

Metin Şengül*

Reflection Modeling Based Broadband Matching Network Design

DOI 10.1515/freq-2016-0040

Received February 12, 2016

Abstract: In this paper, a new broadband matching network design approach based on reflection modeling is proposed, which has two parts: impedance data generation and modeling. In the approach, firstly the output impedance data of the matching network is obtained to get the desired flat transducer power gain in the pass-band. Next the output reflection data are calculated using the obtained impedance data, then they are modeled as a bounded real function. Then this function is synthesized and the desired lossless matching network with initial element values is obtained. A double matching example is solved to illustrate the use of the proposed approach. It is seen that proposed approach provides suitable initials for CAD tools for final trimming.

Keywords: broadband matching, real frequency techniques, matching network, lossless networks, modeling

1 Introduction

Let us investigate the double matching problem seen in Figure 1. Since the matching network is lossless, on the imaginary axis of the complex frequency plane, input and output reflection functions (ρ_1 and ρ_2) are related by the following equation [1]

$$|\rho_1|^2 = |\rho_2|^2. \quad (1)$$

Then the transducer power gain (TPG) can be defined as

$$TPG(\omega) = 1 - |\rho_1|^2 = 1 - |\rho_2|^2. \quad (2)$$

Here the reflection function $\rho_2 = a_2 + jb_2$ at port 2 is defined by

$$\rho_2 = \frac{Z_2 - Z_L^*}{Z_2 + Z_L} \quad (3)$$

where $Z_2 = R_2 + jX_2$ is the output impedance of the matching network and $Z_L = R_L + jX_L$ is the load impedance. Then

*Corresponding author: Metin Şengül, Department of Electrical and Electronics Engineering, Faculty of Engineering and Natural Sciences, Kadir Has University, 34083 Cibali, Fatih, Istanbul, Turkey, E-mail: msengul@khas.edu.tr

by means of (2), (3) and open forms of Z_2 and Z_L , TPG can be expressed in terms of Z_L and Z_2 as follows [2]

$$TPG = \frac{4R_2R_L}{(R_2 + R_L)^2 + (X_2 + X_L)^2} \quad (4).$$

While designing broadband matching networks, the fundamental problem is the determination of a realizable impedance function Z_2 ; which must maximize TPG given by (4) inside an interested frequency band. If such an impedance function can be found, then it is easy to obtain the desired matching network with initial element values.

In the literature, there are lots of works aiming to obtain a realizable impedance function Z_2 . In the real frequency line segment technique (RF-LST), Z_2 is formed as a minimum reactance function and line segments are used to represent its real part R_2 . Then, these line segments are used to optimize the gain performance of the matching network [3], [4].

In direct computational technique (DCT), the real part (R_2) is expressed as a real even rational function [5]. Then the unknown coefficients of this function are optimized to get optimum gain performance.

In another method proposed by Fettweis, Z_2 is expressed as a partial fraction expansion, and then the poles of Z_2 are optimized to obtain optimum gain performance of the system [6].

In the Simplified Real Frequency Technique (SRFT), the lossless matching network is described by means of the scattering matrix which is represented by three real polynomials [7], [8].

In [9], a ratio ($\alpha = R_2/R_L$) is defined under the perfect cancellation condition of the imaginary parts (i.e., $X_2 = -X_L$). Then the output impedance $Z_2 = \alpha R_L - jX_L$ is modeled as a minimum reactance function, and then (if necessary) a Foster impedance is used in series.

In the method proposed in [10], [11], the input or output impedance of the matching network is written in terms of the input or output reflection coefficient, respectively.

But in the proposed approach here, the real part of the output impedance (R_2) is expressed in terms of the real part of the reflection function (a_2), the real part of the load impedance (R_L) and the desired transducer power

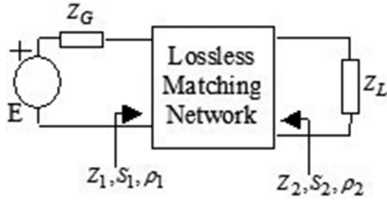


Figure 1: Double matching arrangement.

gain (TPG). In the next section, the rationale of the proposed approach is explained.

2 Rationale of the proposed approach

As mentioned above, a ratio ($\alpha = R_2/R_L$) is defined in [9] between the real part of the output impedance (R_2) and the real part of the load impedance (R_L). In this approach it is assumed that there is a perfect cancellation at the imaginary parts (i. e., $X_2 = -X_L$).

