Solution of Lossless Broadband Matching Problems via Insertion Loss Method

Metin Şengül

Abstract— In this work, lossless broadband matching problem with complex generator and load impedances is solved via insertion loss method that is used to design filters with purely resistive generator and load impedances. In the proposed approach, firstly the power delivered to load impedance before inserting the matching network is calculated. Then the desired power delivered to the load impedance after inserting the matching network is defined. Based on these power values, desired power loss ratio is calculated. Then power loss ratio is calculated in terms of input reflection coefficient, which is written by means of generator impedance and input impedance of the matching network when the load impedance is connected to output port. Finally initialized free parameters are optimized until the power loss ratio of the matching network is equal to the desired power loss ratio. Proposed approach can be used to solve broadband double and single matching problems. Two broadband double matching problems are solved to shown how to use the proposed approach.

Index Terms—Broadband matching, insertion loss, ladder networks, lossless networks, power loss ratio.

I. INTRODUCTION

A two-port network allowing transmission in the passband and attenuating transmission in the stopband is assumed to be a filter.

One of the methods to design filters is image parameter method, which is used to obtain filters with cascaded simpler two-port sections providing desired cutoff frequencies and attenuation characteristics [1].

The other well-known method is the insertion loss method, in which after completely specifying the frequency response of the filter, network synthesis techniques are used [1-3], and the response is written in terms of power loss ratio (P_{LR})

$$P_{LR} = \frac{1}{1 - \left| \Gamma_{in}(\omega) \right|^2} \qquad (1)$$

where Γ_{in} is the reflection coefficient as shown in Fig. 1 [4-6].

Then insertion loss (IL) in dB can be defined as
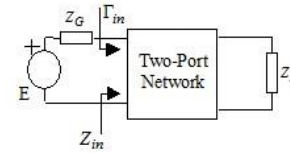$$IL = 10 \log P_{LR} . \qquad (2)$$

Fig. 1. Two-port network between generator and load impedances.

In a Butterworth or Chebyshev filter design process, firstly a low-pass prototype filter is designed in the following manner: The desired power loss ratio of the prototype low-pass filter, which can be found in many books, is formed [1]. After equating the coefficients of the same powers of the desired and formed power loss ratio expressions, element values of the filter are calculated. In the literature, there exist many tables showing the component values for prototype low-pass filter of many different response types, which are formed by using similar design steps [1, 5, 6].

A matching network is assumed to be a two-port network and the power transferred from source to load is desired to be maximum in the passband and attenuated in the stopband. As a result, a matching network is a kind of filter.

Therefore, a two-port can be designed as a filter, single matching and double matching network. In filter designs, both generator and load impedances are purely resistive. If a purely resistive generator and an arbitrary complex load are utilized, then it is a single matching network. A double matching network transfers the power from a generator to a load impedance, which are both complex [7].

In [8], a new approach is proposed to design broadband double or single matching networks with lumped elements. The first comprehensive analysis of globally optimal impedance transformation networks between arbitrary impedances with lossy passives is studied in [9]. In [10], the performance of the distributed active transformer (DAT) structure to that of conventional on-chip impedance-transformations methods is compared.

In the following sections, after briefly explaining the Belevitch representation of a lossless two-port, it is given how to use the insertion loss method to design lossless broadband double or single matching networks.

II. DESCRIPTION OF LOSSLESS TWO-PORT NETWORKS

If Belevitch representation is used to express the scattering parameters of a lossless two-port, they can be written as [7, 8]

$$S_{11}(p) = h(p)/g(p), \qquad S_{12}(p) = \mu f(-p)/g(p)$$
$$S_{21}(p) = f(p)/g(p), \qquad S_{22}(p) = -\mu h(-p)/g(p) \qquad (3)$$

where h is a real polynomial, g is a strictly Hurwitz

polynomial, f is a real (even or odd) monic polynomial for a reciprocal two-port, $p = j\omega$ is the frequency variable. The constant μ is defined as $\mu = f(-p)/f(p) = \pm 1$ for reciprocal networks.

