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Connected Navigation of Non-Communicating Mobile Agents

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***Abstract*—This article discusses the connectivity of autonomous mobile robots that do not have communication capabilities. We show that if the group members follow the proposed Local Steering Strategy, which utilizes information only about the relative positions of neighbor robots, they can sustain their connectivity, even in the case of bounded position measurement errors and the occultation of robots by other robots in the group. To reduce the computational burden in the implementation of the proposed methodology, we used sub-optimal solutions.**

***Index Terms*—Mobile robots, autonomous motion, connectivity**

I. Introduction

One important aspect of the navigation of multiple autonomous mobile agents is how they maintain connectivity. Loosely speaking, the connectivity of a robot group means that each robot can be contacted by any other robot in the group, either directly or via other robots. The method of making contact may differ according to the characteristics of the individual agents. For instance, being able to establish communications via a standard channel of communication, being visible, or being detected by ultrasonic waves are various ways of being contacted, and thus of being connected.

The navigation of autonomous agents may be the primary or secondary task of a group, depending on the application. The transportation of a group of mine-digging robots from one site to another is an example of the latter, in which navigation is a secondary task. However, navigation is the primary task of robots in such missions as defense patrols or underwater exploring. In both cases, connectivity is of vital importance, since it reflects the unity of the group. Thus, connectivity and its maintainability are fundamental concepts in almost any study regarding the decentralized group motion of autonomous agents.

In this paper, we present a methodology for the navigation of autonomous robot groups which maintain group connectivity. We assume that the robots have position sensors of limited range and with bounded measurement errors, but no communication capabilities. In studies related to connected navigation and the group behavior of mobile robots, many authors *assume* group connectivity or communication within the group during the period of motion as a prerequisite for the success of their methods. For example, graph theory or potential field techniques are employed in this way in [1]–[10].

Graph theoretic approaches to maintain the connectivity of mobile agents are mainly based on the maximization of the second smallest eigenvalue (Fiedler value) of the Laplacian matrix of the graph [5]–[8], [11]. Even if this maximization can be accomplished in a distributed manner as suggested in [5], this does not eliminate the necessity of communications between the robots. For example, the method introduced in [5] requires some data to be obtained from neighbor robots to update components of the supergradient of the Laplacian which are computed locally, and [8] provides an extensive literature survey about group connectivity. Only a few studies, however, have focused on the maintenance of connectivity without relying on information exchange or communication between robots [12]–[14]. The algorithmic methodologies in these studies assume that the robots are points, and are designed to work only in $\mathbb{R}^2$ with perfect measurements via sensors.

The approach proposed in a recent work by the authors [15] results in the navigation of a robot group having dynamic topology using only limited-range position sensors with guaranteed connectivity. In [16], an ad hoc method was proposed to resolve possible deadlock cases. In this study, we present an extension of these results by including measurement errors and the occultation of the robots as well as a modification of the navigation strategy to eliminate any deadlock problems.

In the following section, we describe the agents in the group and define the related navigation problems. Section III gives an overview of our theorem on connectivity and discusses a local steering strategy used for maintaining connectivity. A sub-optimal approach to the implementation of this strategy is given in Section IV and tested by simulations in Section V. Lastly, Section VI offers concluding remarks on the study.

II. Problem Formulation

The robots in this study are assumed to be physically identical, and each robot has the capability of moving in all directions and is equipped with limited-range position sensors. These sensors have a known degree of accuracy. We assume that the sensing capability is omnidirectional, but the sensor results can bear both angular and radial measurement errors.

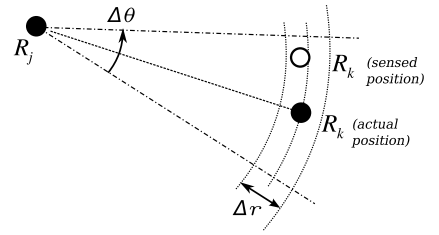

Fig. 1. Angular and radial measurement errors in $\mathbb{R}^2$

Fig. 1 depicts the bounds on the radial and angular components of position errors, $\Delta\theta$ and Δr, respectively.

It is important to note that sensing other robots means obtaining information about the position of the robots in the neighborhood via position sensors. We shall refer to such a mutual visibility between robots as a *link*. However, we should also note that such a link does not imply any communication or information exchange between the robots. As the robots move around, as long as they maintain visibility with their neighboring agents, they can avoid separating from the other robots, even if they do not communicate with them. Also, it should be pointed out that the robots have no labels, and as a result, sensing the other robots does not imply recognizing a specific robot.

