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Artificial neural network based estimation of sparse multipath channels in OFDM systems

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Abstract
In order to increase the transceiver performance in frequency selective fading channel environment, orthogonal frequency division multiplexing (OFDM) system is used to combat inter-symbol-interference. In this work, a channel estimation scheme for an OFDM system in the presence of sparse multipath channel is studied using the artificial neural networks (ANN). By means of ANN's learning capability, it is shown that how to model and obtain a channel estimate and how it allows the proposed technique to give a better system throughput. The performance of proposed method is compared with the Matching Pursuit (MP) and Orthogonal MP (OMP) algorithms that are commonly used in compressed sensing literature in order to estimate delay locations and tap coefficients of a sparse multipath channel. In this work, we propose a performance- efficient ANN based sparse channel estimator with lower computational cost than that of MP and OMP based channel estimators. Even though there is a slight performance lost in a few simulation scenarios in which we have lower computational complexity advantage, in most scenarios, our computer simulations corroborate that our low complexity ANN based channel estimator has better mean squared error and the corresponding symbol error rate performances comparing with MP and OMP algorithms.

Keywords Artificial neural networks · Sparse channel · Channel estimation · Compressed sensing · Matching pursuit · OFDM

1 Introduction

With mounting demands of high data rate mobile communication services, the parallel communication systems have gained popularity in the recent years. OFDM is a digital multi-carrier modulation scheme that divides the available spectrum into multiple equal sub-carriers to transmit the data in parallel over these narrower sub-carriers. These narrowband sub-carriers are mutually orthogonal and can overlap, allowing OFDM systems to use the bandwidth efficiently and be more robust against the ISI caused by the multipath fading. This makes OFDM the most favorable choice for the current and future communication systems like IEEE 802.11a/g/n, IEEE 802.15.3a, IEEE 802.16, IEEE 802.20 in the United States and Digital audio broadcasting (DAB), Digital video broadcasting (DVB), 3-G Long Term Evolution (LTE) in Europe [1,2].

A multipath channel in which most of the channel path coefficients are zero and a few of them are non-zero is called a sparse channel. Some communication problems for OFDM systems, contain channels with large delay spread and a smaller non-zero support [3]. These sparse channels are faced in a number of practical applications like high definition television (HDTV) where there are few echoes but the channel response extends hundreds of data symbols [1]. Underwater acoustics or hilly terrain (HT) delay profiles in broad-band wireless communication systems comprise of a sparsely distributed multipath [4]. A frequency selective channel is a wireless channel whose frequency domain magnitude varies frequency to frequency within the transmission band. Channel estimation of OFDM systems in frequency selective channels with the help of pilot symbols is common and used in many applications [5,6].

Conventional estimation methods applied to nonsparse channels, such as minimum mean square error (MMSE) and least square (LS), exhibit poor performance for sparse channels, therefore in the past few years, sparse channel estimation algorithms have been hot research in compressed

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sensing (CS) area [4]. The pilot assisted channel estimation for OFDM systems aims at reconstructing the channel frequency response from the pilot symbols, therefore, CS theory is considered for pilot-assisted sparse channel estimation with smaller number of pilot symbols [7]. The estimation performance can also be improved and training overhead greatly reduced by using the specially designed, optimized pilot patterns in CS [8,9]. In practice, various pilot designs i.e. block-type, comb-type, and scatter-type are used for different types of channel environments [10]. Estimation of the channel taps is done by one-by-one using MP and OMP algorithms in [3] and the results are collated to thresholded forms of the LS channel estimate. To accurately reconstruct the compressible signal from a few noisy measurements, greedy pursuit (GP) algorithms are preferred. The running speed and reconstruction accuracy of these iterative algorithms are significantly enhanced by refining the selected datasets in each iteration, according to the algorithm known as stage-determined matching pursuit (SdMP) conferred in [11]. Compressive sampling MP (CoSaMP) is used as sparse channel estimator in [12] based on the CoSaMP algorithm devised in [13].

However, all the mentioned multipath sparse channel estimation algorithms either suffer from an error floor or have high computational overhead. One of the machine learning (ML) algorithms known as ANN are performance-efficient systems with low computational complexity that have caught much attention these days for problems in many different fields. The use of ML algorithms in wireless communications is not new and is producing considerably good results in many aspects like fading channel-modeling by using feedforward neural network (FFNN) [14] and, automatic modulation classification using genetic programming (GP) [15] and convolutional neural networks (CNNs) [16]. Four of the ML's multi-classifiers algorithms (machine learning ensemble algorithms) are used and compared to detect if the received noisy signal contains impulse noise or not in an OFDM system; and have shown promising insight [17]. Many studies have been performed on the use of ML for channel estimation in OFDM systems. Recently, advanced ML algorithms known as Deep Learning algorithms have been incorporated for more complicated problems like the doubly selective fading channels in the same setting [18]. In [19], two types of channel estimators based on deep neural networks (DNNs) are proposed for underwater acoustic OFDM communication systems. [20] studies a deep-learning based method that indirectly approximates the CSI (channel state information) and directly retrieves the communicated symbols that allows it to perform better even with smaller number of pilot symbols. In [21], adaptive equalizer for MIMO-OFDM system is designed using neural network with functional expansions and neural weights are adjusted using sparse adaptive filter with block processing of input samples. [22] studies various efficient pilot-based nonsparse channel estimation schemes by neural network technologies for OFDM systems and compares bit error rates of the proposed neural network with that of the other neural network technologies, the least square (LS) algorithm, and the minimum mean square error (MMSE) algorithm in 16QAM environment. [23] uses Genetically Evolved Artificial Neural Network for nonsparse channel estimation in MIMO-OFDM systems and shows that the proposed estimator performs better than LS and MMSE estimators at higher SNR values and close to the MMSE estimator at lower SNR values.

