Phase Shifting Properties of High-Pass and Low-Pass Mixed-Element Two-Ports

Metin Şengül, Gökhan Çakmak, and Rabia Özdemir

***Abstract*—In many applications, circuits containing lumped-elements are preferred because of their small sizes. But it is not possible to avoid losses caused by the connections between lumped-elements at high frequencies. However, the use of these connections as circuit elements will improve the performance of the circuit. Therefore, it becomes inevitable to use circuits with mixed (lumped and distributed) elements at high frequencies. In this brief, phase shifting properties of high-pass and low-pass mixed-element two-ports is explained. Then these two-ports are used to form the initial design of high-pass and low-pass sections of a phase shifter. Finally, the initial design performance can be improved by means of commercially available computer-aided design (CAD) tools.**

***Index Terms*—Distributed elements, high-pass, low-pass, lumped elements, mixed elements, phase shifters.**

I. INTRODUCTION

MIXED element structures are described by means of nonrational transcendental functions based on classical work of cascaded noncommensurate transmission lines [1] or multivariable functions. In [2], synthesis of transcendental input impedance functions formed by cascading ideal transmission lines and lumped-elements is given. In the multivariable function approach, Richards transformation is utilized to describe mixed-element two-ports. Richards transformation can be expressed as $\lambda = j\Omega = j\tan\omega\tau$, here τ is the transmission line delay, and it transforms the transcendental functions of a distributed-element network into rational functions [3]. If this approach is generalized to mixed-element networks, Richards variable ($\lambda = j\Omega$) and original frequency variable ($p = j\omega$) are used for distributed-elements and lumped-elements in the network, respectively.

But a complete theory of design and synthesis for mixed-element networks does not exist. In many works, some restricted network configurations are studied [4]–[7]. In these configurations, cascaded unit elements (UEs) [ideal commensurate transmission lines] and lumped reactances are utilized.

Manuscript received September 21, 2020; accepted November 8, 2020. Date of publication November 17, 2020; date of current version March 26, 2021. This brief was recommended by Associate A. Calimera. *(Corresponding author: Metin Sengul.)*

Metin Şengül is with the Faculty of Engineering and Natural Sciences, Kadir Has University, 34083 Istanbul, Turkey (e-mail: msengul@khas.edu.tr).

Gökhan Çakmak is with the School of Graduate Studies, Kadir Has University, 34083 Istanbul, Turkey (e-mail: gokhan.cakmak@stu.khas.edu.tr).

Rabia Özdemir is with the Faculty of Engineering and Natural Sciences, Kadir Has University, 34083 Istanbul, Turkey (e-mail: 20161706025@stu.khas.edu.tr).

Digital Object Identifier 10.1109/TCSII.2020.3038423

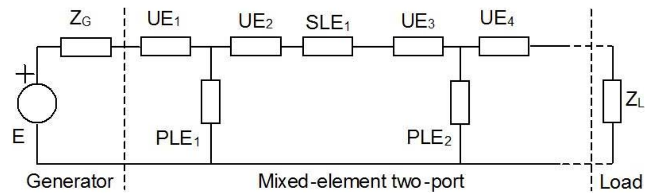

Fig. 1. Mixed-element two-port.

In summary, mixed-element networks consist of cascaded lossless lumped-elements separated by UEs as seen in Fig. 1. In the structure, the first and the last component can be a UE or a series (or parallel) connected lumped-element.

In the following part, characterization of a mixed-element two-port network is briefly described. After explaining the phase shifting properties of high-pass and low-pass mixed-element two-ports, an approach is given to form the initial design of high-pass and low-pass sections of a phase shifter. Then 5-mixed-element high-pass and low-pass two ports of a 180° phase shifter are designed to illustrate the utilization of the proposed appraoch.