Let us denote a group of autonomous mobile robots that are connected by links as discussed above as $\mathcal{G}$ and the robots in the group as R_i, $i = 1, \ldots, N$. Note that the subscripts are arbitrary and for the sake of analysis only. If we consider the robots $R_i, \ldots, R_N$ as vertices and the links between them as the edges of an undirected graph, group $\mathcal{G}$ will be *connected* if there is a path from any robot to any other robot in the group through the links [11]. A group which has at least one pair of robots without a path between them is therefore *disconnected*.

Since the range of the position sensors is limited, a robot may not sense all of the other robots in the group, especially when the total number of robots in the group is large. We refer to the set of robots sensed by R_i as the subgroup $\mathcal{S}_i$. In this way, there are N such subgroups of $\mathcal{G}$, and if $\mathcal{G}$ is connected, $\mathcal{S}_i$ $(i = 1, \ldots, N)$ are nonempty sets.

We denote the radius of the spherical region with R_i at its center and which contains robots in $\mathcal{S}_i$ as d_{max}. In other words, d_{max} is the maximum sensing distance for each robot. On the other hand, if the largest distance between the robots in $\mathcal{G}$ is denoted as D_{max}, $\mathcal{G}$ will be connected if $d_{max} \geq D_{max}$. However, nontrivial and more interesting cases emerge whenever $d_{max} \ll D_{max}$, which corresponds to groups of relatively large number of agents which have limited sensing ranges.

Fig. 2 depicts a group consisting of three robots. It is seen that $R_2 \in \mathcal{S}_1$, and $R_1 \in \mathcal{S}_2$, which means that R_1 and R_2 are linked. The links between R_2 and R_3 are formed likewise. Note that the robot R_2 has the position information of both R_1 and R_3, but R_1 and R_3 cannot sense each other, as the distance between them is larger than d_{max}.

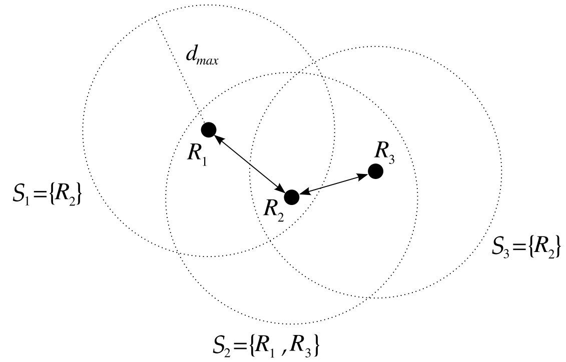

Fig. 2. A group of three robots and their subgroups

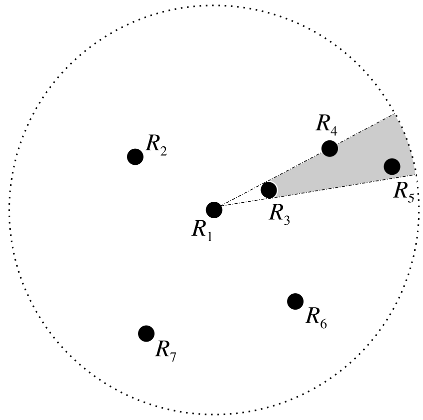

Fig. 3. R_3 occults R_4 and R_5 from R_1

In implementing position measurements, which could be performed using any kind of ultrasonic, laser or vision-based sensors, it is inevitable that some robots might occult others. In such a case, occulted robots are not sensed by another robot, say R_1 (and hence, are not in $\mathcal{S}_1$) although their distances to R_1 are less than d_{max}. Fig. 3 depicts an example of occulting in which the position measurements of R_4 and R_5 cannot be accomplished since R_3 prevents R_4 and R_5 from being "in sight" of R_1. Consequently, whenever occultation occurs, the positions of the occulted robots cannot be taken into account in the computation of local movement at that time instant.

Taking into account these sensory limitations and assuming that a set of mobile agents initially represents a connected group, our objective in this work is to develop a decentralized steering methodology that allows for the navigation of the group while preserving its connectivity without requiring any exchange of information between the robots.

