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PERFORMANCE ANALYSIS OF LOW COMPLEXITY MAXIMAL RATIO TRANSMISSION APPROACHES IN MULTI-RELAY NETWORKS

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PERFORMANCE ANALYSIS OF LOW COMPLEXITY MAXIMAL RATIO TRANSMISSION APPROACHES IN MULTI-RELAY NETWORKS

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Ben, Eylem Erdoğan, bu doktora tezinde sunulan çalışmanın şahsıma ait olduğunu ve başka çalışmalardan yaptığım alıntıların kaynaklarını kurallara uygun biçimde tez içerisinde belirttiğimi onaylıyorum.

Eylem Erdoğan

ABSTRACT

PERFORMANCE ANALYSIS OF LOW COMPLEXITY MAXIMAL RATIO TRANSMISSION APPROACHES IN MULTI-RELAY NETWORKS

Wireless communication networks have to provide more reliability, high data rates with less energy consumption to meet the increasing demand. To this end, this thesis focuses on the employment and analysis of low complexity multi-antenna techniques in amplify-and-forward one-way or two-way multi-relay networks. First, the performance of dual-hop multi-relay maximal ratio transmission over Rayleigh fading channels is studied with both conventional and opportunistic relaying. Analysis starts with the derivation of statistical functions of signal-to-noise ratio such as cumulative distribution function and moment generating function. Then, symbol error rate, outage probability and ergodic capacity are derived. Second work focuses on a two-way relay network with maximal ratio transmission and relay selection. For this network, sum symbol error rate and system outage probability are derived for Nakagami-m fading channels. Finally, a two-way relay network with joint antenna and relay selection is analyzed for Nakagami-m fading channels where system outage probability is obtained.

ÖZET

DÜŞÜK KARMAŞIKLIKLI MAKSİMUM ORANLI İLETİM YAKLAŞIMLARININ ÇOK RÖLELİ AĞLARDA PERFORMANS ANALİZİ

Kablosuz iletişim ağlarının artan talebi karşılamak için yüksek veri hızına sahip, yüksek güvenilirlikli, ve düşük enerji tüketimli olması gerekmektedir. Bu nedenle, bu doktora tezi düşük karmaşıklığa sahip kuvvetlendir-aktar yapısındaki tek yönlü ve iki yönlü çok antenli ve çok röleli sistemlerin uygulama ve analizlerini sunmaktadır. İlk olarak, iki atlamalı maksimum oranlı iletim tekniği kullanılan geleneksel ve fırsatçı yapıdaki çok röleli sistemlerin performansı Rayleigh sönümlemeli kanallarda incelenmektetir. Analiz, kümülatif dağılım fonksiyonu ve moment üreten fonksiyon gibi sinyal-gürültü-oranına ait istatistiksel fonksiyonların türetimi ile başlamaktadır. Sonrasında, sembol hata oranı, kesinti olasılığı ve ergodik kapasite bulunmaktadır. İkinci çalışmada maksimum oranlı iletim tekniği ve röle seçimini içeren iki yönlü röleli ağ modeli üzerinde durulmaktadır. Bu model için, toplam sembol hata oranı ve sistem kesinti olasılığı Nakagami-m sönümlemeli kanallarda türetilmektedir. Son olarak, iki yönlü röleli ağ modeli anten ve röle seçimiyle beraber Nakagami-m sönümlemeli kanallarda analiz edilmekte ve sistem kesinti olasılığı elde edilmektedir.

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Dedicated to my beloved parents,

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LIST OF ABBREVIATIONS

MIMO	multi-input multi-output
Tx/Rx	transmit/receive
STBC	space time block coding
SNR	signal-to-noise ratio
MRT	maximal ratio transmission
MRC	maximum ratio combining
EGC	equal gain combining
WiFi	Wireless Fidelity
LOS	line-of-sight
PDF	probability density function
OP	outage probability
BER	bit error rate
AF	amplify-and-forward
DF	decode-and-forward
PRS	partial relay selection
SER	symbol error rate
OWRN	one-way relay network
TWRN	two-way relay network
SSER	sum symbol error rate
e2e	end-to-end
i.i.d	independent and identically distributed
i.n.i.d	independent and non-identically distributed