Under perfect imaginary part cancellation, TPG can be written via (4) as

$$TPG = \frac{4R_2R_L}{(R_2 + R_L)^2} \quad (5)$$

then the ratio α is obtained as

$$\alpha = \frac{R_2}{R_L} = \frac{(2 - TPG) + 2\mu\sqrt{1 - TPG}}{TPG} \quad (6)$$

where $\mu = \pm 1$ is a uni-modular constant.

A similar ratio can be defined between the real parts of the output and load impedances without any assumption, i. e., perfect imaginary part cancellation. In the derivation, output reflection function ($\rho_2 = a_2 + j b_2$) will be employed.

From (3), the output impedance can be derived as

$$Z_2 = \frac{\rho_2 Z_L + Z_L^*}{1 - \rho_2} \quad (7)$$

If $Z_L = R_L + jX_L$ is substituted in (7), then the real and imaginary parts can be written as

$$Z_2 = R_L \frac{1 + \rho_2}{1 - \rho_2} + jX_L \frac{\rho_2 - 1}{1 - \rho_2} \quad (8)$$

Now, let us substitute $\rho_2 = a_2 + jb_2$ in (8) and use $TPG = 1 - |\rho_2|^2$, then the real and imaginary parts of the output impedance are calculated as

$$R_2 = \frac{TPG}{2(1 - a_2) - TPG} R_L \quad (9a)$$

$$X_2 = \frac{2b_2}{2(1 - a_2) - TPG} R_L - X_L \quad (9b)$$

It is clear that the real part of the output impedance (R_2) depends on the real part of the load impedance (R_L). But the imaginary part of the output impedance (X_2) depends on both the real and imaginary parts of the load impedance (R_L and X_L), not only on X_L .

As can be seen the ratio α is obtained as

$$\alpha = R_2/R_L = \frac{TPG}{2(1 - a_2) - TPG}. \quad (10)$$

Under perfect matching conditions ($TPG = 1$ and $\rho_2 = 0$), $R_2 = R_L$ and $X_2 = -X_L$ can be obtained via (9a) and (9b), respectively, as expected.

Once Z_2 data is generated, the corresponding reflection coefficient data are calculated via $S_2 = (Z_2 - 1)/(Z_2 + 1)$ and modeled as a bounded real function. Finally, the reached reflectance model is synthesized and the desired matching network with initial element values is obtained. If necessary, the performance of the matched system can be optimized using the commercially available CAD packages.

3 Reflection coefficient data modeling

The output impedance $Z_2 = R_2 + jX_2$ must be positive real. Although the real and imaginary part expressions are obtained (eqs (9a) and (9b)), it is not guaranteed that the obtained output impedance data are realizable or belong to a positive real function. So only the calculated real part data will be employed, the imaginary part data will be disregarded. From now on, it is going to be assumed that the output impedance Z_2 is a minimum reactance function and its imaginary part is calculated by using the Hilbert transformation relation such that [1]–[3]

$$X_2(\omega) = H\{R_2(\omega)\}. \quad (11a)$$

The required numerical Hilbert transform can be realized as follows [2]:

$$X_2(\omega) = \sum_{j=1}^{N-1} B_j(\omega) \Delta R_2^{(j)} \quad (11b)$$

where

$$B_j(\omega) = \frac{1}{\pi(\omega_j - \omega_{j+1})} [F_{j+1}(\omega) - F_j(\omega)] \quad (11c)$$

$$\Delta R_2^{(j)} = R_2^{(j)} - R_2^{(j+1)} \quad (11d)$$

and

$$F_j(\omega) = (\omega + \omega_j) \ln(|\omega + \omega_j|) + (\omega - \omega_j) \ln(|\omega - \omega_j|). \quad (11e)$$

Then the reflectance data specified by $S_2 = (Z_2 - 1)/(Z_2 + 1)$ will be considered as the input reflection coefficient of a lossless matching network and it will be modeled as a bounded real scattering coefficient as follows [12]

$$S_2 = S_R + jS_X = S_{22} + \frac{S_{12}S_{21}S_G}{1 - S_{22}S_G} \quad (12)$$

where S_R and S_X are the real and imaginary parts of S_2 , and S_G is the source reflection coefficient written as

$$S_G = \frac{Z_G - 1}{Z_G + 1} \quad (13)$$

where $Z_G = R_G + jX_G$ is the generator impedance.