In fact, once connectivity is assured, the target or navigation trajectory of the mission need not be known by all group members. It suffices if only one agent has this information [13]. We shall call this robot the *leader* of the group and denote it as R_N. Nevertheless, the leader has the same physical properties and capabilities as the other robots. The only difference is that the trajectory to be followed by the group is given to R_N. In fact, the leadership of the group is *hidden*. None of the robots recognize the leader as a distinguished group member. In other words, if R_N is sensed by robot R_j, i.e. $R_N \in \mathcal{S}_j$, R_j can only see it as one of its neighbors and the leadership of R_N does not affect the local steering strategy of R_j.

In the following section, we consider the group of N robots as having one leader, R_N, and $N-1$ *followers*, R_j ($j = 1, \ldots, N-1$). Note that the indexing of robots is irrelevant as regards the problem under discussion and the solution we propose. Nonetheless, we utilize such a numbering of robots for the sake of notational simplicity.

III. Autonomous Motion

Our goal is to develop a methodology for simple autonomous agents, such that a large group of them could navigate as a connected group. We assume that the robots update their information concerning the position of other robots within range of their sensors at every Δt seconds. Also, to take into account measurement errors regarding distance, we define a positive scalar d_m as

$$d_m \stackrel{def}{=} d_{max} - \Delta r$$

where Δr is the bound of the distance measurement error with $d_{max} > \Delta r > 0$. We will denote the position of a robot R_i at time t, as $X_i(t)$, $i = 1, \ldots, N$. Since all robots in the group move autonomously, we will set up local moving rules for each robot.

The motion of each robot is most conveniently described in terms of a coordinate system referring to itself, since each robot is at the center of its own local coordinate system. We denote the position vector in the local coordinates as $x(t)$ and use a notation such that the superscripts in x indicate the robot to which the coordinate frame is attached, and the subscripts indicate which robot's position it represents. For example, x_k^j represents the position vector of R_k in the coordinate frame of R_j. For the robots in $\mathcal{S}_i$, $i = 1, \ldots, N$, we have

$$\|x_k^i(t)\| = \|X_k(t) - X_i(t)\| \leq d_m, \qquad k = 1, \ldots, M$$

where M is the number of robots in $\mathcal{S}_i$. In the next section, we present a result on the sufficient conditions for the maintainability of connectivity.

A. Main Theorem

According to the notation given above, $x_i^i(t+\Delta t)$ is the location, which R_i targets (for the time instant $t+\Delta t$), in R_i's own coordinate system at time t. For any $x_i^i(t+\Delta t)$, let us define two complementary subsets of $\mathcal{S}_i$ as

$$\begin{aligned}\mathcal{S}_{ip} &= \left\{R_p \in \mathcal{S}_i \mid [x_i^i(t+\Delta t)]^{\mathrm{T}} x_p^i(t) \leq 0\right\} \\ \mathcal{S}_{iq} &= \left\{R_q \in \mathcal{S}_i \mid [x_i^i(t+\Delta t)]^{\mathrm{T}} x_q^i(t) > 0\right\}.\end{aligned}$$

This means that if a displacement of R_i to $x_i^i(t+\Delta t)$ will take R_i closer to a robot, then this robot will appear in $\mathcal{S}_{iq}$. Otherwise, it will be a member of $\mathcal{S}_{ip}$. Using, $\mathcal{S}_{ip}$, $\mathcal{S}_{iq}$ and also the notation defined above, we can state the following theorem on group connectivity.

Theorem 1: Consider a group $\mathcal{G}$ of N autonomous mobile robots which are connected at $t = 0$. If the motion of the robots is subject to the constraints

$$\|x_i^i(t+\Delta t)\| \leq \frac{1}{2}\left(d_m - \max_{R_p \in \mathcal{S}_{ip}} \|x_p^i(t)\|\right) \quad (1)$$

and

$$\|x_i^i(t+\Delta t)\|^2 \leq \min_{R_q \in \mathcal{S}_{iq}} \left\{[x_i^i(t+\Delta t)]^{\mathrm{T}} x_q^i(t)\right\} \quad (2)$$

for $i = 1, \ldots, N$, the group preserves its connectivity for $t > 0$.

