A BROADBAND MICROWAVE AMPLIFIER DESIGN BY MEANS OF IMMITTANCE BASED DATA MODELLING TOOL

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ABSTRACT
In this paper a practical broadband microwave amplifier design algorithm is introduced utilizing the immittance data-modelling tool. In the course of design, first, the optimum input and output terminations for the active device are produced employing the real frequency technique. Then, these terminations are modelled utilizing the new immittance-modelling tool to synthesize the front-end and back-end matching networks. An example is included to exhibit the implementation of the proposed design algorithm to construct a single stage BJT amplifier over a wide frequency band. It is expected that the proposed design algorithm will find applications to realize wideband microwave amplifiers put on MMIC for mobile communication.

1. INTRODUCTION

One of the fundamental problems in the design and development of communication systems is to match a given device to the system via coupling circuits so as to achieve optimum performance over the broadest possible frequency band. This problem inherently involves the design of an equalizer network to match the given complex impedances, and usually referred as *impedance matching* or *equalization*.

Recently introduced immittance data modeling tool can be employed successfully to design microwave amplifiers [1]. As indicated in [1], design of microwave amplifier, falls in problems of Type II category. When a broadband microwave amplifier is designed, optimum termination immitances for the active device can be generated point by point employing the Carlin's Real Frequency Line Segment Technique [2-5]. Then, the data for the terminations are modelled by means of the immitance modeling tool. Eventaully, Positive Real (PR) immitances are synthesized to yield the front-end and the back-end matching networks which completes the design.

Therefore, in this presentation, first the immitance based modeling tool is summarized. In section III, Generalized Real Frequency Technique (GRFT) is outlined. The complete design algorithm is given in Section IV. Finally, utilization of the design algorithm is exhibited with an example.

The process described in this paper can easily be extended to design microwave amplifiers with mixed lumped and distributed elements [6]. It is expected that the design technique introduced in this paper will find application to realize microwave amplifiers on MMIC for mobile communication.

2. THE IMMITTANCE BASED DATA MODELLING TOOL [1]

Any positive real rational immittance function $F(s)$ can be written in terms of its minimum and the Foster parts;

$$F(s) = F_m(s) + F_f(s) \tag{1}$$

where $s = \sigma + j\omega$ is the complex domain variable, $F_m(s)$ is the minimum part which is free of $j\omega$ poles, and $F_f(s)$ is the Foster part which includes only $j\omega$ poles. On the real frequency axis $j\omega$, one has

$$\begin{aligned} F(j\omega) &= R(\omega) + jX(\omega) \\ F_m(j\omega) &= R_m(\omega) + jX_m(\omega) \\ F_f(j\omega) &= jX_f(\omega) \end{aligned} \tag{2}$$

It is clear that

$$\begin{aligned} R(\omega) &= R_m(\omega) \\ X(\omega) &= X_m(\omega) + X_f(\omega) \end{aligned} \tag{3}$$

Since $F_m(s)$ is a positive real minimum, which contains no poles on the $j\omega$ axis, its imaginary part $X_m(\omega)$ is related to the real part $R_m(\omega)$ by the Hilbert transformation relation;

$$X_m(\omega) = H\{R(\omega)\} \tag{4}$$

where $H\{.\}$ designates the Hilbert Transformation operation.

In the immittance based modelling technique, the crux of the matter is to decompose the given data into its minimum part and Foster part. Hence, the modelling process is carried out within two major steps: model for the minimum part and the Foster part.

To model the minimum part, it is sufficient to match an analytic form $R(\omega^2)$ for the real part data. Then the complete minimum function *F(s)* can easily be generated from $R(-s^2)$ by means of Gewertz procedure [4].