For a realizable two-port network, the following relation known as Feldtkeller equation must be satisfied [11]

$$g(p)g(-p) = f(p)f(-p) + h(p)h(-p) . \qquad (4)$$

The designer decides where to put the transmission zeros of the two-port network. Then the polynomial $f(p)$ is easily formed by means of the defined transmission zeros. The following practical form may be used in many applications

$$f(p) = p^{k_1} \prod_{i=1}^{k_2} (p^2 + c_i^2) \qquad (5)$$

where k_1 (total number of transmission zeros at DC), k_2 (total number of finite transmission zeros) are nonnegative integers, c_i's specify the finite zeros of transmission [7, 8].

III. PROPOSED APPROACH

Two-port network depicted in Fig. 1 is terminated in a normalized complex load impedance ($Z_L = R_L + jX_L$) and a normalized complex generator impedance ($Z_G = R_G + jX_G$). In this case, load reflection coefficient can be expressed as

$$\Gamma_L = (Z_L - 1)/(Z_L + 1) . \qquad (6)$$

Input reflection coefficient (Γ_{in}) is written as follows

$$\Gamma_{in} = S_{11} + (S_{12}S_{21}\Gamma_L)/(1 - S_{22}\Gamma_L) . \qquad (7)$$

By using (7), normalized input impedance of the two-port network seen in Fig. 1 is computed as

$$Z_{in} = R_{in} + jX_{in} = (1 + \Gamma_{in})/(1 - \Gamma_{in}) . \qquad (8)$$

Input reflection coefficient is computed also in terms of input impedance and generator impedance as follows

$$\Gamma_{in} = (Z_{in} - Z_G^*)/(Z_{in} + Z_G) . \qquad (9)$$

Then power loss ratio of the two-port network can be calculated by substituting (9) in (1).
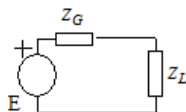

Fig. 2. Generator impedance connected to the load impedance.

Now assume the two-port network between the generator impedance and load impedance is removed, namely the generator impedance is directly connected to the load impedance as seen in Fig. 2. In this case, the power delivered to the load impedance (P_T) can be calculated as

$$PT = \frac{4R_L R_G}{(R_L + R_G)^2 + (X_L + X_G)^2} . \qquad (10)$$

After inserting the two-port network (lossless broadband matching network) between the generator impedance and load impedance as seen in Fig. 1, it is desired to transfer maximum and the flattest power in the passband. In this case, the power delivered to the load impedance (P_R) can be assumed to be unity, namely $P_R = 1$.

Then power loss ratio can also be defined as [12, 13]

$$P_{LR} = P_T / P_R \qquad (15)$$

where P_T is the power delivered to the load impedance before inserting the two-port network, P_R is the power delivered the load impedance after inserting the two-port network.

As the conclusion, the algorithm explained in detail in the next section is proposed to design lossless broadband double and single matching networks with lumped elements via insertion loss method. Mixed or distributed element lossless broadband double and single matching networks can also be designed after modifying the proposed algorithm.

IV. PROPOSED DESIGN ALGORITHM

Inputs:

- ω_i : Sample frequencies.
- $Z_G = R_G + jX_G$, $Z_L = R_L + jX_L$: Measured or calculated normalized generator and load impedance data at sample frequencies, respectively.
- f_{norm} : Frequency normalization number in Hertz.
- R_{norm} : Normalization resistance in ohm.
- $h_n, \ldots, h_2, h_1, h_0$: Initialized coefficients of $h(p)$, where n defines the number of lossless lumped elements in the matching network. It is enough to initialize them as ± 1, or the initialization process described in [14] may be utilized.
- $f(p)$: Designer constructed monic polynomial. Practical form given in (5) can be used to form it.
- $\Delta\delta$: Stopping criteria.

Computational Steps:

Step 1: Strictly Hurwitz polynomial $g(p)$ is formed via (4). Then scattering parameters are calculated via (3).

Step 2: Load reflection coefficient Γ_L is calculated via (6).