The other parameters in (12) are the scattering parameters of the lossless matching network and they are described in terms of three real polynomials as follows [1]:

$$S(p) = \begin{bmatrix} S_{11}(p) & S_{12}(p) \\ S_{21}(p) & S_{22}(p) \end{bmatrix} = \begin{bmatrix} h(p)/g(p) & \mu f(-p)/g(p) \\ f(p)/g(p) & -\mu h(-p)/g(p) \end{bmatrix} \quad (14)$$

where g is a strictly Hurwitz polynomial, h is a real polynomial, f is a real monic polynomial and μ is a unimodular constant ($\mu = \pm 1$), $p = \sigma + j\omega$ is the frequency variable. If the matching network is reciprocal, then the polynomial f is either even or odd and $\mu = f(-p)/f(p)$.

These three polynomials $\{f, g, h\}$ have the following relationship, which is known as the Feldtkeller equation [4]

$$g(p)g(-p) = h(p)h(-p) + f(p)f(-p). \quad (15)$$

It can be concluded from (15) that the polynomial $g(p)$ is a function of the polynomials $h(p)$ and $f(p)$. This means that if the polynomials $f(p)$ and $h(p)$ is known, then the strictly Hurwitz polynomial $g(p)$ is formed by using the left-hand side roots and then the matching network can be completely defined.

Since the designer has an idea about the transmission zero locations, the designer construct the polynomial $f(p)$ on the transmission zeros of the matching network. For practical problems, the transmission zeros can be put on the imaginary axis of the complex p -plane, then the following form of $f(p)$ can be used,

$$f(p) = p^{m_1} \prod_{i=0}^{m_2} (p^2 + a_i^2) \quad (16)$$

where m_1 and m_2 are nonnegative integers and a_i 's are arbitrary real coefficients [1].

4 Proposed approach

The proposed approach can be divided into an impedance data generation algorithm and reflectance modeling algorithm.

4.1 Impedance data generation algorithm

Inputs:

- $Z_{L(measured)} = R_{L(measured)} + jX_{L(measured)}$: Measured load impedance data.
- $\omega_{i(measured)}$: Measurement frequencies, $\omega_{i(measured)} = 2\pi f_{i(measured)}$.
- f_{norm} : Frequency normalization number.
- R_{norm} : Normalization impedance in ohms.
- TPG : Desired flat transducer power gain level in the passband.
- $a_2^{(i)}$: Initial real numbers for the real part of the output reflection function. The upper and lower limits of a_2 can be defined as follows: By using $\rho_2 = a_2 + jb_2$ in (2), transducer power gain is written as $TPG = 1 - a_2^2 - b_2^2$. Then the imaginary part b_2 is expressed as $b_2 = \sqrt{1 - TPG - a_2^2}$. Since a_2 and b_2 numbers must be real, then $1 - TPG - a_2^2 \geq 0$. So a_2 must be between $-\sqrt{1 - TPG}$ and $+\sqrt{1 - TPG}$.
- δ_1 : The stopping criterion of the sum of the square errors.

Outputs:

- The real (R_2) and imaginary part (X_2) data of the output impedance Z_2 . In this approach, Z_2 is assumed to be minimum reactance impedance, since the imaginary part data X_2 are obtained via the Hilbert transform of the real part data R_2 .

Computational Steps:

Step 1: Normalize the measurement frequencies with respect to frequency normalization number f_{norm} and set all the normalized angular frequencies

$$\omega_i = f_{i(measured)} / f_{norm}.$$

Normalize the measured load impedance with respect to impedance normalization number R_{norm} ; $R_L = R_{L(measured)} / R_{norm}$, $X_L = X_{L(measured)} / R_{norm}$ over the entire frequency band.

Step 2: Calculate $R_2(\omega)$ values via (9a).

Step 3: Calculate $X_2(\omega)$ values via Hilbert transform, $X_2(\omega) = H\{R_2(\omega)\}$.

- Step 4:** Calculate transducer power gain (TPG_{Cal}) via (4)
- Step 5:** Calculate the error via $\varepsilon_1(\omega) = TPG - TPG_{Cal}$, then $\delta_Z = \sum |\varepsilon_1(\omega)|^2$.
- Step 6:** If δ_Z is acceptable ($\delta_Z \leq \delta_1$), stop the algorithm and go to the reflectance modeling algorithm. Otherwise, change the initialized $a_2^{(i)}$ values via any constrained optimization routine and go to step 2.

