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SLIDING MODE CONTROLLER SOLUTION FOR THE SHALLOW SUBMERGED OPERATION OF A SUBMARINE

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Abstract: In this paper, a submarine controller is presented which can accommodate the sea wave effects on a submarine at shallow water operation. Sliding mode method is implemented in a way that the robustness of the controller increased with respect to disturbance distribution vector in order to perform the depth control of a shallow submerged submarine under sea wave disturbances. Designed controller kept the submarine performance within acceptable limits. *Copyright © 2003 IFAC*

Keywords: Sliding-mode control, robust control, MIMO linear systems.

1. INTRODUCTION

In this study, the depth control of a submarine at shallow submergence under sea wave disturbances is investigated. Shallow water operation has vital importance for conventional submarines to enable them to use their periscope and charge batteries while cruising in diesel-engine mode. However the depth control becomes more difficult when the vessel is close to the surface due to adverse effects of sea conditions. The vessel at shallow submerge position is effected by sea waves rather than current in the sea that is neglected in this study.

The vehicle in this study (Dumlu and Istefanopulos, 1995) has two control surfaces, namely, the bow and stern planes. Also the content of the trim tanks is used as constant control input. But the content of the trim tanks assumed to be defined and implemented prior to control activity. Depth and pitch angle measurements are performed with a hydrostatic pressure sensor and a gyroscopic system, respectively. The vehicle is assumed to be at shallow submergence and has initially constant low speed motion. The vehicle is stable in roll axis and no control activity required for roll motion. The yaw motion controlled by the rudder is not a function of this controller.

The submarine beneath the sea waves is subject to sea forces and moments. These forces are composed of first and second order parts of sinusoidal wave patterns (Richards, R.J. and D.P. Stoten, 1982). The first order forces tend to cancel each other along the hull of the vehicle and can be neglected for the controller design. Second order part of the wave effect tends to pull the vehicle towards to surface which is also known as suction force. The latter one becomes smaller as the depth increases.

Motion equations of a submarine consists of nonlinear differential equations. These equations are derived in six degrees of freedom. The pitch and heave equations are used for the controller design. The control action is not performed for yaw and roll axes. Since working with a linear model is much simpler than a nonlinear one. The nonlinear equations of the submarine for the pitch and heave axes have been linearized around an equilibrium point.

The adverse effects of the sea waves are modelled and included in the overall submarine model for a more realistic controller design. The sea states from one to six are modelled in order to investigate the control performance for different wave heights.

The vehicle dynamics has been observed by means of a linear observer and in case of rough sea states, the robustness of the sliding mode controller has been adjusted so that it can compensate the effects of sea wave disturbances. It is aimed to keep the vehicle at submerge depth in order to avoid detection due to approaching to surface in case of rough sea conditions.

2. SUBMARINE, SEA AND ACTUATOR MODELS

2.1. Submarine Dynamics

Equation of motion along z-axis (Normal force) is given

$$\dot{w}(t)=\frac{Z'_w U}{Lm'_3}w(t)+\frac{1}{m'_3}(z'_\theta+m')U\dot{\theta}(t)+\frac{Z'_{\dot\theta}L}{m'_3}\ddot{\theta}(t)+\frac{Z'_{\delta B}U^2}{Lm'_3}\delta B(t) \quad (1)$$
$$+\frac{Z'_{\delta S}U^2}{Lm'_3}\delta S(t)+\frac{2}{\rho L^3 m'_3}(Z_{wave}(t)+W_e(t)\cos\theta)$$

where $w(t)$ is the velocity of the submarine along z axis, Q is the rotational velocity, θ is the pitch angle and h is the depth value, ρ is the mass density of sea water, L is the length and m is the weight of the submarine, δB is the bow plane command and δS is the stern plane command. Sea wave disturbances Z_{wave} is the component of sea force along the z-axis of the submarine and M_{wave} is the moment of sea waves about the y axis of the submarine.