Channel estimation based on RBFNN is proposed to estimate channel frequency responses in OFDM interleave division multiple access (OFDM–IDMA) systems. The comparisons are made between the different learning functions used in the neural network training and RBFNN shows better results with an added advantage of it requiring no statistical information of the channel unlike LMMSE [24]. The practice of hybrid ML algorithms that make use of more than one ML algorithm have been of some interest because of their faster convergence and better throughput. In [25], authors combine a back propagation NN for channel estimation and compensation of signals with a genetic algorithm to improve performance and the convergence rate. Various ANN models are adopted for multi-user detection (MUD) in [26]. MUD using NN models have shown to outperform other existing schemes in terms of BER performance and convergence speed. [27] evaluates a channel estimation and equalization strategy based on an online fully complex extreme learning machine (C-ELM), for OFDM systems. This technique is implemented on the fading channels and the nonlinear distortion occuring in high-power amplifier (HPA) and the simulations show that it performs well without pre-training and feedback link between receiver and transmitter.

Sparse channel estimation is a challenging problem since the performance of the channel estimator critically depends on channel tap locations. In this study, we jointly estimate the tap locations and the tap coefficients of the OFDM sparse multipath channel using ANN. The sparse multipath channels used in this work are generated with respect to the specifications in [28]. While training the ANN, as target of ANN, we employ higher resolution equivalent channel that are band limited version of the physical channel. In this method, in order to approximate non-integer tap locations as accurate as possible, we increase the delay resolution in the non-sparse channel expression involving in the receive equation obtained on the receiver side. Although increasing the delay resolution causes higher size of hidden layer in ANN, thanks to offline training (*pre-computation*) property of ANNs, it does not cause computational load. Although the proposed ANN based sparse channel estimation scheme has the advantage of low computing load, the symbol error rate (SER) and the mean-squared error (MSE) performances of the proposed method are also compared with that of well known MP and OMP algorithms in compressed sensing literature. In a general manner, ANN is a nonlinear model which is trained to learn relationship between input and output data. While MP and OMP algorithms use pilot based dictionary matrix in a recursive manner, ANN works as a batch algorithm and use weight coefficients to model the estimator. When compared with MP and OMP algorithms, ANN based estimator uses the advantage of the supervised learning. During the training process, on the contrary of the MP based estimators, due to its supervised learning property, ANN estimator uses input-output pairs to model well the system. Since there is a reasonable relationship between input and output of the estimator, with the help of the supervised learning advantages, ANN based model outperforms the other methods. We show that the proposed ANN sparse multipath channel estimator outperforms MP and OMP based channel estimation algorithms for reasonable higher delay resolution range except a slight performance loss for much higher delay resolution values beyond a certain mid-range SNR levels.

The paper is organized as follows: OFDM signal and channel model is explained in Sect. 2; channel estimation using proposed ANN model is described in Sect. 3, the computational complexity is given in Sect. 4 and finally in Sect. 5 the results of this study obtained from simulations are presented and compared with MP and OMP algorithms in terms of SER and MSE performances. The paper is concluded in the final Sect. 6.

2 Signal and channel models

The efficiency of OFDM lies in the fact that all N sub-carriers are closely spaced, that is allowed because they are orthogonal to each other. The considered OFDM system employ only K active sub-carriers and rest of $N-K$ sub-carriers are reserved for zero-padding. These active sub-carriers are modulated by data symbols and take form of a frequency domain signal that is then converted from serial to parallel. The parallel signal is converted to time domain by taking its K-point inverse Fast Fourier transform (IFFT). After IFFT task, a cyclic prefix (CP) of interval T_{cp} is added to the signal to overcome ISI, such that T_{cp} is larger than maximum delay of multipath channel.The transmitted signal is complex valued and is represented in continuous time-domain as follow

$$s(t)=\frac{1}{N}\sum_{k=-K/2}^{K/2-1} b[k]\,e^{j2\pi k\Delta f t}, \tag{1}$$

where $b[k]$ is the data symbol transmitted over kth sub-carrier, $\triangle f = 1/T$ is the OFDM sub-carrier spacing with $T_{sym} = T + T_{cp}$ as the duration of cyclic prefixed OFDM symbol [29].