4.2 Reflectance modeling algorithm

Inputs:

- R_2, X_2 : The real and imaginary part data obtained from the impedance data generation algorithm.
- $Z_{G(measured)} = R_{G(measured)} + jX_{G(measured)}$: Measured generator impedance data.
- $\omega_{i(measured)}$: Measurement frequencies, $\omega_{i(measured)} = 2\pi f_{i(measured)}$.
- f_{norm} : Frequency normalization number.
- R_{norm} : Normalization impedance in ohms.
- $h_0, h_1, h_2, \dots, h_n$: Initial real coefficients of the polynomial $h(p)$. n is also the degree of the polynomial $h(p)$. h_i can be initialized as ± 1 or the approach given in [13] can be used.
- $f(p)$: This polynomial is formed by means of the transmission zeros of the matching network. The form given in (16) can be used if a ladder type minimum phase structure is desired.
- δ_2 : The stopping criteria, which can be calculated by summing the squared errors.

Outputs:

- $S_2(p) = h(p)/g(p)$: Reflection coefficient in analytic form.

Computational Steps:

- Step 1:** Normalize the measurement frequencies with respect to frequency normalization number f_{norm} and set all the normalized angular frequencies

$$\omega_i = f_{i(measured)} / f_{norm}$$

Normalize the measured generator impedance with respect to impedance normalization number R_{norm} ; $R_G = R_{G(measured)} / R_{norm}$, $X_G = X_{G(measured)} / R_{norm}$ over the entire frequency band.

- Step 2:** Calculate the output impedance data via $Z_2 = R_2 + jX_2$, and then compute $S_2 = \frac{Z_2 - 1}{Z_2 + 1}$ reflection coefficient data.

- Step 3:** Calculate the polynomial $g(p)$ via (15).

- Step 4:** Compute scattering parameters via (14).

- Step 5:** Calculate the source reflection coefficient via (13).

- Step 6:** Compute the reflection coefficient ($S_{2,Cal}$) via (12) by means of the obtained scattering parameters and source reflection coefficient.

- Step 7:** Calculate the error via $\varepsilon_2(\omega) = S_2 - S_{2,Cal}$, then $\delta_S = \sum |\varepsilon_2(\omega)|^2$.

- Step 8:** If δ_S is acceptable ($\delta_S \leq \delta_2$), stop the algorithm. The circuit topology with element values is obtained as the result of the synthesis of $S_2(p) = h(p)/g(p)$ [14]. Also impedance based Foster or Caer methods can be utilized to synthesize the corresponding impedance function $Z_2(p) = (1 + S_2(p))/(1 - S_2(p))$ as explained in [15]. Otherwise, change the initialized h_i values and go to step 3.

In impedance data generation algorithm, the load impedance (Z_L), and in reflectance modeling algorithm, the generator impedance (Z_G) is used. Since both termination impedances are employed, both single and double broadband matching problems can be solved via the proposed approach.

A similar algorithm can be obtained, if the generator impedance (Z_G) is used in the impedance data generation algorithm, and the load impedance (Z_L) is employed in the reflectance modeling algorithm. In this case, the input impedance (Z_1), input reflection function (ρ_1) and input reflection coefficient (S_1) must be used.

5 Example

In this section, a double matching problem is solved. The normalized load and generator impedance data are given in Table 1. If the given load data is modeled, a capacitor

Table 1: Given normalized load and generator impedance data.

| ω | R_L | X_L | R_G | X_G |
|----------|--------|---------|--------|--------|
| 0.0 | 1.0000 | 0.0000 | 1.0000 | 0.0000 |
| 0.1 | 0.8621 | -0.3448 | 1.0000 | 0.1000 |
| 0.2 | 0.6098 | -0.4878 | 1.0000 | 0.2000 |
| 0.3 | 0.4098 | -0.4918 | 1.0000 | 0.3000 |
| 0.4 | 0.2809 | -0.4494 | 1.0000 | 0.4000 |
| 0.5 | 0.2000 | -0.4000 | 1.0000 | 0.5000 |
| 0.6 | 0.1479 | -0.3550 | 1.0000 | 0.6000 |
| 0.7 | 0.1131 | -0.3167 | 1.0000 | 0.7000 |
| 0.8 | 0.0890 | -0.2847 | 1.0000 | 0.8000 |
| 0.9 | 0.0716 | -0.2579 | 1.0000 | 0.9000 |
| 1.0 | 0.0588 | -0.2353 | 1.0000 | 1.0000 |

$C_L = 4$ in parallel with a resistance $R_L = 1$ (i. e., $R_L // C_L$ type of impedance) is obtained, and the generator data can be modeled as an inductor $L_G = 1$ in series with a resistance $R_G = 1$ (i. e., $R_G + L_G$ type of impedance). The same problem is solved here via SRFT.