By substituting the hydrodynamic coefficients,

$$\dot{w}(t)=-2.747\text{x}10^{-3}Uw(t)+0.1429UQ(t)-4.075\dot{Q}(t) \quad (2)$$
$$-6.227\text{x}10^{-4}U^2\delta B(t)-1.245\text{x}10^{-3}U^2\delta S(t)$$
$$+3.053\text{x}10^{-6}Z_{wave}(t)$$

where $Q(t)=\dot{\theta}(t)$.

Equation of motion along y-axis (Pitching Moment) is given

$$\ddot{\theta}(t)=\frac{M'_{\dot w}}{LI'_2}\dot{w}(t)+\frac{M'_w U}{L^2 I'_2}w(t)+\frac{M'_{\dot\theta}U}{LI'_2}\dot{\theta}(t)+\frac{M'_{\delta B}U^2}{LI'_2}\delta B(t) \quad (3)$$
$$+\frac{M'_{\delta S}U^2}{L^2 I'_2}\delta S(t)+\frac{2mg}{\rho L^5 I'_2}(z_G-z_B)\theta+\frac{m_{wave}}{\frac{\rho}{2}L^5 I'_2}$$

By substituting the hydrodynamic coefficients

$$\dot{Q}(t)=-7.219\text{x}10^{-4}\dot{w}(t)+3.786\text{x}10^{-5}Uw(t)-9.024\text{x}10^{-3}UQ(t) \quad (4)$$
$$+6.31\text{x}10^{-6}U^2\delta B(t)-3.155\text{x}10^{-5}U^2\delta S(t)$$
$$+2.64\text{x}10^{-3}(z_G-z_B)\theta(t)+5.40\text{x}10^{-10}M_{wave}(t)$$

If equations (2) and (4) are substituted to each other

$$\dot{w}(t)=-2.91\text{x}10^{-3}Uw(t)+0.18UQ(t)-6.5\text{x}10^{-4}U^2\delta B(t) \quad (5)$$
$$-1.12\text{x}10^{-3}U^2\delta S(t)-1.08\text{x}10^{-2}(z_G-z_B)\theta(t)$$
$$-2.2\text{x}10^{-9}M_{wave}(t)+3.06\text{x}10^{-6}Z_{wave}(t)$$
$$\dot{Q}(t)=4\text{x}10^{-5}Uw(t)-9.15\text{x}10^{-3}UQ(t)+6.78\text{x}10^{-6}U^2\delta B(t)$$
$$-3.07\text{x}10^{-5}U^2\delta S(t)-2.05\text{x}10^{-3}(z_G-z_B)\theta(t)$$
$$+5.42\text{x}10^{-10}M_{wave}(t)-2.20\text{x}10^{-9}Z_{wave}(t)$$

These equations are the states of the submarine dynamics, pitch acceleration and heave velocity. However the depth of the submarine is also required as a state. The depth of the submarine can be written

$$\dot{h}(t)=w(t)\cos\theta(t)-U(t)\sin\theta(t) \quad (6)$$

For small angles, the depth equation can be written as

$$\dot{h}(t)=w(t)-U(t)\theta(t) \quad (7)$$

From Eqns. (5) and (7), the state space realization of the submarine dynamics together with sea wave disturbance effect can be written as

$$\dot{\mathbf{x}}(t)=\mathbf{Ax}(t)+\mathbf{Bu}(t)+\mathbf{Rd}(t), \quad (8)$$
$$\mathbf{y}(t)=\mathbf{Cx}(t).$$

where $\mathbf{x}(t)\in\Re^n$ is the state vector, $\mathbf{u}(t)\in\Re^r$ is the control input vector and $\mathbf{y}(t)\in\Re^m$ is the measurement vector, $\mathbf{d}(t)\in\Re^g$ represents the disturbance vector which is considered as an unknown time function. **A**, **B** and **C** are system parameter matrices and the pair $\{\mathbf{C},\mathbf{A}\}$ is assumed to be observable. Here **R** matrix is the disturbance distribution matrix.