CSI	channel state information
OSTBC	orthogonal space time block coding
CDF	cumulative distribution function
MGF	moment generation function
AWGN	additive white Gaussian noise
BPSK	binary phase shift keying
BFSK	binary frequency shift keying
M-PSK	M-ary phase shift keying
M-PAM	M-ary pulse amplitude modulation
RF	radio-frequency

LIST OF SYMBOLS

$A \to B$	transmission link between A and B
$A \to B \to C$	transmission link between A, B and C
A^T	transpose of A
A^*	conjugate of A
$A^\dagger$	conjugate-transpose of A
$\|A\|$	Frobenius norm of of A
$x \sim \mathcal{CN}(a, \sigma_n^2)$	complex Gaussian rv with mean a and variance σ_n^2
$\Pr[\cdot]$	probability operation
$\mathbb{E}[\cdot]$	expectation operation
$\mathcal{Q}(\cdot)$	Q-function
$f_x(\cdot)$	probability density function of x
$\mathcal{F}_x(\cdot)$	cumulative distribution of x
$\mathcal{M}_x(\cdot)$	moment generating function of x
$\mathcal{L}(\cdot)$	Laplace transform
$\mathcal{G}_a(\cdot)$	array gain
$\mathcal{G}_d(\cdot)$	diversity gain
$\mathcal{X}_a(\cdot)$	multinomial coefficients [1, eqn. 0.314]
$\exp(\cdot)$	exponential function
$\log_2(\cdot)$	logarithm base 2
$\Gamma[\cdot, \cdot]$	upper incomplete Gamma function [1, eqn. 8.350.2]
$\Upsilon[\cdot, \cdot]$	lower incomplete Gamma function [1, eqn. 8.350.1]
$\Gamma[\cdot]$	Gamma function [1, eqn. 8.310.1]

${}_2F_1(\cdot,\cdot;\cdot;\cdot)$	Gauss' hypergeometric function [1, eq.(9.100)]

1. INTRODUCTION

This chapter consists of literature review about wireless channels, MIMO and cooperative communication, motivations, contributions and organization.

1.1. Literature Review

Wireless channels can experience deep fading leading to unreliable communication, thus, increasing diversity order of the system is highly desirable to reduce symbol error rates and outage probabilities. To increase diversity orders, many transmission/reception schemes are proposed and analyzed. The main idea of these techniques is to improve spatial (antenna) diversity by acquiring multiple replicas of the information signal at the receiver. To obtain maximum antenna diversity, multi-input multi-output (MIMO) systems have become a popular solution in wireless networks [2]-[5] in the last decades.

In MIMO structures, transmit/receive diversity techniques can be employed to improve antenna diversity. The popular receive diversity techniques are maximum ratio combining (MRC) and equal gain combining (EGC). MRC provides optimal weight vectors to maximize the received signal-to-noise ratio (SNR). EGC on the other hand, combines each received signal based on an equal gain without using full channel information which makes it more simple than MRC. In addition to these receive diversity techniques, most important transmit diversity schemes are orthogonal space time block coding (OSTBC) [6]-[8]

and maximal ratio transmission (MRT) [9]. OSTBC can provide full-diversity in the system by using simple coding/decoding techniques without the need of full channel information at the transmitter terminal. MRT on the other hand, needs full channel knowledge but can achieve better performance results then OSTBC while requiring low receiver complexity. MRT can be preferable in slow fading wireless systems like wireless mesh or ad-hoc networks with massive number of relays and antennas which prohibits the use of channel coding techniques to obtain high reliability in practice.

Similar to well investigated multiple antenna techniques, "cooperative/relay" transmissions [10]-[13] become popular to obtain reliable high speed wireless transmissions by exploiting nearby mobile units or fixed relays. In practice, available relay terminals can help the transmitted signals to be delivered to destination over independent fading channels at different time or frequency. Research works on the design and analysis of cooperative/relay communication schemes with multiple relays have been increasing tremendously, as there are many transmission scenarios to be studied. The first common multi-relay scenario is well investigated in [14]-[19] and the references therein. References [14] and [15] studies amplify-and-forward (AF) conventional relaying (all relays participate the transmission) and derive symbol error rate (SER) and outage probability (OP) for Rayleigh and Nakagami-m fading channels respectively. Ribeiro *et. al.*, derive SER for general AF cooperative networks in [16]. In [17]-[18], decode-and-forward (DF) conventional relaying is investigated where SER is derived for Rayleigh fading channels whereas [19] derives OP for Nakagami-m fading channels. In addition, EGC based combiner is investigated in DF conventional relaying schemes in [20] where SER and OP are derived for Nakagami-m fading channels. Like conventional schemes, opportunistic relaying (best relay is selected to maximize the received SNR) [21]-[22] is also widely studied in cooperative networks. In [23], OP and SER performance over Nakagami-m fading channels are studied whereas in [24], the performance of ergodic capacity and SEP are examined for Rayleigh fading channels. Also, [25] derives SER and OP expressions for Rayleigh fading channels.