II. TWO-VARIABLE REPRESENTATION OF A LOSSLESS TWO-PORT

A two-port network can be described by means of scattering parameters. In our case, since the two-port contains mixed-elements, it is necessary to use two-variable scattering parameters. Also they must be written in terms of two-variable polynomials g, h, f as follows [7], [8]:

$$S(p,\lambda) = \begin{bmatrix} S_{11}(p,\lambda) & S_{12}(p,\lambda) \\ S_{21}(p,\lambda) & S_{22}(p,\lambda) \end{bmatrix} = \begin{bmatrix} \frac{h(p,\lambda)}{g(p,\lambda)} & \frac{\mu f(-p,-\lambda)}{g(p,\lambda)} \\ \frac{f(p,\lambda)}{g(p,\lambda)} & \frac{-\mu h(-p,-\lambda)}{g(p,\lambda)} \end{bmatrix} \quad (1)$$

where $|\mu| = 1$.

The real coefficient polynomial $g(p,\lambda)$ is a scattering Hurwitz polynomial and its degree is $n_p + n_\lambda$ and $g(p,\lambda)$ can be expressed as $g(p,\lambda) = \mathbf{P}^T\Lambda_g\boldsymbol{\lambda} = \boldsymbol{\lambda}^T\Lambda_g^T\mathbf{P}$ where

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ g_{n_p0} & \cdots & \cdots & g_{n_pn_\lambda} \end{bmatrix}, \quad \begin{matrix} \mathbf{P}^T = \begin{bmatrix} 1 & p & p^2 & \cdots & p^{n_p} \end{bmatrix} \\ \lambda^T = \begin{bmatrix} 1 & \lambda & \lambda^2 & \cdots & \lambda^{n_\lambda} \end{bmatrix}. \end{matrix} \quad (2)$$

Since the polynomial $g(p,\lambda)$ is described in terms of two variables, partial degrees can be written as the highest power of a variable whose coefficient is nonzero, i.e., $n_p = \deg_p g(p,\lambda)$, $n_\lambda = \deg_\lambda g(p,\lambda)$.

In a similar manner, the polynomial $h(p,\lambda)$ is a real coefficients polynomial and its degree is $n_p + n_\lambda$ and $h(p,\lambda)$ can be written as $h(p,\lambda) = \mathbf{P}^T \Lambda_h \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_h^T \mathbf{P}$, where

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_p0} & \cdots & \cdots & h_{n_pn_\lambda} \end{bmatrix}. \quad (3)$$

The polynomial $f(p,\lambda)$ is a real polynomial and it can be formed by means of the transmission zeros of the mixed-element two-port. Then it can be expressed as

$$f(p,\lambda) = f(p)f(\lambda) \quad (4)$$

where the polynomials $f(\lambda)$ and $f(p)$ are formed by utilizing the transmission zeros of the UE and lumped-element two ports, respectively.

If UEs are cascaded in the mixed-element network, $f(\lambda)$ can be written as

$$f(\lambda) = (1-\lambda^2)^{n_\lambda/2}. \quad (5)$$

For lumped-element part, the polynomial $f(p)$ will be an even or an odd polynomial in *p,* if only real-frequency transmission zeros are considered. If the finite imaginary axis zeros, except the zeros at DC, are disregarded, then the polynomial $f(p,\lambda)$ can be obtained as

$$f(p,\lambda) = p^k(1-\lambda^2)^{n_\lambda/2} \quad (6)$$

where k represents the number of transmission zeros at DC.

For $\lambda = 0$, single-variable polynomials $h(p,0)$, $g(p,0)$, and $f(p,0)$ describe the lumped-element section. In a similar manner, for $p = 0$, single-variable polynomials $h(0,\lambda)$, $g(0,\lambda)$ and $f(0,\lambda)$ describe the UE section. As a result it can be said that lumped-element and UE sections can be completely described by means of the corresponding single-variable boundary polynomials.

If the network is to be lossless, then the losslessness condition requires that

$$S(p,\lambda)S^T(-p,-\lambda) = I \quad (7)$$

where I is the identity matrix. Then this expression can be expanded as

$$g(p,\lambda)g(-p,-\lambda) = h(p,\lambda)h(-p,-\lambda) + f(p,\lambda)f(-p,-\lambda). \quad (8)$$

III. Phase Shifting Properties of Mixed-Element Two-Ports

A high-pass / low-pass phase shifter contains a high-pass two-port and a low-pass two-port. In [9], [10], it is said that phase shifts from 0° to +180° or from 0° to −180° are obtained by utilizing the high-pass or a low-pass two-port, respectively. Then if Φ degrees phase shift is desired, then low-pass section must be designed for $-\Phi/2$ degrees phase shift and high-pass section must be designed for $+\Phi/2$ degrees phase shift. So the total phase shift will be Φ degrees.