Channel impulse response (CIR) of the transmission system comprises of delayed impulses triggered by numerous routes of propagation i.e. echoes from the surrounding objects like buildings, trees etc. A characteristic channel impulse response that considers several echoes is the summation of impulses.

$$g(\tau)=\sum_{\ell=1}^{L} h_\ell\,\delta(\tau-\tau_\ell), \quad \ell=1,\dots,L, \tag{2}$$

where, uniformly distributed random variable τ_ℓ in $[0, T_{cp})$ is the delay location of ℓth path and $h_\ell|\tau_\ell \sim \mathcal{CN}(0, \Omega_l^2)$ is the tap coefficients of the ℓth path satisfying normalized channel power such that $\sum_{\ell=1}^{L} E_{\tau_\ell}\{\Omega_l^2\} = 1$. $E_{\tau_\ell}\{\cdot\}$ is the expectation operator with respect to τ_ℓ. So, the continuous-time received signal is obtained as follows

$$\begin{aligned} y(t) &= \int_{-\infty}^{+\infty} g(\tau)\,s(t-\tau)d\tau + w(t)\\ &= \sum_{\ell=1}^{L} h_\ell\,s(t-\tau_\ell)+w(t)\\ &= \frac{1}{N}\sum_{k=-K/2}^{K/2-1}\sum_{\ell=1}^{L} h_\ell\,b[k]\,e^{j2\pi k\Delta f(t-\tau_\ell)}+w(t), \end{aligned} \tag{3}$$

where $w(t)$ is the time-domain zero-mean complex additive white Gaussian noise (AWGN) with variance N_0. Since inverse Fourier transform relationship between discrete frequency and the continuous time is given by

$$y(t)=\frac{1}{N}\sum_{k=-K/2}^{K/2-1} Y[k]\,e^{j2\pi k\Delta f t}, \tag{4}$$

the received signal at the output of FFT is represented using (3) as follows

$$Y[k]=b[k]G[k]+W[k], \tag{5}$$

where the frequency response of the channel is given by

$$G[k]=\sum_{\ell=1}^{L} h_\ell\,e^{-j\frac{2\pi k}{T}\tau_\ell}. \tag{6}$$

The channel taps are generated according to $h_l \sim \mathcal{CN}(0, \Omega_l^2)$ where ℓth path power is defined by

$$\Omega_l^2=\frac{e}{(e-1)L}e^{-\tau_\ell/T_{cp}}. \tag{7}$$

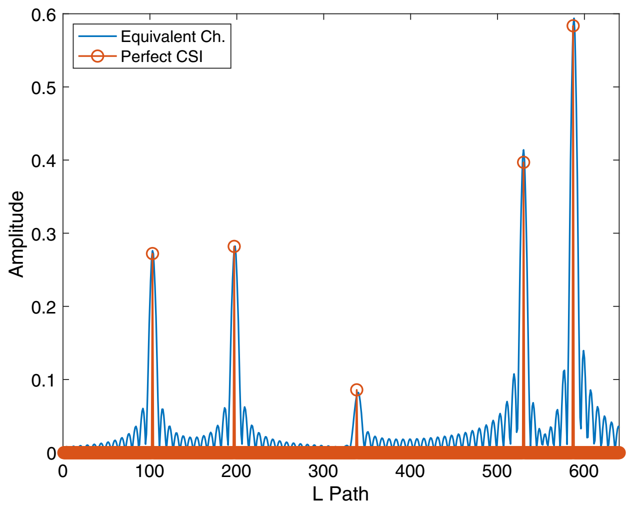

Fig. 1 Sparse multipath channel in time domain for $L = 5$ and its discrete-time equivalent with $\rho = 8$

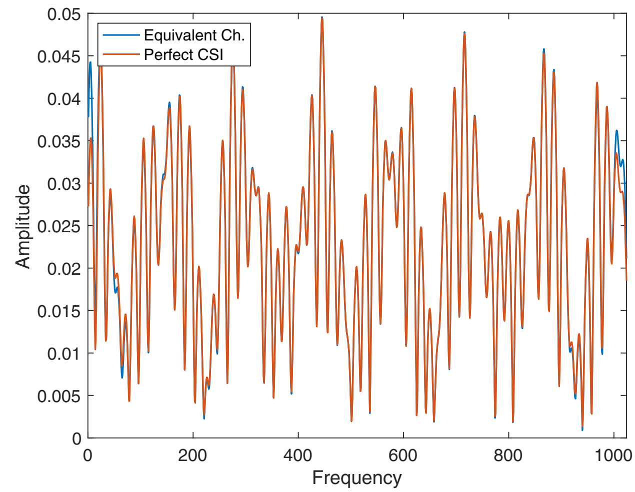

Fig. 2 Sparse multipath channel for $L = 5$ in frequency domain and its discrete equivalent with $\rho = 8$

Given that the estimation considered here is pilot based and the comb-type pilot symbol arrangement is used, the observation model at the pilot symbol locations can be candidly concluded from (5)

$$Y[k_p] = b[k_p]G[k_p] + W[k_p]\,,\ p = 1, 2, \cdots, P \tag{8}$$

Here, P denotes the total number of pilots in the signal. The number of pilot symbols is inversely proportional to the pilot spacing constant that represents the number of data symbols between two successive pilots.

Nevertheless, sparsity of the channel poses difficulties in its discrete-time representation. The discrete-time baseband representation is not reasonable because of the non-integer normalized path delays and results in a poor estimation of the channel. This can be resolved by introducing a finer delay resolution in the Analog-to-Digital (A/D) conversion step of the OFDM receiver; that improves the estimation quality.

