KADIR HAS UNIVERSITY

GRADUATE SCHOOL OF SCIENCE AND ENGINEERING

PROGRAM OF ELECTONICS ENGINEERING

ANALYSIS OF STRUCTURES FORMED WITH SHUNT CAPACITOR SEPERATED BY TRANSMISSION LINES

GÖKHAN ÇAKMAK

MASTER'S THESIS

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Submitted to the Graduate School of Science and Engineering of Kadir Has University in partial fulfillment of the requirements for the degree of Master's in the Program of Electronics Engineering

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This work entitled **ANALYSIS OF STRUCTURES FORMED WITH SHUNT CAPACITORS SEPERATED BY TRANSMISSION LINES** prepared by **GÖKHAN ÇAKMAK** has been judged to be successful at the defense exam held on **08 June 2018** and accepted by our jury as **MASTER'S THESIS.**

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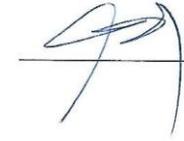
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DATE OF APPROVAL: (8 / 6 /20/8

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ANALYSIS OF STRUCTURES FORMED WITH SHUNT CAPACITORS
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ABSTRACT

There are many works in literature about ladder networks containing inductors and capacitors. But usually it is not desired for the designed circuit to have inductors since they are heavy, bulky and available only for a limited range of values and are difficult to implement at microwave frequencies, they are approximated with distributed components. Richard's transformation is used to convert lumped elements to transmission line sections.

Now consider a low-pass lumped ladder network. If the series inductors between the shunt capacitors are replaced with equal length transmission lines, a practically important mixed structure is obtained. Since the lengths of all the transmission lines are the same, these lines are called commensurate lines or unit elements (UE). It is very practical to fabricate this structure. If the transmission lines are quarter wavelength long, they are referred to as admittance inverters. These structures are useful especially for narrowband (<10%) bandpass and bandstop filters.

In this thesis, as opposed to the structures existing in the literature and explained above briefly, it is not necessary to have quarter wavelength transmission lines. So it is possible to design broadband circuits. Also the transmission lines separating the parallel capacitors are not redundant elements, they are used as circuit elements effective for the desired response. Additionaly if it is preferred not to have shunt capacitors, they can be replaced with open-ended stubs via Richard's transformation. So the resultant circuit is extremely suitable for microstrip fabrication.

In this thesis, the analysis of the mentioned mixed structure has been performed first time in the literature in the following manner. The description of the structure by means of two frequency variables (one for shunt capacitors and one for transmission lines) has been

detailed. Then broadband matching networks for military and commercial applications have been designed by using this practically important mixed structure via the algorithm that has been developed. In the algorithm, the explicit expressions for the coefficients of the descriptive two-variable polynomials in terms of the coefficients of the single variable boundary polynomials have been derived for various numbers of elements. These coefficient relations have been obtained first time in the literature. Since the lumped section contains only shunt capacitors (a degenerate network), it is impossible to use the two-variable polynomials to calculate the capacitor values. So a synthesis algorithm for the structure has been developed to be able to get the capacitor values from the two variable polynomials.

Normalized prototype circuits can be designed via the developed algorithm. So the prototype circuit can be denormalized via the frequency and impedance normalization numbers selected by the designer considering the interested frequency band and impedance level.

Keywords: Ladder networks, Richards transformation, Lumped elements

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İLETİM HATLARI İLE AYRILMIŞ PARALEL KONDANSATÖRLER İÇEREN YAPILARIN ANALİZİ

ÖZET

Bobin ve kondansatörlerden oluşan merdiven devreler hakkında literatürde birçok çalışma bulunmaktadır. Fakat genellikle, ağır ve büyük oldukları için, üretilebilecek değer aralıklarının sınırlı olması ve mikrodalga frekanslardaki üretim güçlüklerinden dolayı, tasarlanan devrelerde bobin olması istenmez dağıtılmış elemanlar (iletim hatları) ile yaklaşık olarak gerçeklenmeye çalışılırlar. Toplu elemanların dağıtılmış elemanlara çevrilmesinde Richards dönüşümü kullanılır.

Paralel bağlı kondansatörlerin arasındaki seri böbinleri, eşit uzunlukta iletim hatları ile değiştirelim. Bu iletim hatları Birim Eleman (BE) olarak isimlendirilir. Eğer hat parçalarının uzunluğu çeyrek dalga boyu olarak seçilirse admitans inverterleri elde edilir. Bu yapılar, özellikle dar bantlı (<10%) band geçiren ve band durduran filter olarak kullanılmaktadır.

Bu tezde, yukarıda kısaca açıklanan yapılardaki iletim hatlarının, literatürdekinin aksine, çeyrek dalga boyu uzunluğunda olma zorunluluğu yoktur. Bu sayede geniş bantlı devreler tasarlanabilmektedir. Ayrıca kondansatörler arasındaki iletim hatları sadece kondansatörleri ayıran, elemanlar olarak yer almayıp, devrenin istenen cevabı vermesi için devre elemanı olarak kullanılmaktadır. Eğer devrede toplu eleman (paralel kondansatör) olması istenmezse, Richards dönüşümü kullanılarak açık-uçlu hat parçaları ile değiştirilebilirler. Sonuç olarak elde edilen devre mikrostrip üretimi için son derece elverişli bir yapı olacaktır.

Bu tezde, yukarıda açıklanan devre yapısının literatürdeki ilk analizi yapılmıştır. Pratik açıdan çok önemli bu yapının iki-değişkenli tanımı detaylı olarak verilmiş, geliştirilen algoritma ile bu yapı kullanılarak bir çok askeri ve ticari uygulamada yer alabilecek genişbant empedans uyumlaştırma devresi tasarımları gerçekleştirilmiştir. Bu

tasarımların yapılabilmesi için, devreyi tanımlayan iki-değişkenli polinomların katsayıları, tek-değişkenli sınır polinomlarının katsayıları kullanılarak hesaplanmıştır. Tüm bu katsayı ifadeleri, devredeki eleman sayısına yani devre derecesine bağlıdır. Devrede en fazla üç kondansatör ve iki hat parçası bulunduğunda katsayı ilişkileri elde edilebilmiştir. Literature bu katsayı ilişkileri kazandırılmıştır. Toplu eleman içeren kısım sadece paralel bağlı kondansatörlerden oluştuğu için (dejenere devre), algoritma sonucunda elde edilen iki-değişkenli polinomlar kullanılarak, kondansatör değerlerinin hesaplanması için yeni bir yaklaşım geliştirilmiştir.

Ayrıca, geliştirilen algoritma ile normalize edilmiş prototip uyumlaştırma devresi tasarımları yapılmaktadır. Dolayısıyla, tasarımcı tarafından, uygulamanın gerektirdiği frekans ve empedans değerlerine uygun normalizasyon sayıları seçilerek tasarlanan devrenin istenen frekans bölgesinde ve empedans seviyesinde çalışması sağlanabilmektedir.

Anahtar Sözcükler: Merdiven devreler, Richards dönüşümü, Toplu elemanlar

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Thanks Professor.

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1. INTRODUCTION

The broadband matching problems are solved by using distributed elements in high frequency applications instead of using lumped components. Because of the implementation problems for lumped components in circuits, the distributed elements are prefered for the circuit designs in microwave frequencies. The lumped elements are used as the equivalence of the distributed components according to the physical size. To receive better performance, ideal lumped and distributed components can be utilized together to form a lossless matching networks with mixed, lumped and distributed components by the help of the Richards transformation.

In this thesis, the research is based on the analysis of the lossless ladder circuits which ones are formed with parallel capacitors separated with the unit elements. Inductors aren't used in this design because of the size and weight. This component causes implementation problems in microwave filters or broadband matching networks. The restrictions about bandwidth in microwave circuits design is another reason for the useless of the inductor. In literature, there are lots of studies about the ladder networks which involve only distributed components, lumped components or both of them. (Yarman, 2008) is a study for ladder networks with only lumped elements. (Aksen, 1994) is a study for low-pass ladder networks which contains inductor and capacitor as lumped elements and the lumped elements are separated by Unit Elements. This study is based on the coefficients of the variables up to 5 components. (Sertbaş, 1997) is an analysis according to the high pass, bandpass and stopband networks which contains distributed components. (Şengül, 2006) is an examination to circuit design which are formed due to the structures and rules of the other three researches.

(Şengül, 2009) is about the mixed elements ladder networks which have the same responses with the distributed components. In order to form mixed element networks,

Richards Transformation (λ=tanh βl) is used to transform the lumped elements to the distributed elements and with the help of the Kuroda identities (Kurokawa, 1965), the way of placement of the transmission lines can be changed .

A low pass ladder network can be formed with the lumped elements and there are transmission lines between these lumped elements. The transmission lines are replaced with the inductors. The resistances can be the first element and the last of these circuits. These transmission lines have quarter-wavelength long. These structures useful for narrowband, bandpass and bandstop filters. In this thesis, all of the wavelengths are equal and the value of the quarter wavelength length of the transmission line is not an indispensable condition to develop wideband circuit designs. These transmission lines are used like a circuit components to get requested response of the circuit. In addition of all these option, the capacitor can be replaced with the open circuit transmission lines with the help of the Richards transformation.

After explanation of the circuit structure, the analysis of these ladder networks with mixed elements. The approach is to explain the mixed and distributed elements in lossless two port network. With the help of the Richards transformation, the multivariable synthesis procedure can be used to generalize the approach due to the mixed lumped and distributed elements. The Richards variable λ is identified for distributed elements and the frequency variable p is identified for lumped elements. To get the values of capacitor and transmission line, the coefficients of the two variable $g(p,\lambda)$ and $h(p,\lambda)$ polynomials must be calculated with the restricted one variable polynomials g(p), h(p) and g(λ), h(λ). The number of the components are started from the 2 and up to 5 in this thesis. The capacitor is the only one to use in this design so there is a need to find a new way to calculate the values of the capacitors. With the selection of proper value for normalization, the circuit can be operated with desired value of frequency and impedance.

2. LUMPED AND DISTRIBUTED COMPONENTS

Growing needs of people causes important and significant developments in communication and circuit technology, especially in MIC technology. Naturally this development face with difficulties about design of networks by using lumped and distributed components. Lumped and distributed components are utilized mostly in RF and microwave engineering due to their frequency parameters. While engineers fabricate these networks according to needs, their main wish is minimization of the loss power or voltage through the transmission lines used as a way for conveying the voltage or powers from the generator to the load components. As you can see, best transmission lines are the lossless ones for designers.

2.1 Transmission Line Components

Lumped and distributed components are used for solving filter and broadband matching distress in RF and microwave circuits. These lumped components are electrical components acted as inductor, capacitor or resistor in low frequencies mostly in RF designs. They use in low frequencies because of the restriction of the small dimension (L_d). This dimension must be less than the ratio of wavelength ($ý$) at specific frequency. The wavelength ($ý$) is calculated by dividing the speed of light to given frequency. There is no general acceptance in literature so $ý/20$ or $ý/10$ can be accepted. The restriction is that the L_d must be less than $ý/20$ or $ý/10$. If the circuit components respond this rules, the network will be called lumped otherwise distributed network. The lumped elements are used in RF owing to some advantages in low frequencies. Some have smaller sizes and smaller coupling, amplitude, phase changes.

One of the transmission line components is the distributed components which are used in high frequencies owing to implementation problem of lumped elements in high frequencies. One of the problem is caused by inductor, inductors don't allow the current

to pass through so it becomes an open-circuit in high frequency. The other one is capacitor. The capacitor becomes a short-circuit so voltage can't be measured on its part. The situation is useless for matching and filter design in microwave circuits. According this situation, distributed components are used instead of lumped elements with transmission lines in distributed circuit. The inductor is converted to the short ended transmission line and the capacitor is converted to open-ended transmission line. The transmission line which one has impedance and delay of transmission line (τ) is the last member of the distributed elements. In Figure 2.1 show the transformation a lump circuit to distributed one. By this way, designers don't have to deal with the complex frequency. The calculation can be done with using impedance, length of the circuit elements values and the others.

Open ended TL

C

Z_0, τ

$C=1/Z_0$ $Z=Z_0 \coth(p, \tau)$
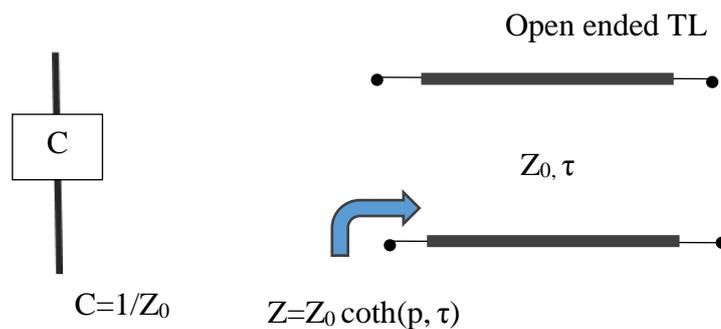
Figure 2.1 Capacitor and transmission line equivalent

Short ended TL

L

Z_0, τ

$L=Z_0$ $Z=Z_0 \tanh(p, \tau)$
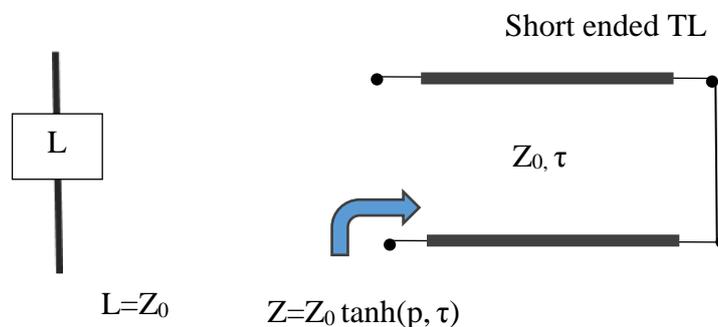
Figure 2.2 Inductor and transmission line equivalent

p is the complex frequency.

$$p = \alpha + jw \tag{2.1}$$

τ is the delay of transmission line. Z is the impedance. Z_0 is the characteristic impedance. $\acute{y}$ is the wavelength. α is the attenuation constant.