Step 3: Input reflection coefficient Γ_{in} is calculated via (7).

Step 4: Normalized input impedance Z_{in} is calculated via (8).

Step 5: Input reflection coefficient Γ_{in} is calculated via (9).

Step 6: Power loss ratio P_{LR} is calculated via (1).

Step 7: P_T (power transmitted to the load before insertion) is calculated via (10), which yields $P_{LR,desired} = \dfrac{P_T}{P_R} = \dfrac{P_T}{1} = P_T$.

Step 8: Error is calculated via $\varepsilon(\omega) = P_{LR,desired} - P_{LR}$, then

$$\delta = \sum |\varepsilon(\omega)|^2 .$$

Power loss ratio defined in (1) can also be expressed as $P_{LR} = 1/TPG$, since transducer power gain at real frequencies is written as $TPG(\omega) = 1 - |\Gamma_{in}(\omega)|^2$. It is shown in [7] that TPG is almost inverse quadratic in the coefficients of $h(p)$. Consequently, optimization becomes a quadratic optimization problem, which has well behaved convergence [15], and a least-square optimization technique provides satisfactory

results. Since the optimization with respect to the coefficients of $h(p)$ is quasi quadratic, in most of the cases ad hoc choice of the coefficients will result satisfactory solutions.

Step 9: Optimization process is stopped when the total squared error difference calculated between two successive iterations is less than $\Delta\delta$, i.e., $|\delta_{i-1} - \delta_i| < \Delta\delta$. Then component values are calculated by synthesizing $S_{11}(p)$ which yields the matching network as a lossless two-port [16]. Alternatively Cauer or Foster methods can be used to synthesize following impedance function $Z_{11}(p) = (1 + S_{11}(p))/(1 - S_{11}(p))$ as detailed in [17].

Initialized coefficients of $h(p)$ are changed via any optimization routine if $|\delta_{i-1} - \delta_i| > \Delta\delta$, and go to Step 1. A least square optimization routine is used in the examples given in the next section. In Fig. 3, the flow chart of the proposed algorithm is given.
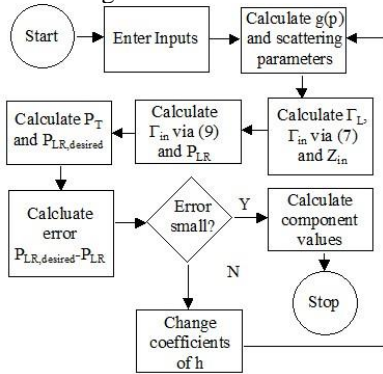

Fig. 3. Flow chart of the proposed algorithm.

V. EXAMPLES

Example 1: In this part, the same lossless broadband double matching problem solved in [8] and solved via SRFT (Simplified real frequency technique [18]) is presented to be able to compare the obtained results. The normalized generator and load impedance values are given in Table I. The given load values are generated from a capacitor $C_L = 4$ in parallel with a resistance $R_L = 1$, and the generator values from an inductor $L_G = 1$ in series with a resistance $R_G = 1$.

TABLE I
CALCULATED GENERATOR AND LOAD IMPEDANCE DATA [8], P_T VALUES
FOR EXAMPLE 1

ω	R_G	X_G	R_L	X_L	P_T
0.0	1.0000	0.0000	1.0000	0.0000	1.0000
0.1	1.0000	0.1000	0.8621	-0.3448	0.9776
0.2	1.0000	0.2000	0.6098	-0.4878	0.9121
0.3	1.0000	0.3000	0.4098	-0.4918	0.8098
0.4	1.0000	0.4000	0.2809	-0.4494	0.6838
0.5	1.0000	0.5000	0.2000	-0.4000	0.5517
0.6	1.0000	0.6000	0.1479	-0.3550	0.4295
0.7	1.0000	0.7000	0.1131	-0.3167	0.3265
0.8	1.0000	0.8000	0.0890	-0.2847	0.2452
0.9	1.0000	0.9000	0.0716	-0.2579	0.1836
1.0	1.0000	1.0000	0.0588	-0.2353	0.1379

The coefficients of $h(p)$ are initialized by ad hoc choices as $h_5 = -1$, $h_4 = 1$, $h_3 = -1$, $h_2 = 1$, $h_1 = -1$, $h_0 = 1$, and $f(p)$ is fixed as $f(p) = 1$ as in [8], since the matching

network is a low-pass two-port. Stopping criteria is selected as $\Delta\delta = 1e - 6$. P_T values calculated via (10) are given in Table I. As explained in Section II, P_R is fixed as unity.