Working with the channel impulse response given in (2) to obtain its discrete-time equivalence with a certain delay resolution, we first find the band limited equivalent of the time-invariant channel in (2) as follows

$$\begin{aligned}\widetilde{g}(\tau) &= \mathcal{F}^{-1}\{G[k]\}\\ &= \frac{1}{N}\sum_{k=-N/2}^{N/2-1} G[k]\, e^{j\frac{2\pi k}{T}\tau}\\ &= \sum_{\ell=1}^{L} h_\ell \left(\frac{1}{N}\sum_{k=-N/2}^{N/2-1} e^{j\frac{2\pi k}{T}(\tau-\tau_\ell)}\right)\\ &= \sum_{\ell=1}^{L} h_\ell\, \frac{N+1}{N}\, \frac{\mathrm{sinc}(\frac{N+1}{N}\,\frac{\tau-\tau_\ell}{T_s})}{\mathrm{sinc}(\frac{1}{N}\,\frac{\tau-\tau_\ell}{T_s})}\end{aligned} \tag{9}$$

for $N \gg 1$ and $\mathrm{sinc}(\frac{\tau-\tau_\ell}{NT_s}) \approx 1$ since $\frac{\tau-\tau_\ell}{NT_s}$ is very close to 0, we approximate

$$\begin{aligned}\widetilde{g}(\tau) &\approx \sum_{\ell=1}^{L} h_\ell\, \mathrm{sinc}(\frac{\tau-\tau_\ell}{T_s})\\ &= \sum_{\ell=1}^{L} h_\ell\, \mathrm{sinc}(\frac{\tau}{T_s} - \frac{\eta_\ell}{\rho})\end{aligned} \tag{10}$$

Defining $\tau = \eta T_s/\rho$, we obtain

$$\widetilde{g}[\eta] = \sum_{\ell=1}^{L} h_\ell\, \mathrm{sinc}(\frac{\eta-\eta_\ell}{\rho})\,,\ \eta = 0, 1, \cdots, \rho N_{cp} - 1, \tag{11}$$

and channel frequency response is approximated with finer delay resolution as follows

$$G[k] = \mathcal{F}\{\widetilde{g}[\eta]\} = \sum_{\eta=0}^{\rho N_{cp}-1} \widetilde{g}[\eta]\, e^{-j\frac{2\pi k}{\rho N}\eta}, \tag{12}$$

where ρ and N_{cp} show resolution constant and length of the cyclic prefix respectively.

A time and frequency domain comparison between the physical channel and its band limited equivalent is presented in Figs. 1 and 2, respectively for $L = 5$, where L shows the number of channel path. From these figures it can be said that, the equivalent channel represents physical channel well. From the received signal at pilot subcarriers, the linear MMSE estimate of $G[k_p]$ can be found as follows

$$\widehat{G}[k_p] = \frac{E\{G[k_p]Y^*[k_p]\}}{E\{|Y[k_p]|^2\}} Y[k_p],\ p = 1, 2, \cdots, P$$
$$= \frac{b^*[k_p]}{|b[k_p]|^2 + N_0} Y[k_p]. \tag{13}$$

By substituting (6) in (8) and collecting $Y[k_p]$ for all $p = 1, 2, ..., P$, we can obtain the observation equation at the pilot locations with the equivalent discrete-time representation of the Linear Time-Invariant Sparse Multipath (LTI-SMP) channel. This observation is further transformed to the vector form

$$\boldsymbol{Y}_p = \boldsymbol{A}\widetilde{\boldsymbol{g}} + \boldsymbol{W}_p, \tag{14}$$

where $\widetilde{\boldsymbol{g}}$ is the time-domain equivalent sparse channel vector given by

$$\widetilde{\boldsymbol{g}} = \left[\widetilde{g}[0], \widetilde{g}[1], \cdots, \widetilde{g}[\rho N_{cp} - 1]\right]^T, \tag{15}$$

$\boldsymbol{Y}_p = \left[Y[k_1], Y[k_2], \ldots, Y[k_P]\right]^T$, $\boldsymbol{W}_p = \left[W[k_1], W[k_2], \ldots, W[k_P]\right]^T$ and $\boldsymbol{A} \in P \times \rho N_{cp}$ is a *so-called* dictionary matrix in sensing theory, with $b[k_p]exp(-j2\pi k_p(q-1)/\rho N)$ as its pth-row, qth-column element.

The sparse estimation problem amounts to the approximately valuating non-zero elements of the sparse coefficient vector $\widetilde{\boldsymbol{g}}$ in (14) that can be achieved by using a sparse signal recovery method. The most efficient methods for such problems are the MP algorithm and its variants.

3 ANN based sparse channel estimation

ANNs are comprised of several extremely linked, adaptive and simple groups of elements that are capable of exceptionally complex and parallel computations for data processing and artificial intelligence (AI). With these characteristic qualities and learning capabilities, ANNs have been used as channel estimators and shown to have better system throughput and lesser computational complexity. A carefully selected and fine-tuned ANN model is used to find the best probable solution to a given problem.

In neural network domain, the wireless channels and the OFDM modulations are perceived as black boxes while training the modeled networks. Many channel models have been established by researchers over the past years for CSI that can express the real channels with reference to the channel statistics. The transmitted symbols are generated by producing a random data sequence and adding the pilot symbols for each corresponding OFDM frame. The channel model defined in Sect. 2 is used to simulate the current random channel state for each transmitted signal. The OFDM signal is transmitted through the channel by adding Gaussian noise. From these simulations, transmitted pilot symbols and received signals are collected to generate the test and train data. The neural network model is trained on this data to minimize the training error.