Although, there is a solution for the calculations in design circuits at high frequencies by the help of the distributed components. The shunt capacitor causes problems in circuits. Because of this problems, Richards transformation (Richards, 1948) approach is used for transforming transcendental functions of the distributed networks to rational functions. By the help of the Richards transformation ($\lambda= \tanh(p\tau)$), the mixed lumped and distributed networks become very important for the development in microwave technology. Richards approach provides multivariable synthesis process where the Richard value λ for the distributed elements and frequency variable p for the lumped elements.
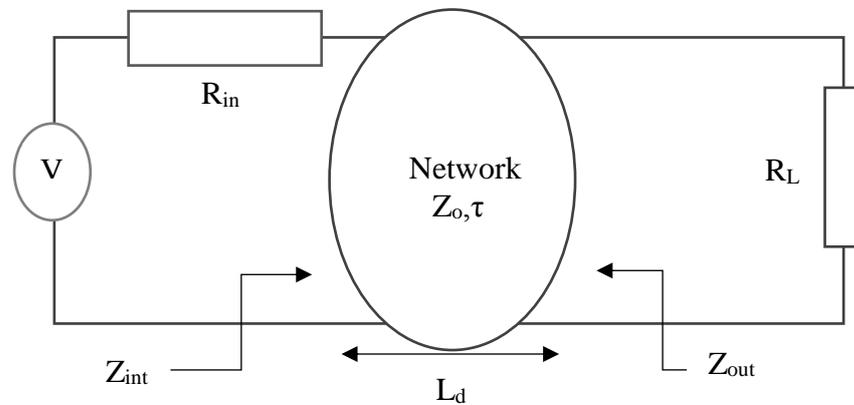
Figure 2.3 Two-port network

Z_{int} is the input impedance. Z_{out} is the output impedance. L_d is the length of transmission unit. Z_0 is the characteristic impedance of transmission line. V represents the source of voltage. R_{in} is the generater resistance. R_L is the load resistance in the load of the network.

The input impedance is calculated like this according to the wave which is provided from the source, follows the transmission line to the load. By using these parameters, the input impedance can be calculated.

$$\lambda = \tanh(p\tau) \tag{2.2}$$

$$Z_{int} = Z_0 \frac{R_L + \lambda Z_0}{Z_0 + \lambda R_L} \tag{2.3}$$

$$Z_{int} = Z_0 \frac{R_{0L} + \tanh(p\tau) Z_0}{Z_0 + \tanh(p\tau) R_L} \tag{2.4}$$

The output impedance can be calculated.

$$Z_{out} = Z_0 \frac{R_{in} + \tanh(p\tau) Z_0}{Z_0 + \tanh(p\tau) R_{in}} \tag{2.5}$$

or

$$Z_{out} = Z_0 \frac{R_{in} + \lambda Z_0}{Z_0 + \lambda R_{in}} \tag{2.6}$$

The open circuit TL is formulated in condition of the value of the terminated impedance (Z_L) is infinity. The formulation shows the calculation:

$$Z_{int} = \lim_{Z_L \to \infty} Z_0 \frac{Z_L + \lambda Z_0}{Z_0 + \lambda Z_L} = Z_0 \tag{2.7}$$

The short circuit TL is formulated with the condition of the value of the terminated impedance (Z_L) is zero. The formulation shows the calculation:

$$Z_{int} = \lim_{Z_L \to 0} Z_0 \frac{Z_L + \lambda Z_0}{Z_0 + \lambda Z_L} = \lambda Z_0 \tag{2.8}$$

2.2 Transmission Line as a Circuit Elements

Transmission line can be explained in electrical parameter. These electrical parameters are characteristic impedance (Z_o), phase constant (β), attenuation constant (α), physical length (L_d) and propagation value ($¥$). By the help of these parameters, parameters can be calculated to use in Richard domain and calculation of efficiency of the networks.

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f is the operation frequency in circuits. ⅄ is the wavelength of the transmission line.

A transmission line analysis can be done with calculating the impedance of a circuit which one is formed with resistance, inductor in serial and admittance and capacitor in parallel like in Figure 2.4.

For a lossy transmission line, the formulation of characteristic impedance of line will be;

$$Z_0 = \sqrt{\frac{R+jwL}{G+jwC}} \tag{2.9}$$

Transmission line   Lumped element representation
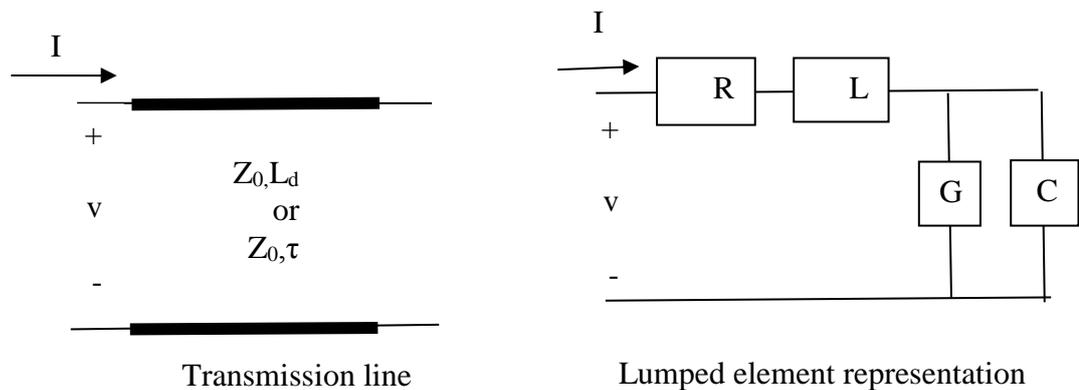
Figure 2.4 Lumped element representation of a transmission line

The calculation of the propagation constant is for lossy transmission line.

The phase constant is represented with β. G is the conductance. L_d is the physical length of the transmission line. P_v is the phase velocity. ϕ is the phase delay measured in radians.

$$⅄ = \sqrt{(R+jwL)(G+jwC)} \tag{2.10}$$

or

$$⅄ = α+jβ \tag{2.11}$$

For a lossless transmission line, R and G are 0 and the formulation of characteristic impedance of line will be:

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$$Z_0 = \sqrt{\frac{L}{C}} \qquad (2.12)$$

The propagation constant is formulated as:

$$¥=\sqrt{LC} \qquad (2.13)$$

2.3 The Utilization of Transmission Lines in Two Port Networks

Impedance and admittance methods are used to calculate the efficiency of gain power transmissions of circuits in low frequencies. These parameters are not useful for high frequencies because they require voltage and current which ones are calculated with the open – short methods (like Thevenin). In high frequencies, these methods are impractical so a new approach is required like scattering parameters. The scattering parameters are calculated according to the flowed waves with forward and backward directions in transmission lines.

The sign of the applied wave to the network show as a and the reflected wave show as b in input and output port of circuits like in Figure 2.5. If Z_0 (the characteristic impedance of transmission line) is equal to the R_L (the load resistance), the reflection is not occured from the load resistance. That means the applied power from the source is entirely dissipated in R_L. In the condition of $Z_0 \neq R_L$, the some amount of the power will dissipate in transmission line or the reflecion wave is occured from the R_L. The efficiency of gain power transmission is not provided as desired.

As seen in Figure 2.5, the normalized applied voltage wave (incident wave) is a_1 in forward direction at the first port and the reflected volage wave b_1 is backward direciton in the first port. The a_2 is the normalized applied voltage wave and the b_2 is the reflected voltage at the secound port of the network.
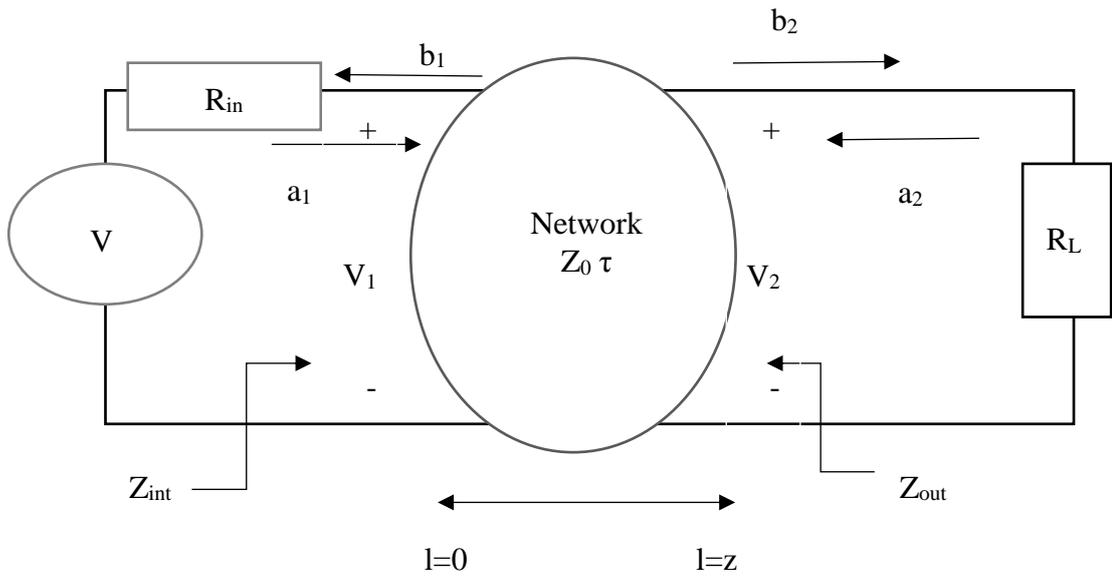
Figure 2.5 Two − port network with waves

The calculations of the normalized incident and reflected waves are a and b in input and output ports. They are defined as:

$$a_1 = \frac{(V_1 + R_{in} I_{int})}{2\sqrt{R_{in}}} \tag{2.14}$$

$$b_1 = \frac{(V_2 - R_{in} I_{int})}{2\sqrt{R_{in}}} \tag{2.15}$$

$$a_2 = \frac{(V_1 + R_L I_0)}{2\sqrt{R_L}} \tag{2.16}$$

$$b_2 = \frac{(V_2 - R_L I_0)}{2\sqrt{R_L}} \tag{2.17}$$

The value of the total voltage V_n and total curent I_n are defined in two-port. n representes the port number.

$$V_1 = (a_1 + b_1)\sqrt{R_{in}} \tag{2.18}$$

$$I_1 = \frac{a_1 - b_1}{\sqrt{R_{in}}} \tag{2.19}$$

$$V_2 = (a_2 + b_2)\sqrt{R_L} \tag{2.20}$$

Due to these waves parameters, the reflection parameters of input, outport ports and load can be calculated

$$I_2 = \frac{a_2 - b_2}{\sqrt{R_L}} \tag{2.21}$$

Sint represents the input reflection coefficient.

$$S_{int} = \frac{b_1}{a_1} = \frac{Z_{int} - R_{in}}{Z_{int} + R_{in}} \tag{2.22}$$

Sout represents the output reflection coefficient.

$$S_{out} = \frac{b_2}{a_2} = \frac{Z_{out} - Z_L}{Z_{out} + Z_L} \tag{2.23}$$

S_L represents the load reflection coefficient.

$$S_L = \frac{Z_L - Z_{out}}{Z_L + Z_{out}} \tag{2.24}$$

S_{in} representes the reflection coefficient from the generater:

$$S_{in} = \frac{Z_{in} - 1}{Z_{in} + 1} \tag{2.25}$$

The reflected power can be calculated by taking square of the reflection coefficient. The reflected power is defined as P_R.

$$P_R = b^2 \tag{2.26}$$

The incident power can be calculated with taking square of the incident coefficient.

The incident power is defined as P_i.

$$P_i = a^2 \tag{2.27}$$

The total power in ports can be calculated with difference of the power incident wave and power reflected wave. P_1 is representation of the power in port 1.

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$$P_1 = P_i - P_R = a_1^2 - b_1^2 \qquad (2.28)$$

P_2 is the total power in port 2.

$$P_2 = P_i - P_R = a_2^2 - b_2^2 \qquad (2.29)$$

The total power of the system is calculated with the summation of the dissipated total powers in two ports. P_T represents the total power of the network.

$$P_T = P_1 + P_2 \qquad (2.30)$$

Transducer power gain of the two port is defined as:

$$TPG = 1 - |S_L|^2 \qquad (2.31)$$

2.4 The Definition of Scattering Parameters and Scattering Matrix

Scattering parameter (Fettweis, 1982) for the explanations of power transfer is pratical and useful tool for networks work at high frequecies. This method is used for finite values at output and input of the network. The tools use open circuits to find values of voltage and current of the network. Another difference between scattering parameter and others are structure of values in networks. In scattering parameters, the waves of voltage and current are utilized to calculate the efficiency of the network.

Previous Seciton 2.3 is about the structure of waves. In this sections, these waves are used to form scattering parameters. In Figure 2.5, the parameters a and b are used for the definition of the scattering parameters. Scattering parameters are the values of the scattering matrix for two port network as:

$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \; a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \; S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad (2.32)$$

S represent the scattering matrix, S_{ij} represent the values of the scattering parameter, i is for the number of the matrix raw, j is for the number of the coloumn of the matrix.

The relations between the scattering matrix and waves are defined as in below.

If a_2 is 0, there is no reflection wave from the terminated load resistance (Z_L) because of the equal match between Z_L and Z_{out}.

Input reflection is:

$$S_{11} = \frac{b_1}{a_1} \tag{2.33}$$

Transducer power gain from input to outport port is:

$$S_{21} = \frac{b_2}{a_1} \tag{2.34}$$

If a_1 is 0, there is an equal match between Z_{int} and R_{int}.

Output reflection is:

$$S_{22} = \frac{b_2}{a_2} \tag{2.35}$$

Transducer power gain in reverse direction is:

$$S_{12} = \frac{b_1}{a_2} \tag{2.36}$$

The formula of dissipated power in two port is written in equation (2.37). This formula can be written with the scattering parameter like:

$$P_d = a_*^T a \left(I - S_*^T S \right) \tag{2.37}$$

I represent the identity matrix, * is the transposed conjugate of a matrix. If scattering matrix is unitary means, P_d is always greater or equal to the zero.

For lossless network,

$$I - S_*^T S = 0 \quad S_*^T S = I \tag{2.38}$$

The equation (2.32) is a link for scattering parameters with each other.

$$S_{11*}S_{11} + S_{21*}S_{21} = 1 \tag{2.39}$$

$$S_{22*}S_{22} + S_{12*}S_{12} = 1 \tag{2.40}$$

$$S_{12*}S_{11}+S_{21}S_{22*}=0 \qquad (2.41)$$

$$S_{21*}S_{22}+S_{12}S_{11}=0 \qquad (2.42)$$

If the network is reciprocal, scattering matrix is symmetrical. Symmetrical means the transposed of a network is equal to its own matrix. For a lossless network, the transmittance of forward and reverse direction are equal each other

$$S^T=S \qquad (2.43)$$

$$S_{21}=S_{12} \qquad (2.44)$$

Scattering transfer matrix (T) is used for cascade connections instead of only scattering matrix (Fetweis, 1970). The relations of normalized waves with the trasfer matrix parameters according this formula:

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = T \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}, \ T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \qquad (2.45)$$

The transfer scattering parameters are:

$$T_{11}=\frac{-\det S}{S_{21}} \quad T_{21}=-\frac{S_{22}}{S_{21}} \quad T_{12}=\frac{S_{11}}{S_{21}} \quad T_{22}=\frac{1}{S_{21}} \qquad (2.46)$$

Reciprocal scattering matrix causes the result of determinant of transfer scattering matrix to be 1 (det T =1).

