FREQUENCY SELECTIVE FADING CHANNEL ESTIMATION IN OFDM SYSTEMS USING KL EXPANSION

Habib Şenol[‡], *Hakan A. Çırpan*[†], *and Erdal Panayırcı*[§]

[‡]Department of Computer Engineering, Kadir Has University, Cibali 34230, Istanbul, Turkey
[†]Department of Electrical Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey
[§]Department of Electronics Engineering, Işık University, Maslak 80670, Istanbul, Turkey

ABSTRACT

This paper proposes a computationally efficient, linear minimum mean square error (MMSE) channel estimation algorithm based on KL series expansion for OFDM systems. Based on such expansion, no matrix inversion is required in the proposed MMSE estimator. Moreover, truncation in the linear expansion of channel is achieved by exploiting the optimal truncation property of the KL expansion resulting in a smaller computational load on the estimation algorithm. The performance of the proposed approach is studied through analytical and experimental results. We provide performance analysis results studying the influence of the effect of SNR and correlation mismatch on the estimator performance. Simulation results confirm our theoretical results and illustrate that the proposed algorithm is capable of tracking fast fading and improving performance.

1. INTRODUCTION

In a wireless orthogonal frequency division multiplexing (OFDM) systems over a frequency selective fading, channel variations arise mainly due to multipath effect [1]. Basis expansion approach could be natural way of modelling the channel variations [2]. Fourier, Taylor series, and polynomial expansion have played a prominent role in deterministic modelling [3]. As an alternative to the deterministic approaches, the variation in the channel can be captured by means of a stochastic modelling [2]. Note that, the random process can be represented as a series expansion involving a complete set of deterministic vectors with corresponding random coefficients. This expansion therefore provides a second order characterization in terms of random variables and deterministic vectors. There are several such series that are widely in use. A commonly used series is the Karhunen-Loeve (KL) expansion [4]. The use of KL expansion with orthogonal deterministic basis vectors and uncorrelated random coefficients has generated interest because of its bi-orthogonal property, that is, both the deterministic basis vectors and the corresponding random coefficients are orthogonal. This allows for the optimal encapsulation of the information contained in the random process into a set of discrete uncorrelated random variables.

In this paper we will focus on OFDM systems over frequency selective fading channel. Channel estimation for OFDM systems has attracted much attention with pioneering works of [5, 6, 7]. Numerous pilot-aided channel estimation methods for OFDM have been developed [5, 6, 7]. In particular, a low-rank approximation is applied to linear MMSE estimator for the estimation of subcarrier channel attenuations by using the frequency correlation of the channel [5]. In [6], a MMSE channel estimator, which makes full use of the time and frequency correlation of the time-varying dispersive channel was proposed. Multipath fading channels have been studied extensively, and several models have been developed to describe their variations [7]. In the case of KL series representation of stochastic channel model, a convenient choice of orthogonal basis set is one that makes the expansion coefficient random variables uncorrelated [8]. When these orthogonal bases are employed to characterize the variation of the channel impulse response, uncorrelated coefficients indeed represent the channel. Therefore, KL representation allows one to tackle the estimation of correlated channel parameters as a parameter estimation problem of the uncorrelated coefficients. Exploiting KL expansion, the main contribution of this paper is to propose a computationally efficient, pilot-aided MMSE channel estimation algorithms. Based on such representation, no matrix inversion is required in the proposed approach. Moreover, optimal rank reduction is achieved by exploiting the optimal truncation property of the KL expansion resulting in a smaller computational load on the estimation algorithm. The performance of the proposed batch approach is explored based on the evaluation of the Bayesian MSE for the random KL coefficients.

2. OFDM SYSTEM

In this section, we introduce a general model for OFDM systems with N subcarriers signaling through a frequency selective fading channel. The channel response is assumed to be constant during one symbol duration. The block diagram in Figure 1. describes of such an OFDM system. The binary information data is grouped and mapped into multiphase signals. In this paper the QPSK modulation is employed. An IDFT is then applied the QPSK symbols $\{X_k\}_{k=0}^{N-1}$, resulting in $\{x_n\}_{n=0}^{N-1}$, i.e., $x_n = \mathrm{IDFT}\{X_k\}$. In order to eliminate intersymbol interference arising due to multipath channel, the guard interval is inserted between OFDM frames. After pulse shaping and parallel to serial conversion, the signals are then transmitted through a frequency selective fading channel. At the receiver, after matched filtering and removing the guard interval, the time-domain received samples $\{y_n\}_{n=0}^{N-1}$, are then sent to the DFT block to demultiplex the multicarrier signals $Y_k = \mathrm{DFT}\{y_n\}$. For OFDM systems with proper cyclic ex-

