
IEEE-30 BUS TEST SYSTEM, SIMULATION RESULTS OF, SIMULTANEOUS OUTAGES OF 3-4 AND 21-22 BRANCHES

Bus No	Bus Voltage (AC in p.u.)	Bus Voltage (HS in p.u.)	$Error\%$
6	1.0343	1.0349	0.0603
7	1.0331	1.0336	0.0487
12	1.0525	1.0530	0.0437
14	1.0390	1.0395	0.0488
21	1.0334	1.0327	0.0683
22	1.0394	1.0401	0.0738
23	1.0288	1.0293	0.0430
24	1.0281	1.0287	0.0557
28	1.0336	1.0341	0.0473
maximum error	-	-	**0.0738**

The second presented double branch outage, outage of transformer $6-9$ and outage of line $14-15$ again does not have a common element in the bounded regions, it is composed of a transmission line and a transformer. The results of the simulation results of this case are given in table (II) for the voltage magnitudes of the buses showing a percentage error higher than 0.3%. Maximum percentage error for this case is obtained as 0.37%.

There is not any case that does not have a common element and composed of two transformers.

TABLE II
IEEE-30 BUS TEST SYSTEM, SIMULATION RESULTS OF, SIMULTANEOUS OUTAGES OF TRANSFORMER 6-9 AND LINE 14-15

Bus No	Bus Voltage (AC in p.u.)	Bus Voltage (HS in p.u.)	$Error\%$
10	1.0410	1.0449	0.3739
17	1.0358	1.0391	0.3143
20	1.0255	1.0287	0.3117
21	1.0294	1.0331	0.3611
22	1.0303	1.0339	0.3519
maximum error	-	-	**0.3739**

The third presented double branch outage, outage of line $19-20$ and outage of line $16-17$ has at least one common element in the bounded regions, it is composed of transmission lines and/or cables. The results of the simulation results of this case are given in table (III) for the voltage magnitudes of the buses showing a percentage error higher than 0.1%. Maximum percentage error for this case is obtained as 0.32%.

TABLE III
IEEE-30 BUS TEST SYSTEM, SIMULATION RESULTS OF, SIMULTANEOUS OUTAGES OF LINES 19-20 AND 16-17

Bus No	Bus Voltage (AC in p.u.)	Bus Voltage (HS in p.u.)	$Error\%$
16	1.0448	1.0461	0.1242
17	1.0464	1.0452	0.1187
18	1.0046	1.0026	0.2054
19	0.9941	0.9909	0.3183
maximum error	-	-	**0.3183**

Simultaneous outages of transformer $4-12$ and line $10-22$ has at least one element in the bounded regions. Bus-6 has a direct connection to Bus-4 and to Bus-10. Therefore, Bus-6 is included both in the neighborhood of the first outaged branch and in the neighborhood of the second outaged branch. Voltage magnitudes of the critical buses showing a percentage error higher than a threshold value (0.5%) are reported in Table ((IV). Note that the maximum percentage error for this representative transformer-line outage is less than 1.0%. In addition, the percentage errors are generally high when compared with those of the previous outages.

TABLE IV
IEEE-30 BUS TEST SYSTEM, SIMULATION RESULTS OF, SIMULTANEOUS OUTAGES OF TRANSFORMER 4-12 AND LINE 10-22

Bus No	Bus Voltage (AC in p.u.)	Bus Voltage (HS in p.u.)	$Error\%$
10	1.0467	1.0407	0.5759
16	1.0288	1.0224	0.6266
17	1.0351	1.0299	0.5019
19	1.0159	1.0108	0.5027
20	1.0229	1.0175	0.5255
21	1.0286	1.0194	0.8901
22	1.0273	1.0171	0.9889
23	1.0110	1.0054	0.5618
24	1.0156	1.0082	0.7225
25	1.0281	1.0209	0.7008
26	1.0106	1.0034	0.7147
27	1.0442	1.0372	0.6734
29	1.0248	1.0178	0.6865
30	1.0136	1.0066	0.6929
maximum error	-	-	**0.9889**

The last representative double branch outage is the outage of transformer $28 - 27$ and transformer $6 - 10$. This case has at least one element in the set of intersecitons of the bounded regions. This can be explained as follows: Bus-8 has a direct connection to Bus-28 and to Bus-6. Therefore, Bus-8 is included both in the neighborhood of the first outaged branch and in the neighborhood of second outaged branch. Moreover, one terminal of the second outaged transformer (Bus-6) is in the bounded region of the first outaged transformer. Therefore, this one is expected to produce one of the worst simulation results in terms of computational accuracy. Due to limited space 0.5 is selected as a threshold value for voltage magnitudes to be presented in table ((V). The maximum percentage error for this transformer-transformer outage is less than 2.98%. The percentage errors are high as being one of the worst double branch outages in the system.