After applying the explained design algorithm, $S_{11}(p)$ function of the designed matching network is reached as $S_{11}(p) = h(p)/g(p)$ where

$h(p) = 0.3289p^5 - 2.2782p^4 - 2.1321p^3 + 0.5572p^2 - 1.5651p + 0.5594,$

$g(p) = 0.3289p^5 + 3.2939p^4 + 6.4730p^3 + 5.8301p^2 + 3.8970p + 1.1458.$

$S_{11}(p)$ is synthesized and the low-pass broadband double matching network depicted in Fig. 4 is obtained.
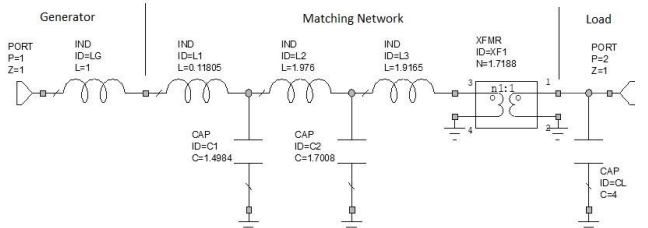

Fig. 4. Designed low-pass broadband matching network; proposed: $C_1 = 1.4984$, $C_2 = 1.7008$, $L_1 = 0.11805$, $L_2 = 1.976$, $L_3 = 1.9165$, $n = 1.7188$, in [8]: $C_1 = 1.4897$, $C_2 = 1.6979$, $L_1 = 0.13233$, $L_2 = 1.9885$, $L_3 = 1.9043$, $n = 1.7135$.

For comparison purpose, TPG curves obtained here, in [8] and via SRFT are given in Fig. 5, which are calculated by using input impedance of the matching network (Z_{in}) and generator impedance (Z_G) in the following expression

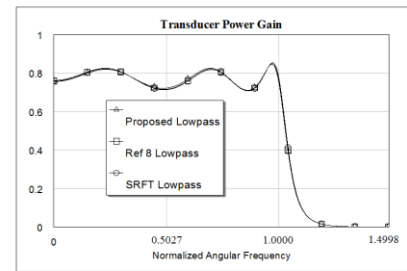$$TPG = \frac{4R_{in}R_G}{(R_{in} + R_G)^2 + (X_{in} + X_G)^2}. \qquad (16)$$

Fig. 5. Performances of the matched systems in example 1.

As seen in Fig. 5, all the performances are very close to each other. As the result, it is clear that the proposed approach, the method given in [8] and SRFT have nearly the same performances. In Fig. 6, frequency responses at selected iterations are given for the proposed approach.
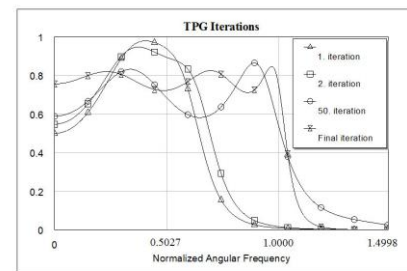

Fig. 6. Frequency responses at selected iterations for example 1.

The proposed algorithm is coded by using Matlab. Average

time for the given example is 15.9012 seconds after three sequential running. It is 16.4019 seconds and 16.3466 seconds in [8] and via SRFT, respectively. The ripple factor ($\tau^2 = (TPG_{max} - TPG_{min})/TPG_{min}$) for the curves in the passband are calculated as 0.1904, 0.1966 and 0.2074 for the method proposed here, given in [8] and SRFT, respectively.