The real part forms are classified based on the selection of the transmission zeros of the matching networks. Let $R(\omega^2)=\dfrac{N(\omega^2)}{D(\omega^2)}$, in this case regarding the zeros of $N(\omega^2)$, the real part forms are described as follows:

For modelling Form-A $\quad N(\omega)=\omega^{2k}$

For modelling Form-B $\quad N(\omega)=\omega^{2k}\prod_{p=1}^{m}(\omega^2-\omega^2{}_p)^2$

For modelling Form-C

$$N(\omega)=\omega^{2k}\prod_{p=1}^{m}(\omega_p{}^2-\omega^2)^2$$

$$\prod_{l=1}^{m_l}(\sigma^2{}_l+\omega^2)\prod_{r=1}^{m_r}\left\{\sigma^4{}_r+2\sigma^2{}_r(\omega^2+\beta^2{}_r)+(\omega^2-\beta^2{}_r)^2\right\}$$

These choices will be picked in accordance with the given data for $R(\omega)$ .In order to extract the Foster part from the original measured data, one has to generate $X_m(\omega)$ using the Hilbert Transformation relation [3]. Eventually, realisable analytical forms for the minimum immittance function and the Foster function are obtained by means of an appropriate curve fitting or interpolation algorithms and they are synthesized to yield the desired model under consideration.

3. GENERALIZED LINE SEGMENT TECHNIQUE FOR MATCHING A COMPLEX LOAD TO A RESISTIVE GENERATOR

Consider the single matching circuit arrangement shown in Figure 1.

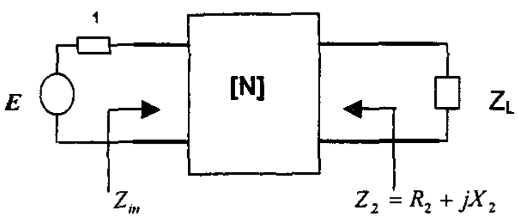

Figure 1: Single matching problem

Let the load impedance Z_L and the equalizer back impedance Z_2 be written in terms of their real and imaginary parts on the real frequency axis as

$$Z_L(j\omega)=R_L(j\omega)+jX_L(j\omega)\ ,$$
$$Z_2(j\omega)=R_2(j\omega)+jX_2(j\omega)\ , \qquad (5)$$

Basic idea is the use of a piecewise linear approximation to represent the unknown real part $R_2(\omega)$ as a number of straight-line segments as shown in Figure 2.

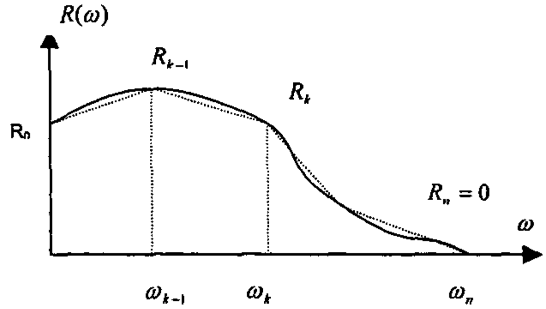

Figure 2: Line segment approximation of the real part

$$R_2(\omega)=R_0+\sum_{i=1}^{n}a_i(\omega)R_i\ ,X_2(\omega)=\sum_{i=1}^{n}b_i(\omega)R_i \qquad (6)$$

The coefficients $a_i(\omega)$ in (6) can be expressed directly in terms of sampling frequencies $(\omega_i, i=1,2,3,......n)$ as follows:

$$a_i(\omega)=\begin{cases}1 & \omega\geq\omega_i,\\ \dfrac{\omega-\omega_{i-1}}{\omega_i-\omega_{i-1}} & \omega_{i-1}\leq\omega\leq\omega_i,\\ 0 & \omega\leq\omega_{i-1},\end{cases}$$

$b_i(\omega)$ in (6) can be expressed using Hilbert transform techniques as

$$b_i(\omega)=\frac{1}{\pi(\omega_i-\omega_{i-1})}\int_{\omega_{i-1}}^{\omega_i}\ln\left|\frac{y+\omega}{y-\omega}\right|dy$$

In the Generalized Real Frequency Technique (GRFT), $Z_2(j\omega)$ can be determined as

$$Z_2(j\omega)=R_2(\omega)+j\left[H\{R_2(\omega)\}+X_{2f}\right] \qquad (7)$$

where, X_{2f} designates the Foster part of the equalizer impedance. It is also noted that X_{2f} is among the unknowns of the problem.