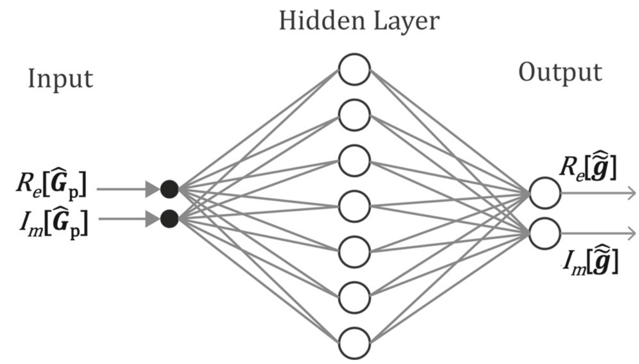

Fig. 3 ANN model

3.1 ANN model and training

In this study, for the estimation of spare multipath channel in an OFDM system, we employ Multilayer Perceptron (MLP) with a single hidden layer. Resilient Backpropagation (Rprop) training algorithm is used as the training function for the network that is a gradient-based, batch update function based on the Manhattan Update rule. Rprop comes after Levenberg Marquardt (LM) training algorithm in terms of speed so it requires lesser memory and hence, is our choice of training function.

The sample space to be used for neural network training is obtained by using the signal and channel models provided in Sect. 2. The signal in (1) is obtained by digitally modulating random bit stream according to observation model in Sect. 2. The complex valued transmit signal passes through the sparse multipath channel and is received as the channel impaired signal. We employ comb-type pilot structure in OFDM system setup in which so-called pilot symbols are known at the both side of the transceiver to estimate the channel.

In this study, the neural network input samples are linear MMSE estimate frequency domain channel in (13). On the other hand, the target set used in the training phase contains the samples of the band limited equivalent channel with higher delay resolution in (11). The channel taps are generated according to equation (7). Total number of (M) samples are collected for the target set in training phase. On the other hand, the length of each OFDM sample of the target set (length of the NN input and output) is directly proportional to the resolution constant ρ because the equivalent channel length is ρN_{cp}, where N_{cp} shows the length of the cyclic prefix. For each SNR level, M different target and input set is generated. Therefore for each SNR level the size of the whole target and input set becomes $\rho N_{cp} \times M$ and $\frac{K}{N_p} \times M$

Table 1 ANN parameters and functions

Parameter	Value	Parameter	Value
Number of hidden layers	1	Number of hidden neurons	512
Input size	$P \times 2$	Number of Samples	5000
Training function	R_{prop}	Performance param.	MSE
Divide function	Random	Divide param.	[0.5 0.2 0.3]

Table 2 OFDM Parameters

Parameter	Value
Number of subcarriers	1024
Number of used subcarriers (K)	$180N/256$
Sub-carrier spacing (Δ_f)	15 KHz
Bandwidth	$10N/1024$ MHz
Carrier frequency (f_c)	2.5 GHz
Modulation type	QPSK, 16QAM
Number of multipaths (L)	5
Pilot spacing (N_p)	8
Resolution factor(ρ)	2, 4, 8

respectively, where N_p shows the length of pilot spacing and K shows the number of used subcarriers.

Another factor in wireless communications is signal-to-noise ratio (SNR) that is a measure of the signal strength compared to the background noise. So, each received signal generated will have a specific SNR. Lower the SNR value, greater the distortion in received signal meaning that the channel estimation becomes more challenging for the smaller values of SNR. To have variety for each SNR level during the training process, input and target vector samples of ANN, $\widehat{\boldsymbol{G}}_{\boldsymbol{p}} = [\widehat{G}[k_1], \widehat{G}[k_2], \cdots, \widehat{G}[k_P]]^T$ and $\widetilde{\boldsymbol{g}}$, are generated using (13) and (15), respectively, and our ANN based estimator provides the estimate of sparse multipath channel vector $\widehat{\widetilde{\boldsymbol{g}}}$ as implemented in an OFDM transceiver whose MSE and SER performances are exhibited by simulation plots in Sect. 5. On the other hand, since the input and output data of ANNs should be real valued, complex valued input and target vector samples, $\widehat{\boldsymbol{G}}_{\boldsymbol{p}}$ and $\widetilde{\boldsymbol{g}}$ are both converted into real valued input and target data concatenating their real and imaginary parts vertically as seen in Fig. 3.

In our neural network structure, we have one hidden layer with 512 neurons. The transfer functions used for the hidden layer and output layer are *tangent sigmoid* and *linear* functions, respectively.

Tables 1 specifies the network parameters and functions for the ANN and Table 2 shows the OFDM parameters. The Divide Function represents the function with which the training set is divided into 3 subsets namely *train*, *validate* and *test* in random fashion. This is used to check goodness of network outputs when it is introduced with an input it has not seen before. The *Divide Parameters* set gives the ratio with which these subsets are obtained. These subsets are necessary for a neural network in the training phase for network to avoid over-training and to keep a check on its MSE based performance. An over-trained network suffers from over-fitting where it memorizes the training sequences instead of finding the input-output relationship and hence is unable to perform efficiently for unseen inputs. The total number of samples in the sample space are set to be 5000. This can be produced in any number as per the requirement in the data production phase discussed in the previous Sect. 3.1.