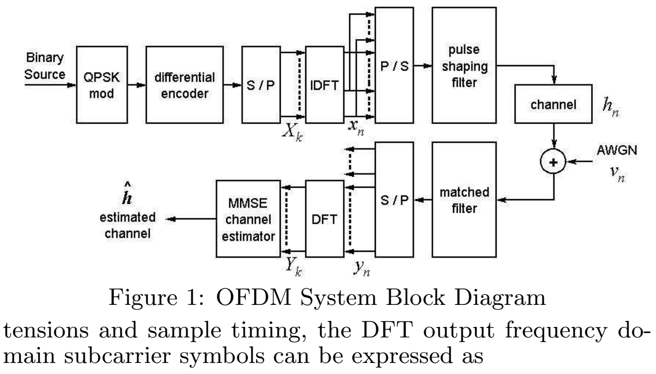

Figure 1: OFDM System Block Diagram

tensions and sample timing, the DFT output frequency domain subcarrier symbols can be expressed as

$$Y_k = X_k H_k + V_k \tag{1}$$

where $V_k = \mathrm{DFT}\{v_n\}$ is frequency domain AWGN samples with zero mean and variance σ^2 and H_k is the channel frequency response given by

$$H_k = \mathbf{w}^\dagger(k)\boldsymbol{h} \quad , \qquad k = 0, 1, .., N-1 \tag{2}$$

where $\boldsymbol{h} = [h_0, h_1, ..., h_{L-1}]$ contains the time response of all L taps, and $\mathbf{w}(k) = [1, e^{-j2\pi k/N}, ..., e^{-j2\pi k(L-1)/N}]^\dagger$ contains the corresponding DFT coefficients and $(\cdot)^\dagger$ denotes the Hermitian transpose. Substituting (2) into (1) yields

$$Y_k = X_k\mathbf{w}^\dagger(k)\boldsymbol{h} + V_k \quad , \qquad k = 0, .., N-1 \ . \tag{3}$$

If we focus at received block $\mathbf{Y} = [Y_0, Y_1, ..., Y_{N-1}]^T$, we can write the following from (3):

$$\mathbf{Y} = \mathbf{X}\mathbf{W}\,\boldsymbol{h} + \mathbf{V} \tag{4}$$

where $\mathbf{X} = \text{diag}[X_0, X_1, ..., X_{N-1}]$ is a diagonal matrix with symbol entries, $\mathbf{W} = [\mathbf{w}(0), ..., \mathbf{w}(N-1)]^\dagger$ is DFT matrix and similarly $\mathbf{V}$ is a zero-mean i.i.d. complex Gaussian vector.

Based on the model (4), our main objective in this paper is to develop batch pilot-aided channel time response estimation algorithm according to MMSE criterion and then explore the performance of the estimators. A proposed approach adapted herein explicitly models the random channel parameters by the KL series representation and estimates the uncorrelated expansion coefficients. Furthermore, the computational load of the proposed MMSE estimation technique is further reduced with the application of the KL expansion optimal truncation property [4]. Let us then introduce random channel model first.