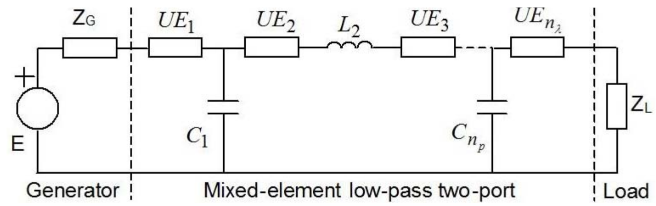

Fig. 2. Low-pass mixed-element network [4].

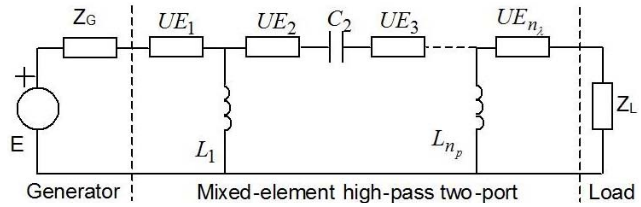

Fig. 3. High-pass mixed-element network [5].

For each section, $S_{21}(p,\lambda)$ can be expressed as

$$S_{21}(p,\lambda) = \frac{f(p,\lambda)}{g(p,\lambda)}. \quad (9)$$

If Φ degrees total phase shift is desired, then the following amplitude and phase constraint must be satisfied

$$S_{21}(j\omega, j\Omega) = \frac{f(j\omega, j\Omega)}{g(j\omega, j\Omega)} = 1\angle\Phi^\circ. \quad (10)$$

As mixed-element networks, many different networks can be selected. For instance, if parallel inductors and series capacitors with unit elements are used, then a high-pass mixed-element network is obtained. But if parallel capacitors and series inductors with unit elements are utilized, then a low-pass mixed-element network is formed.

Let us explain the fundamental properties of a low-pass two-port with mixed-elements as seen in Fig. 2. Detailed analysis of this structure can be found in [4].

Since the transmission zeros of the lumped-element two-port are at infinity, the polynomial $f(p,\lambda)$ is $f(p,\lambda) = (1-\lambda^2)^{n_\lambda/2}$ where n_λ is the number of unit elements in the low-pass structure. Then $S_{21}(j\omega, j\Omega)$ is obtained as

$$\begin{aligned} S_{21}(j\omega, j\Omega) &= \frac{f(j\omega, j\Omega)}{g(j\omega, j\Omega)} = \frac{(1+\Omega^2)^{n_\lambda/2}}{|g(j\omega, j\Omega)|\angle g(j\omega, j\Omega)} \\ &= \frac{(1+\Omega^2)^{n_\lambda/2}}{|g(j\omega, j\Omega)|}\angle - g(j\omega, j\Omega) = 1\angle -\Phi^\circ/2. \quad (11) \end{aligned}$$

As a result, the following two constraints must be satisfied

$$|g(j\omega, j\Omega)| = (1+\Omega^2)^{n_\lambda/2}, \quad (12a)$$

$$\angle g(j\omega, j\Omega) = \Phi^\circ/2. \quad (12b)$$

Now let us explain the fundamental properties of a high-pass two-port with mixed-elements as seen in Fig. 3. Detailed analysis of this structure can be found in [5].