OWRNs (cooperative transmission) with two source terminals suffer from the loss of spectral efficiency as the transmission is completed in 4 time slots. Two way relay networks (TWRNs) can be a desirable solution for the loss of spectral efficiency. Motivated from the advantages of bidirectional relaying, TWRNs with single antenna is investigated considerably in [26]-[29] and the references therein. In [26] and [27], symbol error rate and system OPs are derived for Rayleigh fading channels respectively whereas [28] derives SER, OP and ergodic sum rate for Rayleigh fading channels. Besides, [29] investigates both outage and SER performance over Nakagami-m fading channels.

1.2. Motivation and Contributions

To enhance the capabilities of cooperative structures, design and analysis of multiple-antenna techniques in single or multi-relay/cooperative transmissions have become essential. Specifically, to lower cost and complexity, transmit, receive or joint transmit/receive antenna selection [30]-[32] have been widely investigated in the literature to maximize full-diversity with reduced complexity i.e., low radio-frequency (RF) chains in cooperative networks. In [33], transmit antenna selection (TAS)/MRC scheme is proposed where SER and OP are derived for Nakagami-m fading channels. In [34] and [35], transmit/receive AS is investigated for Nakagami-m fading channels whereas [36] and [37] analyze joint antenna and relay selection in dual-hop networks for Rayleigh and Nakagami-m fading channels. Like antenna/relay selection schemes, MRT can be a promising option in relay transmissions as it performs better than AS while requiring low receiver complexity. In [38], MRT/MRC scheme is employed at both hops where SER and OP are derived for Rayleigh fading channels. Duong et. al., on the other hand considers a similar system model in [39] where SER is derived over Nakagami-m fading channels in the presence of co-channel interference. Motivated from the advantages of aforementioned diversity techniques, this thesis focuses on the employment and analysis of MRT and AS techniques in one-way or two-way multi-relay networks. To this end, (i) new scenarios are proposed to meet the requirements of enhanced data rates and high reliability, (ii) new methods are introduced to lower the computation complexity in relay transmissions and (iii) simple/useful expressions of well-known performance indicators are obtained in which system designer can easily understand the performance without the need for complex prototyping. The main contributions of this thesis can be summarized as follows

- Dual-hop conventional and opportunistic relaying schemes with MRT is investigated where closed form SER, OP and ergodic capacity expressions are derived for Rayleigh fading channels.
- A new tractable SNR bound is proposed for an MRT-based opportunis-

tic TWRN where sum symbol error rate and system OP are derived for Nakagami-m fading channels.

- By using the same SNR bound, joint antenna and relay selection scheme is analyzed in TWRNs where system OP is derived for Nakagami-m fading channels.

1.3. Organization

The thesis is organized as follows;

- Chapter 2 gives brief description about wireless systems, MIMO, cooperative and two-way relay networks.
- Chapter 3 proposes dual-hop multi-relay conventional and opportunistic schemes with MRT. The analysis of the proposed system starts with the derivation of the probability density function, cumulative distribution function and moment generating function of the signal-to-noise ratio (SNR). Then, symbol error rate, outage probability and ergodic capacity are derived. By obtaining asymptotic expressions of symbol error rate and outage probability, diversity and array gains are obtained. In addition, impact of imperfect channel estimations are investigated and optimum power fraction values are calculated.
- Chapter 4 investigates the performance of an amplify-and-forward multi-input multi-output two-way relay network where two sources are equipped with multiple antennas employing MRT and the communication is carried

through the selected relay resulting in the largest received power. For this network, sum symbol error rate, user and system outage probabilities are derived for independent and non-identically distributed (i.n.i.d) Nakagami-m fading channels. By obtaining asymptotic expressions of these performance indicators, diversity and array gains are obtained. With the help of asymptotic system outage expression, the optimum location of relay is found by solving the convex optimization problem. Furthermore, the impact of limited feedback and imperfect channel estimations are investigated which are critical on the performance of MRT.