$$\mathbf{x}(t)=[w(t),Q(t),\theta(t),h(t)]^T,$$
$$\mathbf{u}(t)=[\delta B(t),\delta S(t)]^T,$$
$$\mathbf{d}(t)=[Z_{wave}(t),M_{wave}(t)]^T.$$

2.2. Sea Model

The adverse effects of the sea waves are modelled to include in the overall submarine model for a more realistic controller design. The sea model given in this paper is the one accepted in International Towing Tank Conference (ITTC). There is only one single parameter in that model, the significant wave height (Sükan, 1985),

$$S(\omega)=\frac{8.1\times10^{-3}\cdot g^2}{\omega^5}\cdot\exp\left[\frac{-3.11}{H_s^2\cdot\omega^4}\right] \tag{9}$$

where H_s is the significant wave height in meters, ω is the frequency in rad/sec and $g=9.81\frac{m}{\sec^2}$. It is clear from these units that the dimension of the sea state turns out to be $(m^2-\sec)$. But this dimension is converted into $(ft^2-\sec)$ to be consistent with the submarine model.

Sea waves have two types of effects on ship dynamics as disturbance, one is the disturbance on force dynamics and the other one is the disturbance on moment dynamics.

$$Z_{wave}(t)=\left[2.2772\times10^5-1.4552\times10^4\sum_{i=1}^{N}F_{li}\sin\omega_{ei}t\right]\cdot\sum_{i=1}^{N}F_{li}\sin\omega_{ei}t$$

$$M_{wave}(t)=1.7780\times10^7\cdot\sum_{i=1}^{N}F_{li}\cos\omega_{ei}t$$

2.3. Actuator Dynamics

The submarine simulation model also includes actuator dynamics. There are three control inputs and three actuators. Two of the actuators are used as bow and stern hydroplanes that are electro-hydraulic systems. The actuator for the third input is a pump to fill or empty the auxiliary tank. As the actuators are mechanical devices their control action is limited.

Limit values for bow and stern hydroplanes are $\pm30^\circ$. A digital filter can represent the dynamics of the bow and stern hydroplanes as,

$$X_h(k+1)=0.885X_h(k)+0.115U_a(k)$$

where X_h is the ordered hydroplane deflection and U_h is the actual hydroplane deflection.

3. CONTROLLER DESIGN

The controller is a modified version of standard sliding-mode controller (Slotine and Li, 1991). Submarine dynamics are reduced to 2 input 2 output form by discarding depth state from the state space representation in Eqn. (8) and treating pitch angle state as a known disturbance. Reduced state space realization of the submarine dynamics turns out to be

$$\dot{\mathbf{x}}_n(t)=\mathbf{A}_n\mathbf{x}(t)+\mathbf{B}_n\mathbf{u}(t)+\mathbf{R}_n\mathbf{d}(t)+\mathbf{F}\theta(t),$$
$$\mathbf{y}_n(t)=\mathbf{C}_n\mathbf{x}_n(t).$$

where $\mathbf{C}_n$ is 2x2 identity matrix which means states are observable and

$$\mathbf{x}_n(t)=\left[w(t),Q(t)\right]^T,$$
$$\mathbf{u}(t)=\left[\delta B(t),\delta S(t)\right]^T.$$

By means of an observer, the output error vector can be generated as,

$$\dot{\hat{\mathbf{x}}}_n(t)=(\mathbf{A}_n-\mathbf{LC}_n)\hat{\mathbf{x}}_n(t)+\mathbf{B}_n\mathbf{u}(t)+\mathbf{F}\theta(t)+\mathbf{Ly}_n(t),$$
$$\hat{\mathbf{y}}_n(t)=\mathbf{C}_n\hat{\mathbf{x}}_n(t),$$
$$\mathbf{e}(t)=\mathbf{y}_n(t)-\hat{\mathbf{y}}_n(t)=\mathbf{C}_n\left[\mathbf{x}_n(t)-\hat{\mathbf{x}}_n(t)\right].$$