2.5 Canonic Representation of Scattering and Scattering Transfer Matrix

f, g and h are three canonic polynomials and these parameters are for the explanation of the scattering matrix parameters and the scattering transfer matrix. As seen in the formulas of scattering matrix and transfer matrix:

$$S = \begin{bmatrix} \dfrac{h}{g} & \dfrac{\sigma f_*}{g} \\ \dfrac{f}{g} & \dfrac{-\sigma h_*}{g} \end{bmatrix} \quad T = \begin{bmatrix} \dfrac{\sigma g_*}{f} & \dfrac{h}{f} \\ \dfrac{\sigma h_*}{f} & \dfrac{g}{f} \end{bmatrix} \qquad (2.47)$$

f_* means f(-p) where p is the complex frequency (p=α+jβ). These are the properties of the parameters of the 3 canonic polynomials.

f(p), h(p) and g(p) are real polynomials in complex frequency p. g is strictly Hurwitz polynomial (Fettweis, 1982). f, g and h parameters have a relation according to this formula:

$$g\ g_* = hh_* + ff_*$$
(2.48)

σ is the constant 1 or -1.

Supposing the two-port network is resiprocal, the value of the f polynomial can be even or odd. This situation effects the value of σ canstant 1 or -1 with f is even and f is odd. With this properties the formula of (2.48) can be changed to this:

$$gg_* = hh_* + \sigma f^2$$
(2.49)

According to another role, degree of g polynomial is equal or larger than the degree of h polynomial and the degree of polynomial f. The transmission zeros at infinity is calculated with taking difference of the degree of polynomial g and the degree of polynomial f.. The directions are S_{21} and S_{12} scattering parameters and the calculation will be done with 3 canonic polynomials (f, g, h). Due to the (2.49) formula, S_{12} is equal to f/g and S_{21} is equal to f_*/g. With the (2.49), this formula can be written:

$$\frac{ff_*}{g^2} = \frac{gg_* - hh_*}{g^2}$$
(2.50)

If ff_* is a real even polynomial, locations of zeros are symmetry with the jw axis. By the help of the g's strictly Hurwits property, any cancellation can not be occured in ff_* in the closed right half plane. Real part of p is equal to zero or a possitive number. Due to the these conditions, the number of finite transmission zeros can be received from the degree of f and the infinity transmission zeros can be received with difference of deg g and deg f. Number of the finite and infinity transmission zeros are determined from the degree of g polynomial on condition of $0 \le \mathrm{Re}(p)$.

The input impedance of Figure 2.3 with output is terminated with a resistance (Z_0) can be calculated with polynomials.

$$Z_{\mathrm{int}} = \frac{1 + S_{11}}{1 - S_{11}} = \frac{g + h}{g - h} = \frac{n}{d}$$
(2.51)

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By calculating the even part of the Z_{int}, all of the transmission zeros can be identified. Even part of the Z_{int} is calculated as:

$$Z_{int} = \frac{1}{2}(Z_{int} + Z_{int*}) = \frac{1}{2}\left(\frac{n}{d} + \frac{n_*}{d_*}\right) = \frac{1}{2}\left(\frac{nd_* + n_* d}{dd_*}\right) = \frac{ff_*}{(g-h)(g_* + h_*)} \qquad (2.52)$$

Because of the numerator and denominator terms in last part of the formula, the jw zeros are eliminated. The $0.5(nd_* + n_* d)$ is equal to the ff_* and the finite transmission zeros can be found in this part. Under this situation, all of the transmission zeros can be determined by equation (3.4).

3.TWO VARIABLE CHARACTERIZATION OF MIXED STRUCTURES

For implementation of the lumped and distributed components together in cascaded lossless two port (Şengül, 2010), the two variable characterization method is needed. By the help of the Richard's transformation, one of the variable is Richards variable λ and the other one is the complex frequency variable p. Lumped sections can be formed according to the p and the distributed sections can be formed due to Richards variable. If these two variable are assumed indipendent variable with each other, the implementation of the lumped and distributed components can be achieved by the using multivariable methods (Şengül, 2009).

The relation between the f, g, h two variable polynomials with scatter parameters is shown as:

$$S(p,\lambda)=\frac{1}{g(p,\lambda)}\begin{bmatrix} h(p,\lambda) & \sigma f(p,\lambda) \\ f(p,\lambda) & -\sigma h(p,\lambda) \end{bmatrix} \tag{3.1}$$

$$S_{11}=\frac{h(p,\lambda)}{g(p,\lambda)}, \; S_{12}=\frac{\sigma f(p,\lambda)}{g(p,\lambda)}, \; S_{21}=\frac{f(p,\lambda)}{g(p,\lambda)}, \; S_{22}=\frac{-\sigma h(p,\lambda)}{g(p,\lambda)} \tag{3.2}$$

Scatttering parameters in (3.1) are formed with one variable scattering parameters s(p) and s(λ). S(p) and S(λ) are the explanation of the scattering parameters of lumped (p) and distributed (λ) sections. In a project, the number of the components are the determination of transmission zeros in networks.

$g(p,\lambda)$ is a scattering Hurtwitz polynomial with two variable. The formula of $g(p,\lambda)$ is formed with the coefficients:

$$g(p,\lambda)=\sum_{i=0}^{N_\lambda}\sum_{j=0}^{N_p}g_{ij}p^i\lambda^j \tag{3.3}$$

Variable p is for the lumped and λ is for the distributed sections. N_λ is the number of the the distributed elements (Unit Element) and N_p is the number of the lumped elements in cascaded network.

Another representation of the $g(p,\lambda)$ is for the thesis which has the first row for the distributed coefficients and the last column is for the coefficients of lumped. The others are unknown coefficients whichs are solved with the algorithm improved in this thesis to find accurate values of capacitors and impedance of UEs:

$$g(p,\lambda)=p^T M_g \lambda \tag{3.4}$$

$$M_g=\begin{bmatrix} g_{00} & \cdots & g_{0N_\lambda} \\ \vdots & \ddots & \vdots \\ g_{N_p 0} & \cdots & g_{N_p N_\lambda} \end{bmatrix} \tag{3.5}$$

In (2.49) formula, $G(p,\lambda)=gg_*$ means multiplication of the g function with pozitif variable (p,λ) and the same function with negative variable $(-p,-\lambda)$. The degree of result function can be used to identify the finite and infinity transmission zeros of the network.

$g(p,\lambda)$ is one of the polynomial used in equation (3.4). The first row is for the distributed coefficients and the last column is for the coefficients of the lumped elements. These are the known coefficients in this thesis. There are other known coefficients whichs values are zero in matrix. All accept these known coefficients are the unkown coefficients and can be calculated with using algorithm identificated in this thesis.

The formulation of the zeros coefficients:

$$h(p,\lambda)=p^T M_h \lambda \tag{3.6}$$

$$M_h = \begin{bmatrix} h_{00} & \cdots & h_{0N_\lambda} \\ \vdots & \ddots & \vdots \\ h_{N_p0} & \cdots & h_{N_pN_\lambda} \end{bmatrix}$$ (3.7)

In (2.48) formula, $H(p,\lambda)=hh_*$ means multiplication of the h function with pozitif variable (p,λ) and the same function with negative variable $(-p,-\lambda)$.

$h(p,\lambda)$ is one of the polynomial used in equation (2.48). The first row is for the distributed coefficients and the last column is for the coefficients of the lumped elements. These are the known coefficients in this thesis. There are other known coefficients whichs values are zero in matrix. Accept these known coefficients, the others are the unknown coefficients calculated by using algorithm identificated in this thesis.

If a two-port network has UEs in cascaded mode, the $f(p,\lambda)$ is defined as:

$$f(p,\lambda)=p^{k1}\left(1-\lambda^2\right)^{u/2}$$ (3.8)

In formula (3.8), k1 is the number of transmission zeros at dc and it is equal 0 in this thesis because there is no serial capacitor and paralel inductor components in network. U represents the number of cascaded UEs in networks.

For k1 =0,

$$f(p,\lambda)=p^{k1}\left(1-\lambda^2\right)^{u/2}=f(p)f(\lambda)$$ (3.9)

In this thesis, $g_{00}=1$, $h_{00}=0$ and $f_{00}=1$ are chosen for the same input and output normilization for $f_{00}=0^{k2}(1-0^2)^{u/2}$ no matter value of the u in this condition, the answer is always 1.

In (2.48) formula, $F(p,\lambda)=ff_*$ means multiplication of the f function with pozitif variable $f(p,\lambda)$ and the same function with negative variable $f(-p,-\lambda)$. F function is used for the identification of the finite transmission zeros in network.

In low pass ladder, the number of k1 is equal zero at infinity because there are no capacitior in series and inductor in paralel at the circuits.

3.1 Analysis of Mixed Element Structures

In this thesis, the low pass ladders have paralel capacitors separated with unit elements. The capacitors are the lumped components (p) and the unit elements (λ) with impedance (Z_0) and transmission delay (τ) are distributed elements. The aim of this algorithm to find the unknown coefficients in M_h and M_g. By using G(p,f) and H(p,f), the proper value of capacitors and unit elements can be calculated for broadband matching examples. The total degree (N_p+N_λ) of the networks are 2 to 5 in this thesis.

3.1.1 Mixed element structures formed with one capacitor and one UE
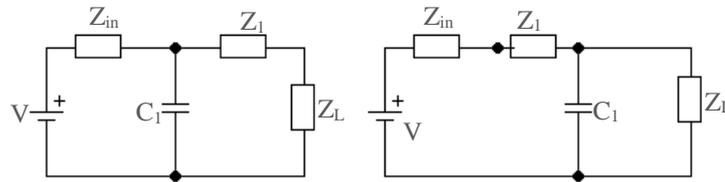
Figure 3.1 Mixed element structures formed with one capacitor and one unit element

This network has one capacitor ($N_p=1$) and one unit element ($N_\lambda=1$). According to equation (2.48), two variable functions f, g, h are shown as:

For u=1 and f(p)=1,

$$f(p,\lambda)=p^{k1} \left(1-\lambda^2\right)^{u/2}=p^0 \left(1-\lambda^2\right)^{1/2} \tag{3.10}$$

For (3.10),

$$F(p,\lambda)= f(p,\lambda)f(-p,-\lambda)= \left(1-\lambda^2\right) \tag{3.11}$$

$$H(p, \lambda)=h(p, \lambda)h(-p,-\lambda) \tag{3.12}$$

$$h(p, \lambda)=0+h_{01}\lambda+h_{10}p+h_{11}p\,\lambda \tag{3.13}$$

The matrix of M_h has 2×2 dimensions and h_{00} is 0. According to the equation (3.8), the matrix is:

$$M_h = \begin{bmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{bmatrix} \tag{3.14}$$

The known parameters are h_{00}, h_{01}, h_{11} and the unknown coefficient is h_{10}. The boundaries are the last column for distributed and the first row for lumped elements.

$$G(p, \lambda) = g(p, \lambda)g(-p,-\lambda) \tag{3.15}$$

The matrix of M_g has 2×2 dimensions and g_{00} is 1. According to the equation (3.5), the matrix is:

$$M_g = \begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{bmatrix} \tag{3.16}$$

The known parameters are g_{00}, g_{01}, g_{11} and the unknown coefficient is g_{10}. The boundaries are the last column for distributed and the first row for lumped elements.

$$G(p, \lambda) = H(p, \lambda) + F(p,\lambda) \tag{3.17}$$

According to these properties and explanation about F, G, H polynomials. The unknown coefficients of G and H can be calculated by algorithm which one is the main goal of the thesis.

$$G(p, \lambda) - H(p, \lambda) - F(p,\lambda) = 0 \tag{3.18}$$

The equations are formed with using Matlab according to the result of equation (3.18). The result is the main equation which has parameters like powered λ and p (disjoint p,λ and adjacents pλ onces) and powered coefficients. This equation is needed to be paranthesized due to the same powered λ for getting new equations which ones have only parameters contains powered p and powered coefficients. Finally, this equation is paranthesized according to the same powered p and get equations containing powered coefficients or parameters of M_h and M_g. These last equations are all equal to zero. By the help of these equations, the unknown coefficients can be calculated.

The equations are provided with using Matlab. These equations are:

$$g_{11}^2 - h_{11}^2 = 0 \tag{3.19}$$

$$g_{01}^2 - h_{01}^2 = 0 \qquad (3.20)$$

$$g_{11} - g_{10}g_{01} + h_{01}h_{10} = 0 \qquad (3.21)$$

$$g_{10}^2 - h_{10}^2 = 0 \qquad (3.22)$$

The unknown coefficients g_{10} and h_{10} are calculated with these equations.

From the equation (3.22),

$$g_{10} = |h_{10}| \rightarrow m_1 = \frac{h_{10}}{g_{10}} \qquad (3.23)$$

From the equation (3.19),

$$g_{11} = |h_{11}| \rightarrow m_2 = \frac{h_{11}}{g_{11}} \qquad (3.24)$$

From the equation (3.21),

$$g_{10} = \frac{g_{11}}{g_{01} - mh_{01}} \qquad (3.25)$$

If m_2 is equal to 1, the first component is capacitor. If m_2 is equal to -1, the first component is unit element. The constant value of m_1 is -1.

3.1.2 Mixed element structures formed with two capacitors and one UE

This network has two capacitors ($N_p = 2$) and one unit element ($N_\lambda = 1$). The matrix of M_h has 3×2 dimensions and h_{00} is 0. According to the equation (3.7), the matrix is:

$$M_h = \begin{bmatrix} 0 & h_{01} \\ h_{10} & h_{11} \\ 0 & h_{21} \end{bmatrix} \qquad (3.26)$$

The known parameters are h_{00}, h_{01}, h_{11}, h_{21}, h_{20} and the unknown coefficient is h_{10}. The boundaries are the last column for distributed and the first raw for lumped elements.

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The matrix of M_g has 3×2 dimensions and g_{20} is 0 and g_{00} is1.

$$M_{g=}\begin{bmatrix} 1 & g_{01} \\ g_{10} & g_{11} \\ 0 & g_{12} \end{bmatrix} \qquad (3.27)$$

The known parameters are $g_{00}, g_{01}, g_{11}, g_{12}$ and the unknown coefficient is g_{10}. The boundaries are the last column for distributed and the first raw for lumped elements.
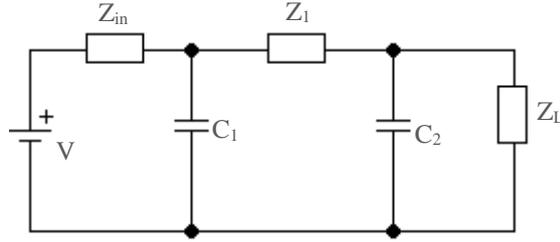
Figure 3.2 Mixed element structures formed with two capacitors and one UE

According to these properties and explanation about F, G, H polynomials.The unknown coefficients of G and H can be calculated with algorithm.