TABLE V
IEEE-30 Bus Test System, simulation results of, simultaneous outages of transformers 28-27 and 6-10

Bus No	Bus Voltage (AC in p.u.)	Bus Voltage (HS in p.u.)	$Error\%$
10	1.0263	1.0319	0.5502
19	1.0066	1.0117	0.5008
20	1.0107	1.0159	0.5176
21	1.0088	1.0152	0.6407
22	1.0078	1.0144	0.6588
23	0.9958	1.0022	0.6340
24	0.9737	0.9833	0.9865
25	0.9139	0.9304	1.7981
26	0.8942	0.9122	2.0145
27	0.8888	0.9069	2.0392
29	0.8656	0.8881	2.6015
30	0.8521	0.8775	2.9800
maximum error	-	.-	**2.9800**

In the following tables we present results of speed tests we have conducted for IEEE 14, 30, 57, 118 and 300 Bus Test Systems. The number of double branch outages simulated are 260 for IEEE 14 Bus Test System, 1214 for IEEE 30 Bus Test System, 4148 for IEEE 57 Bus Test System, 15380 for IEEE 118 Bus Test System and 87614 for IEEE 300 Bus Test System.

Table (VI) gives average time per double outage for different test systems are given for both harmony search based simulation and AC load flow. We use 3000 iterations as maximum number of iterations, and if the best value does not change for 300 consecutive iterations the program stops. From the table while the average time for harmony search behaves nearly linearly, the average time for AC load flow increases quadratically. For IEEE 118 Bus Test System, mean error value is computed as 0.45 and standart deviation is computed as 0.74.

If we decrease maximum number of iterations to 1500, and if the best value does not change for 150 consecutive iterations, the new average times per outage are shown in table (VII). Again the average time for harmony search method behaves linearly and for IEEE 300 Bus system the time obtained is less than that of AC load flow. For IEEE 118 Bus Test System, mean error value is computed as 0.47 and standart deviation is computed as 0.84.

TABLE VI
Speeds per double outage simulation by using HS method and AC load flow in seconds, Max. iteration number=3000, control of change for last 300 iterations.

Test System	HS	AC
IEEE 14 Bus	0.0485	0.0107
IEEE 30 Bus	0.0505	0.0133
IEEE 57 Bus	0.0499	0.0175
IEEE 118 Bus	0.0386	0.0204
IEEE 300 Bus	0.0592	0.0439

TABLE VII
Speeds per double outage simulation by using HS method and AC load flow in seconds, Max. iteration number=1500, control of change for last 150 iterations.

Test System	HS	AC
IEEE 14 Bus	0.0395	0.0107
IEEE 30 Bus	0.0403	0.0133
IEEE 57 Bus	0.0344	0.0175
IEEE 118 Bus	0.0325	0.0204
IEEE 300 Bus	0.0424	0.0439

If the number of consecutive iterations that does not change is decreased to 75, the new average times per outage are shown in table (VIII). Again the average time for harmony search method behaves linearly and faster than the previous examples. For this case, for IEEE 118 Bus Test System, mean error value is computed as 0.77 and standart deviation is computed as 1.64.

TABLE VIII
Speeds per double outage simulation by using HS method and AC load flow in seconds, Max. iteration number=1500, control of change for last 75 iterations.

Test System	HS	AC
IEEE 14 Bus	0.0256	0.0107
IEEE 30 Bus	0.0275	0.0133
IEEE 57 Bus	0.0229	0.0175
IEEE 118 Bus	0.0259	0.0204
IEEE 300 Bus	0.0256	0.0439

V. Conclusions

In this study, the constrained optimization problem representing the double branch outage phenomena in an electric power system is solved by using the harmony search algorithm. Accuracy results of sample double branch outage simulations of IEEE 30 Bus Test systems are given. IEEE 14,30,57, 118 and 300 Bus Test systems are used for speed tests to compare AC load flow computational speed to harmony search based double branch outage simulation. Simulation results show that harmony search based double branch outage simulation is accurate and fast.

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