Let us now transform the designed low-pass prototype broadband matching network to a band-pass matching network. If $f_{norm} = 1\,GHz$, $R_{norm} = 50\,\Omega$ and $\Delta = 0.1$ (fractional bandwidth, which is defined as the absolute bandwidth (BW) divided by the center frequency (f_c), so $\Delta = BW/f_c$) is selected, then L_1, L_2, L_3 must be replaced with series LC pairs and C_1, C_2 must be replaced with parallel LC pairs. After calculating the component values in LC pairs by using the formulae given in many books [1], transducer power gain 1 given in Fig. 7 is obtained. Since the generator inductor (L_G) and load capacitor (C_L) seen in Fig. 4 cannot be replaced with series and parallel LC pairs, respectively, the gain response is not the desired one. If L_G and C_L could be replaced with the necessary LC pairs, transducer power gain 2 given in Fig. 7 would be obtained. Fractional bandwidth of the designed band-pass matching network would be $\Delta = (1.0531\,GHz - 0.9501\,GHz)/1\,GHz = 0.103$. Since it is not possible to change the given load and generator impedances, it is necessary to design the broadband matching network with high-pass, band-pass or band-stop response than to transform the designed low-pass prototype broadband matching network. In the second example given below, a high-pass broadband matching network is designed.
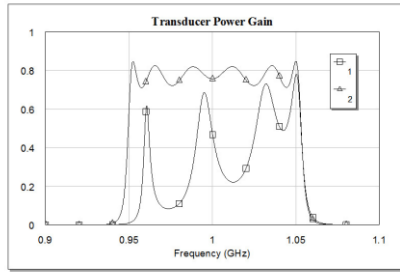

Fig. 7. Performances of the transformed system.

Example 2: In this example, a high-pass lossless broadband matching network is designed between a load impedance (formed by an inductor $L_L = 2$ in parallel with the series connection of a capacitor $C_L = 2$ and a resistance $R_L = 1$) and a generator impedance (formed by an inductor $L_G = 1$ in parallel with a resistance $R_G = 1$). Calculated load and generator impedance data and P_T values are given in Table II. As explained in Section II, P_R is fixed as unity.

The coefficients of $h(p)$ are initialized by ad hoc choices as $h_4 = 1$, $h_3 = -1$, $h_2 = 1$, $h_1 = -1$, $h_0 = 1$. Since a high-pass broadband matching network is desired, then $f(p)$ is fixed as $f(p) = p^4$, which means that all the transmission zeros are put to DC. Stopping criteria is selected as $\Delta\delta = 1e - 6$.

After applying the explained design algorithm, $S_{11}(p)$ function is reached as $S_{11}(p) = h(p)/g(p)$ where

$$h(p) = -0.1337p^4 + 0.0787p^3 + 0.7931p^2 + 0.6384p + 1.1483,$$

$$g(p) = 1.0089p^4 + 2.6286p^3 + 3.3161p^2 + 2.4904p + 1.1483.$$

TABLE II
CALCULATED GENERATOR AND LOAD IMPEDANCE DATA, P_T VALUES FOR EXAMPLE 2

ω	R_G	X_G	R_L	X_L	P_T
1.0	0.5000	0.5000	1.1096	0.0411	0.7696
1.1	0.5475	0.4977	1.0910	0.0304	0.8062
1.2	0.5902	0.4918	1.0767	0.0231	0.8351
1.3	0.6283	0.4833	1.0654	0.0180	0.8582
1.4	0.6622	0.4730	1.0565	0.0143	0.8769
1.5	0.6923	0.4615	1.0493	0.0115	0.8922
1.6	0.7971	0.4494	1.0433	0.0094	0.9048
1.7	0.7429	0.4370	1.0384	0.0078	0.9154
1.8	0.7642	0.4245	1.0343	0.0066	0.9243
1.9	0.7831	0.4121	1.0307	0.0056	0.9319
2.0	0.8000	0.4000	1.0278	0.0048	0.9385