The Transducer Power Gain (TPG) of the system can be written in terms of the reflection coefficient at port 2 as

$$T(\omega)=1-|\rho_2|^2\ ;\quad \rho_2=\frac{Z_2-Z_L^*}{Z_2+Z_L} \qquad (8)$$

(8) can directly be expressed in terms of the real and imaginary parts of the load impedance Z_L and the back-end impedance Z_2 of the equalizer [2].

$$T(\omega)=\frac{4R_2(\omega)R_L(\omega)}{(R_2(\omega)+R_L(\omega))^2+(X_2(\omega)+X_L(\omega))^2} \qquad (9)$$

If the real frequency load data is given as

$Z_L(j\omega)=R_L(j\omega)+jX_L(j\omega)$, then the matching problem becomes essentially to that of finding $Z_2(j\omega)$ point by point such that $T(\omega)$ is maximized over the band of operation.

Once $Z_2(j\omega)=R_2(\omega)+jX_2(\omega)$ is determined point by point employing the Generalized Real Frequency Technique, it is modelled as a positive real function by means of the *"immittance based data modelling tool[1]"*.

In the following section, we will introduce the new microwave amplifier design technique via the immitance modelling tool.

3.1 Extension of Immittance Based Data Modelling Tool to the Design of Amplifiers

Let us consider the single stage amplifier configuration shown in Figure 3 where the active two-port device is denoted by [A]. The lossless two-ports N_1 and N_2 designate the front-end and back-end matching networks respectively.

A single stage microwave amplifier can conceptually be constructed within two steps by using the Real Frequency Technique. In the first step, the optimum immitance data Z_{q1} for the front-end matching network is generated point by point over the band of operation. In this step, we presume that the output port of the active is closed with unit termination (i.e. 50 ohms) (Figure 4). Hence, the input impedance of the transistor is given by

$$Z_{in}=\frac{1+S_{11}}{1-S_{11}} \tag{10}$$

and it is considered as the termination of the input matching network.

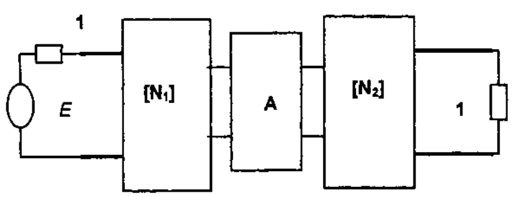

Figure 3: Single stage amplifier equalized at both input and output

In this case, we face a single matching problem. Thus, employing the GRFT, optimum impedance data Z_{q1} for the front-end matching network is generated. The gain of the system shown in Figure 4, is given by

$$T_1(\omega)=\left\{\frac{|S_{21}|^2}{1-|S_{11}|^2}\right\}\frac{4R_{q1}R_{L1}}{\left[(R_{q1}+R_{L1})^2+(X_{q1}+X_{L1})^2\right]} \tag{11}$$

In (11), the driving point input impedance of the front-end equalizer is

$$Z_{q1}(\omega)=R_{q1}(\omega)+jX_{q1}(\omega) \tag{12}$$

The load impedance $Z_{L1}(\omega)=R_{L1}(\omega)+jX_{L1}(\omega)$ is set to Z_{in} which is specified by (10).

The term $P_1(\omega)=\left\{\frac{|S_{21}|^2}{1-|S_{11}|^2}\right\}$ in front of the gain function can be regarded as a weight factor. Thus,

$$T_1(\omega)=P_1(\omega)\frac{4R_{q1}R_{L1}}{\left[(R_{q1}+R_{L1})^2+(X_{q1}+X_{L1})^2\right]} \tag{13}$$

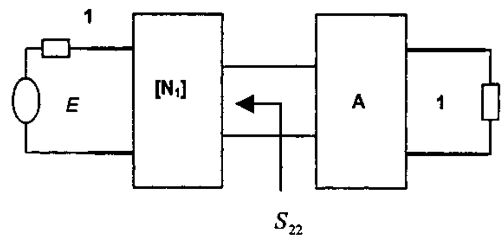

Figure 4: Single stage amplifier equalized at the input

In this step, the negative slope of the gain is compensated by optimizing T_1 to a flat gain level T_{01}.

