INTERNATIONAL JOURNAL OF CIRCUIT THEORY AND APPLICATIONS
Int. J. Circ. Theor. Appl. 2017; **45**:133–140
Published online 30 May 2016 in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/cta.2232

LETTER

Broadband matching via reflection function optimization

Metin Şengül*,†

Department of Electrical and Electronics Engineering, Kadir Has University, Cibali-Fatih 34083 Istanbul, Turkey

SUMMARY

In this paper, a practical approach is presented for designing broadband matching networks via reflection function optimization. In the proposed algorithm, the input or output reflection function of the matching network is expressed in terms of three real polynomials describing the matching network, load and generator reflection coefficients. Next one of the polynomials is optimized to get minimum reflection function values in the passband. Then matching network topology and element values are obtained via the formed input reflection coefficient expression. Two examples are presented to explain the usage of the new approach. Copyright © 2016 John Wiley & Sons, Ltd.

Received 23 February 2016; Accepted 28 April 2016

KEY WORDS: broadband matching; lossless networks; matching networks; real frequency techniques

1. INTRODUCTION

The broadband matching problem is regarded as the construction of a lossless two-port network between a resistive or complex generator and complex load impedance, and where the transferred power from generator to load is maximized in the passband [1, 2]. Usually transducer power gain (*TPG*), which is defined as the ratio of power delivered to the load to the available power from the generator, is used to measure the power transfer capacity of the matching network (Figure 1).

Broadband matching problems can be collected in two groups as single matching and double matching problems. In the first type of problems, a resistive generator is matched to a complex load [3]. On the other hand, if the power is transferred from a complex generator to a complex load, then the problem is referred to as a double matching problem [4, 5].

In literature, there are lots of different techniques to design broadband matching networks. But they can be grouped basically as the methods based on *TPG* optimization and the methods based on modeling. In the first group, the selected free parameters are optimized until the desired gain level is reached [6–15]. In the other group, first the values of any selected function are calculated, and then a model is formed for these data [16–18].

But in the proposed approach, the input or output reflection function of the matching network is optimized in the passband. In the next section, the rationale of the proposed approach is described.

2. RATIONALE OF THE PROPOSED APPROACH

Consider the double matching arrangement shown in Figure 1. Because the two-port is lossless, on the imaginary axis of the complex frequency plane, input and output reflection functions (ρ_1 and ρ_2) are related by the following equation

*Correspondence to: Metin Şengül, Department of Electrical and Electronics Engineering, Kadir Has University, Cibali-Fatih, 34083 Istanbul, Turkey.
†E-mail: mtnsngl@gmail.com

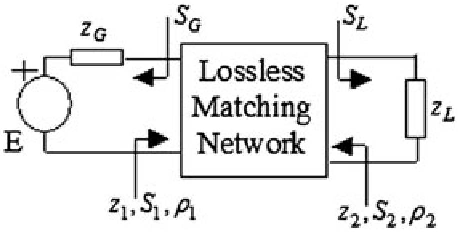

Figure 1. Double matching arrangement.

$$|\rho_1|^2 = |\rho_2|^2. \tag{1}$$

Then the transducer power gain at real frequencies can be defined as

$$TpG(\omega) = 1 - |\rho_1|^2 = 1 - |\rho_2|^2. \tag{2}$$

This equation means that to get maximum flat *TPG* curve in the passband corresponds to get minimum flat $|\rho_1|^2$ or $|\rho_2|^2$ curve. So it is necessary for ρ_1 or ρ_2 to be expressed in terms of any parameters related to the matching network, load and generator. Now let us obtain these expressions.

Input reflection function (ρ_1) can be defined as

$$\rho_1 = \frac{z_1 - z_G^*}{z_1 + z_G} \tag{3}$$

where z_1 is the normalized input impedance seen at port 1 when port 2 is terminated by the normalized load (z_L), z_G is the normalized generator impedance and the upper asterisk denotes complex conjugation.