4 Computational complexity

The computational complexity of MP, OMP and ANN is derived from the number of the total additions and multiplications. $\mathcal{O}(\cdot)$ notation is used to express the complexity of the methods. General OMP and MP algorithms contain the following steps:

- **Step1**: Initialize $\boldsymbol{r}_0 = \boldsymbol{Y}_p$, $\widehat{\widetilde{\boldsymbol{g}}}_{\eta_i} = 0$,$\boldsymbol{\Phi_0} = \emptyset$ and $i = 0$
- **Step2**: Find $\hat{\eta}_i = \arg\max_{\eta_i} \frac{|\boldsymbol{a}_{\eta_i}^{\dagger} \boldsymbol{r}_i|}{||\boldsymbol{a}_{\eta_i}||^2}$
- **Step3**: Update $\boldsymbol{\Phi}_{i+1} = \boldsymbol{\Phi}_i \cup \{\boldsymbol{a}_{\hat{\eta}_i}\}$
- **Step4**: Calculate $\widehat{\widetilde{\boldsymbol{g}}}_{\hat{\eta}_i} = (\boldsymbol{\Phi}_{i+1}^{\dagger}\boldsymbol{\Phi}_{i+1})^{-1}\boldsymbol{\Phi}_{i+1}^{\dagger}\boldsymbol{Y}_p$ (OMP)

  Calculate $\widehat{\widetilde{\boldsymbol{g}}}_{\hat{\eta}_i} = (\boldsymbol{\Phi}_{i+1}^{\dagger}\boldsymbol{\Phi}_{i+1})^{-1}\boldsymbol{\Phi}_{i+1}^{\dagger}\boldsymbol{r}_i$ (MP)
- **Step5**: Update $\boldsymbol{r}_{i+1} = \boldsymbol{Y}_p - \boldsymbol{\Phi}_{i+1}\widehat{\widetilde{\boldsymbol{g}}}_{\hat{\eta}_i}$ (OMP)

  Update $\boldsymbol{r}_{i+1} = \boldsymbol{r}_i - \boldsymbol{\Phi}_{i+1}\widehat{\widetilde{\boldsymbol{g}}}_{\hat{\eta}_i}$ (MP)
- **Step6**: Stop the algorithm if the stopping condition is achieved ($||\boldsymbol{r}_i|| < \epsilon$), otherwise set $i = i + 1$ and go to Step2

In this algorithm, $\boldsymbol{r}_i$ is residue vector, $\boldsymbol{\Phi}_i$ is a set in which updated columns are concatenated and $\boldsymbol{a}_{\eta_i}$ is n_ith column vector of dictionary matrix $\boldsymbol{A}$. According to the given OMP algorithm, computational complexity calculated and results are given in Table 3. Where, the λ represents the iteration index of the algorithm. Considering at least L iteration requirement, the complexity of MP and OMP algorithms can be given as $\mathcal{O}(4\rho N_{cp} P L)$ for multiplications and as

Table 3 Computational complexity of OMP

Steps	Complex multiplication	Complex addition
Step2	$(\rho N_{cp} - \lambda)P$	$(\rho N_{cp} - \lambda)(P-1)$
Step4	$2P\lambda^2 + \lambda^3 + P\lambda$	$2(P-1)\lambda^2 + \lambda^3 + (P-1)\lambda$
Step5	$P\lambda$	$P + (P-1)\lambda$

Table 4 Computational complexity comparison for $\lambda = 5$

	MP & OMP		ANN	
	Multiplication	Addition	Multiplication	Addition
$\rho = 2$	288000	216000	256000	256000
$\rho = 4$	576000	432000	419840	419840
$\rho = 8$	1152000	864000	747520	747520

$\mathcal{O}(3\rho N_{cp} PL)$ for additions. Where P and L show the number of the pilots and paths respectively.

In a single layer ANN, while N_H shows the number neuron in the hidden layer, the number of the multiplications are $2PN_H$ and $2N_H\rho N_{cp}$ from input to hidden layer and from hidden layer to output respectively and the total number of the multiplications are $2N_H(P + \rho N_{cp})$. Especially for large hidden layer numbers, since the complexity of the addition is almost the same with that of multiplication, the complexity of ANN can be represented as $\mathcal{O}(2N_H(P + \rho N_{cp}))$ for both multiplications and additions. By considering the parameters shown in Table 1 and 2 , the total number of the multiplications and additions are given in Table 4 for ρ values studied in this work. From Table 4, it can be seen that, the total number of the multiplications and additions of MP and OMP are greater than that of ANN, especially for ρ values greater than 2.

5 Simulation results

In this section, we present computer simulations to evaluate the performance of the proposed ANN based sparse channel estimation algorithm. In these simulations the digital modulation schemes are Quadrature Phase Shift Keying (QPSK) and 16 Quadrature Amplitude Modulation (16QAM).

The total number of sub-carriers are $N = 1024$ and after zero-padding the number of used sub-carriers is $K = 180N/256$. The MSE and SER performances for MP, OMP and ANN based estimators are studied considering an LTI-SMP channel with $L = 5$ paths and the other OFDM system parameters are presented in Table 2.