3. LINEAR EXPANSION OF THE CHANNEL

The series expansion referred to as KL expansion provides a second moment characterization in terms of uncorrelated random variables and deterministic orthogonal vectors. In the KL expansion method, the orthogonal deterministic basis vectors and its magnitude are, respectively the eigenfunction and eigenvalue of the covariance matrix. Since channel impulse response $\boldsymbol{h}$ is a zero-mean Gaussian process with covariance matrix $\mathbf{C}_{\boldsymbol{h}}$, the KL transformation rotates the vector $\boldsymbol{h}$ so that all its components are uncorrelated. Thus the vector $\boldsymbol{h}$, representing the channel impulse response during OFDM block, can be expressed as a linear combination of the orthonormal basis vectors as $\boldsymbol{h} = \sum_{l=0}^{L-1} g_l\boldsymbol{\psi}_l = \boldsymbol{\Psi}\boldsymbol{g}$, where $\boldsymbol{\Psi} = [\boldsymbol{\psi}_0, \boldsymbol{\psi}_1, \cdots, \boldsymbol{\psi}_{L-1}]$, $\boldsymbol{\psi}_l$'s are the orthonormal basis vectors, $\boldsymbol{g} = [g_0, g_1, \cdots g_{L-1}]^T$, and g_l is the weights of the expansion. If we form the covariance matrix as $\mathbf{C}_{\boldsymbol{h}} = \boldsymbol{\Psi}\boldsymbol{\Lambda}_{\boldsymbol{g}}\boldsymbol{\Psi}^\dagger$, where $\boldsymbol{\Lambda}_{\boldsymbol{g}} = E\{\boldsymbol{g}\boldsymbol{g}^\dagger\}$, the KL expansion is the one in which $\boldsymbol{\Lambda}_{\boldsymbol{g}}$ of $\mathbf{C}_{\boldsymbol{h}}$ is a diagonal matrix (i.e., the coefficients are uncorrelated). If $\boldsymbol{\Lambda}_{\boldsymbol{g}}$ is diagonal, then the form $\boldsymbol{\Psi}\boldsymbol{\Lambda}_{\boldsymbol{g}}\boldsymbol{\Psi}^\dagger$ is called an *eigendecomposition* of $\mathbf{C}_{\boldsymbol{h}}$. The fact that only the eigenvectors diagonalize $\mathbf{C}_{\boldsymbol{h}}$ leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem in this application is equivalent to estimating the i.i.d. complex Gaussian vector $\boldsymbol{g}$ KL expansion coefficients.

4. MMSE ESTIMATION OF KL COEFFICIENTS

A low-rank approximation to the frequency-domain linear MMSE channel estimator is provided by [5] to reduce the complexity of the estimator. Optimal rank reduction is achieved in this approach by using the SVD of the channel attenuations covariance matrix $\mathbf{C}_{\mathbf{H}}$ of dimension $N \times N$. In contrast, we adapt the MMSE estimator for the estimation of multipath channel parameters $\boldsymbol{h}$ that uses covariance matrix of dimension $L \times L$. The proposed approach employs KL expansion of multipath channel parameters and reduces the complexity of the SVD used in *eigendecomposition* since L is usually much less than N. We will now develop MMSE batch estimator for pilot assisted OFDM system in the sequel.

4.1 MMSE Ch[illegible]n

Considering (4), we now assume that N_p pilot symbols are uniformly inserted at known locations of the i^{th} OFDM block, the $N_p \times 1$ vector corresponding to the DFT output at the pilot locations becomes

$$\mathbf{Y}_p = \mathbf{X}_p\mathbf{W}_p\boldsymbol{h} + \mathbf{V}_p \tag{5}$$

where $\mathbf{X}_p = \text{diag}[\mathbf{X}_i(0), \mathbf{X}_i(\Delta), \cdots, \mathbf{X}_i((N_p-1)\Delta)]$ is a diagonal matrix with pilot symbol entries, Δ is pilot spacing interval, $\mathbf{W}_p$ is an $N_p \times L$ FFT matrix generated based on pilot indices, and similarly $\mathbf{V}_p$ is the under-sampled noise vector.

For the estimation of $\boldsymbol{h}$, the new linear signal model can be formed by premultiplying both sides of (5) by $\mathbf{X}_p^\dagger$ and assuming pilot symbols are taken from a QPSK constellation $\mathbf{X}_p^\dagger\mathbf{X}_p = \mathbf{I}_{N_p}$, then the new form of (5) becomes

$$\tilde{\mathbf{Y}} = \mathbf{W}_p\boldsymbol{h} + \tilde{\mathbf{V}} \tag{6}$$

where $\tilde{\mathbf{Y}} = \mathbf{X}_p^\dagger\mathbf{Y}_p$ and $\tilde{\mathbf{V}} = \mathbf{X}_p^\dagger\mathbf{V}_p$ and $\tilde{\mathbf{V}}$ is statistically equivalent to $\mathbf{V}_p$.