In a similar manner, the reflection function at port 2 (ρ_2) can be defined as

$$\rho_2 = \frac{z_2 - z_L^*}{z_2 + z_L} \tag{4}$$

where z_2 is the normalized output impedance seen at port 2 when port 1 is terminated in z_G.

Now let us define the impedances z_G and z_1 at port 1 as

$$z_G = \frac{1 + \Gamma_G}{1 - \Gamma_G}, \quad z_1 = \frac{1 + \Gamma_1}{1 - \Gamma_1}. \tag{5}$$

Also let us define the impedances z_L and z_2 at port 2 as

$$z_L = \frac{1 + \Gamma_L}{1 - \Gamma_L}, \quad z_2 = \frac{1 + \Gamma_2}{1 - \Gamma_2}. \tag{6}$$

Γ_1 and Γ_2 can be written as a function of the scattering parameters (S_{ij}, $i,j = 1, 2$) of the matching network and the reflection coefficient of the load and generator, respectively, as

$$\Gamma_1 = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}, \tag{7}$$

$$\mathit{\Gamma}_2 = S_{22} + \frac{S_{12}S_{21}\Gamma_G}{1 - S_{11}\Gamma_G}. \tag{8}$$

Here the scattering parameters of the matching network can be expressed in Belevitch form as a function of three real polynomials as follows [12]:

DOI: 10.1002/cta

$$S(p) = \begin{bmatrix} S_{11}(p) & S_{12}(p) \\ S_{21}(p) & S_{22}(p) \end{bmatrix} = \frac{1}{g(p)} \begin{bmatrix} h(p) & \mu f(-p) \\ f(p) & -\mu h(-p) \end{bmatrix} \tag{9}$$

where $p = \sigma + j\omega$ is the complex frequency variable, h is a real coefficient polynomial, g is a strictly Hurwitz polynomial, f is a real monic polynomial which is constructed via the transmission zeros of the matching network and μ is a constant ($\mu = \pm 1$).

These three polynomials $\{f, g, h\}$ are related by the following Feldtkeller equation [7]

$$g(p)g(-p) = h(p)h(-p) + f(p)f(-p). \tag{10}$$

It can be concluded from (10) that the strictly Hurwitz polynomial $g(p)$ is a function of the polynomials $h(p)$ and $f(p)$. If $f(p)$ and $h(p)$ are specified, then the scattering parameters of the matching network can be obtained via (9).

In most applications, locations of transmission zeros of the matching network are decided by the designer. Thus, the polynomial $f(p)$ is usually formed by the designer who may use the following form of $f(p)$

$$f(p) = p^{m_1} \prod_{i=0}^{m_2} \left(p^2 + a_i^2\right) \tag{11}$$

where m_1 and m_2 are nonnegative integers and a_i's are arbitrary real coefficients. In this form, the transmission zeros are on the imaginary axis of the complex p-plane.

Finally substituting the relationships from (5), (7) and (9) in (3) yields the reflection function at port 1 as follows:

$$\rho_1 = \frac{(1 - \Gamma_G)\left[\mu \Gamma_L \left(h_* \Gamma_{G^*} - g_*\right) + \left(g \Gamma_{G^*} - h\right)\right]}{\left(\Gamma_{G^*} - 1\right)\left[\mu \Gamma_L (h_* - g_* \Gamma_G) + (g - h\Gamma_G)\right]}. \tag{12}$$

In a similar manner, substituting the relationships from (6), (8) and (9) in (4) yields the reflection function at port 2 as follows:

$$\rho_2 = \frac{(1 - \Gamma_L)\left[\mu (h_* - g_* \Gamma_G) + \Gamma_{L^*}(g - h\Gamma_G)\right]}{\left(\Gamma_{L^*} - 1\right)\left[\mu \Gamma_L (h_* - g_* \Gamma_G) + (g - h\Gamma_G)\right]}. \tag{13}$$

So if the polynomial f is formed, and the polynomial h is initialized, then the polynomial g is obtained via (10), which yields the calculation of the reflection function at port 1 or port 2 via (12) or (13) with the help of S_G and S_L.