where $\mathbf{e}(t)\in\Re^p$ is the output error vector, $\hat{\mathbf{x}}_n$ and $\hat{\mathbf{y}}_n$ are state and output estimates of the decomposed state space realization of the submarine dynamics. We can write the output error dynamics as,

$$\dot{\mathbf{e}}(t)=\dot{\mathbf{y}}_n(t)-\dot{\hat{\mathbf{y}}}_n(t)=\mathbf{C}_n\left[\dot{\mathbf{x}}_n(t)-\dot{\hat{\mathbf{x}}}_n(t)\right],$$
$$\dot{\mathbf{e}}(t)=\left[(\mathbf{A}_n-\mathbf{LC}_n)\mathbf{e}(t)+\mathbf{C}_n\mathbf{R}_n\mathbf{d}(t)\right], \tag{10}$$

From (10), sea wave disturbance distribution vector can be extracted as

$$\mathbf{C}_n\mathbf{R}_n\mathbf{d}(t)=\dot{\mathbf{e}}(t)-(\mathbf{A}_n-\mathbf{LC}_n)\mathbf{e}(t) \tag{11}$$

Sliding-mode controller is robust to uncertainties when the upper boundary of the uncertainty is given. Assume there is no information for the upper boundary of the uncertainty caused by model mismatch or sea wave disturbances. In that case, the proposed methodology places the disturbance distribution vector as an additive term to the equivalent control of the controller.

In order to achieve all states of the system in (8) to track the given desired trajectories at the same time, the switching surface function (Edwards and Spurgeon, 1998) is defined as follows,

$$\mathbf{s}(t)=\tilde{\mathbf{x}}(t)+\Lambda\int\tilde{\mathbf{x}}(t)dt \tag{12}$$

where $\mathbf{s}$ is the sliding surface vector of components, Λ is a scalar vector which defines the slopes of the sliding surfaces, $\tilde{\mathbf{x}}$ is the state error vector and defined as,

$$\tilde{\mathbf{x}}=\mathbf{x}_n-\mathbf{x}_d$$

A Lyapunov function is defined as,

$$\mathbf{V}=\frac{1}{2}\mathbf{s}^T\mathbf{s}$$

which is positive definite and it is required that the following condition must be satisfied for overall system response to be stable,

$$\dot{\mathbf{V}} < 0 \qquad \forall t > 0$$

which states the attractive condition for the sliding mode control. First derivative of sliding surface function follows from (8) and (12),

$$\dot{\mathbf{s}}(t) = \mathbf{A}_n\mathbf{x}_n(t) + \mathbf{B}_n\mathbf{u}(t) + \mathbf{R}_n\mathbf{d}(t) + \mathbf{F}\theta(t) - \dot{\mathbf{x}}_d(t) + \Lambda\tilde{\mathbf{x}}(t),$$

Assuming $\mathbf{B}_n$ is invertible,

$$\mathbf{B}_n\mathbf{u}_{eq}(t) = \left[-\mathbf{A}_n\mathbf{x}_n(t) - \mathbf{R}_n\mathbf{d}(t) - \mathbf{F}\theta(t) + \dot{\mathbf{x}}_d(t) - \Lambda\tilde{\mathbf{x}}(t)\right],$$

$$\mathbf{u}_{eq}(t) = \mathbf{B}_n^{-1}\left[-\mathbf{A}_n\mathbf{x}_n(t) - \mathbf{R}_n\mathbf{d}(t) - \mathbf{F}\theta(t) + \dot{\mathbf{x}}_d(t) - \Lambda\tilde{\mathbf{x}}(t)\right]. \quad (13)$$

where $\mathbf{u}_{eq}$ is the equivalent control term of the overall controller which guarantees system states to track the desired trajectories. Here $\mathbf{R}_n\mathbf{d}$ can be replaced from (11).