Equation (6) offers a Bayesian linear model representation. Based on this representation, the minimum variance estimator for the time-domain channel vector $\boldsymbol{h}$ for the ith OFDM block, i.e., conditional mean of $\boldsymbol{h}$ given $\tilde{\mathbf{Y}}$, can be obtained using MMSE estimator. We should clearly make the assumptions that $\boldsymbol{h} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{\boldsymbol{h}})$, $\tilde{\mathbf{V}} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{\tilde{\mathbf{V}}})$ and $\boldsymbol{h}$ is uncorrelated with $\tilde{\mathbf{V}}$. Therefore, MMSE estimate of $\boldsymbol{h}$ is given by [9]:

$$\hat{\boldsymbol{h}} = (\mathbf{W}_p^\dagger\mathbf{C}_{\tilde{\mathbf{V}}}^{-1}\mathbf{W}_p + \mathbf{C}_{\boldsymbol{h}}^{-1})^{-1}\mathbf{W}_p^\dagger\mathbf{C}_{\tilde{\mathbf{V}}}^{-1}\tilde{\mathbf{Y}} \ . \tag{7}$$

We now us assume that $\mathbf{C}_{\tilde{\mathbf{V}}} = E\left[\tilde{\mathbf{V}}\tilde{\mathbf{V}}^\dagger\right] = \sigma^2\mathbf{I}_{N_p}$ and uniformly spaced pilot symbols are inserted with pilot spacing interval Δ and $N = \Delta \times N_p$, correspondingly, $\mathbf{W}_p^\dagger\mathbf{W}_p$ reduces to $\mathbf{W}_p^\dagger\mathbf{W}_p = N_p\mathbf{I}_L$,and we can therefore express (7) by

$$\hat{\boldsymbol{h}} = (N_p\mathbf{I}_L + \sigma^2\mathbf{C}_{\boldsymbol{h}}^{-1})^{-1}\mathbf{W}_p^\dagger\tilde{\mathbf{Y}} \ . \tag{8}$$

Since MMSE estimation still requires the inversion of $\mathbf{C}_{\boldsymbol{h}}$, it therefore suffers from a high computational complexity. However, it is possible to reduce complexity of the MMSE algorithm by diagonalizing channel covariance matrix with a linear KL expansion.

4.2 [illegible] KL Coeffi[illegible]

In contrast to (6) in which only $\boldsymbol{h}$ is to be estimated, we now assume the KL series expansion coefficients $\boldsymbol{g}$ is unknown. Substituting $\boldsymbol{h} = \boldsymbol{\Psi}\boldsymbol{g}$ in (6), the data model (6) is then rewritten for each OFDM block as

$$\tilde{\mathbf{Y}} = \mathbf{W}_p\boldsymbol{\Psi}\boldsymbol{g} + \tilde{\mathbf{V}} \tag{9}$$

which is also recognized as a Bayesian linear model, and recall that $\boldsymbol{g} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Lambda}_{\boldsymbol{g}})$. As a result, the MMSE estimator of $\boldsymbol{g}$ is

$$\hat{\boldsymbol{g}} = \boldsymbol{\Gamma}\boldsymbol{\Psi}^\dagger\mathbf{W}_p^\dagger\tilde{\mathbf{Y}} \tag{10}$$

where

$$\begin{aligned} \boldsymbol{\Gamma} &= \boldsymbol{\Lambda}_{\boldsymbol{g}}(N_p\boldsymbol{\Lambda}_{\boldsymbol{g}} + \sigma^2\mathbf{I}_L)^{-1} \qquad (11) \\ &= \text{diag}\left\{\frac{\lambda_{g_0}}{\lambda_{g_0}N_p + \sigma^2}, \cdots, \frac{\lambda_{g_{L-1}}}{\lambda_{g_{L-1}}N_p + \sigma^2}\right\} \end{aligned}$$

and $\lambda_{g_0}, \lambda_{g_1}, \cdots, \lambda_{g_{L-1}}$ are the singular values of $\boldsymbol{\Lambda}_{\boldsymbol{g}}$.

It is clear that the complexity of the MMSE estimator in (8) is reduced by the application of KL expansion. However, the complexity of the $\hat{\boldsymbol{g}}$ can be further reduced by exploiting the optimal truncation property of the KL expansion [4].