In the second step of the conceptual design, the back-end matching network will be generated as set of data. In this case, the gain $T_2(\omega)$, which is subject to optimization, is expressed in terms of the driving point impedance Z_{q2} of the output- matching network N_2.

$$T_2(\omega)=\frac{T_1(\omega)}{1-\left|\hat{S}_{22}\right|^2}\frac{4R_{q2}R_{L2}}{\left[(R_{q2}+R_{L2})^2+(X_{q2}+X_{L2})^2\right]} \tag{14}$$

In (14), the term $\hat{S}_{22}$ is the reflection coefficient of the active device seen at the output when the front-end matching network is present. Hence,

$$\hat{S}_{22}=S_{22}+\frac{S_{12}S_{21}S_{q1}}{1-S_{11}S_{q1}} \tag{15}$$

In (15), S_{q1} is the input reflection coefficient of the front-end equalizer and it is given by

$$S_{q1}=\frac{Z_{q1}+1}{Z_{q1}-1} \tag{16}$$

Furthermore,

$$Z_{L2}=\frac{1+\hat{S}_{22}}{1-\hat{S}_{22}}=R_{L2}+jX_{L2} \tag{17}$$

Defining a new weight factor $P_2(\omega)$ as

$$P_2(\omega)=\frac{T_1(\omega)}{1-\left|\hat{S}_{22}\right|^2} \tag{18}$$

the gain of the overall system is given by

$$T_2(\omega)=P_2(\omega)\frac{4R_{q2}R_{L2}}{\left[(R_{q2}+R_{L2})^2+(X_{q2}+X_{L2})^2\right]} \tag{19}$$

Finally, optimization of $T_2(\omega)$ to a flat gain level T_{02} yields the Thevenin's impedance Z_{q2} as set of points.

In the first step, it would be wise to select T_{01} as the minimum value of $\frac{|S_{21}|^2}{1-|S_{11}|^2}$ over the operation band.

Similarly in the second step, one can choose T_{02} as the minimum value of the term $\frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$ over the specified frequencies.

In the course of the optimizations $Z_{q1}(j\omega)=R_{q1}(\omega)+jX_{q1}(\omega)$ and $Z_{q2}(j\omega)=R_{q2}(\omega)+jX_{q2}(\omega)$ are computed point by point as described in the Generalized Real Frequency Technique (GRFT). To improve the optimization, the imaginary parts X_{qi} can be computed as $X_{qi}=H\{R_{qi}\}+X_{qf}, i=1,2$ where X_{qf} designates the Foster parts of driving point impedance Z_{qi}.

The above-mentioned process is summarized in the following algorithm.

4. THE ALGORITHM: DESIGN OF A SINGLE STAGE MICROWAVE AMPLIFIER

Part I: Design of front-end equalizer

Inputs:
$S_{11}, S_{12}, S_{21}, S_{22}$: The Scattering parameters of the active element over the prescribed frequency band.

Computation steps:
Step I: Construct the weight function

$$P_1(\omega)=\left\{\frac{|S_{21}|^2}{1-|S_{11}|^2}\right\} \tag{20}$$

Step II: Construct the gain function

$$T_1(\omega)=P_1(\omega)\frac{4R_{q1}R_{L1}}{\left[(R_{q1}+R_{L1})^2+(X_{q1}+X_{L1})^2\right]} \tag{21}$$

Here, the terms R_L and X_L refer to the real and imaginary parts of the input impedance of the transistor when its output is loaded by 1-ohm resistor, which is given by the equation $Z_L=\frac{1+S_{11}}{1-S_{11}}=R_L+jX_L$. The terms R_{q1} and X_{q1} refer to the real and imaginary parts of the output impedance of the front-end matching circuit respectively.