- In Chapter 5, a simple end-to-end SNR bound is used in the performance of MIMO amplify-and-forward two-way relay network with joint antenna and relay selection. Both approximate and asymptotic system outage probability expressions are derived for i.n.i.d Nakagami-m fading channels. In addition, by using asymptotic system outage probability, diversity and array gains are obtained for arbitrary number of antennas, relays and fading severity.

- Chapter 6 provides the conclusions of the dissertation where future works are also discussed.

1.4. Notations

Throughout the dissertation, bold letters denote vectors where italic symbols specify scalar variables. The following symbols $(\cdot)^T$, $(\cdot)^\dagger$ and $\|\cdot\|$ are used for transpose, conjugate-transpose and Frobenius norm respectively. A complex

Gaussian random variable with mean a and variance σ_n^2 is denoted as $\mathcal{CN}(a, \sigma_n^2)$. A $n \times n$ identity matrix is shown as $\boldsymbol{I}_n$. Furthermore, $\Gamma[\cdot]$ denotes Gamma function [1, eqn. 8.310.1], $\Gamma[\cdot, \cdot]$ specifies upper incomplete Gamma function [1, eqn. 8.350.2], $\Upsilon[\cdot, \cdot]$ denotes lower incomplete Gamma function [1, eqn. 8.350.1] and ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ stands for Gauss' hypergeometric function [1, eqn. 9.100]. Symbols $\Pr[\cdot]$, $\mathbb{E}[\cdot]$ stand for probability and expectation operations respectively and $\mathcal{Q}(\cdot)$ specifies the Q-function.

2. BACKGROUND

In this section, background information which can be helpful to understand the thesis better, is presented. Specifically, wireless channels, MIMO, wireless channel models, system performance metrics, cooperative/relay networks are discussed.

2.1. Wireless Communication Systems

Wireless communications can be described as the information transmission from one place to another through wireless medium by using electromagnetic waves [40]-[41]. The received signal for a single-input single-output channel can be mathematically expressed as follows

$$y = hx + n, \tag{2.1}$$

where h is the channel coefficient, n is the additive noise and x is the source signal. In general, wireless medium or wireless channel is affected from two phenomena, large-scale and small-scale propagation effects [42]. The former is path loss and shadowing. Path loss depends on the distance between transmitter and receiver and may result in the reduction of received power. Shadowing deteriorates the signal power due to the obstacles between transmitter and receiver. Small scale effect which is known as short term or multipath fading is the deviation of the attenuation varies with time or frequency and results in the deviation of transmitted signal in multiple dimensions.

Multipath fading can be modeled by using various channel models. Throughout the dissertation, fading coefficients are assumed to be distributed with Rayleigh or Nakagami-m fading channels [43]. To characterize the multipath fading with no line-of-sight (LOS) path, Rayleigh fading is used. In this case, magnitude of channel coefficients is distributed as

$$f_h(h) = \frac{2h}{\bar{\gamma}} e^{\frac{-h^2}{\bar{\gamma}}}, h \geqslant 0, \tag{2.2}$$

and the probability density function (PDF) of SNR becomes

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{\frac{-\gamma}{\bar{\gamma}}}, \gamma \geqslant 0, \tag{2.3}$$

where $\gamma = \bar{\gamma}|h|^2$ and $\bar{\gamma}$ is average SNR. When there are K antennas in the system, the PDF of SNR can be obtained as

$$f_\gamma(\gamma) = \frac{\gamma^{K-1}}{\Gamma(K)\bar{\gamma}^K} e^{\frac{-\gamma}{\bar{\gamma}}}, \gamma \geqslant 0, \tag{2.4}$$

where K is the number of transmit or receive antennas.

Nakagami-m is a general distribution to model small scale fading depending on the fading severity parameter m. In Nakagami-m fading, PDF of SNR is characterized as

$$f_{\gamma}(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} e^{\frac{-m\gamma}{\bar{\gamma}}}, \gamma \geqslant 0. \tag{2.5}$$

where m differs from 1/2 to ∞, $m = 1/2$ corresponds to one-sided Gaussian distribution and $m = 1$ for Rayleigh fading. For K transmit or receive antennas, Nakagami-m fading can be distributed as

$$f_{\gamma}(\gamma) = \frac{m^{mK} \gamma^{mK-1}}{\bar{\gamma}^{mK} \Gamma(mK)} e^{\frac{-m\gamma}{\bar{\gamma}}}, \gamma \geqslant 0. \tag{2.6}$$

In wireless networks, performance analysis depends on the averaging of error function. Some important performance metrics can be listed as follows [43].