Since the transmission zeros of the lumped-element two-port are at zero, the polynomial $f(p,\lambda)$ is $f(p,\lambda) = p^{n_p}(1-\lambda^2)^{n_\lambda/2}$ where n_p is the number of lumped-elements and n_λ is the number of unit elements in the high-pass structure. Then,

TABLE I
AMPLITUDE AND PHASE CONSTRAINTS OF $g(j\omega, j\Omega)$ FOR $n_p = 1$ TO $n_p = 4$

n_p	$\lvert g(j\omega, j\Omega)\rvert$	$\angle g(j\omega, j\Omega)$	Equation no
1	$\omega(1+\Omega^2)^{n_\lambda/2}$, $n_\lambda = 0,1,2$	$90° - \Phi°/2$	(14)
2	$\omega^2(1+\Omega^2)^{n_\lambda/2}$, $n_\lambda = 1,2,3$	$180° - \Phi°/2$	(15)
3	$\omega^3(1+\Omega^2)^{n_\lambda/2}$, $n_\lambda = 2,3,4$	$-90° - \Phi°/2$	(16)
4	$\omega^4(1+\Omega^2)^{n_\lambda/2}$, $n_\lambda = 3,4,5$	$-\Phi°/2$	(17)

$S_{21}(j\omega, j\Omega)$ is obtained as

$$S_{21}(j\omega, j\Omega) = \frac{f(j\omega, j\Omega)}{g(j\omega, j\Omega)} = \frac{(j\omega)^{n_p}(1+\Omega^2)^{n_\lambda/2}}{|g(j\omega, j\Omega)|\angle g(j\omega, j\Omega)} = 1\angle\Phi°/2. \quad (13)$$

The constraints given in Table I must be satisfied for $n_p = 1$ to $n_p = 4$, where $|n_p| - |n_\lambda| = 0$ *or* 1.

For $n_p > 4$, the same phase constraints are valid; (14) must be satisfied for $n_p = 5$, (15) must be satisfied for $n = 6$, and so on. The following generalized form can be used for amplitude constraints

$$|g(j\omega, j\Omega)| = \omega^{n_p}(1+\Omega^2)^{n_\lambda/2}, \ \left|n_p\right| - |n_\lambda| = 0 \ or \ 1. \quad (18)$$

While the lumped-element version of the above derivations can be found in [11], the distributed-element version can be found in [12]. In [11], the connections between lumped-elements are not considered, so they will destroy the performance of the network at high frequencies. In [12], only distributed elements are utilized, so the circuit size will be larger. But in this brief, lumped elements are preferred because of their small sizes, and also the connections between lumped elements are considered as circuit elements.

The approach given in the following section can be utilized to form the initial design of high-pass and low-pass mixed-element two-ports of a phase shifter. Eventually, the initial design can be improved by optimizing the performance of the phase shifter employing the commercially available computer-aided design (CAD) packages.

IV. PROPOSED APPROACH

In this brief, high-pass and low-pass mixed-element two-ports are described by using scattering parameters written in terms of two-variable polynomials g, h, f. So firstly these descriptive polynomials of high-pass and low-pass mixed-element two-ports are obtained, then normalized components values are calculated. Finally by using the selected normalization frequency and resistance values, real component values are computed.

Inputs:

- $\pm\Phi/2$: If Φ degrees phase shift is desired, then $-\Phi/2$ degrees for low-pass section, and $+\Phi/2$ degrees for high-pass section must be selected.
- ω_i : Normalized frequencies, $i = 1, 2, \ldots, m$.
- n_p : Number of lumped-elements in the low-pass or high-pass two ports.
- n_λ : Number of cascaded unit elements in the low-pass or high-pass two ports. It must be noted that $|n_p| - |n_\lambda| = 0$ *or* 1.
- $f(p, \lambda) = (1-\lambda^2)^{n_\lambda/2}$ for low-pass case and $f(p, \lambda) = p^{n_p}(1-\lambda^2)^{n_\lambda/2}$ for high-pass case.
- For low-pass two-port: $h_{00}, h_{01}, \ldots, h_{0n_\lambda}$(coefficients of $h(\lambda)$) and $h_{00}, h_{10}, \ldots, h_{n_p0}$ (coefficients of $h(p)$).
- For high-pass two-port: $h_{n_p0}, h_{n_p1}, \ldots, h_{n_pn_\lambda}$ (coefficients of $h(\lambda)$) and $h_{00}, h_{10}, \ldots, h_{n_p0}$ (coefficients of $h(p)$).
- τ : Initial delay of the unit elements.
- δ : Acceptable error level.