In this example, using Fano’s or Youla’s relations [16], [17], the ideal flat gain level is computed as

$$TPG = 1 - \exp(-2\pi/R_L C_L \omega_C)$$

$$= 1 - \exp(-2\pi/1 \cdot 4 \cdot 1) = 0.7921.$$

The real part of the output reflection function is initialized as $a_2 = \sqrt{1 - TPG} = 0.4560$. Also the upper and lower limits of a_2 are set as 0.4560 and -0.4560 , respectively. If the proposed impedance data generation algorithm is run, the data seen in Table 2 are obtained.

Table 2: calculated R_2 and X_2 data.

| ω | R_2 | X_2 |
|----------|--------|---------|
| 0.0 | 0.3737 | 0.0000 |
| 0.1 | 0.3842 | -0.0007 |
| 0.2 | 0.4214 | 0.0037 |
| 0.3 | 0.5104 | 0.0346 |
| 0.4 | 0.7432 | 0.3728 |
| 0.5 | 0.0957 | 0.4959 |
| 0.6 | 0.0996 | 0.2406 |
| 0.7 | 0.1943 | 0.1879 |
| 0.8 | 0.2376 | 0.2775 |
| 0.9 | 0.0268 | 0.2550 |
| 1.0 | 0.1220 | 0.1758 |

The coefficients of the polynomial $h(p)$ are initialized as ± 1 , so $h(p) = -p^4 + p^3 - p^2 + p - 1$. So the number of lossless lumped elements in the matching network is four. The polynomial $f(p)$ is selected as $f(p) = 1$; which means a low-pass matching network. If the proposed reflectance modeling algorithm is run, the following reflection coefficient is obtained

$$S_2(p) = \frac{h(p)}{g(p)} \text{ where}$$

$$h(p) = 1.4020p^4 - 0.6787p^3 - 0.4552p^2 - 1.2898p - 0.5145$$

$$g(p) = 1.4020p^4 + 3.6987p^3 + 4.2595p^2 + 3.2826p + 1.1246.$$

After synthesizing the obtained scattering parameter or the corresponding impedance function, the matching network and normalized element values depicted in Figure 2 are obtained. For the proposed approach, a transformer with a turn ratio of 0.61 must be connected between Z_G and the input of the matching network. But for SRFT, a transformer with a turn ratio of 1.671 must be connected between the output of the matching network and Z_L .

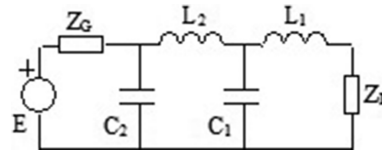


Figure 2: Designed lumped-element double matching network; proposed: $L_1 = 0.64056$, $L_2 = 0.57531$, $C_1 = 5.0001$, $C_2 = 2.4938$, $n = 0.61$, SRFT: $L_1 = 1.8021$, $L_2 = 1.8697$, $C_1 = 1.7871$, $C_2 = 1.5504$, $n = 1.671$.

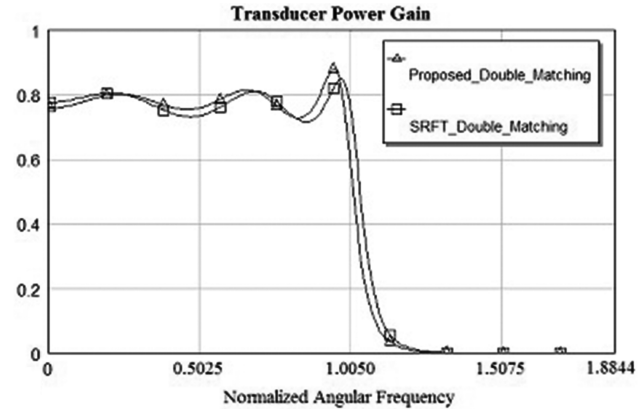


Figure 3: Performance of the matched system designed with lumped elements.

It is clear from Figure 3 that the obtained performance of the matched system is satisfactorily good. However, it can be further improved via optimization utilizing Microwave Office [18]. For comparison purpose, the performances obtained via the proposed approach and via SRFT are shown in Figure 3.

The performances are very close to each other as seen in Figure 3. Consequently, it can be concluded that the proposed approach is an alternative way for producing initial element values for broadband matching problems.