Average SNR: Among other performance indicators, average SNR is the easiest one to compute and serves as an indicator about the overall reliability of the system performance. Average SNR can be obtained as

$$\bar{\gamma} = \int_0^{\infty} \gamma f_{\gamma}(\gamma) d\gamma \tag{2.7}$$

Outage Probability: Outage probability (OP) is a widely used performance indicator in wireless communications and can be defined as the probability of SNR falls below a certain threshold γ_{th}

$$P_{out} = \int_0^{\gamma_{th}} f_{\gamma}(\gamma) d\gamma = F_{\gamma}(\gamma_{th}) \tag{2.8}$$

Average Bit Error Rate: Bit error rate (BER) is the most important and most difficult performance indicator in wireless communications. The difficulty of BER lies on the conditional BER which is a nonlinear function of SNR. On the other hand, BER is important because it provides an understanding of system performance based on the received SNR, modulation and coding techniques used in the system. BER can be calculated as

$$P_b(e) = \int_0^{\infty} P_b(e|\gamma) f_\gamma(\gamma) d\gamma, \tag{2.9}$$

where $P_b(e|\gamma)$ is the conditional error rate.

Ergodic Capacity: Ergodic capacity can be specified as the maximum mutual information or expectation of information rate between transmitter and receiver.

$$C_{erg} = \mathbb{E}\left[\log_2(1+\gamma)\right], \tag{2.10}$$

where γ is the received SNR.

Asymptotic Analysis: As the exact expressions of BER and OP do not reveal any information about diversity and array gains, simple forms of BER and OP are obtained by taking $\bar{\gamma} \to \infty$ and keeping the dominant terms. Hence, diversity

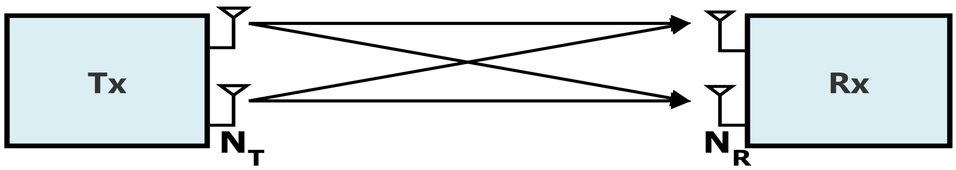

Figure 2.1. Block diagram of MIMO channel

and array gains are obtained as [44]

$$\begin{aligned} \mathcal{G}_d &= -\lim_{\bar{\gamma}\to\infty} \frac{\log(P_b(\bar{\gamma}))}{\log(\bar{\gamma})} \\ \mathcal{G}_a &= -\lim_{\bar{\gamma}\to\infty} (\bar{\gamma}^{\mathcal{G}_d} P_b)^{-\frac{1}{\mathcal{G}_d}} \end{aligned} \tag{2.11}$$

Wireless channels can be affected from shadowing, path loss and multipath fading. To mitigate the detrimental effects of these impairments and to improve reliability and data rates, multi-antenna structures have become a popular solution in the last decades.

2.2. MIMO Structures

MIMO channel is the wireless medium between a multiple-antenna transmitter and receiver. MIMO structures can improve reliability and connectivity by sending multiple copies of the source signal to the receiver. In other words, the detrimental effects of multipath fading can be mitigated by exploiting time, frequency or space dimension [45]-[46]. The block diagram of a MIMO network is depicted in Fig.2.1. Under slow fading, the received signal can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2.12}$$

where $\mathbf{y}$ is $N_R \times 1$ received signal, $\mathbf{H}$ is $N_T \times N_R$ channel matrix and $\mathbf{n}$ is $N_R \times 1$ additive noise vector.