Computations:

Step 1: Obtain the strictly Hurwitz polynomials $g(p)$ and $g(\lambda)$ from $g(p)g(-p) = h(p)h(-p) + f(p)f(-p)$ and $g(\lambda)g(-\lambda) = h(\lambda)h(-\lambda) + f(\lambda)f(-\lambda)$ by using the initialized polynomials $f(p)$, $h(p)$ and $f(\lambda)$, $h(\lambda)$, respectively.

Step 2: The remaining unknown terms of the coefficients matrices Λ_h and Λ_g can be computed by means of the explicit expressions derived in [4] for low-pass two-port and in [5] for high-pass two-port.

Step 3: For a low-pass two-port, the total error is calculated by means of the following equation

$$\delta_c = \sum_{i=1}^{m}\left((1+\Omega_i^2)^{n_\lambda/2} - |g(j\omega_i, j\Omega_i)|\right)^2 + \sum_{i=1}^{m}\left(\Phi°/2 - \angle g(j\omega_i, j\Omega_i)\right)^2. \quad (19)$$

where m is the total number of frequency points as indicated in the second item of Inputs.

For a high-pass two-port, a similar δ_c equations can be formed by considering the constraints seen in Table I and by noting the comments given in the previous section under Table I.

Step 4: If $\delta_c \leq \delta$, stop the design and calculate the component values of the designed low-pass and high-pass two-port networks. For low-pass case, $S_{11}(p) = h(p)/g(p)$ (where $h(p) = h_{n_p0}p^{n_p} + \cdots + h_{10}p + h_{00}$ and $g(p) = g_{n_p0}p^{n_p} + \cdots + g_{10}p + g_{00}$) must be synthesized to obtain the lumped-element values of Cauer I circuit. In this step, zero shifting method, simple continuous fraction expansion or decomposition technique of Fettweis [13] can be used. For high-pass case, in a similar manner, $S_{11}(p)$ must be synthesized to obtain the lumped-element values of Cauer II circuit. For both cases, the approach proposed in [14], [15] can be used to compute the characteristic impedances of the cascaded unit elements.

If $\delta_c > \delta$, go to the next step.

Step 5: In this step, change the initialized coefficients of polynomials $h(p)$ and $h(\lambda)$, and delay of the unit elements (τ) via any optimization routine and go to the first step. Nonlinear least square optimization approach is employed in the following example.

V. EXAMPLE

In this example, 5-mixed-element (3 lumped-element and 2 unit elements) low-pass and 5- mixed-element (3 lumped-element and 2 unit elements) high-pass two ports of a 180° phase shifter are formed. The polynomial $f(p, \lambda)$ is $f(p, \lambda) = (1-\lambda^2)$ for low-pass section. For $n_p = 3$ and $n_\lambda = 2$, the polynomial $h(p)$ is defined as $h(p) = h_{30}p^3 + h_{20}p^2 + h_{10}p + h_{00}$, where the coefficients are initialized as $h_{30} = -0.01$, $h_{20} = 0.01$, $h_{10} = 0.01$ and $h_{00} = 0$. The polynomial $h(\lambda)$ is defined as $h(\lambda) = h_{02}\lambda^2 + h_{01}\lambda + h_{00}$, where the coefficients are initialized as $h_{02} = 0$, $h_{01} = 0$ and $h_{00} = 0$, and initial delay of UEs is $\tau = 0.6$. The coefficient h_{00} is not optimized to be able to obtain a transformer free two-port network.

After applying the proposed approach, the following coefficient matrices and delay are obtained

$$\Lambda_h = \begin{bmatrix} 0 & 0.00000 & 0.00000 \\ 0.00988 & 0.05554 & -0.34612 \\ 0.00989 & -0.08734 & 0 \\ -0.00489 & 0 & 0 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 & 2.00000 & 1.00000 \\ 0.34612 & 0.69223 & 0.34612 \\ 0.05985 & 0.08734 & 0 \\ 0.00489 & 0 & 0 \end{bmatrix},$$
$$\tau = 0.59940.$$

Then normalized lumped and unit element values are calculated as $C_1 = 0.14035$, $L_2 = 0.356$, $C_3 = 0.19588$ and $Z_1 = 1$, $Z_2 = 1$, respectively.