Proof: Note that the position of each robot in $\mathcal{S}_i$ can constrain the motion of R_i either via (1) or via (2), based on whether this robot appears in $\mathcal{S}_{ip}$ or $\mathcal{S}_{iq}$. Let R_a and R_b be any two robots within their mutual sensing range, that is, $R_a \in \mathcal{S}_b$ and $R_b \in \mathcal{S}_a$ at time t.

First, suppose that $R_b \in \mathcal{S}_{ap}$ and $R_a \in \mathcal{S}_{bp}$. Then, it follows from (1)

$$2\|x_a^a(t+\Delta t)\| + \max_p \|x_p^a(t)\| \leq d_m \quad (3)$$

and

$$2\|x_b^b(t+\Delta t)\| + \max_p \|x_p^b(t)\| \leq d_m. \quad (4)$$

Noting that $\max_p \|x_p^a(t)\| \geq \|x_b^a(t)\|$, $\max_p \|x_p^b(t)\| \geq \|x_a^b(t)\|$, and $\|x_b^a(t)\| = \|x_a^b(t)\|$, we obtain from (3) and (4),

$$\|x_a^a(t+\Delta t)\| + \|x_b^b(t+\Delta t)\| + \|x_b^a(t)\| \leq d_m.$$

Further, by triangle inequality, we get

$$\|x_a^a(t+\Delta t) - [x_b^a(t) + x_b^b(t+\Delta t)]\| \leq d_m. \quad (5)$$

Note that the term $x_b^a(t) + x_b^b(t+\Delta t)$ is the position of R_b at time $t+\Delta t$ as expressed in the local coordinate frame attached to R_a at time t. Therefore, (5) shows that the distance between the robots $R_a$and R_b will not be larger than d_m at time $t+\Delta t$.

Next, we assume that $R_b \in \mathcal{S}_{aq}$ and $R_a \in \mathcal{S}_{bq}$. In this case, we have to proceed using the constraint in (2). Namely,

$$\|x_a^a(t+\Delta t)\|^2 \leq [x_a^a(t+\Delta t)]^{\mathrm{T}} x_b^a(t). \quad (6)$$

Since

$$\left\|x_a^a(t+\Delta t) - \frac{x_b^a(t)}{2}\right\|^2 = \|x_a^a(t+\Delta t)\|^2 + \left\|\frac{x_b^a(t)}{2}\right\|^2 - [x_a^a(t+\Delta t)]^{\mathrm{T}} x_b^a(t), \quad (7)$$

using (6), we obtain

$$\left\|x_a^a(t+\Delta t) - \frac{x_b^a(t)}{2}\right\| \leq \frac{\|x_b^a(t)\|}{2}. \quad (8)$$

Noting that $x_b^a(t) = -x_a^b(t)$ and using the triangle inequality along with (8), it follows that

$$\begin{aligned} &\|x_a^a(t+\Delta t) - [x_b^a(t) + x_b^b(t+\Delta t)]\| \\ &= \left\| \left(x_a^a(t+\Delta t) - \frac{x_b^a(t)}{2} \right) - \left(x_b^b(t+\Delta t) - \frac{x_a^b(t)}{2} \right) \right\| \\ &\leq \left\| x_a^a(t+\Delta t) - \frac{x_b^a(t)}{2} \right\| + \left\| x_b^b(t+\Delta t) - \frac{x_a^b(t)}{2} \right\| \\ &\leq \|x_b^a(t)\| \\ &\leq d_m, \end{aligned} \tag{9}$$

which asserts the link between R_a and R_b at time $t + \Delta t$ in the same way as (5).

To complete the proof, we also must analyze cases where $R_b \in \mathcal{S}_{aq}$ while $R_a \in \mathcal{S}_{bp}$, and $R_b \in \mathcal{S}_{ap}$ while $R_a \in \mathcal{S}_{bq}$. Without loss of generality, we consider only the former, since the proof for the latter can be obtained by an interchange of subscripts a and b only.

In other words, the motion of R_a and R_b will be constrained by (6) and (4), respectively. Similar to (7), we can state:

$$\begin{aligned} &\|x_a^a(t+\Delta t) - x_b^a(t)\|^2 \\ &= \|x_a^a(t+\Delta t)\|^2 + \|x_b^a(t)\|^2 - 2[x_a^a(t+\Delta t)]^{\mathrm{T}} x_b^a(t). \end{aligned}$$

In light of (6), we get

$$\|x_a^a(t+\Delta t) - x_b^a(t)\| \leq \|x_b^a(t)\|. \tag{10}$$

Therefore, (4) with (10) yields

$$2\|x_b^b(t+\Delta t)\| + \|x_a^a(t+\Delta t) - x_b^a(t)\| \leq d_m$$

or

$$\|x_a^a(t+\Delta t) - x_b^a(t) - x_b^b(t+\Delta t)\| \leq d_m - \|x_b^b(t+\Delta t)\|. \tag{11}$$

Hence, the validity of (5) is maintained in this case as well.