In the simplified real frequency technique (SRFT), *TPG* is expressed in terms of the descriptive polynomials (h, g and f), generator and load reflection coefficients Γ_G and Γ_L, respectively. Then *TPG* is optimized to get maximum power transfer [10, 11]. But in the proposed approach, the reflection function at port 1 and port 2 have been written in terms of the same variables as seen in (12) and (13). Then in light of (2), $|\rho_1|^2$ or $|\rho_2|^2$ is minimized to get maximum power transfer from generator to load.

As the result, the following algorithm can be offered to design broadband matching networks with lumped elements for both single and double matching problems. But the same algorithm can easily be adapted to design distributed or mixed element broadband matching networks. The algorithm given in [12–14] can also be used to design broadband matching networks. But it is substantially different from the algorithm proposed here. In [12–14], transducer power gain is expressed in terms of impedances and then it is maximized. *TPG* expression used in [12] can be obtained if (3) or (4) is substituted in (2). But here, input or output reflection function is expressed in terms of three real polynomials describing the matching network, load and generator reflection coefficients and then the selected reflection function is minimized. Because (2) has not been used in the derivation of (12) and (13), it is clear that the presented method here and the one in (12) are completely different from each other.

DOI: 10.1002/cta

3. ALGORITHM

Inputs:

• $Z_{L(given)} = R_{L(given)} + jX_{L(given)}$, $Z_{G(given)} = R_{G(given)} + jX_{G(given)}$: Given (measured or calculated) load and generator impedance data, respectively.
• $\omega_{i(given)}$: Given measurement frequencies, $\omega_{i(given)} = 2\pi f_{i(given)}$.
• f_{norm}: Normalization frequency.
• R_{norm}: Impedance normalization number in ohms.
• $h_0, h_1, h_2, \ldots, h_n$: Initialized real coefficients of $h(p)$. Here n is the degree of the polynomial and it is also equal to the number of lossless elements in the broadband matching network. These coefficients can be initialized as ± 1 in an ad hoc manner, or the approach explained in [19] can be used.$f(p)$: A monic polynomial. As explained this polynomial is formed by the designer via the transmission zeros of the matching network. The form given in (11) is practical.δ_c: The stopping criteria of the algorithm.

Outputs:

• Analytic form of the input scattering parameter of the two-port network given in the Belevitch form of $S_{11}(p) = h(p)/g(p)$.
• Matching network topology with element values are obtained as the result of the synthesis of $S_{11}(p)$. Synthesis is carried out in the Darlington sense. That is, $S_{11}(p)$ is synthesized as a lossless two-port [20]. Also the synthesis process can be carried out by using impedance based Foster or Cauer methods via $z_{11}(p) = (1 + S_{11}(p))/(1 - S_{11}(p))$ as explained in [21].

Computational Steps:

Step 1: Normalize the given frequencies with respect to f_{norm} and set all the normalized angular frequencies $\omega_i = f_{i(given)}/f_{norm}$.Normalize the given load and generator impedances by using the impedance normalization number R_{norm}; $r_L = R_{L(given)}/R_{norm}$, $x_L = X_{L(given)}/R_{norm}$, $r_G = R_{G(given)}/R_{norm}$, $x_G = X_{G(given)}/R_{norm}$ over entire the passband.
Step 2: Compute the reflection coefficients $\Gamma_G = \frac{z_G - 1}{z_G + 1}$ and $\Gamma_L = \frac{z_L - 1}{z_L + 1}$.
Step 3: Form the strictly Hurwitz polynomial $g(p)$ from (10).
Step 4: Calculate the values of the input or output reflection functions via (12) or (13), respectively.
Step 5: Calculate the error via $\varepsilon(\omega) = |\rho_1|^2$ or $\varepsilon(\omega) = |\rho_2|^2$, then $\delta = \sum \varepsilon(\omega)$.
Step 6: If δ is acceptable ($\delta \le \delta_c$), stop the algorithm and synthesize $S_{11}(p)$. Otherwise, change the initialized coefficients of the polynomial $h(p)$ via any optimization routine and return to step 3.