The polynomial $f(p, \lambda)$ is $f(p, \lambda) = p^3(1-\lambda^2)$ for high-pass section. For $n_p = 3$ and $n_\lambda = 2$, the polynomial $h(p)$ is defined as $h(p) = h_{30}p^3+h_{20}p^2+h_{10}p+h_{00}$, where the coefficients are initialized as $h_{30} = 0$, $h_{20} = 0.01$, $h_{10} = 0.01$ and $h_{00} = 0.01$. The polynomial $h(\lambda)$ is defined as $h(\lambda) = h_{32}\lambda^2+h_{31}\lambda+h_{30}$, where the coefficients are initialized as $h_{32} = 0$, $h_{31} = 0$ and $h_{30} = 0$, and initial delay of UEs is $\tau = 0.2$. The coefficient h_{30} is not optimized to be able to obtain a transformer free two-port network.

After applying the proposed approach, the following coefficient matrices and delay are obtained

$$\Lambda_h = \begin{bmatrix} 0.45106 & 0 & 0 \\ 0.02015 & 1.76470 & 0 \\ -0.01167 & 0.02599 & 1.53797 \\ 0 & 0.00000 & 0.00000 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 0.45106 & 0 & 0 \\ 1.18249 & 1.76470 & 0 \\ 1.53790 & 3.07579 & 1.53797 \\ 1 & 2.00000 & 1.00000 \end{bmatrix},$$
$$\tau = 0.06008.$$

Then normalized lumped and unit element values are calculated as $C_4 = 1.2885$, $L_5 = 0.64534$, $C_6 = 1.3331$ and $Z_3 = 1$, $Z_4 = 1$, respectively.

Now assume 180° phase shift is desired at 1 GHz, namely denormalization frequency is $f_{norm} = 1\,GHz$, and if denormalization resistance is selected as $R_{norm} = 50\Omega$. Then ideal component values are $C_1 = 0.44675pF$, $L_2 = 2.83296nH$, $C_3 = 0.62351pF$, $Z_1 = 50\Omega$, $Z_2 = 50\Omega$, $C_4 = 4.10142pF$, $L_5 = 5.13545nH$, $C_6 = 4.24339pF$, $Z_3 = 50\Omega$, $Z_4 = 50\Omega$.

The initial design of high-pass and low-pass mixed-element two-ports of 180° phase shifter with ideal component values is given in Fig. 4.

Phase variations and transducer power gain curves of the initial design of mixed-element 180° phase shifter are given in Fig. 5.

Although *TPG*is very close to unity over 10% bandwidth (0.9898 at 0.95GHz and 0.9832 at 1.05GHz for low-pass case and 0.9183 at 0.95GHz and 0.9566 at 1.05GHz for high-pass case), it is measured that the phase shift is 177.88° at 0.95GHz and 181.31° at 1.05GHz. The initial design performance can be improved by means of commercially available computer-aided design (CAD) tools.

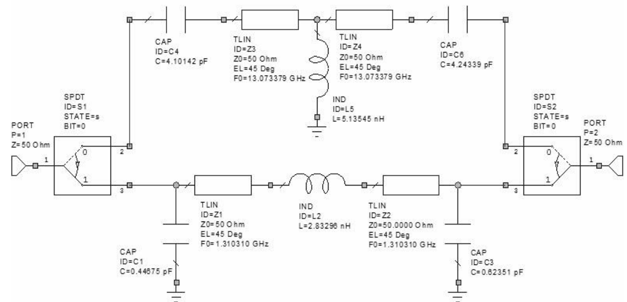

Fig. 4. Designed mixed-element 180° phase shifter.