The results in (5), (9) and (11) show that any two robots $R_a \in \mathcal{S}_b$ and $R_b \in \mathcal{S}_a$ sensing each other at time t will still be linked when they move to their new locations at $t + \Delta t$. Hence, we conclude that if the group is connected at $t = 0$, it will also be connected for $t > 0$. ■

Note that if a robot R_i is occulted by another robot in $\mathcal{S}_i$, the number of robots in $\mathcal{S}_i$ might decrease. Nevertheless, this does not disturb the overall connectivity, as the existence of the occulting robot itself is the evidence of the connection between R_i and the occulted robot.

B. Local Steering Strategy

As long as the constraints in (1) and (2) are satisfied when a given navigation trajectory is followed, formation control and other mission-oriented tasks can be accomplished by minimizing suitable cost functions. Therefore, in view of Theorem 1, once group connectivity is assured by (1) and (2), the following Local Steering Strategy can be applied for navigation by a group which is composed of follower robots and a leader.

Local Steering Strategy: Subject to the constraints (1) and (2)

- The follower robots R_j ($j = 1, \ldots, N-1$) move towards a target location $x_j^j(t+\Delta t)$, which minimizes the cost function $J(x_j^j(t+\Delta t))$ related to the positions of the robots in $\mathcal{S}_j$.
- The leader R_N follows the navigation trajectory.

Several types of cost functions can be used in implementing the local steering strategy. An example may be given as

$$J(x_j^j(t+\Delta t)) = \max_k \|x_j^j(t+\Delta t) - x_k^j(t)\|, \tag{12}$$

which makes the j^{th} robot try to decrease the distance to the farthest robot that it senses. Another possible approach could be to employ

$$J(x_j^j(t+\Delta t)) = \sum_{k=1}^{M} \Big(\|x_j^j(t+\Delta t) - x_k^j(t)\| - d_0 \Big)^2 \tag{13}$$

as the cost function in order to force the robots to keep their distances with the robots in their subgroups as close to a desired distance as possible. Here, d_0 ($d_0 < d_m$) denotes the desired distance.

Note that both (12) and (13) are defined in terms of local coordinates to ensure a distributed algorithm. The choice of suitable cost functions will depend on the requirements of the mission. It is possible to take into account fixed as well as time-varying cost functions, and these can incorporate the position information of all or only some of the neighbor agents. Furthermore, it is possible for each agent to minimize a different cost function.

Note that the constraint in (1) delimits the distance to be travelled by each robot at every sampling interval. It is a direct consequence of the requirement that the distance between any two robots that are connected at time t must be less than d_m at $t+\Delta t$, even in the worst case, which happens when the robots are moving in opposite directions. Therefore, (1) alone is sufficient to maintain connectivity. Nevertheless, it was shown in [15] and [16] that if the motion of the robots is constrained only by (1), the group may get stuck in situations where none of the robots can move. Such situations are called *deadlock*. A typical example of a deadlock is when all inter-robot distances are d_{max}, so that $\|x_i^i(t+\Delta t)\| = 0$, $i = 1, \ldots, N$ and, hence, none of the robots can move. In avoiding deadlocks, it is essential to allow the outermost robots in the group to move towards their neighbors. However, (1) depends only on the magnitude of $x_i^i(t+\Delta t)$ and does not account for its direction. The direction of $x_i^i(t+\Delta t)$ can be utilized by considering the neighbor robots in two distinct subsets as implied by (1) and (2). In fact, it is shown in [17] that the constraints (1) and (2) prevent deadlocks under reasonably mild conditions as regards the cost functions, which are fulfilled by, for example, those in (12) and (13).

IV. A SUBOPTIMAL SOLUTION TO A LOCAL STEERING PROBLEM

The robots in this study are quite simple and limited devices especially from the computational point of view. Our purpose is to provide a decentralized control methodology which can be applied to such simple robots yet still lead to satisfactorily good group navigation. Below, we propose a gradient-descent-based iterative method to reduce the computational burden in the implementation of the Local Steering Strategy with the cost function in (13).