4. EXAMPLES

4.1. Example 1

In the first example, a double-matching problem is solved. The normalized load and generator impedance data are given in Table I. It should be noted that the given load data can easily be

Table I. Given normalized load and generator impedance data.

ω	r_L	x_L	r_G	x_G
0.0	1.0000	0.0000	1.0000	0.0000
0.1	0.8621	−0.3448	1.0000	0.1000
0.2	0.6098	−0.4878	1.0000	0.2000
0.3	0.4098	−0.4918	1.0000	0.3000
0.4	0.2809	−0.4494	1.0000	0.4000
0.5	0.2000	−0.4000	1.0000	0.5000
0.6	0.1479	−0.3550	1.0000	0.6000
0.7	0.1131	−0.3167	1.0000	0.7000
0.8	0.0890	−0.2847	1.0000	0.8000
0.9	0.0716	−0.2579	1.0000	0.9000
1.0	0.0588	−0.2353	1.0000	1.0000

DOI: 10.1002/cta

modeled as a capacitor $C_L = 4$ in parallel with a resistance $R_L = 1$ (i.e. $R_L//C_L$ type of impedance), and the generator data as an inductor $L_G = 1$ in series with a resistance $R_G = 1$ (i.e. $R_G + L_G$ type of impedance). Because the given impedance data are already normalized, skip step 1. For comparison purposes, the same example is solved here via the simplified real frequency technique (SRFT) and the method proposed in [12].

The polynomial $h(p)$ is initialized as $h(p) = -p^4 + p^3 - p^2 + p - 1$ in an ad hoc manner. Also the polynomial $f(p)$ is selected as $f(p) = 1$, because a low-pass matching network is desired. Then the proposed algorithm is run, and the following input scattering parameter of the broadband matching network is obtained

$$S_{11}(p) = \frac{h(p)}{g(p)} \text{ where}$$
$$h(p) = -2.8451p^4 - 2.6280p^3 - 0.0913p^2 - 1.7304p + 0.4744,$$
$$g(p) = 2.8451p^4 + 6.0921p^3 + 5.3999p^2 + 3.8774p + 1.1068.$$

If the obtained input scattering parameter or the corresponding impedance function is synthesized, the matching network seen in Figure 2 is obtained. The normalization frequency and impedance normalization number is selected as $f_{norm} = 1\ GHz$ and $R_{norm} = 50\ \Omega$, respectively, then the real element values are calculated as $L_1 = 14.033nH$, $L_2 = 13.025nH$, $C_1 = 5.2286pF$, $C_2 = 6.0578pF$, $n = 0.6315$, $C_L = 12.732pF$, $R_L = 50\Omega$, $L_G = 7.9577nH$ and $R_G = 50\Omega$.

As seen in Figure 3, the obtained performance of the system looks very good. But, if it is desired, it can be improved by considering the losses via any CAD tool having realistic element models [22]. For comparison purposes, the performance obtained by means of the offered algorithm here, via SRFT and via the proposed method in [12] are depicted in Figure 3. Also the input and output reflection curves are given in Figure 3. Because there is a transformer in the matching network, it is natural that there will be no power transfer at DC. If this section of Figure 3 were drawn by zooming in, it could be seen.

The algorithm is coded in Matlab, and the problem is solved ten times. The average elapsed time is 1.9665 s. It is 1.9011 s via SRFT and 1.6434 s via the method proposed in [12].