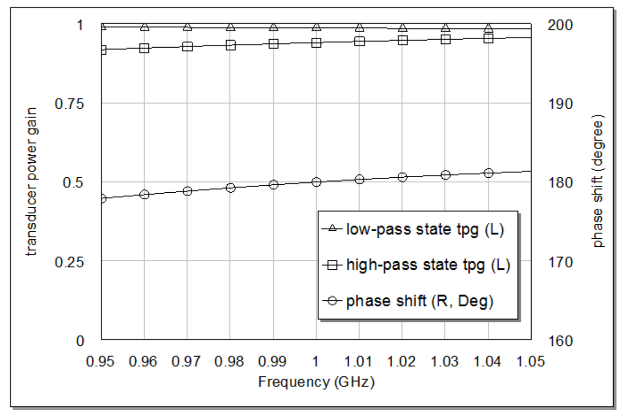

Fig. 5. Transducer power gain curves for low-pass and high-pass states and phase shifts of 180° mixed-element phase shifters.

Let us use the closest standard lumped element values as $C_1 = 0.4pF$, $L_2 = 2.8nH$, $C_3 = 0.6pF$ and $C_4 = 4.3pF$, $L_5 = 5.1nH$, $C_6 = 4.3pF$. Now characteristic impedance values of unit elements are optimized to improve the performance of the phase shifter and the following values are found; $Z_1 = 61.0555\Omega$, $Z_2 = 30.3673\Omega$, $Z_3 = 28.3735\Omega$, $Z_4 = 24.1192\Omega$. After optimization, transducer power gain is found as 0.9559 at 0.95GHz and 0.9396 at 1.05GHz for low-pass case and 0.9414 at 0.95GHz and 0.9751 at 1.05GHz for high-pass case, phase shift is found as 177.82° at 0.95GHz and 181.12° at 1.05GHz. Phase shift difference over 10% bandwidth is 3.43° and 3.3° for the initial and optimized design, respectively. As a result, it is clear that the performance of the optimized phase shifter is very close to that of the initial design. So the proposed approach provides excellent results as a front-end when utilized together with CAD tools.

A smaller phase shift difference (about 1° over 10% bandwidth) is obtained for the optimized design. But in this case, transducer power gain level is between 0.8 and 0.9. So the designer must decide which one is more important: flat phase shift or high transducer power gain.

In [11], 5-lumped-element 180° phase shifter is designed, and obtained phase shift is 180°±1.5° over 10% bandwidth. In [12], 5-stub 180° phase shifter is designed, and obtained phase shift is 180°±2° in the same bandwidth. Also the phase shift of 180° phase shifter designed in [16] is 178.9°±1.4°.

Microwave Office software tool is employed to simulate the phase shifter [17]. In the simulations, TLIN is used for a UE as seen in Fig. 4. One of the parameters to define TLIN is *EL* (electrical length or phase length) in degrees. It is selected as 45° in the example. Also there is another parameter, $F0$ (frequency used to specify *EL*). This frequency is calculated by using the delay of UE. After completing the initial design, delay of UE is found as $\tau = 0.59940$ and $\tau = 0.06008$ for low-pass and high-pass sections, respectively.

Normalized $F0$ is calculated as $F0_{normalized} = EL/\tau \cdot 360^o = 0.2085$. Then it is denormalized by using denormalization frequency as $F0 = F0_{normalized} \cdot f_{norm} \cdot 2 \cdot \pi = 1.3103 GHz$. In a similar manner, for high-pass two-port, $F0$ is calculated as $F0 = 13.0733 GHz$.

In the proposed design algorithm, normalized angular frequencies (ω_i) are utilized. In the example, phase shift is desired to be 180° at $\omega = 1$ (which corresponds to $f \cong 0.1592$). In the denormalization process, the frequency where phase shift will be 180° is selected as $1\,GHz$. So to be able to shift $f \cong 0.1592$ to $1\,GHz$, it must be multiplied by $f_{norm} \cdot 2\pi$. So in the calculations of $F0$ above, $F0_{normalized}$ is multiplied by $f_{norm} \cdot 2\pi$.

VI. Conclusion

It is not possible to avoid losses caused by the connections between lumped-elements at high frequencies. However, the use of these connections as circuit elements will improve the performance of the circuit. Therefore, it becomes inevitable to use circuits with mixed-elements at high frequencies.

If the circuit topology and initial component values are supplied, commercially available computer-aided design tools are excellent to optimize the performance of the circuit by working on the component values. Therefore, initialization process is very important, since the performance of a circuit is highly nonlinear in terms of the component values. So in this brief, after explaining the phase shifting properties of high-pass and low-pass mixed-element two-ports, an initialization approach is proposed to construct the high-pass and low-pass mixed-element two-ports of a phase shifter.