The equations are formed with using matlab according to the resut of this equation (3.18). The result is the main equation which has parameters like powered λ and p (disjoint p, λ and adjacents pλ onces) and powered coefficients. This equation is needed to be paranthesized due to the same powered λ for getting new equations which ones have only parameters contains powered p and powered coefficients. Finally, this equation is paranthesized according to the same powered p and get equations contain powered coefficients or parameters of M_h and M_g. These last equations are all equal to zero. By the help of these equations, the unknown coefficients can be calculated.

These equations are provided with using matlab. These equations are:

$$g_{01}^2 - h_{01}^2 = 1 \qquad (3.28)$$

$$g_{21}^2 - h_{21}^2 = 0 \qquad (3.29)$$

$$g_{20}^2 - h_{20}^2 = 0 \tag{3.30}$$

$$g_{10}^2 - h_{10}^2 - 2(g_{20}) = 0 \tag{3.31}$$

$$g_{11}^2 - h_{11}^2 - 2(g_{01}g_{21} - h_{01}h_{21}) = 0 \tag{3.32}$$

$$g_{11}g_{20} - h_{11}h_{20} - g_{10}g_{21} + h_{10}h_{21} = 0 \tag{3.33}$$

$$g_{11} - g_{10}g_{01} + h_{01}h_{10} = 0 \tag{3.34}$$

The unknown parameters g_{10} and h_{10} are calculated with these equations.

For this structure, $m_1 = -1$, $m_2 = -1$.

From the equation (3.31):

$$g_{10} = |h_{10}| \rightarrow m_1 = \frac{h_{10}}{g_{10}} \tag{3.35}$$

From the equation (3.29):

$$g_{21} = |h_{21}| \rightarrow m_2 = \frac{h_{21}}{g_{21}} \tag{3.36}$$

From the equation (3.34):

$$g_{10} = \frac{g_{11}}{g_{01} - m_1 h_{01}} \tag{3.37}$$

From the equation (3.31):

$$m_1 = m_2 \tag{3.38}$$

3.1.3 Mixed element structures formed with one capacitor and two UEs
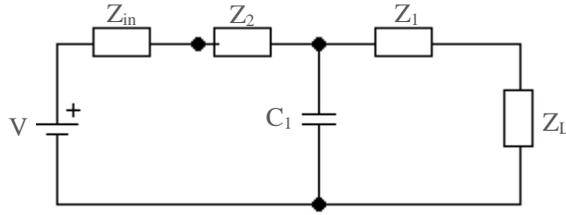
Figure 3.3 Mixed element structures formed with one capacitor and two UEs

This network has one capacitor ($N_p=1$) and two Unit Elements ($N_\lambda=2$). The matrix of M_h has 2×3 dimensions and h_{00} is 0. According to the equation (3.12), the matrix is:

$$M_h = \begin{bmatrix} 0 & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \end{bmatrix} \tag{3.39}$$

The known parameters are $h_{00}, h_{01}, h_{02}, h_{12}$, and the unknown coefficients are h_{10}, h_{11}. The boundaries are the last column for distributed and the first row for lumped elements.

The matrix of $\mathbf{M_g}$ has 3×2 dimensions and g_{00} is 1.

$$M_g = \begin{bmatrix} 1 & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \end{bmatrix} \tag{3.40}$$

The known parameters are $g_{00}, g_{01}, g_{02}, g_{12}$ and the unknown coefficients are g_{10}, g_{11}. The boundaries are the last column for distributed and the first row for lumped elements.

According to these properties and explanation about F, G, H polynomials. The unknown coefficients of G and H can be calculated with algorithm.

The equations are formed with using Matlab according to the resut of this equation (3.18). The result is the main equation which has parameters like powered λ and p (disjoint p,λ and adjacents pλ onces) and powered coefficients. This equation is needed to be paranthesized due to the same powered λ for getting new equations which ones have only parameters contains powered p and powered coefficients. Finally, this equation is paranthesized according to the same powered p and get equations contain powered

coefficients or coefficients of M_h and M_g. These last equations are all equal to zero. By the help of these equations, the unknown coefficients can be calculated.

These equations are provided by using Matlab. These equations are:

$$g_{12}^2 - h_{12}^2 = 0 \tag{3.41}$$

$$g_{02}^2 - h_{02}^3 = 0 \tag{3.42}$$

$$g_{11}g_{02} - h_{11}h_{02} - g_{01}g_{12} + h_{01}h_{12} = 0 \tag{3.43}$$

$$g_{11}^2 - h_{11}^2 - 2(g_{12}g_{10} - h_{12}h_{10}) = 0 \tag{3.44}$$

$$g_{01}^2 - h_{01}^2 - 2g_{02} = 2 \tag{3.45}$$

$$g_{11} - g_{10}g_{01} + h_{01}h_{10} = 0 \tag{3.46}$$

$$g_{10}^2 - h_{10}^2 = 0 \tag{3.47}$$

For this structure, $m_1 = -1$, $m_2 = +1$.

The unknown parameters g_{11}, g_{10}, h_{11} and h_{10} are calculated by these equations.

From the equation (3.47):

$$g_{10} = |h_{10}| \rightarrow m_1 = \frac{h_{10}}{g_{10}} \tag{3.48}$$

From the equation (3.41),

$$g_{12} = |h_{12}| \rightarrow m_2 = \frac{h_{12}}{g_{12}} \tag{3.49}$$

From the equation (3.46),

$$g_{10} = \frac{g_{11}}{g_{01} - m_1 h_{01}}$$ (3.50)

From the equation (3.43),

$$h_{11} = \frac{g_{11}\, g_{02} - g_{01}\, g_{12} + h_{01}\, h_{12}}{h_{02}}$$ (3.51)

From the equation (3.44) with using (3.51),

$$g_{11} = g_{02}(g_{01} g_{12} - h_{01} h_{12}) - h_{02}^2 \frac{g_{12} - m_1 h_{12}}{g_{01} - m_1 h_{01}}$$ (3.52)

3.1.4 Mixed element structures formed with two capacitors and two UEs
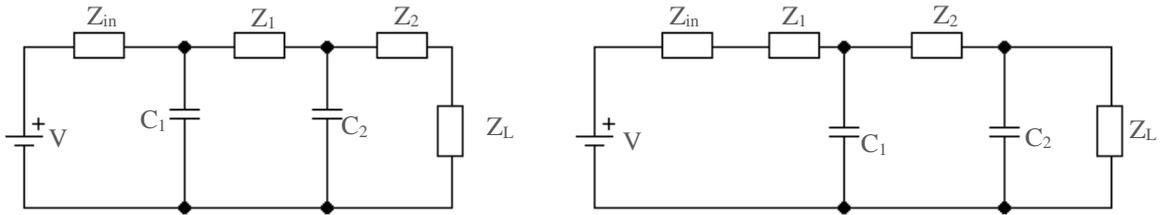
Figure 3.4 Mixed element structures formed with two capacitors and two UEs

This network has two capacitors (N_p=2) and two unit elements (N_λ=2). The matrix of M_h has 3×3 dimensions and h_{00} is 0. According to the equation (3.8), the matrix is:

$$M_h = \begin{bmatrix} 0 & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ 0 & h_{21} & h_{22} \end{bmatrix}$$ (3.53)

The known parameters are $h_{00}, h_{01}, h_{02}, h_{12}, h_{22}, h_{20}$ and the unknown coefficients are h_{10}, h_{11}, h_{21}. The boundaries are the last column for distributed and the first row for lumped elements.

The matrix of M_g has 3×3 dimensions and g_{20} is 0 and g_{00} is 1.

$$M_g = \begin{bmatrix} 1 & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ 0 & g_{21} & g_{22} \end{bmatrix}$$ (3.54)

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The known parameters are g_{00}, g_{01}, g_{02}, g_{12}, g_{22}, g_{20} and the unknown coefficients are g_{10}, g_{11}, g_{21}. The boundaries are the last column for distributed and the first row for lumped elements.

According to these properties and explanation about F, G, H polynomials. The unknown coefficients of G and H can be calculated with algorithm.

The equations are formed with using Matlab according to the resut of this equation (3.18). The result is the main equation which has parameters like powered λ and p (disjoint p, λ and adjacents pλ onces) and powered coefficients. This equation is needed to be paranthesized due to the same powered λ for getting new equations which ones have only parameters contains powered p and powered coefficients. Finally, this equation is paranthesized according to the same powered p and get equations contain powered coefficients or coefficients of M_h and M_g. These last equations are all equal to zero. By the help of these equations, the unknown coefficients can be calculated.

These equations are provided by using Matlab. These equations are:

$$g_{10}^2 - h_{10}^2 = 0 \tag{3.55}$$

$$g_{11} - g_{10}g_{01} + h_{01}h_{10} = 0 \tag{3.56}$$

$$g_{21}g_{10} - h_{21}h_{10} = 0 \tag{3.57}$$

$$g_{01}^2 - h_{01}^2 - 2g_{02} = 2 \tag{3.58}$$

$$g_{11}^2 - h_{11}^2 - 2\left(g_{12}g_{10} - g_{22} - h_{12}h_{10} + g_{01}g_{21} - h_{01}h_{21}\right) = 0 \tag{3.59}$$

$$g_{21}^2 - h_{21}^2 = 0 \tag{3.60}$$

$$g_{11}g_{02} - h_{11}h_{02} - g_{01}g_{12} + h_{01}h_{12} = 0 \qquad (3.61)$$

$$g_{11}g_{22} - h_{11}h_{22} - g_{21}g_{12} + h_{21}h_{12} = 0 \qquad (3.62)$$

$$g_{02}^2 - h_{02}^2 = 1 \qquad (3.63)$$

$$g_{12}^2 - h_{12}^2 - 2(g_{02}g_{22} - h_{02}h_{22}) = 0 \qquad (3.64)$$

$$g_{22}^2 - h_{22}^2 = 0 \qquad (3.65)$$

The unknown parameters g_{10}, g_{11}, g_{21} and h_{10}, h_{11}, h_{21} are calculated with these equations.

From the equation (3.55),

$$g_{10} = |h_{10}| \rightarrow m_1 = \frac{h_{10}}{g_{10}} \qquad (3.66)$$

From the equation (3.65),

$$g_{22} = |h_{22}| \rightarrow m_2 = \frac{h_{22}}{g_{22}} \qquad (3.67)$$

From the equation (3.60),

$$g_{21} = |h_{21}| \rightarrow m_3 = \frac{h_{21}}{g_{21}} \qquad (3.68)$$

From the equation (3.57),

$$m_1 = m_3 \qquad (3.69)$$

From the equation (3.56),

$$g_{10} = \frac{g_{11}}{g_{01} - m_1 h_{01}} \qquad (3.70)$$

From the equation (3.61),

$$h_{11} = \frac{g_{11}\,g_{02} - g_{01}\,g_{12} + h_{01}\,h_{12}}{h_{02}} \tag{3.71}$$

From the equation (3.56),

$$g_{21} = \frac{g_{11}\,g_{22} - h_{11}\,h_{22}}{g_{12} - m_1 h_{12}} \tag{3.72}$$

From the equation (3.59) with using (3.72),

$$g_{11} = g_{02}\left(g_{01} g_{12} - h_{01} h_{12}\right) - h_{02}^2 \frac{g_{12} - m_1 h_{12}}{g_{01} - m_1 h_{01}} + h_{02}^2 \frac{(g_{01} - m_1 h_{01})\left(\frac{g_{02} h_{22}}{h_{02}} - g_{22}\right)}{g_{12} - m_1 h_{12}} \tag{3.73}$$

For this structure, $m_1 = -1$.

If m_2 is equal to -1, the first component is capacitor.

If m_2 is equal to 1, the first component is Unit Element.

3.1.5 Mixed element structures formed with three capacitors and two UEs

This network has three capacitors ($N_p = 3$) and two unit elements ($N_\lambda = 2$). The matrix of M_h has 4×3 dimensions and h_{00} is 0. According to the equation (3.12), the matrix is:

$$M_h = \begin{bmatrix} 0 & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ 0 & h_{21} & h_{22} \\ 0 & 0 & h_{32} \end{bmatrix} \tag{3.74}$$

The known parameters are $h_{00}, h_{01}, h_{02}, h_{12}, h_{22}, h_{20}, h_{32}, h_{30}, h_{31}$ and the unknown coefficients are h_{10}, h_{11}, h_{21}. The boundaries are the last column for distributed and the first row for lumped elements.
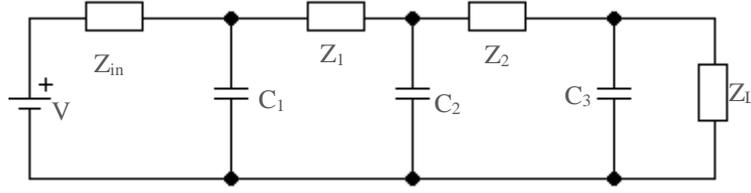
Figure 3.5 Low order structure formed with three capacitors and two UEs

The matrix of M_g has 4×3 dimensions and g_{20}, g_{30}, g_{31} are 0 and g_{00} is 1.

$$M_g = \begin{bmatrix} 1 & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ 0 & g_{21} & g_{22} \\ 0 & 0 & g_{32} \end{bmatrix} \tag{3.75}$$

According to these properties and explanation about F, G, H polynomials. The unknown coefficients of G and H can be calculated by algorithm.

The equations are formed with using Matlab according to the resut of this equation (3.18). The result is the main equation which has parameters like powered λ and p (disjoint p, λ and adjacents pλ onces) and powered coefficients. This equation is needed to be paranthesized due to the same powered λ for getting new equations which ones have only parameters contains powered p and powered coefficients. Finally, this equation is paranthesized according to the same powered p and get equations contain powered coefficients or parameters of M_h and M_g. These last equations are all equal to zero. By the help of these equations, the unknown coefficients can be calculated.

These equations are provided with using Matlab. These equations are:

$$g_{32}^2 - h_{32}^2 = 0 \tag{3.76}$$

$$g_{22}^2 - h_{22}^2 - 2(g_{12}g_{32} - h_{12}h_{32}) = 0 \tag{3.77}$$

$$g_{12}^2 - h_{12}^2 - 2(g_{02}g_{22} - h_{02}h_{22}) = 0 \tag{3.78}$$

$$g_{02}^2 - h_{02}^2 = 1 \tag{3.79}$$

$$g_{21}g_{32} - h_{21}h_{32} = 0 \tag{3.80}$$

$$g_{11}g_{22} - h_{11}h_{22} - g_{01}g_{32} + h_{01}h_{32} - g_{12}g_{21} + h_{12}h_{21} = 0 \tag{3.81}$$

$$g_{11}g_{02} - h_{11}h_{02} - g_{01}g_{12} + h_{01}h_{12} = 0 \tag{3.82}$$

$$g_{21}^2 - h_{21}^2 + 2(g_{10}g_{32} - h_{10}h_{32}) = 0 \tag{3.83}$$

$$g_{11}^2 - h_{11}^2 + 2\left(g_{22} - g_{12}g_{10} + h_{12}h_{10} - g_{01}g_{21} + h_{01}h_{21}\right) = 0 \tag{3.84}$$

$$g_{01}^2 - h_{01}^2 - 2g_{02} = 2 \tag{3.85}$$

$$g_{21}g_{10} - h_{21}h_{10} = 0 \tag{3.86}$$

$$g_{11} - g_{10}g_{01} + h_{01}h_{10} = 0 \tag{3.87}$$

$$g_{10}^2 - h_{10}^2 = 0 \tag{3.88}$$

For this situation, $m_1 = -1 \; and \; m_2 = -1$.