Step III: Optimize the gain function $T_1(\omega)$ to obtain a flat gain level of $min\left\{\frac{|S_{21}|^2}{1-|S_{11}|^2}\right\}$

And determine the break points for Z_{q1} as described in the GRTF.

Step IV: Having obtained the data for Z_{q1}, generate the analytic form for it using immittance based modelling tool and synthesize it.

Step V: By using this analytic form of the impedance Z_{q1}, compute the front-end and the back-end matching networks reflection coefficients as follows.

(a) $S_{q1}=\frac{Z_{q1}-1}{Z_{q1}+1}$

(b) $S_2=S_{22}+\frac{S_{12}S_{21}S_{q1}}{1-S_{11}S_{q1}}$

Using (b) calculate the load impedance

(c) $Z_{L2}=\frac{1+S_2}{1-S_2}=R_{L2}+jX_{L2}$

Save the terms $T_1(\omega), Z_{L2}, S_2$ that have just been computed . The first part of the algorithm is completed.

Part II: Design of back-end equalizer

Inputs:
$S_{11}, S_{12}, S_{21}, S_{22}$: The Scattering parameters of the active element over the prescribed frequency band.
S_2 and Z_{L2} (calculated at the end of the first step)

Computation steps:
Step I: Construct the weight function

$$P_2(\omega)=\frac{T_1(\omega)}{1-|S_2|^2}$$

Step II: Construct the gain function

$$T_2(\omega) = P_2(\omega)\frac{4R_{q2}R_{L2}}{\left[(R_{q2}+R_{L2})^2 + (X_{q2}+X_{L2})^2\right]}$$

The terms R_L and X_L refer to the real and imaginary parts of the output impedance of the transistor when its input is loaded by the front-end matching circuit, which is given by the equation

$Z_L = \dfrac{1+S_2}{1-S_2} = R_L + jX_L$. The terms R_{q1} and X_{q1} refer to the real and imaginary parts of the output impedance of the back-end matching circuit respectively.

Step III: Optimize the gain function $T_1(\omega)$ to obtain a flat gain level of $min\left\{\dfrac{|S_{21}|^2}{1-|S_{11}|^2}\dfrac{1}{1-|S_{22}|^2}\right\}$

And determine Z_{q2} point by point to optimize T_2 employing GRFT.

Step IV: Having obtained the data for Z_{q2} generate the analytic form for it using immittance based modelling tool and synthesize it.

Now, let us introduce an example to design a single stage amplifier.

5. EXAMPLE

In this example, we wish to design a microwave amplifier employing the immittance-based data-modelling tool. For this purpose commercially available transistor HP-AT41511 was selected and its biasing conditions are

$$V_{CE} = 8V, I_C = 10mA, Z_0 = 50\Omega$$

Bandwidth = 500 MHz. (500MHz-1GHz)

Table 1: Typical Scattering Parameters for HP-AT41511

F	S_{11}		S_{21}		S_{12}		S_{22}	
GHz	$\|S_{11}\|$	$\angle\theta$	$\|S_{21}\|$	$\angle\theta$	$\|S_{12}\|$	$\angle\theta$	$\|S_{22}\|$	$\angle\theta$
0.5	0.57	-121	11.7	106	0.039	44	0.52	-37
0.6	0.54	-132	10.0	100	0.042	43	0.48	-38
0.7	0.52	-139	8.7	96	0.044	43	0.45	-39
0.8	0.51	-146	7.7	92	0.046	43	0.44	-40
0.9	0.50	-151	6.9	89	0.049	43	0.43	-41
1.0	0.46	-155	6.3	86	0.051	44	0.42	-41