The ripple factor τ^2 for the curves in the passband can be calculated as

$$\tau^2_{proposed} = \frac{TpG_{\max} - TpG_{\min}}{TpG_{\min}} = \frac{0.8712 - 0.6707}{0.6707} = 0.2989,$$
$$\tau^2_{SRFT} = \tau^2_{\mathrm{Re}f\,9} = \frac{0.8520 - 0.7142}{0.7142} = 0.1929.$$

It is clear that the method proposed in [12] and SRFT have nearly the same performance. On the other hand, the proposed method here has the largest ripple factor and elapsed time. But it can be concluded that the proposed method here as an alternative method generates pretty good initials for the commercially available design packages.

4.2. Example 2

In this example, a single matching problem is solved. The load is selected as a monopole antenna. The normalized impedance data for the antenna are provided over $20 - 100\,MHz$ in Table II. f_{norm} is selected as $100\,MHz$.

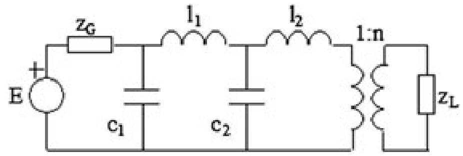

Figure 2. Designed lumped-element double-matching network; proposed: $l_1 = 1.7635$, $l_2 = 1.6368$, $c_1 = 1.6426$, $c_2 = 1.9031$, $n = 0.6315$, SRFT and Ref [12]: $l_1 = 1.8698$, $l_2 = 1.7935$, $c_1 = 1.5506$, $c_2 = 1.7878$, $n = 0.5997$.

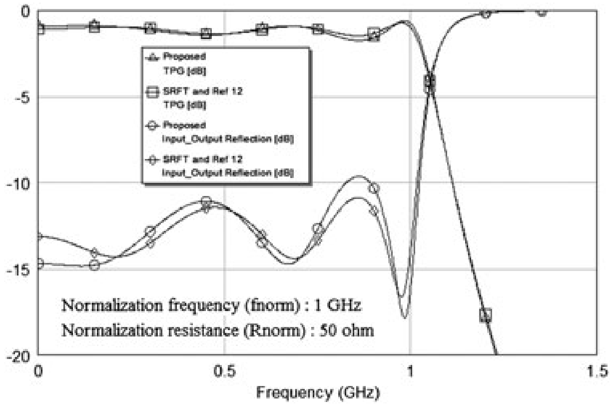

Figure 3. Performance of the matched system designed with lumped elements.

Table II. Normalized impedance data for the antenna.

ω	r_L	x_L
0.20	0.60	−6.00
0.30	0.80	−2.20
0.40	0.80	0.00
0.45	1.00	1.40
0.50	2.00	2.80
0.55	3.40	4.60
0.60	7.00	7.60
0.65	15.0	8.80
0.70	22.4	−5.40
0.75	11.0	−13.0
0.80	5.00	−10.8
0.90	1.60	−6.80
1.00	1.00	−4.40

The polynomial $h(p)$ is initialized as $h(p) = -p^3 - p^2 - p - 1$ in an ad hoc manner. Also the polynomial $f(p)$ is selected as $f(p) = 1$. Then the proposed algorithm is run, and the following input scattering parameter of the broadband matching network is obtained

$$S_{11}(p) = \frac{h(p)}{g(p)}$$

where

$$h(p) = 1.9591p^3 - 2.8216p^2 + 2.6432p - 1.3231,$$
$$g(p) = 1.9591p^3 + 3.1625p^2 + 3.1639p + 1.6585.$$

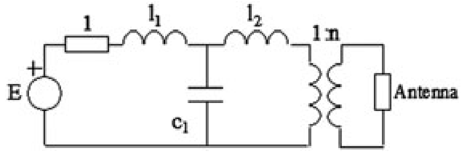

Figure 4. Designed lumped-element single-matching network; proposed: $l_1 = 0.65477$, $l_2 = 1.2929$, $c_1 = 1.5524$, $n = 2.9814$, SRFT and Ref [12]: $l_1 = 0.85236$, $l_2 = 1.542$, $c_1 = 1.3962$, $n = 2.4902$.