Some important structures of MIMO systems can be categorized as beamforming, spatial multiplexing gain and spatial diversity. To enhance the capabilities of MIMO, transmit/receive beamforming can be employed at the transmitter or receiver sides to maximize (i) the received SNR, (ii) minimize error rate or (iii) to maximize the channel capacity. Spatial multiplexing gain which can be created by sending multiple data streams from each antenna improves space dimension reuse and channel reliability [47]. Spatial diversity which can mitigate the adverse effects of multipath fading, can be created by sending multiple copies of the source signal. In this section, two important transmit diversity methods, MRT and MRC are introduced.

Maximum Ratio Combining : MRC weights and combines the signals from different diversity branches according to their signal-to-noise raito (SNR) [48]-[49]. The received signal at the MRC combiner can be expressed as

$$y_{MRC} = \sum_{r=1}^{N_R} w_r (h_r x + n_r), \tag{2.13}$$

where x is the transmitted signal, h_r is the channel coefficient, w_r is the MRC weight and n_r is the additive noise with N_0 variance. The received SNR can be expressed as

$$\gamma_{MRC} = \frac{P\left|\sum_{r=1}^{N_R} w_r h_r\right|^2}{N_0 \sum_{r=1}^{N_R} |w_r|^2}, \tag{2.14}$$

where P is transmit power and optimal weights can be found by using Cauchy-Swartz inequality

$$\left|\sum_{r=1}^{N_R} w_r h_r\right|^2 \leq \sum_{r=1}^{N_R} |w_r|^2 \times \sum_{r=1}^{N_R} |h_r|^2. \tag{2.15}$$

To maximize the received SNR, weights can be obtained as $w_r = h_r^*$ and the received SNR can be written as

$$\gamma_{MRC} = \frac{P}{N_0} \sum_{r=1}^{N_R} |h_r|^2. \tag{2.16}$$

Maximum Ratio Transmission : In MRT, all paths are scaled according to the channel information at the transmitter side. The received signal can be expressed as

$$y_{MRT} = \sum_{t=1}^{N_T} w_t h_t x + n, \tag{2.17}$$

where w_t and h_t is the weight and channel coefficient respectively. The received

SNR can be written as

$$\gamma_{MRT} = \frac{P\left|\sum_{t=1}^{N_T} w_t h_t\right|^2}{N_0}. \tag{2.18}$$

To maximize the received SNR, weights can be obtained as $w_t = h_t^*/\sqrt{\sum_{t=1}^{N_T}|h_t|^2}$. By substituting w_t into (2.18), received SNR becomes

$$\gamma_{MRT} = \frac{P}{N_0}\sum_{t=1}^{N_T}|h_t|^2. \tag{2.19}$$

From (2.19) and (2.16), we understand that both MRT and MRC have equal received SNRs. However, MRT has low receiver complexity compared to MRC or OSTBC and performs better than well-known OSTBCs which can be important in practical wireless high speed systems e.g. WiFi or wireless sensor networks.

2.3. Cooperative Communications

Similar to multiple antenna techniques, cooperative transmissions which is presented in Fig. 2.2, have become a popular solution to improve reliability, coverage and spatial diversity. In cooperative communication, the transmission is completed in two time slots which is referred as half-duplex. In the first time slot, source transmits its message to relay and destination, in the second time slot, relay transmits the processed source signal to the destination. At the destination, important diversity combining techniques such as MRC, EGC can be used to obtain considerable amount of diversity. The process at relay

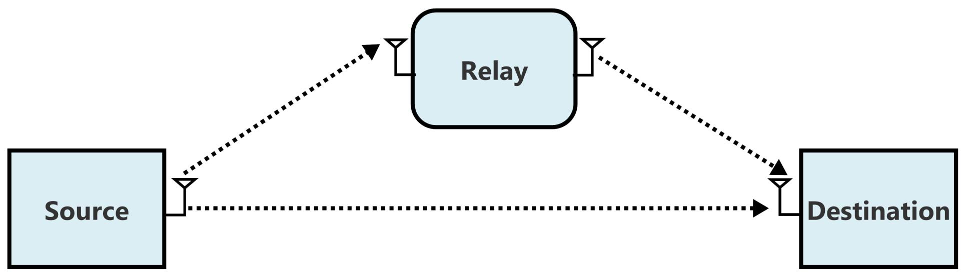

Figure 2.2. Block diagram of cooperative communication

terminals can be divided into two major categories; AF and DF. These two approaches are described below