From the equation (3.88):

$$g_{10} = m_1|h_{10}| \rightarrow m_1 = \frac{h_{10}}{g_{10}} \tag{3.89}$$

From the equation (3.89):

$$h_{10} = m_1\, g_{10} \tag{3.90}$$

From the equation (3.76):

$$g_{32}=|h_{32}| \rightarrow m_2=\frac{h_{32}}{g_{32}} \tag{3.91}$$

From the equation (3.91):

$$h_{32}= m_2\, g_{32} \tag{3.92}$$

From the equation (3.86):

$$g_{21}=|h_{21}| \rightarrow m_1=\frac{h_{21}}{g_{21}} \tag{3.93}$$

From the equation (3.93):

$$h_{21}= m_1\, g_{21} \tag{3.94}$$

From the equation (3.80-86):

$$m_1 = m_2 \tag{3.95}$$

From the equation (3.87):

$$g_{10}=\frac{g_{11}}{g_{01} - m_1 h_{01}} \tag{3.96}$$

From the equation (3.82):

$$h_{11}=\frac{g_{11}\, g_{02} - g_{01}\, g_{12}+ h_{01}\, h_{12}}{h_{02}} \tag{3.97}$$

From the equation (3.81):

$$g_{21}=\frac{g_{11}\, g_{22} - h_{11}\, h_{22} - g_{01}g_{32} + h_{01}h_{32}}{g_{12} - m_1 h_{12}} \tag{3.98}$$

From the equation (3.84, 3.97, 3.98):

$$g_{11}=g_{02}\left(g_{01}g_{12} - h_{01}h_{12}\right) - h_{02}^{2}\frac{g_{12} - m_1 h_{12}}{g_{01} - m_1 h_{01}} + h_{02}^{2}\frac{(g_{01} - m_1 h_{01})(\frac{g_{02}h_{22}}{h_{02}} - g_{22})}{g_{12} - m_1 h_{12}} \tag{3.99}$$

The order of the components are parallel capacitor, unit element, parallel capacitor, unit element and parallel capacitor, unit element in networks.

3.2 Characteristic Impedance and Capacitance Calculations

After the calculation of the unknown coefficients of g and h polynomials, the value of the cascaded lossless commensurate lines (Şengül, 2008) and the value of the capacitors are calculated due to the scattering transfer parameters.

3.2.1 Characteristic impedance calculations of unit elements

The realization problem of the lumped components in circuits with using microwave frequencies cause needs to use another approach. Distributed components are the solution for this problem. With the Richards transformation, distributed components can be seen like lumped element networks. The formulation of λ is in equation (2.2). $p=\sigma+jw$ is the complex frequency.

In this thesis, the networks are assumed as lossless, reciprocal two ports. Definition of these networks are provided of using f, g, h polynomials in Belevitch form. The scattering transfer matrix (T) is formed according to a cascaded two networks (N_1 and N_2) and the formula of the scattering transfer matrix is in equation (2.46).

The formation of the T with N_1 transfer matrix T_1 and N_2 scattering transfer matrix T_2 are represented as:

$$T=T_1 \times T_2 \tag{3.100}$$

The matrix formation of (3.74) is:

$$T_1 = \frac{1}{f_1}\begin{bmatrix} m_1 g_{1*} & h_1 \\ m_1 h_{1*} & g_1 \end{bmatrix} \quad T_2 = \frac{1}{f_2}\begin{bmatrix} m_2 g_{2*} & h_2 \\ m_2 h_{2*} & g_2 \end{bmatrix} \tag{3.101}$$

The properties of f, g, h polynomials in (T, T_1, T_2) are same and the properties must be proper for the Feldtkeller equation. According the equations (3.100) and (3.101), the relation of the g, f, h polynomials in (T, T_1, T_2) can be showed with these equations.

$$\frac{g}{f} = g_1 g_2 + m_1 h_{1*} h_2 \tag{3.102}$$

$$\frac{h}{f} = m_1 m_2 h_{1*} h_2 + m_1 g_1 g_2 \tag{3.103}$$

$$f = f_1 f_2 \tag{3.104}$$

$$m = m_1 m_2 \tag{3.105}$$

With using these 4 equations and $T_2 = T_1^{-1} T$, the f, g, h polynomials coefficients of the second cascaded network (N_2) can be calculated. With these results, the impedance of the N_2 can be calculated.

$$h_2 = \frac{hg_1 - gh_1}{m_1 f_1 f_{1*}} \qquad g_2 = \frac{g_{1*}g - hh_{1*}}{f_1 f_{1*}} \tag{3.106}$$

In Figure 3.1, the calculation of the impedances of unit element Z_1 in network N_1 is:

$$Z_1 = \frac{g_1(1) + h_1(1)}{g_1(1) - h_1(1)} = \frac{1 + S_{11}(\lambda)}{1 + S_{11}(\lambda)} \tag{3.107}$$

Z_2 is the impedance of second network N_2. For the calculation, $g_2(\lambda)$ and $h_2(\lambda)$ polynomials whichs are the polynomials of the two variable polynomials $g_2(p, \lambda)$ and $h_2(p, \lambda)$ are needed. The formula is:

$$Z_2 = Z_1 \frac{g_2(1) + h_2(1)}{g_2(1) - h_2(1)} \tag{3.108}$$

N_λ is the number of cascaded Unit Elements in two-port network.

The definition of the $g_2(\lambda)$ and $h_2(\lambda)$ are;

$$g(\lambda) = \sum_{k=1}^{N_\lambda} M_k \lambda^{k-1} \tag{3.109}$$

$$h(\lambda) = \sum_{k=1}^{N_\lambda} N_k \lambda^{k-1} \tag{3.110}$$

The coefficients are formed with these equations.

$$M_k = \sum_{s=1}^{N_\lambda} (-1)^{s/N_\lambda} \, y_s \tag{3.111}$$

$$N_k = \sum_{s=1}^{N_\lambda} x_s \tag{3.112}$$

and,

$$x_s = h_{s-1} g(1) - g_{s-1} h(1) \tag{3.113}$$

$$y_s = g_{s-1} g(1) - h_{s-1} h(1) \tag{3.114}$$

3.2.2 Capacitance calculations

In this section, the values of shunt capacitors are calculated with synthesis algorithm explained in (Şengül, 2018). Shunt capacitors are the only components from the lumped sections because they offer low rate for range and difficulty for utilization at microwave frequencies.

For low order networks, inductors can be only located in serial to the capacitor. In this network, transmission lines (unit elements) which have the same length, are used instead of the inductors. Unit elements are practical to receive the wanted response with shunt capacitor. Without the unit elements, there will be no filter or matching networks because the capacitors will be parallel to the source and the terminated impedance. In that situation, the gain of transmitted power will not depend on the frequency anymore.

The lumped elements are defined in the last column of M_g and M_h as seen in equations (3.4) and (3.6). The scattering transfer matrix T in equation (2.46) contained two-variable functions f, g, h is the definition to use for calculating the value of the capacitors. These variables are p ($p=\alpha+jw$) for capacitors and λ for the unit elements. The polynomial $g(p,\lambda)$ is a scattering Hurwitz real coefficients polynomial.

The formation of the scattering transfer matrix of mixed elements is:

$$T(p,\lambda)=T(p)T_R(p, \lambda)$$ (3.115)

$T(p)$ is the scattering transfer matrix identification of the shunt capacitor and $T_R(p, \lambda)$ is the transfer matrix identification of the remaining matrix without the extraction of the shunt capacitor identification. Remaining transfer scattering network can be calculated with getting inverse of the transfer matrix of shunt capacitor and multiply with the transfer matrix of the all network.

$$T_R(p,\lambda)=T^{-1}(p)T(p,\lambda)$$ (3.116)

$T(p)$ is formed with $h(p)$, $g(p)$, $f(p)$ functions. These functions are identify as:

$$h_i(p)= -\frac{C_i}{2}p, \quad g_i(p)=\frac{C_i}{2}p+1, \quad f_i(p)=1$$ (3.117)

The utilization of only capacitor in lumped sections, the circuit become a degenerated network. Because of this, the use of the first column is impractical to get all values of capacitors. Only the sum of the capacitors' values can be calculated. To find the each of the capacitors' values, the last column of the M_g and M_h are used in this thesis.

The formulation of the total capacitors' value with using the first column of M_g and M_h is:

$$C_t=\frac{g_{10} + mh_{10}}{g_{00} - mh_{00}}$$ (3.118)

First value of i is zero, before the calculation of capacitor is initialized.

First step, $a = N_p-i$, $b=N_p-(i+1)$ and $c= N_\lambda$. N_p is the number of capacitors and N_λ is the number of Unit Elements in two-port network. The formulation (3.119) shows the first value of capacitor in two-port network.

For i=i+1,

$$C_i=\frac{g_{ac} + mh_{ac}}{g_{bc} - mh_{bc}}$$ (3.119)

$$m = \frac{g_{ac}}{h_{ac}} \tag{3.120}$$

Second step, with the calculation of C_1, $T(p)$ can be constituted according to equation (3.117).

$$T_i(p) = \begin{bmatrix} \frac{mg(-p)}{f(p)} & \frac{h(p)}{f(p)} \\ \frac{mh(-p)}{f(p)} & \frac{g(p)}{f(p)} \end{bmatrix} \tag{3.121}$$

The next step, T_R is calculated due to equation (3.116) and we get M_{Rg} and M_{Rh} due to equation (3.116).

$$g_R(p,\lambda) = p^T M_{Rg} \lambda, \quad h_R(p,\lambda) = p^T M_{Rh} \lambda \tag{3.122}$$

Final step, if the value of the last capacitor is C_2, C_2 is calculated with the coefficients of M_{Rg} and M_{Rh} which are formed in equation (3.117).

$$C_2 = \frac{g_{10} + m_R h_{10}}{g_{00} - m_R h_{00}} \tag{3.123}$$

Third step, the capacitor is calculated as equation (3.124) $a = N_p - i$, $b = N_p - (i+1)$, $c = N_\lambda$.

$$C_{i+1} = \frac{g_{ac} + m h_{ac}}{g_{bc} - m h_{bc}} \tag{3.124}$$

$$m = \frac{g_{ac}}{h_{ac}} \tag{3.125}$$

A new scattering transfer matrix $T_{i+1}(p)$ is formed with equation (3.121).

$$h_{i+1}(p) = -\frac{C_{i+1}}{2}p, \quad g_{i+1}(p) = \frac{C_{i+1}}{2}p+1, \quad f_{i+1}(p) = 1 \tag{3.126}$$

The $T_{(i+1)}(p)$ is used in equation (3.126) to form a new remaining scattering transfer matrix

$T_{R(i+1)}(p,\lambda)$ as:

$$T_{R(i+1)}(p,\lambda) = T_{(i+1)}^{-1}(p) T_{(i+1)}(p,\lambda) \tag{3.127}$$

If $N_p - i = 1$, the last capacitor will be calculated.

$$C_{i+1} = \frac{g_{10} + m_R h_{10}}{g_{00} - m_R h_{00}} \qquad (3.128)$$

$$m_R = \frac{g_{10}}{h_{10}} \qquad (3.129)$$

Otherwise, the third step is used again until the $N_p - i = 1$.

4. BROADBAND MATCHING NETWORKS

The aim of the broadband matching is to design an equalizer network to get equal impedance between generated (Z_{in}) and input (Z_{int}) or load (Z_L) and output (Z_{out}) (Bowik, Byler and Ajluni, 2008). These equivalence is used to get maximum power gain which is the ratio of power transferred to the load and the power constituted at the generator due to the specified frequency band.

In ideal matching, the ratio is one without reflections and dissipated power in equalizer network between the generator and load impedance.

There are three classifications in broadband matching problems (Yarman, 1985) due to the difference of passive elements at input and output ports. These classifications are single, double matching and active two port problems. If there is a purely resistance in input port and a complex load in output port, the matching problem will be classified as single matching problem. If the generator is the complex instead of a purely resistance, the matching problem will be classified as double matching. If the equalizer network consists active elements with complex in generator and load, the problem is classified as active two-port.

The lossless network in matching is provided with perfect match impedance of generator resistance and input impedance or the load impedance and output impedance. That means, the reflection parameters in port 1 (S_{in}) and port 2 (S_{out}) are equal to zero according to equations (2.22) and (2.23). That results are valid for complex load, output impedance or generator impedance and input impedance. The transducer power gain is calculated with these equations (2.22) and (2.23) and get this equation:

$$TPG(w) = 1 - |S_{int}|^2 = 1 - |S_{out}|^2 \qquad (4.1)$$

For a lossless network, S_{in} and S_{out} are zero and TPG(w) is 1. The main goal of broadband matching is to be received maximum transducer power gain from the network in a frequency band. As seen in equation (4.1), the minimization value of reflections parameters effects the value of transducer power gain.

After these explanations, the next sections are about the techniques for simple and double matching problems.

4.1 Line Segment Technique for a Single Matching Problem

In a single matching problem, the circuit has a resistance at input port and a complex load at output port. To get a transducer power gain, an equalizer network must be calculated.

The representation of load impedance (Z_L) and output impedance (Z_{out}) are:

$$Z_L(jw) = R_L(w) + j_{X_L}, \qquad Z_{out}(jw) = R_{out} + jX_{out} \qquad (4.2)$$

$$S_{out} = \frac{Z_{out}(jw)-Z_L(jw)}{Z_{out}(jw)-Z_L(jw)} \qquad TPG(w)=1-|S_{out}|^2 \qquad (4.3)$$

As seen in equation (4.4), transducer power gain is obtained with the real and imaginary parts of load $Z_L(jw)$ and the output impedance $Z_{out}(jw)$.

$$TPG(w) = \frac{4\,R_{int}R_{in}}{(R_{int}+R_{in})^2+(X_{int}+X_{in})^2} = \frac{4\,R_{out}R_L}{(R_{out}+R_L)^2+(X_{out}+X_L)^2} \qquad (4.4)$$

As seen in equation (4.4), the parameters of output impedances (R_{out}, X_{out}) should be calculated properly to get maximum TPG value. The real frequency approach (Carlin and Yarman, 1983) approach can be used to get those Z_{out} value.