The minimum of the cost function given in (13) is the location to which follower robot R_j aims to arrive at each sampling time. In a general case, the minimization of (13) subject to the constraints in (1) and (2) requires solving a set of nonlinear equations and this approach may result in several local minima, which should be further checked to ensure they are the absolute minimum. In other words, it requires high computational power, especially when the number of the robots in $\mathcal{S}_j$ increases.

It should be noted that the optimal points are computed for each follower robot R_j at every sampling time. The location of the optimal points depends on the positions of the robots in the subgroup $\mathcal{S}_j$. Since the sensed robots in $\mathcal{S}_j$ also move autonomously, these local targets will be updated every Δt sec, possibly before reaching them. Since (1) and (2) constrain the magnitude of $x_i^i(t+\Delta t)$, they will not be violated as the robots are moving towards their local targets. Hence, the solution will only provide a direction to the optimal points, because the solution will be updated before R_j arrives at that location.

This fact can be exploited to introduce an iterative method to implement the Local Steering Strategy in a sub-optimal way. Rather than solving for the minimum points of the cost function, each robot R_j can move in the direction of the negative gradient of the cost function, evaluated at the position of R_j for each sampling instant. That is,

$$x_j^j(t+\Delta t) = x_j^j(t) - \gamma \left. \frac{\partial J(x_j^j(t+\Delta t))}{\partial x_j^j(t+\Delta t)} \right|_{x_j^j(t+\Delta t)=x_j^j(t)} \quad (14)$$

where $\gamma > 0$ is a positive gain, and x_j^j is the position vector of R_j in its local coordinates. From (13), (14) and the fact that $x_j^j(t) = 0$, it follows that

$$x_j^j(t+\Delta t) = 2\gamma \sum_{k=1}^{M} \left(\|x_k^j(t)\| - d_0 \right) \frac{x_k^j(t)}{\|x_k^j(t)\|}. \quad (15)$$

The gain γ in (15) should be treated as a parameter by which one can choose the distance of the local target so as to satisfy the constraints in (1) and (2), rather than as a constant to be determined a priori. In that respect, there is no reason why γ must be kept constant during navigation. Therefore, the direction of the next movement can be obtained by (15) and then implemented in this direction with a magnitude so as to satisfy the inequalities (1) and (2).

V. Simulation Results

In this section, we present our computer simulations to verify the theoretical results of the previous sections. In the simulations, the working space was taken as a section of xy-plane in $\mathbb{R}^2$. The sensor range (d_{max}) was 15 units. The bounds on the measurement errors were $\Delta\theta = 12^\circ$ for angle and $\Delta r = 0.03 d_{max}$ for distance measurements. The following scenario was applied: The leader was given a trajectory and as the leader started navigation, the rest of the group also moved so as to stay connected under the Local Steering Strategy. The simulation was done for a group consisting of 20 robots where (15) is used with $\gamma = 0.2$.

The snapshots of the simulation can be seen in Fig. 4. With the initial locations shown in Fig. 4 and $d_{max} = 15$, the group was connected and accommodated 62 links at the beginning. The trajectory of the leader is shown by the solid line through the graph. As soon as the simulation started with $d_0 = 11$, the group widened but kept its connectivity. Occulting happened rarely. For 20 robots, the maximum possible number of links is $20 \cdot (20-1)/2 = 190$. Fig. 5 indicates that at least 63 of 190 possible links were preserved until the end of the navigation. The sharp turns in the trajectory of the leader are important, as they could disrupt the shape of the group.

Also, the impact of d_0 on connectivity can be assessed by Fig. 5, in which the number of links in the group throughout the entire simulation is plotted for three different values of d_0. It is seen that for $d_0 = 5$, the number of links lies between those for $d_0 = 11$ and $d_0 = 8$. This is an interesting result and deserves some interpretation. Whenever d_0 is small, the group is dense and occupies a smaller area. Conversely, the group widens with increasing d_0. However, when the robots are confined to a smaller volume due to a low value in d_0, the number of occulted robots increases significantly. Hence, the number of links decreases because visibility is reduced by occulting. However, as was noted in Section III, this does not imply any weakness for overall group connectivity.