- Amplify-and-Forward Relaying: Amplify-and-forward (AF) is the simplest relaying technique in cooperative transmissions. In AF approach, the source signal at the relay is amplified with fixed or variable gain and then forwarded to destination.
  - Variable Gain: In variable gain relaying, relay choses the amplification factor based on the instantaneous channel information between source and relay ($S \to R$).
  - Fixed Gain: In fixed gain relaying, relay uses the statistics of channel to choose amplification gaain [50]-[51]. This approach is simpler but performs worse than variable gain relaying.
- Decode-and-Forward Relaying: In DF approach, relay decodes the source signal and then re-encodes and retransmits to the destination. However, this approach may result in significant error propagation due to detection

errors at the relay(s) and thus reduces the cooperation advantages.

To leverage the advantages of cooperative communication and to improve degrees of freedom, reliability, capacity and diversity gains further, multiple relays can be employed as shown in Fig. 2.3. In multi-relay cooperative networks, the transmission (between source and destination) can be carried though all relays (conventional), best relay (opportunistic) by considering all channel coefficients between $S \rightarrow R$ and relay-destination ($R \rightarrow D$) paths or the selection can only be done at $S \rightarrow R$ path (partial relay selection).

- Conventional Relaying: In conventional relaying, the transmission from source to destination is carried through all-relays using AF or DF protocols and the diversity combining techniques can be used at the destination. Conventional relaying achieves best performance as is known, but using all relays improve cost and complexity.
- Opportunistic Relaying: In opportunistic relaying, the transmission is completed through the best relay resulting in the largest end-to-end (e2e) SNR.
- Partial Relay Selection: Different from opportunistic, in partial relay selection (PRS), only $S \rightarrow R$ paths are considered which means PRS is simpler but performs worse than opportunistic relaying. PRS scheme is widely investigated in [52]-[54].

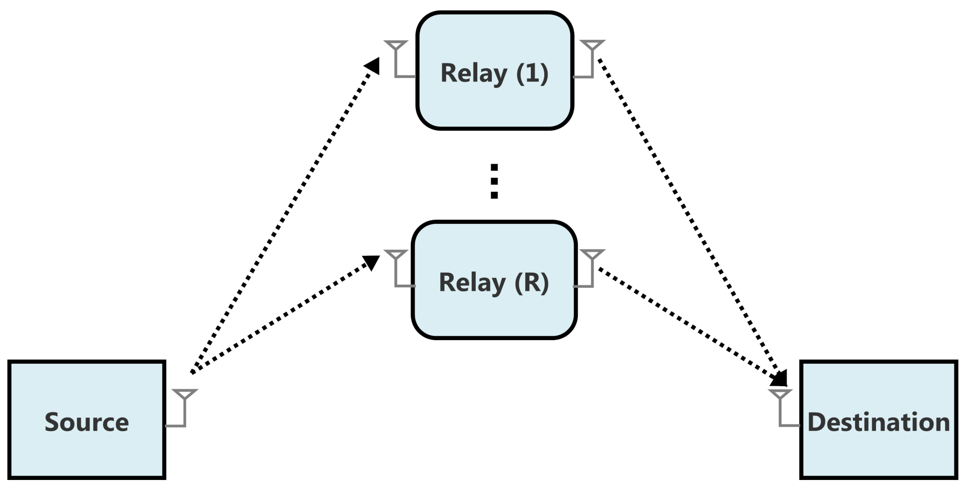

Figure 2.3. Block diagram of multi-relay transmission

One-way relay networks: In wireless systems, the transmission is in half-duplex mode to lower the complexity of transmissions and receptions. In half-duplex relaying, if the transmission is one-sided, this network type is referred as one-way relay network (OWRN). In Fig 2.4, OWRN is depicted in which two source nodes are communicating with each other in 4 time slots. However, in practical MIMO systems, only 2 time slots is needed to complete the transmission. To lower this spectral efficiency loss (due to 4 time slots), two-way relay networks are proposed.