To find the R_{out}, the unknown real parts of Z_{out} is represented as a number of line segments (Carlin, 1977). The formulation is:

$$R_{out}=k_0+\sum_{j=1}^{n} b_j(w)k_j \qquad (4.5)$$

$b_j(w)$ can be identified in R_{out} according to the sampling frequency (w_j, j=1,2,…..n).For $w \geq w_j$, $b_j(w)$ is equal to zero. For $w_{j-1} \leq w \leq w_i$, $b_j(w)$ is equal to $(w - w_{j-1})/(w_i - w_{j-1})$. For $w \leq w_{j-1}$, $b_j(w)$ is equal to zero.

After calculation of R_{out}, the imaginary part of the out impedance is calculated with Hilbert transformation (Carlin, 1977). The same line segments representation is used to identify the X_{out}.

$$X_{out} = \sum_{j=1}^{n} c_j(w)k_j \tag{4.6}$$

$c_j(w)$ is calculated due to Hilberts transformation technique as:

$$c_j(w) = \frac{1}{\pi(w_j - w_{j-1})} I(w) \tag{4.7}$$

$I(w)$ is calculated as:

$$I(w) = \int_{w_{j-1}}^{w_j} \ln \left| \frac{y+w}{y-w} \right| \, dy \tag{4.8}$$

After the calculation of the unknown output impedance, the transducer power gain equation (4.4) can be calculated. The actual result may not be desired one. The least square method can be utilized to minimize the difference between the target and actual power gain. The target value of transducer power gain is represented with T_d. E is the difference of the actual and desired one. N_w is the number of sampling frequencies.

$$E = \sum_{j=1}^{N_w} \left(T(w_j, k_j) - T_d \right)^2 \tag{4.9}$$

4.2 Solution for Double Matching Problems

Direct computational technique (Carlin and Yarman, 1983) can be solution for double matching problem. In this method, the real part is identified as a real even rational function with the unknown coefficients to optimize the gain characteristic over a specified passband.

The direct computational technique is used with the output impedance which is unknown parameter. The TPG equation is shown in equation (4.13) and the complex normalized input reflection coefficient (S_{in}) is used for the TPG.

S_{int} is showed as:

$$S_{int} = \frac{Z_{int} - Z_{in}}{Z_{int} + Z_{in}} \tag{4.10}$$

In these equation, Z_{int} is the input impedance and Z_{in} is the generator impedance at port one. Z_L is the load impedance and Z_{out} is the output impedance in network.

The real normalized reflection coefficient of the generator (S_{in}) is show as:

$$S_{in} = \frac{Z_{in}-1}{Z_{in}+1} \tag{4.11}$$

The real normalized reflection coefficient of the port 1 is showed as:

$$S_{int} = \frac{Z_{int}-1}{Z_{int}+1} \tag{4.12}$$

Transducer power gain is shown as with S_{in} and S_{int}:

$$TPG(w) = \frac{(1-|S_{in}|^2)(1-|S_{int}|^2)}{|1-S_{in}S_{int}|^2} \tag{4.13}$$

The goal is identication of S_{int} as a function of the impedance Z_2. S_{int} is identified due to the scattering parameters of scattering matrix as:

$$S_{int} = S_{11} + \frac{S_{12}^2 S_L}{1-S_{22}S_L} = \frac{S_{11}-S_L k}{1-S_L S_{22}} \qquad k = S_{11}S_{22}-S_{12}^2 \tag{4.14}$$

With the help of the equations of (2.39-42), it can be showed that:

$$k = \frac{S_{12}}{S_{21}} = \frac{S_{22}}{S_{11}} \tag{4.15}$$

Using this identification, S_{int} is:

$$S_{int} = \frac{S_{12}}{S_{12*}} \frac{S_L - S_{22*}}{1-S_L S_{22}} \tag{4.16}$$

$$S_{22}=\frac{Z_{out}-1}{Z_{out}+1} \quad Z_{out}=\frac{n}{d} \tag{4.17}$$

With the help of the equation (4.17), the even part of the Z_{out} is

$$ev(Z_{out})=\frac{1}{2}(Z_{out}+Z_{out*})=\frac{n\,n_*}{d\,d_*}=HH_* \quad H=\frac{f_*}{d} \tag{4.18}$$

With using equations (2.40), (4.17) and (4.18), this equation can be written:

$$S_{12}S_{12*}=1-S_{22}S_{22*}=\frac{2(Z_{out}+Z_{out*})}{(Z_{out}+1)(Z_{out*}+1)}=\frac{4HH_*}{(Z_{out}+1)(Z_{out*}+1)} \tag{4.19}$$

With this equation, S_{12} is equal to this equation:

$$S_{12}=\frac{2H}{(Z_{out}+1)(Z_{out*}+1)} \tag{4.20}$$

Finally, S_{int} is equalt to:

$$S_{int}=\frac{H}{H_*}=\frac{Z_L-Z_{out*}}{Z_L+Z_{out}} \tag{4.21}$$

$$H(jw)=\frac{f(-jw)}{g(jw)}=\sqrt{R_{out}}e^{rmj\emptyset_H(w)} \quad \emptyset_H(w)=\arg f(-jw)-\arg d(jw) \tag{4.22}$$

$$S_{int}=e^{rmj\emptyset_H(w)}\frac{Z_L-Z_{out*}}{Z_L+Z_{out}} \tag{4.23}$$

The equation (4.23) shows that Z_{out} has an effect to determine the ratio of transducer power gain with Z_{in} and Z_L.

4.3 Parametric Representation of Brune Functions

Brune functions are developed by Fettweis (Fettweis, 1979). This method is essentially proposed for single matching problems, and is depended on the parametric representation of the positive real impedance $Z_{out}(p)$ of a lossless network. The positive real impedance Z_{out} which is identification of impedance when looked from the second port to the generator, can be explained in a partial fraction expansion. Moreover the parameters

which ones are used to identify the the poles of Z_{out} are based on the optimizaiton of the gain performance of the system for wanted matching load.

$Z_{out}(p)$ is a positive real functions which has simple poles and it is assumed that it is a minimum reactance function because of it, it can be identify from its even part. In other words, the $Z_{out}(p)$ is equal to $Z_{out}(p)=even(p)+odd(p)$ and $even(p)$ is equal to $R_{out}(p)$. For $even(p)=R_{out}(p)$, the $R_{out}(p)$ is Hilbert transformation of the $Z_{out}(p)$.

The representation of the pozitif real impedance function $Z_{out}(p)$ is :

$$Z_{out}=C_0+\sum_{i=1}^{k}\frac{C_i}{p-p_i} \tag{4.24}$$

p is the complex constant which is the distinct poles of $Z_{out}(p)$ with $Re(p_i)<0$ and B_i is the complex residues, C_0 is a real constant.

$$Even\ Z_{out}=\frac{Z_{out}(p)+Z_{out}(p)}{2}=\frac{f(p)f(-p)}{n(p)n(-p)} \tag{4.25}$$

n(p) is the hurtwitz denominator of $Z_{out}(p)$ and f(p) is a real polynomial. It can be even or odd polynomial for lossless reciprocal two-ports. If the Z_{out} is a minimum reactance function, its poles are located in the left half of the complex p plane. The Hurwitz denominator polynomial d(p) can be identified as:

$$d(p)=D_k\prod_{i=1}^{k}(p-p_i) \tag{4.26}$$

D_k is a real nonzero constant.

The equations (4.24) and (4.25) are used to get this formula:

$$C_0+\sum_{i=1}^{k}\frac{C_i}{p-p_i}=\frac{f(p)f(-p)}{n(p)n(-p)} \tag{4.27}$$

With using equations (4.26) and (4.27), the C_i can be calculated with this formula:

$$C_i=-\frac{f(p_i)f(-p_i)}{p_l D_k^2 \prod_{i=1}^{k}(p_i^2-p_l^2)} \tag{4.28}$$

If degree of the f polynomial is smaller than k, C_0 is equal to 0. If the degree of the polynomial f is equal to k, the value of C_0 is $1/D_k^2$.

With this equation (4.28), the definition of the Z_{out} can be written with terms of the polynomial f(p) and the roots of d(p).

The monic poynomial can be written as:

$$f(p) = p^{k_1} \sum_{i=0}^{k_2} b_i p^{2i}$$ (4.29)

The k_1 and k_2 are nonnegative integers, b_i is an arbitrary real coefficients. If the zeros of f(p) is located on the real frequency axis of the p plane, f can be identified as:

$$f(p) = p^{k1} \prod_{i=0}^{k2} \left(p^2 + a_i^2\right)$$ (4.30)

If the number of poles is even, the poles can be assumed as conjugate pairs. If the number of poles is odd, the pole can be chosen as real.

For i=1...... num, $p_i = -\alpha_i + j\beta_i$, $p_i = -\alpha_i - j\beta_i$. If the number of poles are even, the num is equal half of number of holes for α is bigger than zero. If the number of poles are odd, num is equal to (n-1)/2 for α is bigger than zero.

According to these explanations, the parameters of output impedance can be written as:

$$R_{out} = -\sum_{i=1}^{k} \frac{p_i C_i}{w^2 + p_i^2} + C_0 \qquad X_{out}(w) = -w \sum_{i=1}^{k} \frac{C_i}{w^2 + p_i^2}$$ (4.31)

4.4 Real Frequency Matching with Scattering Parameters

The matching problem can be formulated in terms of scattering parameters of the lossless equalizer network. The name of this frequency scattering approach is Simplified Real Frequency Technique (SRFT) (Yarman, 1985). A lossless matching network is identified with the scattering parameters which ones are formed with canonic polynomials f, g, h.

The canonic polynomials are identified with the Belevitch representation.

$$S_{11}=\frac{h(p)}{g(p)} \quad S_{12}=\sigma\frac{f(-p)}{g(p)} \quad S_{21}=\frac{f(p)}{g(p)} \quad S_{22}=\frac{-\sigma h(-p)}{g(p)} \quad \sigma=\frac{f(p)}{f(-p)} \tag{4.32}$$

In equation (4.32), g(p) is a Hurwitz polynomial, f(p) is a real monic polynomial and σ is a constant (σ= -1 or +1). If the two port N is reciprocal as well, then f is either even or odd.

The relation of g, f, h polynomial with each other is:

$$g(p)g(-p)=h(p)h(-p)+f(p)f(-p) \tag{4.33}$$

The degree relation of g, f, h polynomial, degree of g polynomial can be equal or bigger than degree of h and f.

As seen in these properties, f and h polynomials are the parameters of Hurwitz polynomial g. The definition of a network can be done with h(p) and f(p) polynomials. f(p) is the zeros of transmission of the matching two-port network and depend on the number of distributed elements and lumped elements which are chosen by the designer.

As the equalizer network type of components and numbers of the components are determined, f(p) can be calculated with the degree n due to those selections. After the coefficients of h(p) are initialized, g(p) can be calculated according to equation (4.33). The calculations of the polynomials are used to calculate the value of scattering parameters as in equation (4.32). With these scattering parameters, the input and output reflections can be calculated also to get the transducer power gain in equation (4.1).

TPG can also be shown with combination of the equations (4.13) and (4.14).

$$TPG(w)=\frac{(1-|S_{in}|^2)|S_{21}|^2(1-|S_L|^2)}{|1-S_{11}S_{in}|^2|1-S_{out}S_L|^2} \tag{4.34}$$

S_{in} is the input reflection coefficient and terminated in Z_L. S_L is the load reflection coefficient. S_{out} is the output reflection coefficient and terminated in Z_{in}. S_{11} is the reflection coefficient of port one. S_{22} is the reflection coefficient of the port 2.

To calculate the transducer power gain, these are the needs and orders. Firstly, the real coefficients are needed to be initialized. Secondly, selection of the polynomial form of f(p) and its degree must equal or less than the degree of g(p). Thirdly, calculation of gg_* according to $ff_* + hh_*$ and the find the roots of $G(p) = gg_*$. Fourthly, the location of LPH of roots are chosen and g(p) is formed with them. Finally, with the known parameters of f, g and h polynomials, the scattering parameters in equation (4.32) and reflection coefficients are calculated to use them to find transducer power gain in equation (4.34).

The transducer power gain is become more efficient with less value of scattering parameter of input, output and resistance in port one and load and increasing value of forward gain (S_{21}) according to the equation (4.34).

5. EXAMPLES

5.1 Broadband Matching Network Formed with Mixed Element Structure

To design two-port broadband matching networks with lumped elements and distributed elements, two variable functions are needed to generalize the method. This method consists of Richards variable λ for distributed sections and frequency variable p for lumped sections. The purpose of the broadband matching is to get efficient design for the power transfer from the source to the load resistance. The efficiency can be measured in a specific frequency range with the transducer power gain which is the ratio of the power at the load to the available power at the source.

The description of the transducer power gain is as:

$$TPG(w) = \frac{4\,R_{int}R_{in}}{(R_{int}+R_{in})^2+(X_{int}+X_{in})^2} = \frac{4\,R_{out}R_L}{(R_{out}+R_L)^2+(X_{out}+X_L)^2} \tag{5.1}$$

The input impedance is represented in terms of $Z_i = R_i + jX_i$, the load impedance is represented with $Z_0 = R_0 + jX_0$, source impedance is represented as $Z_s = R_s + jX_s$ and the output impedance is formulated as $Z_{out} = R_{out} + jX_{out}$ as seen equation (5.1).

As seen in equation (5.1), to calculate the transducer power gain, the value of the input and output impedance of the network are needed. First, the scattering parameters in equation (3.2) are needed to calculate the reflection coefficients from the load impedance.

$$S_L = \frac{Z_L-1}{Z_L+1} \tag{5.2}$$

Then, with the reflection coefficients, the input reflection coefficient can be identified with these coefficients and the scattering parameters as in equation (3.2).

$$S_i = S_{11} + \frac{S_{12}S_{21}S_L}{1 - S_{22}S_L} \tag{5.3}$$

The output reflection coefficient is terminated at Z_{in} (the generator impedance in port 1).

$$S_o = S_{22} + \frac{S_{12}S_{21}S_{in}}{1 - S_{22}S_{in}} \tag{5.4}$$

With the scattering parameters, transducer power gain can be calculated with these formula:

$$TPG = (1 - |S_i|^2)\frac{|S_{21}|^2(1 - |S_0|^2)}{|1 - S_{11}S_i|^2 \ |1 - S_{22}S_o|^2} \tag{5.5}$$

Finally, the input and output impedances are shown as:

$$Z_{int} = \frac{1 + S_i}{1 - S_i} \tag{5.6}$$

$$Z_{out} = \frac{1 + S_o}{1 - S_o} \tag{5.7}$$

The example is about getting an efficient network for transfer power by calculating equalized network. The efficiency is measured by getting the objective Transducer power gain TPG(w)=0.98 in this example.

The circuit has a serial inductor (L_L=2) to a parallel combination of resistance (R=1) and capacitor (C_L=1) at load. The normalized load and generator impedance values are shown in Table 5.1.