Part I: In this part of the algorithm we selected 6 break frequencies ω_1=500Mhaz, ω_2=600Mhz, ω_3=700 Mhz, ω_4=800Mhz, ω_5=900Mhz and ω_6=1 GHz. The Transducer power gain T_1 is compensated to a flat gain level T_{01} =17dB. In this design, there was no need to employ foster part for Z_{q1}. Hence, as the result of optimization R_{q1} is computed

$$R_{q1} = [1.036168e-1 \quad 7.956833e-2 \quad 1.189832e-1 \quad 1.943650e-1 \quad 3.186138e-1 \quad 8.179332e-1]$$

Part II: In this part the back-end matching network is constructed when the front-end is present. Similarly, supplying the initial guess values for the resistive excursions R_{q2}, T_2 is optimised to a flat gain level T_{02}=15dB. As the result of optimisation R_{q2} is found as

$$R_{q2} = [7.633953e-1 \quad 1.131313 \quad 9.398499e-1 \quad 7.549760e-1 \quad 7.556511e-1 \quad 7.707590e-1]$$

Evaluation of $X_2(\omega)$ at the break frequencies yields

$$X_{q1} = [8.627670e-3 \quad 3.909525e-2 \quad 9.118513e-2 \quad 1.265858e-1 \quad 1.727060e-1 \quad -2.587056e-1]$$

$$X_{q2} = [-7.650150e-2 \quad -2.538549e-1 \quad -5.712840e-1 \quad -6.412332e-1 \quad -7.012788e-1 \quad -1.031250]$$

By using the immittance based data modelling tool, the minimum reactance functions can be calculated analytically and this leads to the synthesis of the equalizer circuits. For both front-end and back-end matching networks, modelling form A is selected for $R(\omega^2)$

The program code was run and at the end the minimum reactance functions for the input and the output equalizers were found to be

$$Z_{out-front} = \frac{2.087s^3 + 5.589s^2 + 4.595s + 1.363}{10.955s^4 + 29.342s^3 + 34.030s^2 + 33.695s + 19.400}$$

$$Z_{in-back} = \frac{4.411s^3 + 2.599s^2 + 4.502s + 0.668}{10.069s^4 + 5.934s^3 + 15.858s^2 + 4.814s + 4.261}$$

The final amplifier configuration and overall performance curve are given in Figure 5 and Figure 6 respectively.

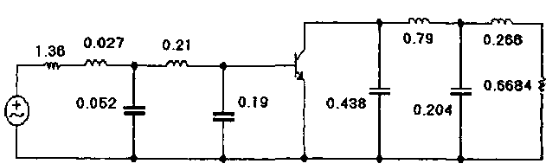

Figure 5: Designed amplifier configuration

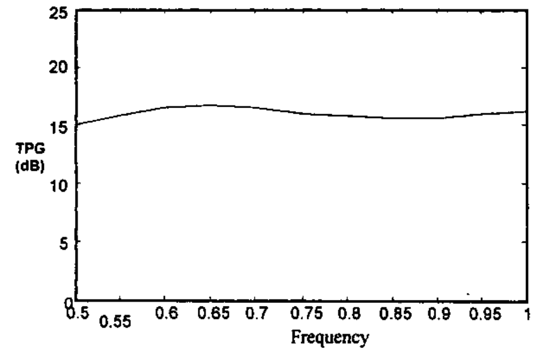

Figure 6: Overall TPG performance of the amplifier

6. CONCLUSION

In this paper, the immitance data-modelling tool is applied to design single stage microwave amplifiers. On the other hand, optimum immitance terminations for the active device are generated employing the Generalized Real Frequency Technique. An algorithm is presented to ease the understanding of the design process introduced in this paper. Implementation of the algorithm has been exhibited by means of an example. It can readily be seen that the single stage microwave amplifier design algorithm involves only simple linear arithmetic computations during optimization routine, while processing numerically defined load impedances of any complexity. The gain function is quadratic in the unknowns, and hence the problem reduces to that of a quadratic optimisation. The design algorithm presented here, can easily be extended to construct microwave amplifiers with mixed, lumped and distributed elements, employing realizable, two variable, driving point network functions [6].

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