$S_{11}(p)$ function is synthesized and the high-pass broadband matching network seen in Fig. 8 is obtained.
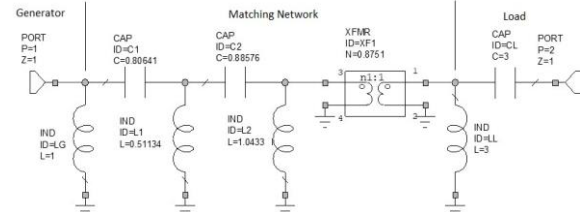

Fig. 8. Designed high-pass broadband matching network, proposed: $C_1 = 0.80641$, $C_2 = 0.88576$, $L_1 = 0.51134$, $L_2 = 1.0433$, $n = 0.8751$, in [8]: $C_1 = 0.85865$, $C_2 = 0.84573$, $L_1 = 0.50465$, $L_2 = 0.8589$, $n = 0.9364$.

In Fig. 9, TPG curves obtained here, in [8] and via SRFT are given for comparison purpose, which are calculated via (16). As seen in Fig. 9, all the performances are nearly the same. In Fig. 10, frequency responses at selected iterations are given for the proposed approach.
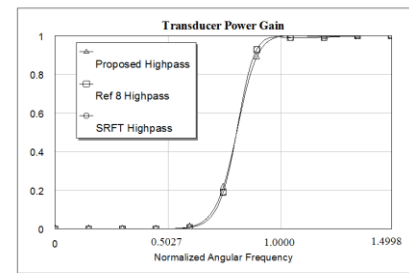

Fig. 9. Transducer power gain curves of the matched systems in example 2.

The proposed algorithm is coded via Matlab. Average time for the given example is 10.6008 seconds after three sequential running. It is 10.6271 seconds and 11.4861 seconds via the proposed approach in [8] and via SRFT, respectively. The ripple factor for the curves in the passband are calculated as 0.0040, 0.0114 and 0.0114 for the method proposed here, given in [8] and SRFT, respectively.

A comparison of the proposed method, approach in [8] and SRFT is given in Table III, where number of iterations, elapsed times and ripple factors for the examples are noted.
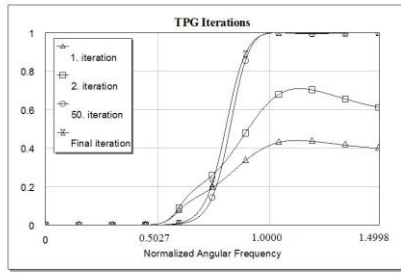
Fig. 10. Frequency responses at selected iterations for example 2.

Obtained lossless broadband double matching networks in both of the examples are simulated via Microwave Office of Applied Wave Research Inc. (AWR) [19].

TABLE III
COMPARISON OF THE PROPOSED METHOD, APPROACH IN [8] AND SRFT.

		Example 1	Example 2
Number of Iterations	Proposed	289	316
	[8]	296	27
	SRFT	296	27
Elapsed Time (sec.)	Proposed	15.9012	10.6008
	[8]	16.4019	10.6271
	SRFT	16.3466	11.4861
Ripple Factor	Proposed	0.1904	0.0040
	[8]	0.1966	0.0114
	SRFT	0.2074	0.0114

In the examples, the same matching problems are solved also by using SRFT, which is a proven method in the literature. All the component values and performances of the designed matching networks via the proposed approach and SRFT are very close to each other. As the result, this can be accepted to be the verification of the proposed approach.

VI. CONCLUSION

For microwave engineers, broadband matching network design is one of the most important problems. Therefore, in this work, a new approach based on insertion loss method to design lossless broadband matching networks is proposed.

In the given algorithm, firstly desired power loss ratio is calculated, which is the ratio of the power delivered to the load before inserting the matching network to the power delivered to the load after inserting the matching network. Then power loss ratio of the designed matching network is computed in terms of input reflection coefficient, which is written by using generator impedance and input impedance when load impedance is connected to output port. Finally selected free parameters are optimized until the error between the desired power loss ratio and the power loss ratio of the matching network is acceptable.