DOI: 10.1002/cta

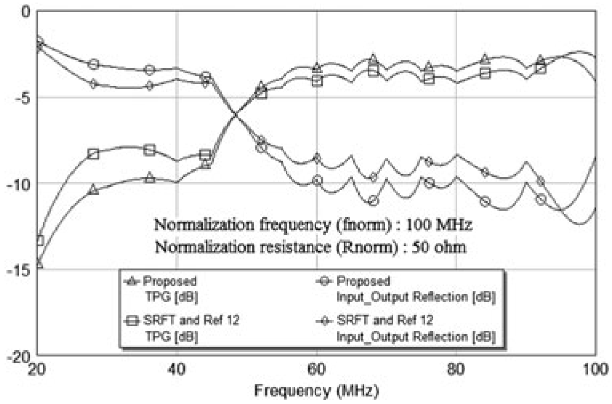

Figure 5. Performance of the matched antenna.

Table III. Comparison of the alternative solutions.

Method	Element type	Need transformer	Convergence rate	Need modeling	Ripple factor
Proposed	Lumped, distributed, mixed	Yes	Fast	No	Modest
SRFT	Lumped, distributed, mixed	Yes	Fast	No	Low
Ref [12–14]	Lumped, distributed, mixed	Yes	Fast	No	Low
Ref [16]	Lumped, distributed, mixed	Yes	Slow	Yes	High

After synthesizing the obtained input scattering parameter or the corresponding impedance function, the matching network seen in Figure 4 is obtained. The normalization frequency and impedance normalization number is selected as $f_{norm} = 100\ MHz$ and $R_{norm} = 50\ \Omega$, respectively, then the real element values are calculated as $L_1 = 1.0421nH$, $L_2 = 2.0577nH$, $C_1 = 2.4707nF$, $n = 2.9814$ and $R_G = 50\Omega$.

The same problem is solved via the method proposed in [12] and SRFT with the same initials. For comparison purposes, the performance obtained by means of the offered algorithm here, via SRFT and via the proposed method in [12], are depicted in Figure 5. Also the input and output reflection curves are given in Figure 5.

From Figure 5, it is clear that the method proposed in [12] and SRFT have the same performance, which is better than the performance of the proposed method from 20 MHz to 48 MHz frequency band. On the other hand, the performance of the proposed method is better from 48 MHz to 95 MHz. So it can be concluded that the proposed method here generates pretty good initials for the commercially available CAD tools for final optimization by using practical element models.

In Table III, the proposed method is compared with some different solutions in the literature for the same examples; for the sake of comparing the different methods, element type used in the matching networks, need for a transformer, the relative convergence rates, need for modeling and ripple factor levels are given in the table.

5. CONCLUSION

Usually commercially available computer-aided design (CAD) tools are utilized to design broadband matching networks. Because the matching network topology and initial element values are unknown and the system performance is highly nonlinear in terms of the element values, these packages may not generate acceptable solutions. Therefore, in this paper, a new initialization method is proposed for CAD tools.

In the proposed method, the input or output reflection function of the matching network is expressed as a function of the descriptive polynomials of the matching network and load and generator reflection

DOI: 10.1002/cta

coefficients. Then this function is minimized over the interested frequency band via the offered algorithm.

Finally, the formed input scattering parameter (or the corresponding input impedance function) is synthesized, and the desired matching network topology with initial element values is obtained. Obviously, the performance of the matching network may be improved by considering the losses via any CAD tool having realistic element models.

In the proposed method, the polynomial $f(p)$ is constructed by using the transmission zeros of the matching network, so they are under the control of the designer. Single and double matching problems can also be solved via the proposed method.

Two examples have been presented to design broadband matching networks with lumped elements. It was shown that the proposed method generates pretty good initials for CAD tools. So it is concluded that the offered algorithm can be used to generate initials for commercially available CAD packages to design broadband matching networks for microwave communication systems.