It is clear that all terms are known except disturbance distribution matrix in (13). To satisfy the sliding condition a corrective control term is used for sliding mode controllers. The overall controller with the corrective control term will be derived as (Slotine and Li, 1991) ,

$$\mathbf{u}(t) = \mathbf{u}_{eq}(t) - \mathbf{k}\text{sat}(\mathbf{s}). \quad (14)$$

where $\mathbf{k}$ is the corrective gain vector which is used to guarantee a sliding regime on the switching surface vector $\mathbf{s}$(t) and sat($\mathbf{s}$) is the saturation function with respect to each variable of $\mathbf{s}$ sliding surface vector.

4. DESIGN EXAMPLE AND SIMULATION RESULTS

Each disturbance distribution vector term has been inserted into the standard sliding-mode controller as an additive gain as can be seen in (13). In other words, the controller transformed to run in an adaptive manner in case a sea state change in the system dynamics.

Matlab-SIMULINK software has been used for simulations. The submarine depth control for sea state 1 situation with respect to a given desired trajectory upto 30 m.depth is performed with the proposed sliding mode control scheme and the results are evaluated.

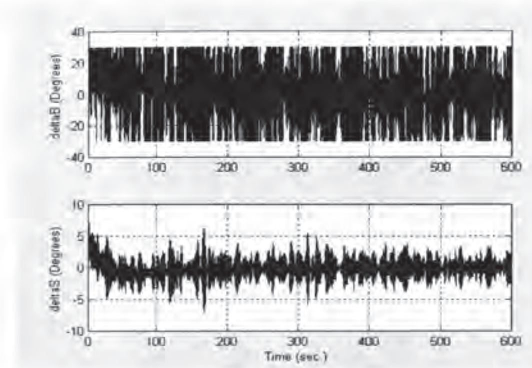

Fig. I. Bow and stern plane commands for sea state 1 with proposed scheme.

It can be assumed that there is chattering on the control planes when the control commands are observed in Fig.I. This is not true if a small portion of the simulation results are investigated. It can be seen in Fig. II that the proposed scheme with saturation switching function does not cause chattering between 100^{th} and 105^{th} seconds.

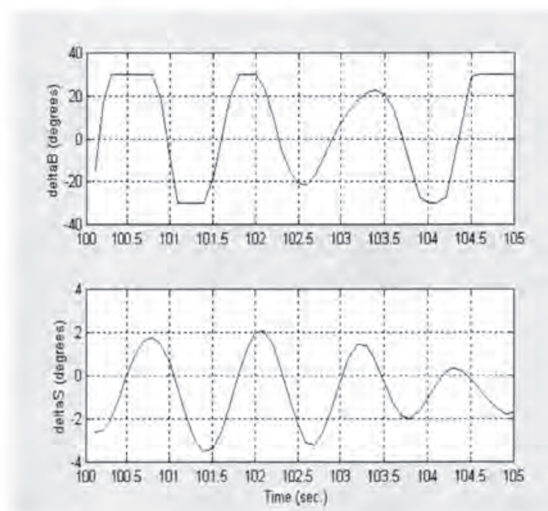

Fig. II. Bow and stern plane commands for sea state 1 with proposed scheme in a smaller scale.

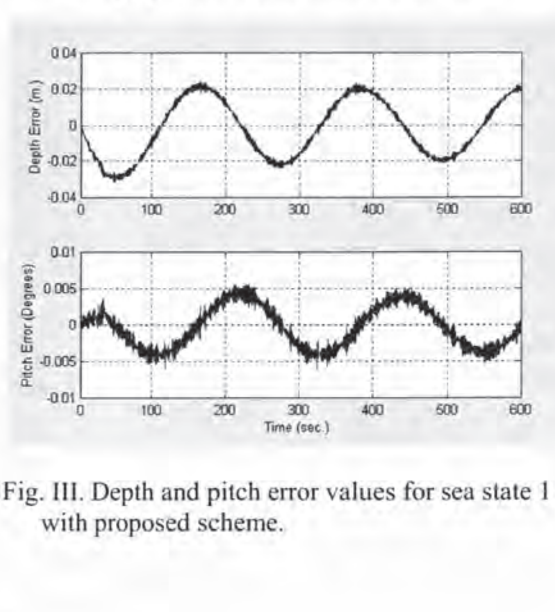

Fig. III. Depth and pitch error values for sea state 1 with proposed scheme.