VI. Conclusions

This work examined the navigation of mobile robot groups and the methodology presented does not require communication between the robots. Rather, a local steering strategy, which uses only information regarding the position of neighbor agents, is employed to sustain the connectivity of group members. The limited-range sensors are modeled in such a way that they produce angular and radial position measurement errors to better reflect a realistic situation. Moreover, robots may be occulted by other robots. In this way, the methodology accounts for all of the fundamental difficulties that can arise in real-life implementation.

This study demonstrates that once the robots start their motion as a connected group, the steering strategy assures their connectivity without any risk of deadlock. Also, the fact that no communication or hierarchy among the robots is required makes it possible for new members to be easily accepted into the group. The simulations verified the success of the methodology.

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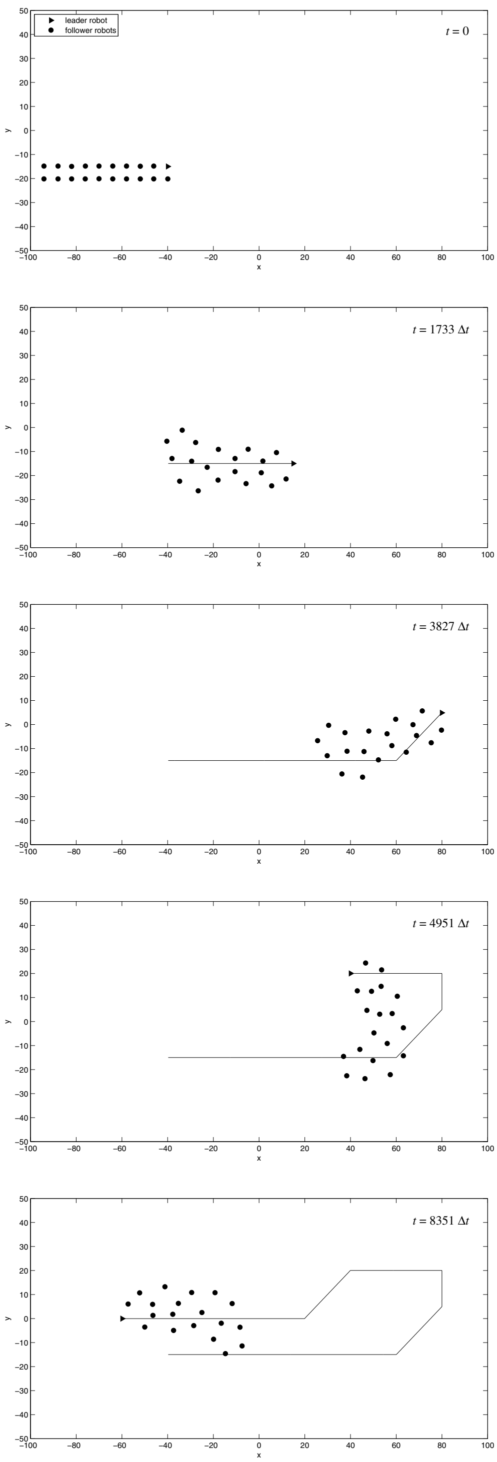

Fig. 4. Navigation of 20 robots ($d_{max} = 15$, $d_0 = 11$)

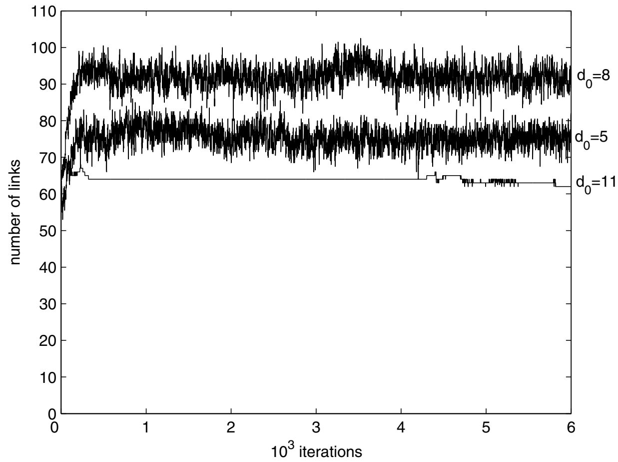

Fig. 5. Total number of links in a group of 20 robots ($d_{max} = 15$)

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