Table 5.1 Given Normalized Load and Generator Impedance Data

w	R_L	X_L	R_{in}	X_{in}
0.0	1.0000	0.0000	1.0000	0.0000
0.1	0.9901	0.1010	1.0000	0.1000
0.2	0.9615	0.2077	1.0000	0.2000
0.3	0.9174	0.3248	1.0000	0.3000
0.4	0.8621	0.4552	1.0000	0.4000
0.5	0.8000	0.6000	1.0000	0.5000
0.6	0.7353	0.7588	1.0000	0.6000
0.7	0.6711	0.9302	1.0000	0.7000
0.8	0.6098	1.1122	1.0000	0.8000
0.9	0.5525	1.3028	1.0000	0.9000
1.0	0.5000	1.5000	1.0000	1.0000

The circuit has three capacitors and two unit elements. The capacitors are defined in last column of M_g and M_h matrices. The unit elements are defined in first row of M_g and M_h matrices.

The last column and the first row coefficients of M_h matrix are $h(p)=p^3-p^2+p+1$ and $h(\lambda)= -\lambda$ respectively. The coefficients of the h and g functions due to distributed and lumped elements are chosen in random 1 or -1. After the choose of the coefficients value, the G(p) is formed as:

$$G(p)=g(p)g(-p) \tag{5.8}$$

The LPH roots of the G(p) is g(p) part. After the chose of the LPH roots, the initial parameters will be ready. With these parameters, the calculation of the unknown parameters of $f(p,\lambda)$ and $g(p,\lambda)$ which are described in third section are calculated (M_g and M_h) due to the algorithm is developed in this thesis. After the scattering parameters, input reflection coefficient (S_i) and output reflection coefficient (S_o) are calculated, the input and output impedances are calculated. Later, these parameters are used in equation (5.1) to calculate TPG. The result is named as actual transducer power gain and represented as T. In this example, the desired transducer power gain is represented as T_d. This process is repeated until getting the targeted min error calculated in least square equation (4.9) or end of given optimization number (in these example 4000). In the end, the process is finished with reaching maximum optimization number (4000). The elapsed time is 155.4 seconds.

$$M_h = \begin{bmatrix} 0 & 0.8982 & -0.0991 \\ -1.4657 & -0.0880 & -0.1066 \\ 0 & -3.0708 & -0.0164 \\ 0 & 0 & -1.0855 \end{bmatrix} \qquad (5.9)$$

$$M_g = \begin{bmatrix} 1 & 2.1947 & 1.0049 \\ 1.4657 & 4.5333 & 2.0281 \\ 0 & 3.0708 & 2.0425 \\ 0 & 0 & 1.0855 \end{bmatrix} \qquad (5.10)$$

After getting M_g and M_h, the value of capacitors are calculated according to section (3.2.2). These parallel capacitor normalized values are $C_1=1.0715$, $C_2=0.80556$, $C_3 =1.0543$. The values of the unit elements are calculated with synthesis approach proposed in section (3.2.1). These values are $Z_1=1.47$, $Z_2=1.6229$, $\tau=1.8059$.

The transducer power gain curve of the designed mixed element broadband matching network is given in Figure 5.2.
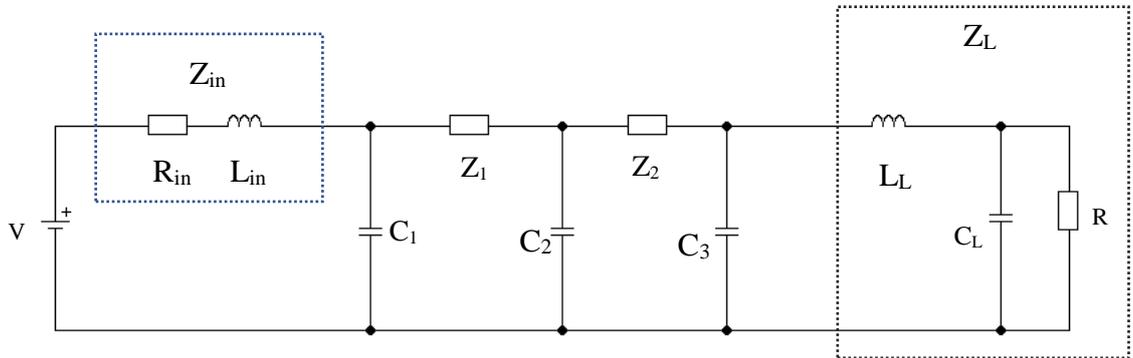
Figure 5.1 Designed broadband matching network
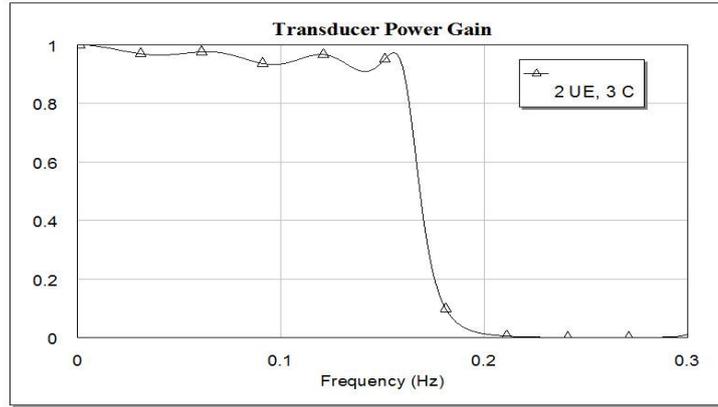
Figure 5.2 Performance of the designed mixed element matching network

5.2 Capacitance Value Calculations

It is about the calculation of values of capacitors in a network as seen in Figure 3.4. The identification of M_g and M_h are represented in equations (3.54) and (3.53), respectively. These value are needed to be calculated the values as explained in section (3.2.2).

The initial parameters are the first raw and the last column of M_g and M_h, respectively. The first row is represented the distributed components with two elements. The last column is the lumped components with two elements for this example. The equations in section (3.1.4) are used to calculate g_{11}, g_{10}, g_{21} and h_{11}, h_{10}, h_{21} coefficients. The f polynomial is calculated according to $f(p,\lambda)=p^{k1}(1-\lambda^2)^{N\lambda/2}$ and for this example $f(p,\lambda)=p^0(1-\lambda^2)^1$. The calculated coefficients are shown as (Şengül and Çakmak, 2018):

$$M_g=\begin{bmatrix} 1 & 110/21 & 29/21 \\ 11/2 & 55 & 63/2 \\ 0 & 63 & 189 \end{bmatrix} \quad M_h=\begin{bmatrix} 0 & 100/21 & 20/21 \\ -11/2 & -41 & 21/2 \\ 0 & -63 & -189 \end{bmatrix}$$

In these network, there are also two transmission lines to separate two capacitors.

From equation (3.118), the total value of two capacitors is calculated.

$$C_t=\frac{g_{10}+mh_{10}}{g_{10}-mh_{10}}=\frac{11/2+(-1)(-11/2)}{1-(-1)0}=11 \tag{5.11}$$

Where $m=\frac{g_{10}}{h_{10}}=-1$.

After this process, the following parameters can be formed via equation (3.117).

$$h_i(p)= -\frac{11}{2}p, \quad g_i(p)=\frac{11}{2}p+1, \quad f_i(p)=1 \qquad (5.12)$$

With using the last column instead of first one, the capacitor can be calculated via equation (3.119).

$$C_1=\frac{g_{22}+mh_{22}}{g_{12}-mh_{12}}=\frac{189+(-1)(-189)}{63/2-(-1)(21/2)}=9 \quad m=\frac{g_{22}}{h_{22}}= -1 \qquad (5.13)$$

With using equation (3.117) and the value C_1, parameters of $T(p)$ can be written as:

$$h(p)= -\frac{9}{2}p, \quad g(p)=\frac{9}{2}p+1, \quad f(p)=1 \qquad (5.14)$$

Next step is the formation of remaining scattering transfer matrix via equation (3.116).

$$T_R(p,\lambda)=T^{-1}(p)T(p,\lambda) \qquad (5.15)$$

$$T_R(p,\lambda)=\begin{bmatrix} (-9p/2)+1 & -9p/2 \\ 9p/2 & \frac{9p}{2}+1 \end{bmatrix}^{-1} \times T(p,\lambda) \qquad (5.16)$$

The value of M_{Rg} and M_{hr} are parameters of $T_R(p,\lambda)$ calculation according to the equation (5.14) with $f(p,\lambda)=f(p)f(\lambda)=1(1-\lambda^2)$. The results are:

$$M_{Rg}=\begin{bmatrix} 1 & 110/21 & 29/21 \\ 1 & 10 & 21 \end{bmatrix} \quad M_{Rh}=\begin{bmatrix} 0 & 100/21 & 20/21 \\ -1 & 4 & 21 \end{bmatrix}$$

With using first column of these matrices, the last capacitor value can be calculated because the lumped section is not a degenerate network anymore.

$$C_2=\frac{g_{10}+mh_{10}}{g_{00}-mh_{00}}=\frac{1+(-1)(-1)}{1-(1)0}=2, \quad m= g_{10}/h_{10}=-1 \qquad (5.17)$$

The characteristic impedance value of transmission lines are $Z_1=7$ and $Z_2=3$ via (Şengül, 2008). The details are as:

$$g(\lambda)=1+\frac{110}{21}\lambda+\frac{29}{21}\lambda^2 \quad \text{and} \quad h(\lambda)=0+\frac{100}{21}\lambda+\frac{20}{21}\lambda^2 \qquad (5.18)$$

For $\lambda=1$, the first impedance in network is calculated as:

$$Z_1=\frac{g(1)+h(1)}{g(1)-h(1)}=\frac{280/21}{40/21}=7 \qquad (5.19)$$

The calculation of the second impedance is according to equations (3.107-114).

$$x_1=h_0g(1)-g_0h(1)= 0\times7.61-1\times5.71= -5.71 \qquad (5.20)$$

$$x_2=h_1g(1)-g_1h(1)= 4.76\times7.61-5.23\times5.71=6.36 \qquad (5.21)$$

$$x_3=h_2g(1) -g_2h(1)= 0.95\times7.61-1.38\times5.71= -0.65 \qquad (5.22)$$

$$y_1=g_0g(1)-h_0h(1)=1\times7.61-0\times5.71=7.61 \qquad (5.23)$$

$$y_2=g_1g(1)-h_1h(1)=5.23\times7.61-4.76\times5.71=12.62 \qquad (5.24)$$

$$y_3=g_2g(1)-h_2h(1)=1.38\times7.61-0.95\times5.71=5.07 \qquad (5.25)$$

$$N_1= x_1= -5.71 \qquad (5.26)$$

$$N_2= x_1+x_2= 0.65 \qquad (5.27)$$

$$N_3= x_1+x_2+x_3= 0 \qquad (5.28)$$

$$D_1= y_1= 7.61 \qquad (5.29)$$

$$D_2 = y_2 - y_1 = 5.01 \tag{5.30}$$

$$D_3 = y_3 - y_2 + y_1 = 0.06 \tag{5.31}$$

$$g^2(\lambda) = D_1 + D_2\lambda + D_3\lambda^2 = 12.68 \tag{5.32}$$

$$h^2(\lambda) = N_1 + N_2\lambda + N_3\lambda^2 = -5.06 \tag{5.33}$$

$$Z_2 = Z_1 \frac{g^{(2)}(1) + h^{(2)}(1)}{g^{(2)}(1) - h^{(2)}(1)} = 7\frac{7.62}{17.74} = 3 \tag{5.34}$$

All of the impedance values are calculated.

6. CONCLUSION

In this thesis, the analysis of the mixed element structure formed with shunt capacitors separated by equal length transmission lines has been performed first time in the literature. The description of the structure by means of two frequency variables (one for shunt capacitors and one for transmission lines) has been given. Then broadband matching networks for many applications have been designed by using this practically important mixed structure via the algorithm that has been developed. In the algorithm, the explicit coefficient relations of the descriptive two-variable polynomials in terms of the coefficients of the single variable boundary polynomials have been derived for various numbers of elements. These coefficient relations have been obtained first time in the literature. Since the lumped section is a degenerate network, it is impossible to use the two-variable polynomials to calculate the capacitor values. So a synthesis algorithm for the structure has been developed to be able to calculate the capacitor values from the two variable polynomials.

If it is preferred not to have shunt capacitors, they can be replaced with open-ended stubs via Richard's transformation and the resultant circuit will be extremely suitable for microstrip fabrication.

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APPENDIX A

A.1 Participation Certificate

APPENDIX B

B.1 Matlab Codes Main Program

```
clc
tic
clear
syms L fr
global m2 dist lump w mu SG SL T0 ZG ZL f_poly f_p nC nUE


%*****source and load impedance, souce and load relection coefficient
calculation*****
w=0:0.1:1;
z=(i.*fr.*2)+1/(1+i*fr*1);
ZL=subs(z,fr,w);
r11=1;
r33=i.*fr.*1;
z11=r11+r33;
z11=simplify(z11);
ZG=subs(z11,fr,w);
% ZG=ones(1,length(w));
SG=(ZG-1)./(ZG+1);
SL=(ZL-1)./(ZL+1);
%****************************************************************

%*****Initial values****************************************
nC=input('Enter number of Cs:'); % örnek için 3
nUE=input('Enter number of UEs:'); %örnek için 2
if nUE>1
    hj0=[-1 1 -1 1]; %lumped, son kolon elemanларý
    h0i=[-1]; %dist, ilk satýr elemanларý (en düþük ve en yüksek
dereceli girilmiyor)
else
    hj0=[1 1 -1]; %lumped, son kolon elemanларý
end
T0=0.98; %gain
thau=0.6; %delay
%****************************************************************
```

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```matlab
%*****Optimisation vector construction**************************
dist=nUE;
lump=nC;
dimension=dist+lump;


if dist==lump
    m2=input('Enter m2 value (+1/-1):'); %örnek için +1 gir
end


for a=1:lump+1;
    v(a)=hj0(a);
end
if nUE>1
    for a=1:dist-1;
        v(lump+1+a)=h0i(a);
    end
    v(dimension+2)=thau;
else
    v(dimension+1)=thau;
end
%****************************************************************
f_p=(1-L^2)^(dist/2);
mu=1;
%*****optimisation part*****************************************
const=length(v);
LB=[ones(1,const-1).*(-Inf) 0];
UB=ones(1,const).*Inf;


OPTIONS=optimset('MaxFunEvals',1000,'MaxIter',4000);
v_new = lsqnonlin('error_find',v,[],[],OPTIONS);
%****************************************************************


%*****Gettin h0i, hj0 and thau after optimisation**************
for a=1:lump+1;
    hj0(a)=v_new(a);
end
if nUE>1
```

```
        h0i(1)=hj0(length(hj0));
        for a=1:dist;
            h0i(a+1)=v_new(lump+1+a);
        end
        h0i(dist+1)=0;
        thau=v_new(dist+lump+2);
    else
        h0i(1)=hj0(length(hj0));
        h0i(dist+1)=0;
        thau=v_new(dist+lump+1);
    end