6 Conclusion

Commercially available computer-aided design tools are very useful to optimize system performance by working on the element values. But system performance is highly nonlinear in terms of the element values. So proper initial element value generation is a vital problem. In this paper, a new approach has been proposed to generate proper initial element values.

In lots of the existing broadband matching methods, the output impedance of the matching network is described in terms of a parameter, then this parameter is optimized until maximum possible flat transducer

power gain in the interested frequency band is obtained. But in the proposed method, firstly the real and imaginary part data of the output impedance are obtained. Then the corresponding reflection coefficient data are modeled as a bounded real function.

Finally, this function is synthesized and the designed matching network with initial element values is reached. But the gain performance of the matched system can be further improved by means of a CAD tool.

Also in this paper it is shown that the real part of the output impedance of a matching network depends on the real part of the load impedance, but the imaginary part of the output impedance depends on both real and imaginary parts of the load impedance, except in the perfect match condition.

Distributed or mixed element matching networks can be designed via the proposed approach if the necessary modifications are made in the reflectance modeling algorithm.

The proposed approach has two different algorithms: Impedance data generation and reflectance modeling algorithms. So there are two optimization parts in the approach. This can be regarded as the disadvantage of the approach.

References

- [1] A. Aksen, "Design of lossless two-port with mixed, lumped and distributed elements for broadband matching," Ph.D. dissertation, Ruhr University, Bochum, Germany, 1994.
- [2] B. S. Yarman, *Broadband Networks*. New York, NY: Wiley Encyclopedia of Electrical and Electronics Engineering, 1999, vol. II, pp. 589–604.
- [3] H. J. Carlin, "A new approach to gain-bandwidth problems," *IEEE Trans. CAS*, vol. 23, pp. 170–175, 1977.
- [4] H. J. Carlin and P. P. Civalleri, *Wideband Circuit Design*. Boca Raton, FL: CRC Press LLC, 1998. ISBN: 0-8483-7897-4, Library of Congress Card Number:97 26966.
- [5] H. J. Carlin and B. S. Yarman, "The double matching problem: Analytic and real frequency solutions," *IEEE Trans. Circuits Syst.*, vol. 30, pp. 15–28, Jan. 1983.
- [6] A. Fettweis, "Parametric representation of brune functions," *Int. J. Circuit Theory Appl.*, vol. 7, pp. 113–119, 1979.
- [7] B. S. Yarman and H. J. Carlin, "A simplified real frequency technique applied to broadband multi-stage microwave amplifiers," *IEEE Trans. Microw. Theory Tech.*, vol. 30, pp. 2216–2222, Dec. 1982.
- [8] B. S. Yarman, "A simplified real frequency technique for broadband matching complex generator to complex loads," *RCA Review*, vol. 43, pp. 529–541, Sept. 1982.
- [9] B. S. Yarman, M. Şengül, and A. Kılıç, "Design of practical matching networks with lumped elements via modeling," *IEEE Trans. CAS-I: Regular Papers*, vol. 54, no. 8, pp. 1829–1837, Aug. 2007.
- [10] M. Şengül, "Design of practical broadband matching networks with mixed lumped and distributed elements," *IEEE Trans. CAS-II: Express Briefs*, vol. 61, no. 11, pp. 875–879, Nov. 2014.
- [11] M. Şengül, "Design of practical broadband matching networks with lumped elements," *IEEE Trans. CAS-II: Express Briefs*, vol. 60, no. 9, pp. 552–556, Sept. 2013.
- [12] D. M. Pozar, *Microwave Engineering*, 3rd ed. Hoboken, NJ: Wiley, 2005.
- [13] M. Şengül, B. S. Yarman, C. Volmer, and M. Hein, "Design of distributed-element RF filters via reflectance data modeling," *Int. J. Electron. Commun. (AEU)*, vol. 62, pp. 483–489, Aug. 2008.
- [14] M. Şengül, "Synthesis of resistively terminated LC ladder networks," *Istanbul Univ. J. Electrical Electronics Eng.*, vol. 11, no. 2, pp. 1407–1412, 2011.
- [15] W. C. Yengst, *Procedures of Modern Network Synthesis*. New York: The Macmillian Company, 1964.
- [16] R. M. Fano, "Theoretical limitations on the broadband matching of arbitrary impedances," *J. Franklin Inst.*, vol. 249, pp. 57–83, 1950.
- [17] D. C. Youla, "A new theory of broadband matching," *IEEE Trans. Circuit Theory*, vol. 11, pp. 30–50, Mar. 1964.
- [18] AWR: Microwave Office of Applied Wave Research Inc.: www.appwave.com.