4.3 TheKL Ep

A truncated expansion $\boldsymbol{g}_r$ can be formed by selecting orthonormal basis vectors associated with the first largest r eigenvalues. For the problem at hand, truncation property of the KL expansion results in a low-rank approximation as well. Thus, a rank-r approximation to $\boldsymbol{\Lambda}_{\boldsymbol{g}_r}$ is defined as

$$\boldsymbol{\Lambda}_{\boldsymbol{g}_r} = diag\left\{\lambda_{g_0}, \lambda_{g_1}, \cdots, \lambda_{g_{r-1}}, 0, \cdots, 0\right\} \ . \tag{12}$$

Since the trailing $L-r$ variances $\{\lambda_{g_l}\}_{l=r}^{L-1}$ are small compared to the leading r variances $\{\lambda_{g_l}\}_{l=0}^{r-1}$, then the trailing $L-r$ variances are set to zero to produce the approximation. However, typically the pattern of eigenvalues for $\boldsymbol{\Lambda}_{\boldsymbol{g}}$ splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious. The optimal truncated KL (rank-r) estimator of (10) now becomes

$$\hat{\boldsymbol{g}}_r = \boldsymbol{\Gamma}_r \ \boldsymbol{\Psi}^{\dagger}\mathbf{W}_p^{\dagger}\tilde{\mathbf{Y}} \tag{13}$$

where

$$\begin{aligned} \boldsymbol{\Gamma}_r &= \boldsymbol{\Lambda}_{\boldsymbol{g}_r}(K_p\boldsymbol{\Lambda}_{\boldsymbol{g}_r} + \sigma^2\mathbf{I}_L)^{-1} \qquad (14) \\ &= \mathrm{diag}\left\{\frac{\lambda_{g_0}}{\lambda_{g_0}N_p + \sigma^2}, \cdots, \frac{\lambda_{g_{r-1}}}{\lambda_{g_{r-1}}N_p + \sigma^2}, 0, \cdots, 0\right\} \ . \end{aligned}$$

5. PERFORMANCE ANALYSIS

We turn our attention to analytical performance results of the MMSE approach. We exploit the performance of the truncated MMSE KL estimator under SNR and correlation mismatches. With these performance results, then the estimator performance can also be obtained for some special cases, i.e., the case of the MMSE KL estimator under no mismatch.

5.1 ByMSEfTeMMSEEn UeSNRMn

Bayesian MSE is computed for the truncated (low-rank) case as follows. Substituting (9) in (13), truncated MMSE KL estimator now becomes

$$\hat{\boldsymbol{g}}_r = N_p \ \boldsymbol{\Gamma}_r \ \boldsymbol{g} + \boldsymbol{\Gamma}_r \ \boldsymbol{\Psi}^{\dagger}\mathbf{W}^{\dagger}\tilde{\mathbf{V}} \ . \tag{15}$$

The estimation error

$$\hat{\boldsymbol{\epsilon}}_r = \boldsymbol{g} - \hat{\boldsymbol{g}}_r = (\mathbf{I}_L - N_p \ \boldsymbol{\Gamma}_r)\boldsymbol{g} - \boldsymbol{\Gamma}_r \ \boldsymbol{\Psi}^{\dagger}\mathbf{W}_p^{\dagger}\tilde{\mathbf{V}} \tag{16}$$

and then taking as $\sigma^2 = 1/SNR$ and $\tilde{\sigma}^2 = 1/\widetilde{SNR}$, the average Bayesian MSE is

$$\begin{aligned} \mathbf{B}_{MSE}(\hat{\boldsymbol{g}}_r) &= \frac{1}{L}\,\mathrm{tr}\left(\mathbf{C}_{\hat{\boldsymbol{\epsilon}}_r}\right) \\ &= \frac{1}{L}\sum_{i=0}^{r-1}\frac{\lambda_{g_i}(1 + N_p\lambda_{g_i}\frac{SNR^2}{\widetilde{SNR}})}{(1 + N_p\lambda_{g_i}SNR)^2} + \frac{1}{L}\sum_{i=r}^{L-1}\lambda_{g_i} \qquad (17) \end{aligned}$$

Based on the result obtained in (17), Bayesian estimator performance can be further elaborated for the following scenarios:

- By taking $\widetilde{SNR} = SNR$, the performance result for the case of no SNR mismatch is

$$\mathbf{B}_{MSE}(\hat{\boldsymbol{g}}_r) = \frac{1}{L}\sum_{i=0}^{r-1}\frac{\lambda_{g_i}}{1 + N_p\lambda_{g_i}SNR} + \frac{1}{L}\sum_{i=r}^{L-1}\lambda_{g_i} \ . \tag{18}$$

  Notice that, the second term in (18) is the sum of the powers in the KL transform coefficients not used in the truncated estimator. Thus, truncated $\mathbf{B}_{MSE}(\hat{\boldsymbol{g}}_r)$ can be lower bounded by $\frac{1}{L}\sum_{i=r}^{L-1}\lambda_{g_i}$ which will cause an irreducible error floor in the SER results.