Proposed algorithm is used to design lossless broadband double and single matching networks with lumped elements. Mixed or distributed element broadband matching networks can also be designed after modifying the proposed algorithm.

Two broadband double matching networks are designed as examples and it was shown that proposed approach has a very close performance to the performances of SRFT and the method given in [8]. Therefore, practical broadband matching networks for microwave applications can be designed by utilizing the proposed insertion loss method based approach.

REFERENCES

[1] D. M. Pozar, *Microwave engineering*. 3rd ed., NJ, John Wiley & Sons Inc., 2005.
[2] M. Şengül, "Shifted-modified Chebyshev filters," *Turk J Elec Eng & Comp Sci*, vol. 21, pp. 1351-1358, May 2013.
[3] M. T. Chryssomallis, J. N. Sahalos, "Filter synthesis using products of Legendre polynomials," *Electrical Engineering*, vol. 81, pp. 419-424, Jun. 1999.
[4] A. Thube, M. Chattopadhyay, RF filter design using insertion loss method and genetic optimization algorithm, *International journal of Engineering Research and Applications*, vol. 3, no. 4, pp. 207-211, Jul-Aug. 2013.
[5] D. M. Pozar, *Microwave and RF design of wireless systems*, John Wiley & Sons, Inc., NY, 2001.
[6] J. F. White, *High frequency techniques, an introduction to RF and microwave engineering*, IEEE Press and Wiley Interscience, NJ, 2004.
[7] A. Aksen, "Design of lossless two-port with mixed, lumped and distributed elements for broadband matching," Ph.D. dissertation, Ruhr University, Bochum, Germany, 1994.
[8] M. Şengül, "Design of practical broadband matching networks with lumped elements," *IEEE Trans. CAS II: Exp. Briefs*, vol. 60, no. 9, pp. 552–556, Sept. 2013.
[9] C. R. Chappidi, K. Sengupta, "Globally optimal matching metworks with lossy passives and efficiency bounds," *IEEE Trans. CAS I: Regular Papers*, vol. 65, no. 1, pp. 257-269, Jan. 2018.
[10] I. Aoki, S. D. Kee, D. B. Rutledge, A. Hajimiri, "Distributed active transformer-a new power-combining and impedance-transformation technique," *IEEE T Microw Theory*, vol. 50, no. 1, pp. 316-331, Jan. 2002.
[11] H. J. Carlin, P. P. Civalleri, *Wideband circuit design*. CRC Press LLC, 1998, ISBN: 0-8483-7897-4, Library of Congress Card Number: 97 26966.
[12] L. Tihanyi, *Electromagnetic compatibility in power electronics*, The IEEE Press, 1995, ISBN: 0-7803-0416-0.
[13] K. Kostov, J. Kyyra, "Insertion loss in terms of four-port network parameters," *IET Sci Meas Technol*, vol. 3(3), pp. 208-216, June 2009.
[14] M. Şengül, B. S. Yarman, C. Volmer, M. Hein, "Desing of distributed-element RF filters via reflectance data modeling," *Int. J. Electron. Commun. (AEU)*, vol. 62, pp. 483-489, Aug. 2008.
[15] C. L. Lawson, R. J. Hanson, *Solving least-squares problems*. Prentice-Hall, 1974, ISBN: 0-8987-1356-0.
[16] M. Şengül, "Synthesis of resistively terminated LC ladder networks," *Istanbul University-Journal of Electrical & Electronics Engineering*, vol. 11, no. 2, pp. 1407-1412, Jul. 2011.
[17] W. C. Yengst, *Procedures of modern network synthesis*. New York, The Macmillian Company, 1964.
[18] B. S. Yarman, H. J. Carlin, "A simplified real frequency technique applied to broadband multistage microwave amplifiers," *IEEE Trans MTT*, vol 30, pp. 2216–2222, Dec. 1982.
[19] AWR : Microwave Office of Applied Wave Research Inc.: [Online]. Available: www.awrcorp.com