Employing Monte-Carlo simulations with these settings, the MSE and the corresponding SER performance curves of the proposed ANN based sparse channel estimator are plotted under different scenarios. While plotting the performance

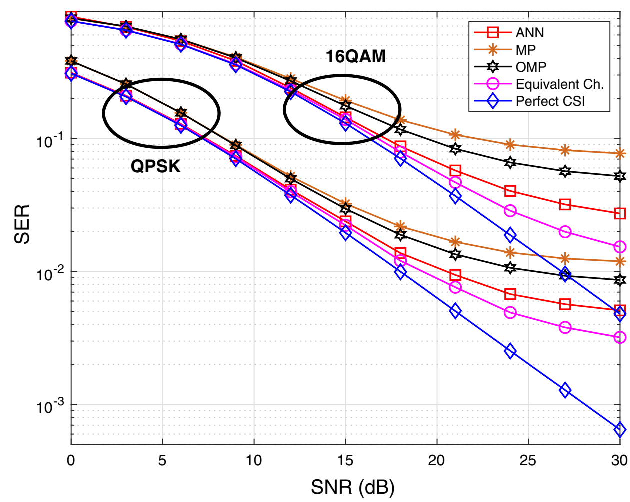

Fig. 4 SER performance comparisons of MP, OMP and ANN channel estimators ($\rho = 2$)

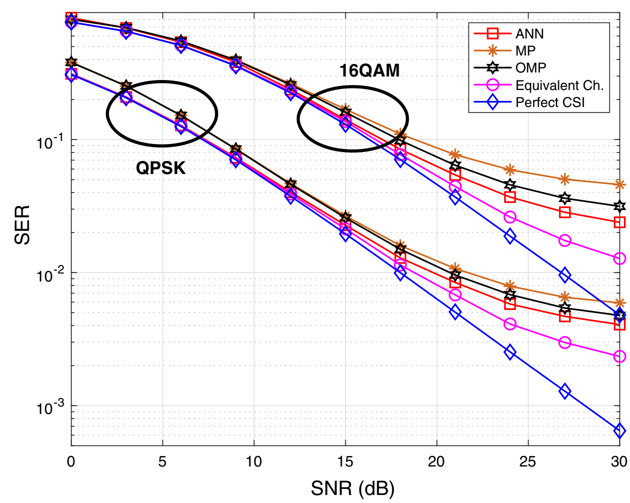

Fig. 5 SER performance comparisons for MP, OMP and ANN channel estimators ($\rho = 4$)

curves for the perfect CSI cases in these figures, we use the perfect channel expression in (6).

Figures 4, 5, and 6 show SER versus SNR comparison of the MP, OMP and ANN estimators for different ρ values considering the perfect CSI and equivalent CSI cases as well. From the figures, it can be seen that, for ρ values 2 and 4 the ANN outperforms MP and OMP algorithms for all SNR levels. On the another hand, at the receiver side the euclidean distance between the location of noisy received sig-

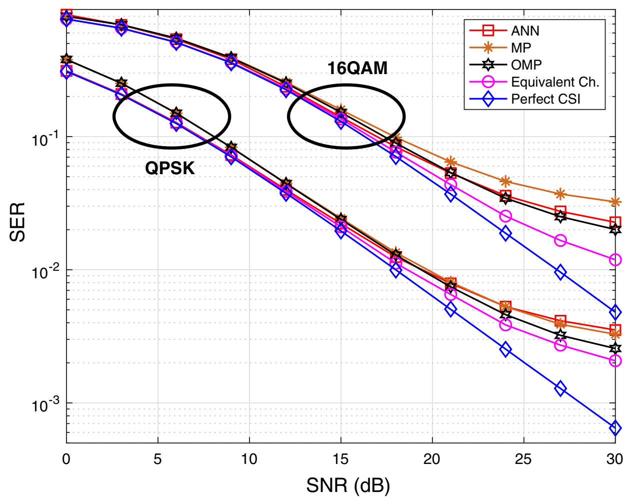

Fig. 6 SER performance comparisons for MP, OMP and ANN channel estimators ($\rho = 8$)

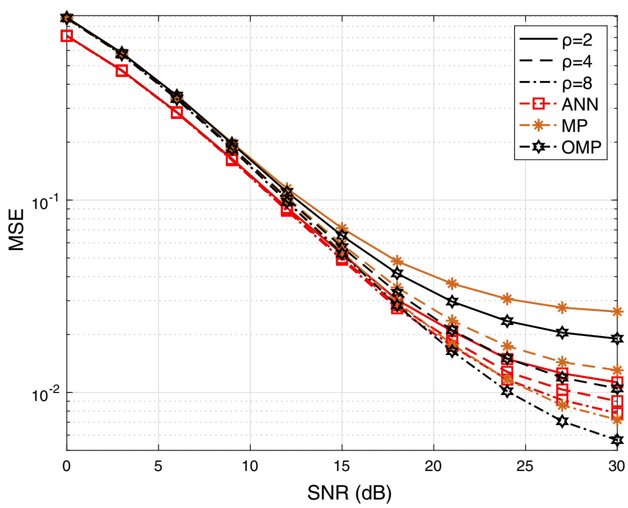

Fig. 7 MSE performance comparisons for QPSK signaling and different ρ values

nal and constellation points (every possible transmit signals) are calculated, and therefore, the constellation point which is closest to the received signal is decided as the transmit signal. 16QAM, as compared to QPSK, is more susceptible to additive noise because the constellation points are closer to each other (i.e.,narrower decision boundary) so that a less power of noise is required for correct symbol detection. Therefore, QPSK gives a better SER performance at the same SNR value.