- Finally, as $r \rightarrow L$ in (17), the Bayesian MSE in the case of no SNR mismatch is also be obtained as,

$$\mathbf{B}_{MSE}(\hat{g}) = \frac{1}{L}\sum_{i=0}^{L-1}\frac{\lambda_{g_i}}{1 + N_p\lambda_{g_i}SNR} \tag{19}$$

5.2 ByMSEfTeMMSEKL Es teUeChMn

In this section we derive the Bayesian MSE of the truncated MMSE KL estimator under correlation mismatch. Although the real multipath channel $\tilde{\boldsymbol{h}}$ has the expansion correlation $\mathbf{C}_{\tilde{\boldsymbol{h}}}$, we designed the estimator for the multipath channel $\boldsymbol{h} = \boldsymbol{\Psi}\boldsymbol{g}$ with correlation $\mathbf{C}_{\boldsymbol{h}}$. To evaluate the estimation error $\tilde{\boldsymbol{g}} - \hat{\boldsymbol{g}}_r$ in the same space, we expand the $\tilde{\boldsymbol{h}}$ onto the eigenspace of $\boldsymbol{h}$ as $\tilde{\boldsymbol{h}} = \boldsymbol{\Psi}\tilde{\boldsymbol{g}}$ resulting in correlated expansion coefficients.

For the real channel, data model in (9) can be rewritten as

$$\tilde{\mathbf{Y}} = \mathbf{W}_p\boldsymbol{\Psi}\tilde{\boldsymbol{g}} + \tilde{\mathbf{V}} \tag{20}$$

and substituting in (13), truncated MMSE KL estimator now becomes

$$\hat{\boldsymbol{g}}_r = N_p \ \boldsymbol{\Gamma}_r \ \tilde{\boldsymbol{g}} + \boldsymbol{\Gamma}_r \ \boldsymbol{\Psi}^{\dagger}\mathbf{W}_p^{\dagger}\tilde{\mathbf{V}} \tag{21}$$

For the truncated MMSE estimator, the error is

$$\hat{\boldsymbol{\epsilon}}_r = \tilde{\boldsymbol{g}} - \hat{\boldsymbol{g}}_r = (\mathbf{I}_L - N_p \ \boldsymbol{\Gamma}_r)\tilde{\boldsymbol{g}} - \boldsymbol{\Gamma}_r \ \boldsymbol{\Psi}^{\dagger}\mathbf{W}_p^{\dagger}\tilde{\mathbf{V}} \tag{22}$$

As a result, taking as $\sigma^2 = 1/SNR$, the average Bayesian MSE is

$$\begin{aligned} \mathbf{B}_{MSE}(\hat{\boldsymbol{g}}_r) &= \frac{1}{L}\,\mathrm{tr}\left(\mathbf{C}_{\hat{\boldsymbol{\epsilon}}_r}\right) \qquad (23) \\ &= \frac{1}{L}\sum_{i=0}^{r-1}\frac{\tilde{\lambda}_{g_i} + N_p \ SNR \ \lambda_{g_i}(\tilde{\lambda}_{g_i} + \lambda_{g_i} - 2\beta_i)}{1 + N_p \ SNR \ \lambda_{g_i}} + \frac{1}{L}\sum_{i=r}^{L-1}\tilde{\lambda}_{g_i}, \end{aligned}$$

where $\boldsymbol{\beta}$ is the real part of $E[\tilde{\boldsymbol{g}}\boldsymbol{g}^{\dagger}]$ and β_i's are the diagonal elements of $\boldsymbol{\beta}$. With this result, we will now highlight some special cases:

- Letting $\beta_i = \lambda_{g_i} = \tilde{\lambda}_{g_i}$ for the case of no mismatch in the correlation of KL expansion coefficients, truncated Bayesian MSE is identical to that obtained in (18).
- As $r \rightarrow L$ in (23), Bayesian MSE under no correlation mismatch is identical to that in (19).