As an example, 5-mixed-element low-pass and high-pass two ports of a 180° phase shifter are formed to illustrate the utilization of the proposed approach.

It is shown that the proposed initialization approach provides very suitable initials to improve the performance of the phase shifter by working on the component values. Therefore, it is expected that the proposed approach be used as a front-end for the commercially available CAD tools to design high-pass and low-pass mixed-element two-ports of a phase shifter for wireless or in general microwave communication systems.

References

[1] B. K. Kinarivala, "Theory of cascaded structures: Lossless transmission lines," *Bell Syst. Tech. J.*, vol. 45, no. 4, pp. 631–649, Apr. 1966.

[2] L. A. Riederer and L. Weinberg, "Synthesis of lumped-distributed networks: Lossless cascades," *IEEE Trans. Circuits Syst.*, vol. CS-27, no. 10, pp. 943–956, Oct. 1980.

[3] P. I. Richards, "Resistor-transmission-line circuits," *Proc. Inst. Radio Eng.*, vol. 36, no. 2, pp. 217–220, Feb. 1948.

[4] A. Aksen, "Design of lossless two-port with mixed, lumped and distributed elements for broadband matching," Ph.D. dissertation, Dept. Elect. Eng., Ruhr Univ., Bochum, Germany, 1994.

[5] A. Sertbaş, "Two-variable scattering formulas to describe some classes of lossless two-ports with mixed lumped elements and commensurate stubs," *Turkish J. Elect. Eng. Comput. Sci.*, vol. 13, no. 2, pp. 231–239, 2005.

[6] M. Şengül, "Construction of lossless ladder networks with simple lumped elements connected via commensurate transmission lines," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 56, no. 1, pp. 1–5, Jan. 2009.

[7] M. Şengül, "Design of practical broadband matching networks with mixed lumped and distributed elements," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 61, no. 11, pp. 875–879, Nov. 2014.

[8] S. Basu and A. Fettweis, "On synthesizable multidimensional lossless two-ports," *IEEE Trans. Circuits Syst.*, vol. CAS-35, no. 12, pp. 1478–1486, Dec. 1988.

[9] B. S. Yarman, "Novel circuit configurations to design loss balanced 0°–360° digital phase shifters," *Int. J. Electron. Commun.*, vol. 45, pp. 96–104, Mar. 1991.

[10] B. S. Yarman, "New circuit configurations for designing 0°- 180° digital phase shifters," *IEE Proc. H Microw. Antennas Propag.*, vol. 134, no. 3, pp. 253–260, Jun. 1987.

[11] M. Şengül, "High-pass/low-pass section design of 0°–360° lumped-element phase shifters via the real frequency technique," *Turkish J. Elect. Eng. Comput. Sci.*, vol. 25, pp. 1922–1931, Nov. 2017.

[12] M. Şengül, "Design of phase shifters with ladder stubs via real frequency technique," *J. Circuit Syst. Comp.*, to be published.

[13] A. Fettweis, "Factorization of transfer matrices of lossless two-ports," *IEEE Trans. Circuits Syst.*, vol. CAS-17, no. 2, pp. 86–94, Feb. 1970.

[14] M. Şengül, "Synthesis of cascaded commensurate transmission lines," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 55, no. 1, pp. 89–91, Jan. 2008.

[15] M. Şengül, "Cascaded lossless commensurate line synthesis," in *Proc. Eur. Conf. Circuit Theory Design (ECCTD)*, Antalya, Turkey, Aug. 2009, pp. 295–298.

[16] X. Tang and K. Mouthaan, "A broadband 180° phase shifter with a small phase error using lumped elements," in *Proc. Asia–Pac. Microw. Conf. (APMC)*, Singapore, Dec. 2009, pp. 1315–1318.

[17] *AWR: Microwave Office of Applied Wave Research Inc.* Accessed: Dec. 1, 2020. [Online]. Available: http://www.awrcorp.com/