In Fig. 6 for $\rho = 8$, we observe a slight loss in the SER performance of ANN estimator beyond 20dB SNR values. This is because of the fact that the unnecessarily increasing number of unknown coefficients with higher ρ values effects the estimator performance negatively and dominates after a certain higher ρ value. In these figures, perfect CSI plots are plotted for SER performance benchmark in order to compare

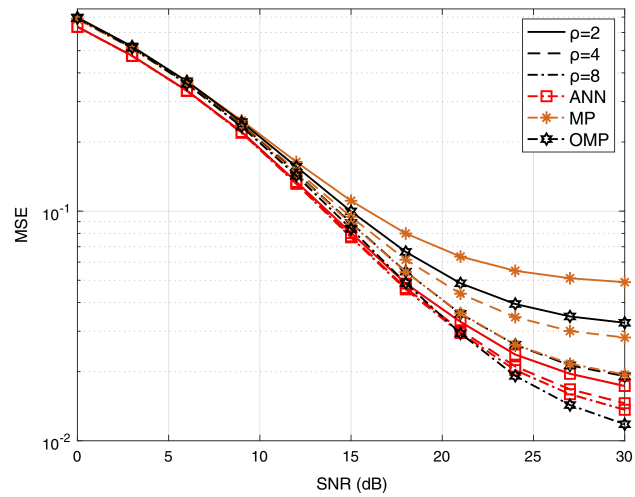

Fig. 8 MSE performance comparisons for 16QAM signaling and different ρ values

the performances of MP, OMP and ANN channel estimators. On the other hand, the performance of the equivalent CSI is a little bit lower than that of perfect CSI since tap locations are approximated but not exactly expressed by the equivalent channel. For the sake of the clarity, SER performances for SNR values greater than 20 dB are also given in Table 5.

Figures 7 and 8 show the MSE performance plots of the proposed ANN based channel estimator for different ρ values. From these figures it is clearly seen that for $\rho = 2$ and $\rho = 4$ values, ANN outperforms MP and OMP for all SNR levels. However, for the value of $\rho = 8$, there is a slight loss in the SER performance of ANN estimator beyond 20dB SNR values. This justifies the slight loss in the SER performance for $\rho = 8$ as stated in the previous paragraph. It is straightforward that the computational complexity of ANN is lower than that of MP and OMP algorithms since offline training process of the ANN is a precomputation.

6 Conclusions

In this work, a low complexity ANN based channel estimation algorithm is proposed for OFDM systems operating over sparse multipath channels. For better modeling of sparse multipath channel, we use the finer delay resolutions to be able to represent sparse multipath delay positions within one OFDM symbol duration. Thus, we obtain equivalent multipath channel by approximating non-integer tap locations as accurate as possible with the finer delay resolutions. However, using finer resolutions to approximate non-integer delay locations causes higher computational complexity in channel estimator algorithm since finer resolution increases the size of delay grid and consequently the dimension of the delay search space. MP and OMP algorithms, commonly

Table 5 SER values of the estimators

		SNR (QPSK)				SNR (16QAM)			
		21 dB	24 dB	27 dB	30 dB	21 dB	24 dB	27 dB	30 dB
$\rho=2$	MP	0.0167	0.0139	0.0125	0.0119	0.1066	0.0899	0.0817	0.0772
	OMP	0.0135	0.0107	0.0093	0.0086	0.0837	0.0660	0.0567	0.0521
	ANN	0.0094	0.0068	0.0057	0.0051	0.0575	0.0403	0.0319	0.0273
$\rho=4$	MP	0.0107	0.0079	0.0065	0.0059	0.0769	0.0592	0.0503	0.0459
	OMP	0.0096	0.0068	0.0054	0.0048	0.0641	0.0457	0.0362	0.0315
	ANN	0.0084	0.0058	0.0047	0.0041	0.0542	0.0370	0.0284	0.0239
$\rho=8$	MP	0.0081	0.0053	0.0039	0.0033	0.0645	0.0461	0.0370	0.0323
	OMP	0.0074	0.0046	0.0032	0.0026	0.0537	0.0344	0.0250	0.0201
	ANN	0.0079	0.0053	0.0041	0.0035	0.0532	0.0360	0.0274	0.0227

used in compressed sensing literature, are basically search algorithms employed for determining the channel tap delay locations, and therefore, they suffer from the higher computational complexity due to finer delay resolution. In this work, in order not to deal with the higher computational load during estimation process, we propose an ANN based sparse multipath channel estimator exploiting its offline training property. The computer simulations, except a slight performance loss for $\rho = 8$ beyond 20dB SNR levels, have demonstrated that the proposed algorithm has much better channel estimation and symbol error rate performances than that of the MP and OMP algorithms as very popular sparse signal recovery methods. As a future work, this slight performance loss emerging with much higher delay resolutions can be eliminated by investigating different ANN structures. Finally, comparing with MP and OMP algorithms, we conclude that ANN based sparse multipath channel estimation algorithms may have pretty much potential to be good candidates to meet better performance and smaller latency between transmitters and receivers in sparse channel environment.

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Compliance with ethical standards

Conflicts of interest The authors declare that they have no conflict of interest.

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