6. SIMULATIONS

In this section, the merits of our channel estimators is illustrated through simulations. We choose average mean square error (MSE) as our figure of merit. We consider the fading multipath channel with L paths given by (2) with an exponentially decaying power delay profile [5].

The scenario for our simulation study consists of a wireless QPSK OFDM system employing the pulse shape as a unit-energy Nyquist-root raised-cosine shape with rolloff $\alpha = 0.2$, with a symbol period(T_s) of 0.120 μs, corresponding to an uncoded symbol rate of 8.33 Mbit/s. Transmission bandwidth(5 MHz) is divided into 1024 tones. We assume that the fading multipath channel has $L = 40$ paths with an exponentially decaying power delay profile (2) with an $\tau_{rms} = 5$ sample (0.6 μs) long.

A QPSK-OFDM sequence passes through channel taps and is corrupted by AWGN (10dB, 20dB, 30dB and 40dB respectively). We use a pilot symbol for every twenty (Δ=20) symbols.

In order to evaluate the performance of the proposed full-rank MMSE estimator to mismatch only in SNR design, the estimator is tested when SNRs of 10 and 30 dB are used in the design. The MSE curves for a design SNR of 10, 30dB are shown in Figure 3. The performance of the MMSE estimator for high SNR (30 dB) design is better than low SNR (10 dB) design across a range of SNR values (0 - 30 dB). This results confirm that channel estimation error is concealed in noise for low SNR whereas it tends to dominate for high SNR. Thus, the system performance degrades especially for low SNR design.

To analyze full-rank MMSE estimator's performance further, we need to study sensitivity of the estimator to design errors, i.e., correlation mismatch. We therefore designed the estimator for a uniform channel correlation which gives the worst MSE performance among all channels [5] and evaluated for an exponentially decaying power-delay profile. As it can be seen from Figure 4 only small performance loss is observed for low SNRs when the estimator is designed for mismatched channel statistics. This justifies the result that a design for worst correlation is robust to mismatch.

The truncated estimator performance is also studied as a function of the number of KL coefficients. Figure 5 presents the MSE result of the truncated MMSE estimator. If only a few expansion coefficients is employed to reduce the complexity of the proposed estimator, then the MSE between channel parameters becomes large. However, if the number of parameters in the expansion is increased, the irreducible error floor still occurs.

7. CONCLUSION

We consider the design of low complexity MMSE channel estimator for OFDM systems in unknown wireless dispersive fading channels. We derive the batch MMSE estimator based on the stochastic orthogonal expansion representation of the channel via KL transform. Based on such representation, we show that no matrix inversion is needed in the MMSE algorithm. Therefore, the computational cost for implementing the proposed MMSE estimator is low and computation is numerically stable. Moreover, the performance of our proposed batch method was studied through the derivation of minimum Bayesian MSE. Since the actual channel statistics and SNR may vary within OFDM block, we have also analyzed the effect of modelling mismatch on the estimator performance and shown both analytically and through simulations that the performance degradation due to such mismatch is negligible for low SNR values.

REFERENCES

[1] E. Biglieri, J. Proakis and S. Shamai, "Fading Channels: Information-Theoretic and Communications Aspects," *IEEE Transactions on Information Theroy*, Vol. 44, No. 6, pp. 2619-2692, Oct. 1998.

[2] M. K. Tsatsanis, G. B. Giannakis, and G. Zhou, Estimation and equalization of fading channels with random coefficients, *Signal Processing*, vol. 53, pp. 211229, 1996.

[3] H. A. Cirpan and M. K. Tsatsanis, "Maximum Likelihood Blind Channel Estimation in the Presence of Doppler Shifts," *IEEE Transactions on Signal Processing*, Vol. 47, No. 6, pp. 1559-1569, June 1999.

[4] K. Yip and T. Ng, "Karhunen-Loeve Expansion of the WSSUS Channel Output and its Application to Efficient Simulation," *IEEE Journal on Selected Areas in Communications,* vol. 15, no 4, pp.640-646, May 1997.