Depth and pitch error values are within acceptable limits as can be seen in Fig. III.

The same simulations for depth control is performed with sea state 6. A small scale of the simulation results are shown for meaningful results as can be seen from Fig. IV, the control action is increased and the saturation of the bow plane inputs caused an increase and saturation for the stern plane inputs.

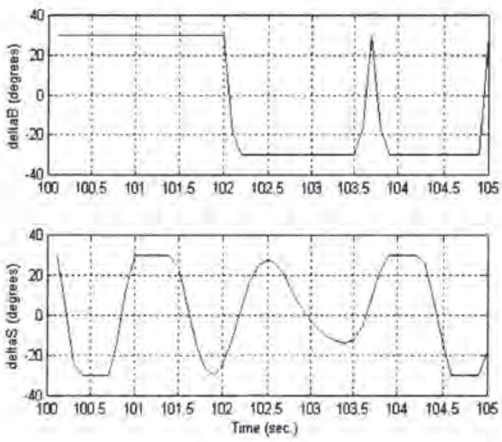

Fig. IV. Bow and stern plane commands for sea state 6 with proposed scheme in a smaller scale.

This response in Fig. IV. shows that the proposed scheme reconfigures the controller structure in order to compensate adverse sea affects.

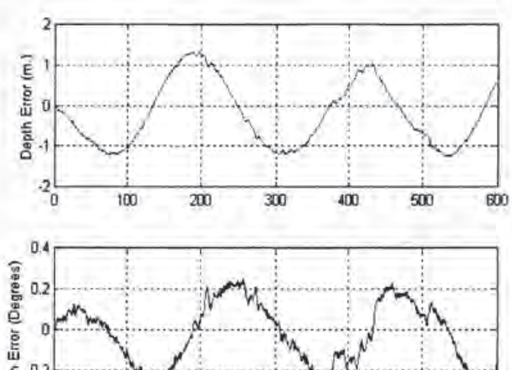

Fig. V. Depth and pitch error values for sea state 6 with proposed scheme.

Depth and pitch error values are still within acceptable limits as can be seen in Fig.V. Depth error is around 1m. and pitch error is around 0.2 degree. On the other hand, a standard sliding mode controller without disturbance rejection property performs unsatisfactorily which can be seen in Fig. VI.

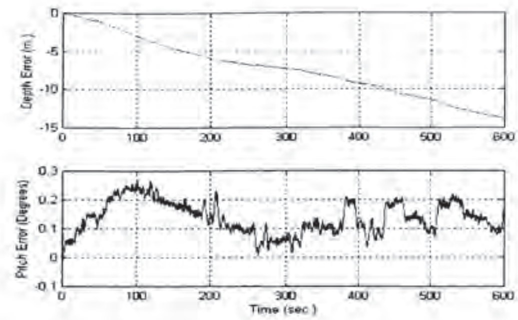

Fig. VI. Depth and pitch error values for sea state 6 with standard scheme.

CONCLUSIONS

A sliding mode controller has been proposed to compensate primarily the sea wave disturbances on the submarine. Sea wave disturbances or other disturbances caused by model mismatch have been observed and a disturbance distribution vector is obtained with a linear observer.

Standard sliding mode controller can not cope with excessive environmental changes but the proposed scheme gives satisfactory results under excessive disturbances. Standard sliding mode controller designed with Lyapunov approach is modified by updating the controller with respect to disturbance information. Hence, the robustness of the controller adjusted dynamically and the overall performance of the controller enhanced to overcome the disturbances caused by sea waves during shallow water operation.

Such an approach is very useful for real case applications of this field. In return, a reconfiguring robust controller can be obtained in order to perform the tasks expected from a submarine.

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