%****************************************************************
gj0=LLEL(hj0,[zeros(1,length(hj0)-1) 1]); %lumped
g0i=LLELd(h0i,f_p); %dist

%*****Calculation of optimised h and g matrices*******************
if dist==1 & lump==1;
    m1=-1;
    [Ah,Ag]=hg11n(h0i,hj0,m1,f_p);
elseif dist==1 & lump==2;
    m1=-1;
    m2=-1;
    [Ah,Ag]=hg12n(h0i,hj0,m1,f_p);
elseif dist==2 & lump==1;
     m1=-1;
     [Ah,Ag]=hg21n(h0i,hj0,m1,f_p);
elseif dist==2 & lump==2;
    m1=-1;
    [Ah,Ag]=hg22n(h0i,hj0,m1,m2,f_p);
elseif dist==2  & lump==3;
    m1=-1;
    [Ah,Ag]=hg23n(h0i,hj0,m1,f_p);
end


Ah
Ag
thau
%****************************************************************
```

```
%****Calculation of load and source impedances, source and load
reflection coefficients over new frequency range*****
w=0:0.1:2;
ZL=subs(z,fr,w);
ZG=subs(z11,fr,w);
% ZG=ones(1,length(w));
SG=(ZG-1)./(ZG+1);
SL=(ZL-1)./(ZL+1);


%****************************************************************


%*****According to optimised h and g matrices, getting the values of
h, hpara, g, gpara and f*****
for a=1:length(w);
    d=i*tan(w(a)*thau);
    hv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            hv(a)=hv(a)+Ah(b,c)*((i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end


for a=1:length(w);
    d=-i*tan(w(a)*thau);
    hpv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            hpv(a)=hpv(a)+Ah(b,c)*((-i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end


for a=1:length(w);
    d=i*tan(w(a)*thau);
    gv(a)=0;
    for b=1:lump+1;
```

```
        for c=1:dist+1;
            gv(a)=gv(a)+Ag(b,c)*((i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end


for a=1:length(w);
    d=-i*tan(w(a)*thau);
    gpv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            gpv(a)=gpv(a)+Ag(b,c)*((-i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end


%***************************************************************
fv=subs(f_p,L,i.*tan(w.*thau));
fpv=conj(fv);


%*****Calculation of tpg over the frequency range*****************
S22=-mu.*hpv./gv;
S12=mu.*fpv./gv;
S21=fv./gv;
S11=hv./gv;


SL=(ZL-1)./(ZL+1);
S1=S11+(S12.*S21.*SL)./(1-S22.*SL);
Z11=(1+S1)./(1-S1);
r1=(Z11-conj(ZG))./(Z11+ZG);
SG=(ZG-1)./(ZG+1);
S2=S22+(S12.*S21.*SG)./(1-S11.*SG);
Z22=(1+S2)./(1-S2);


tpg=(4.*real(ZL).*real(Z22))./((real(ZL)+real(Z22)).^2+(imag(ZL)+imag(
Z22)).^2);
%***************************************************************
```

```
%*****Plotting the result*****
hold on
renk=(round(rand(3,1)))';
div=(round(rand(3,1)))'+1;
color=[renk(1)/div(1) renk(2)/div(2) renk(3)/div(3)];
plot(w,T0,'r*',w,tpg,'color',color)
axis([0 2 0 1])
%***************************************************************
toc
% %*************************************************************
```

B.2 Matlab Codes Error Calculation

```
function eps=error(v,m2,dist,lump,w,mu,SG,SL,T0,ZG,ZL,f_poly,f_p)
%*****error sub-program*****
syms L fr


global m2 dist lump w mu SG SL T0 ZG ZL f_poly f_p nC nUE


%*****calculation of h0i, hj0 and thau from optimisation vector*****
dimension=dist+lump;


for a=1:lump+1;
    hj0(a)=v(a);
end
if nUE>1
    h0i(1)=hj0(length(hj0));
    for a=1:dist;
        h0i(a+1)=v(lump+1+a);
    end
    h0i(dist+1)=0;
    thau=v(dist+lump+2);
else
    h0i(1)=hj0(length(hj0));
    h0i(dist+1)=0;
    thau=v(dist+lump+1);
end
%***************************************************************
```

```matlab
%*****Calculation of h and g matrices****************************
if dist==1 & lump==1;
    m1=-1;
    [Ah,Ag]=hg11n(h0i,hj0,m1,f_p);
elseif dist==1 & lump==2;
    m1=-1;
    m2=-1;
    [Ah,Ag]=hg12n(h0i,hj0,m1,f_p);
elseif dist==2 & lump==1;
    m1=-1;
    [Ah,Ag]=hg21n(h0i,hj0,m1,f_p);
elseif dist==2 & lump==2;
    m1=-1;
    [Ah,Ag]=hg22n(h0i,hj0,m1,m2,f_p);
elseif dist==2  & lump==3;
    m1=-1;
    [Ah,Ag]=hg23n(h0i,hj0,m1,f_p);
end
%****************************************************************


%*****calculation of h, hpara, g, gpara and f values**************
for a=1:length(w);
    d=i*tan(w(a)*thau);
    hv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            hv(a)=hv(a)+Ah(b,c)*((i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end

for a=1:length(w);
    d=-i*tan(w(a)*thau);
    hpv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            hpv(a)=hpv(a)+Ah(b,c)*((-i*w(a))^(b-1))*(d)^(c-1);
        end
```

```matlab
        end
    end


for a=1:length(w);
    d=i*tan(w(a)*thau);
    gv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            gv(a)=gv(a)+Ag(b,c)*((i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end


for a=1:length(w);
    d=-i*tan(w(a)*thau);
    gpv(a)=0;
    for b=1:lump+1;
        for c=1:dist+1;
            gpv(a)=gpv(a)+Ag(b,c)*((-i*w(a))^(b-1))*(d)^(c-1);
        end
    end
end


fv=subs(f_p,L,i.*tan(w.*thau));
fpv=conj(fv);
%**************************************************************

%*****calculation of tpg*************************************
S22=-mu.*hpv./gv;
S12=mu.*fpv./gv;
S21=fv./gv;
S11=hv./gv;


SL=(ZL-1)./(ZL+1);
S1=S11+(S12.*S21.*SL)./(1-S22.*SL);
Z11=(1+S1)./(1-S1);
r1=(Z11-conj(ZG))./(Z11+ZG);
SG=(ZG-1)./(ZG+1);
```

```matlab
S2=S22+(S12.*S21.*SG)./(1-S11.*SG);
Z22=(1+S2)./(1-S2);


tpg=(4.*real(ZL).*real(Z22))./((real(ZL)+real(Z22)).^2+(imag(ZL)+imag(
Z22)).^2);
%*************************************************************


eps=double(sum(((tpg-T0)./tpg).^2))
%*************************************************************
return
```

B.3 Matlab Codes Last Column of g Matrix

```matlab
function gj0=gj0find(hj0);
H_S=hj0;
boy=length(H_S);
Fara=zeros(1,(boy-1));
Fara(1,boy)=1;
F_S=Fara;
HS=paraconjugate(H_S);
FS=paraconjugate(F_S);
% Constructing the even polynomial
G = conv(H_S , HS);
F = conv(F_S,FS);


G_S = G + F;
    RO = roots(G_S);
  s = length(RO);
  for h = 1 : s
    if real (RO(h)) < 0
        rootlhp(h)= RO(h);
    else
        rootlhp(h)= 0;
    end
end
[k , l , m ] = find(rootlhp);
g_s = poly (m);
g0iara=g_s;
```

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```
b=length(g_s);
for a=1:b;
    g_s(a)=g_s(a)/g0iara(b);
end
gj0=g_s;
```

B.4 Matlab Codes First Row of g Matrix

```
%**********************************************************
%  function for calculating the Hurwitz polynomial    *
%                     g_Lambda                          *
%                                                       *
%    the arguments are passed from the main code        *
%                                                       *
%                                                       *
%**********************************************************
function g0i=g0ifind(h0i)
%h0i=[-0.824 -4.325 0];
h_Lambda=h0i;
n_Lambda=length(h0i)-1;
F_temp=[-1 0 1];
F_Lambda=[-1 0 1];
for i=1:(n_Lambda - 1)
    F_Lambda = conv(F_Lambda,F_temp);
end
HP_Lambda = paraconjugate(h_Lambda);
% Constructing the even polynomial
G = conv(h_Lambda , HP_Lambda);
GP_Lambda = G + F_Lambda;
RO = roots(GP_Lambda);
s = length(RO);
for h = 1 : s
    if real (RO(h)) < 0
        rootlhp(h)= RO(h);
    else
        rootlhp(h)= 0;
    end
end
[k , l , m ] = find(rootlhp);
```

```
g_Lambda = poly (m);
g_Lambda = g_Lambda./g_Lambda(length(g_Lambda));
g0i=g_Lambda;
```

B.5 Matlab Codes Paraconjugate Calculations

```
%**********************************************************
%function for calculating paraconjugate polynomial        *
%
%the arguments are passed from the main code                    *
%                                                                  *
%this function finds the paraconjugate of PS polynomial  *
%**********************************************************


function P_S=paraconjugate (PS)
c = length (PS)-1;
for k = 0:c
   if rem(k , 2)== 0
      P_S(c-k+1) = (+1)*PS(c-k+1);
   else
      P_S(c-k+1) = (-1)*PS(c-k+1);
   end
end
return
```

B.6 Matlab Codes One Capacitor and One UE

```
function [Ah,Ag]=hg11n(h0i,hj0,m1,f_p);
%*****hg11 sub-program*****

%*****calculation of h and g matrices of the networks contains one
distributed and lumped elements*****
gj0=LLEL(hj0,[zeros(1,length(hj0)-1) 1]); %lumped
g0i=LLELd(h0i,f_p); %dist

h00=h0i(2);
h01=h0i(1);
g00=g0i(2);
```

```
g01=g0i(1);
h11=hj0(1);
g11=gj0(1);


alfa=g01-m1*h01;
g10=g11/alfa;
h10=m1*g10;


Ah=[h00 h01;h10 h11];
Ag=[g00 g01;g10 g11];
%****************************************************************


return
```

B.7 Matlab Codes Two Capacitors and One UE

```
function [Ah,Ag]=hg12n(h0i,hj0,m1,f_p)
%*****hg12 sub-program*****


%*****calculation of h and g matrices of the networks contains one
distributed and two lumped elements*****
gj0=LLEL(hj0,[zeros(1,length(hj0)-1) 1]); %lumped
g0i=LLELd(h0i,f_p); %dist


h00=h0i(2);
h01=h0i(1);
h11=hj0(2);
h21=hj0(1);
g00=g0i(2);
g01=g0i(1);
g11=gj0(2);
g21=gj0(1);
alfa=g01-m1*h01;
g10=g11/alfa;
h10=m1*g10;
h20=0;
g20=0;
```

```
Ah=[h00 h01;h10 h11;h20 h21];
Ag=[g00 g01;g10 g11;g20 g21];
%**************************************************************
 Return
```

B.8 Matlab Codes One Capacitor and Two UEs

```
function [Ah,Ag]=hg21n(h0i,hj0,m1,f_p)
%*****hg21 sub-program*****

%*****calculation of h and g matrices of the networks contains two
distributed and one lumped elements*****
gj0=LLEL(hj0,[zeros(1,length(hj0)-1) 1]); %lumped
g0i=LLELd(h0i,f_p); %dist

h00=h0i(3);
h01=h0i(2);
h02=h0i(1);
h12=hj0(1);
g00=g0i(3);
g01=g0i(2);
g02=g0i(1);
g12=gj0(1);
alfa=g01-m1*h01;
beta=g12-m1*h12;
g11=g02*(g01*g12-h01*h12)-h02^2*(beta/alfa);
g10=g11/alfa;
h11=(g11*g02-g01*g12+h01*h12)/(h02);
h10=m1*g10;

Ah=[h00 h01 h02;h10 h11 h12];
Ag=[g00 g01 g02;g10 g11 g12];
%**************************************************************

return
```

B.9 Matlab Codes Two Capacitors and Two UEs

```
function [Ah,Ag]=hg22n(h0i,hj0,m1,m2,f_p)
%*****hg22 sub-program*****


%*****calculation of h and g matrices of the networks contains two
distributed and two lumped elements*****
gj0=LLEL(hj0,[zeros(1,length(hj0)-1) 1]); %lumped
g0i=LLELd(h0i,f_p); %dist


h00=h0i(3);
h01=h0i(2);
h02=h0i(1);
h12=hj0(2);
h22=hj0(1);
g00=g0i(3);
g01=g0i(2);
g02=g0i(1);
g12=gj0(2);
g22=gj0(1);
alfa=g01-m1*h01;
beta=g12-m1*h12;
g11=g02*(g01*g12-h01*h12)-
h02^2*(beta/alfa)+h02^2*((alfa*((g02*h22/h02)-g22))/beta);
g10=g11/alfa;
h11=(g11*g02-g01*g12+h01*h12)/(h02);
g21=(g11*g22-h11*h22)/beta;
h10=m1*g10;
h22=m2*g22;
h21=m1*g21;
h20=0;
g20=0;


Ah=[h00 h01 h02;h10 h11 h12;h20 h21 h22];
Ag=[g00 g01 g02;g10 g11 g12;g20 g21 g22];
return
```

B.10 Matlab Codes Three Capacitors and Two UEs

```
function [Ah,Ag]=hg23n(h0i,hj0,m1,f_p)
%*****hg23 sub-program*****


%*****calculation of h and g matrices of the networks contains two
distributed and three lumped elements*****
gj0=LLEL(hj0,[zeros(1,length(hj0)-1) 1]); %lumped
g0i=LLELd(h0i,f_p); %dist


h00=h0i(3);
h01=h0i(2);
h02=h0i(1);
h12=hj0(3);
h22=hj0(2);
h32=hj0(1);
g00=g0i(3);
g01=g0i(2);
g02=g0i(1);
g12=gj0(3);
g22=gj0(2);
g32=gj0(1);


alfa=g01-m1*h01;
beta=g12-m1*h12;
g11=g02*(g01*g12-h01*h12)-
h02^2*(beta/alfa)+h02^2*((alfa*((g02*h22/h02)-g22))/beta);
g10=g11/alfa;
h11=(g11*g02-g01*g12+h01*h12)/(h02);
g21=(g11*g22-h11*h22-g01*g32+h01*h32)/beta;
h10=m1*g10;
h21=m1*g21;
h20=0;
g20=0;
h30=0;
g30=0;
h31=0;
g31=0;
```

```
Ah=[h00 h01 h02;h10 h11 h12;h20 h21 h22;h30 h31 h32];
Ag=[g00 g01 g02;g10 g11 g12;g20 g21 g22;g30 g31 g32];
%**************************************************************


return
```