REFERENCES

1. Fano RM. Theoretical limitations on the broadband matching of arbitrary impedances. *Journal of the Franklin Institute* 1950; **249**:57–83.
2. Youla DC. A new theory of broadband matching. *IEEE Transactions on Circuit Theory* 1964; **11**:30–50.
3. Carlin HJ, Civalleri PP. A general theory for broadband matching of two-ports with application to fet amplifiers with internal feedback. *International Journal of Circuit Theory and Applications* 1991; **19**(1):51–64.
4. Youla DC, Carlin HJ, Yarman BS. Double broadband matching and the problem of reciprocal reactive 2n-port cascade decomposition. *International Journal of Circuit Theory and Applications* 1984; **12**(3):269–281.
5. Dehollain C, Neirynck J. Simplified impedance broadband matching by the image parameter method: the RC case. *International Journal of Circuit Theory and Applications* 1995; **23**(2):91–116.
6. Carlin HJ. A new approach to gain-bandwidth problems. *IEEE Transactions on Circuits and Systems* 1977; **23**:170–175.
7. Carlin HJ, Civalleri PP. Wideband circuit design. CRC Press LLC: ISBN: 0-8483-7897-4, Library of Congress Card Number: 97 26966, 1998.
8. Carlin HJ, Yarman BS. The double matching problem: analytic and real frequency solutions. *IEEE Transactions on Circuits and Systems* 1983; **30**:15–28.
9. Fettweis A. Parametric representation of brune functions. *International Journal of Circuit Theory and Applications* 1979; **7**:113–119.
10. Yarman BS. A simplified real frequency technique for broadband matching complex generator to complex loads. *RCA Review* 1982; **43**:529–541.
11. Yarman BS, Carlin HJ. A simplified real frequency technique applied to broadband multi-stage microwave amplifiers. *IEEE Transactions on Microwave Theory and Techniques* 1982; **30**:2216–2222.
12. Şengül M. Design of practical broadband matching networks with lumped elements. *IEEE Transaction CAS-II:Express Briefs* 2013; **60**(9):552–556.
13. Şengül M. Design of practical broadband matching networks with commensurate transmission lines. *International Journal of Electronics and Communications (AEU)* 2013; **67**(8):676–680.
14. Şengül M. Design of practical broadband matching networks with mixed lumped and distributed elements. *IEEE Transaction CAS-II:Express Briefs* 2014; **61**(11):875–879.
15. Dedieu H, Dehollain C, Neirynck J, Rhodes G. New broadband matching circuit. *International Journal of Circuit Theory and Applications* 1994; **22**(1):61–69.
16. Yarman BS, Şengül M, Kılınç A. Design of practical matching networks with lumped elements via modeling. *IEEE Transaction CAS-I: Regular Papers* 2007; **54**(8):1829–1837.
17. Şengül M. Modeling based real frequency technique. *International Journal of Electronics and Communications (AEU)* 2008; **62**(2):77–80.
18. Yarman BS, Kılınç A, Aksen A. Immittance data modeling via linear interpolation techniques: a classical circuit theory approach. *International Journal of Circuit Theory and Applications* 2004; **32**(6):537–563.
19. Şengül M, Yarman BS, Volmer C, Hein M. Design of distributed-element RF filters via reflectance data modeling. *International Journal of Electronics and Communications (AEU)* 2008; **62**:483–489.
20. Şengül M. Synthesis of resistively terminated LC ladder networks. *Istanbul University-Journal of Electrical & Electronics Engineering* 2011; **11**(2):1407–1412.
21. Yengst WC. *Procedures of Modern Network Synthesis*. The Macmillian Company: New York, USA, 1964.
22. AWR: Microwave Office of Applied Wave Research Inc. (Available from: http://www.appwave.com) [accessed on 19 May 2016].

DOI: 10.1002/cta