[5] O. Edfords, M. Sandell, J.J. Van de Beek, S.K.. Wilson, and P.O. Borjesson, "OFDM Channel estimation by singular value decomposition," *IEEE Trans. on Commun.* vol. 46, pp. 931-938 july 1998.

[6] Y. (G.) Li, L. J. Cimini, and N. R. Sollenberger, "Robust channel estimation for OFDM systems with rapid dispersive fading channels," *IEEE Trans. Commun.*, Vol. 46, No.7, pages 902-914, July 1998.

[7] P. Schniter, "Low-Complexity Estimation of Doubly-Selective Channels," IEEE workshop on Signal Processing Advances in Wireless Communications, SPAWC-2003, Rome, Italy, 15-18 June 2003.

[8] E. Panayırci and H. A. Çırpan, "Maximum A Posteriori Multipath Fading Channel Estimation for OFDM Systems," *European Transactions on Telecommunications: Special Issue on Multi Carrier Spread Spectrum & Related Topics*, Vol. 13, No. 5, pp.487-494, September/October 2002.

[9] S.M. Kay, "Fundamentals of Statistical Signal Processing: Estimation Theory," *Prentice Hall* 1993.

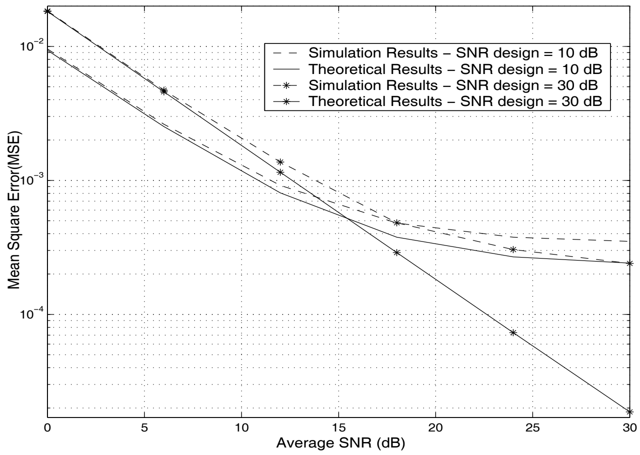

Figure 2: Effects of SNR design mismatch on MSE

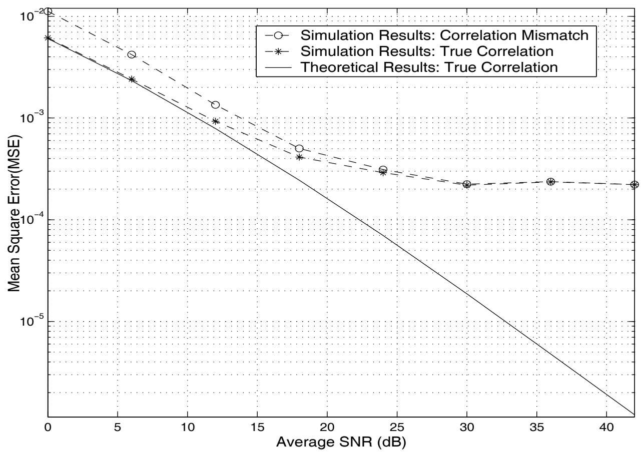

Figure 3: Effects of Correlation mismatch on MSE

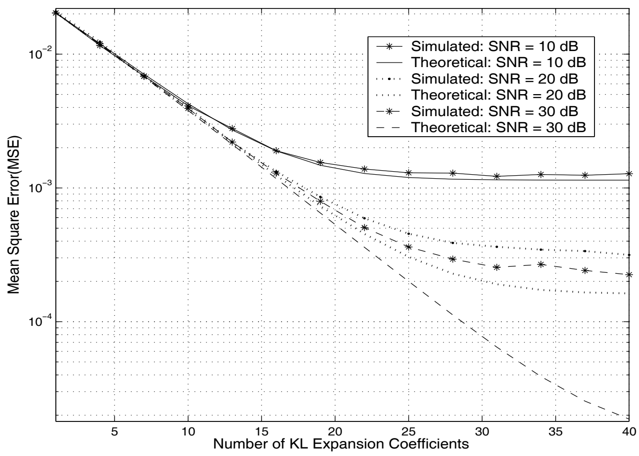

Figure 4: MSE vs number of KL Expansion Coefficients