Two-way relay networks: Block diagram of a two-way relay network (TWRN) is illustrated in Fig. 2.5. In TWRNs, two terminals can concurrently transmit their messages to a relay in the first time slot, and then the relay broadcasts the processed total signal in the second time slot so that each terminal can get the transmitted message by subtracting its own message from the total

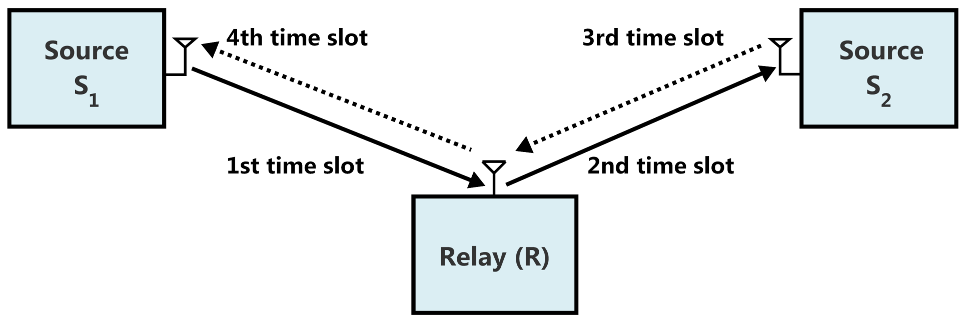

Figure 2.4. Block diagram of one-way relay network

signal. This technique has become popular since it can be a desirable solution for the loss of spectral efficiency occurring in one-way cooperative networks [55]-[59]. To analyze the performance of TWRNs, new performance indicators such as system OP and sum SER are proposed.

System outage probability: In TWRNs, user OP can be defined as $\gamma_{S_1 \to R \to S_2}$ or $\gamma_{S_2 \to R \to S_1}$ transmission is in outage. System OP on the other hand, considers two source nodes and can be defined if at least one of the source nodes is in outage. System OP can be written as

$$P_{out} = \Pr\Big(\min\left(\gamma_{S_1 \to r \to S_2}, \gamma_{S_2 \to r \to S_1}\right) \leqslant \gamma_{th}\Big), \tag{2.20}$$

where $\gamma_{S_1 \to R \to S_2}$ and $\gamma_{S_2 \to r \to S_1}$ are the e2e SNRs between $S_1 \to R \to S_2$ and $S_2 \to r \to S_1$ transmission links.

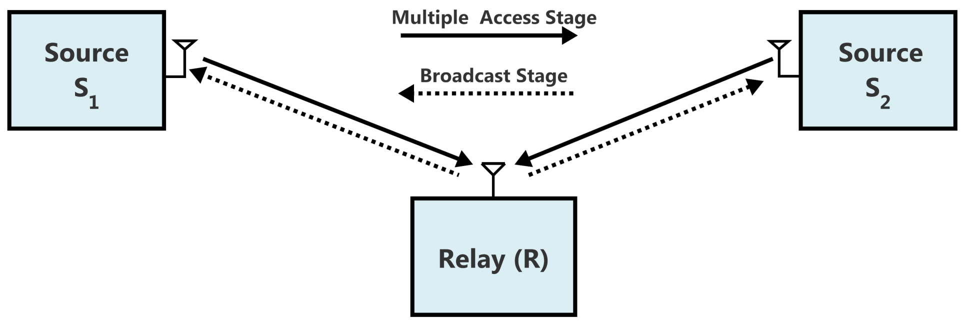

Figure 2.5. Block diagram of two-way relay network

Sum symbol error rate: Sum symbol error rate (SSER) can be defined as the summation of SER at S_1 and S_2 nodes and can be expressed as

$$P_s(e) = P_{s_1}(e) + P_{s_2}(e). \tag{2.21}$$

3. DUAL-HOP MULTI-RELAY MAXIMUM RATIO TRANSMISSION

In this chapter, the performance of dual-hop multi-relay MRT over Rayleigh flat fading channels is studied with both conventional and opportunistic relaying. Performance analysis starts with the derivation of the PDF, cumulative distribution function and moment generating function of the SNR. Then, both approximate and asymptotic expressions of SER and OP are derived for arbitrary numbers of antennas and relays. With the help of asymptotic SER and OP, diversity and array gains are obtained. In addition, impact of imperfect channel estimations is investigated and optimum power allocation factors for source and relay are calculated. Finally, the analytical findings are validated by numerical examples.

3.1. Introduction

Wireless networks can experience deep fading due to unreliable communication. In an attempt to mitigate the effects of fading while increasing degrees of freedom, reliability, capacity and diversity gains further, using multiple-antenna techniques in relay/cooperative transmissions can be attractive, although the mathematical analysis can get quite complicated [60]-[61]. Reference [62] explores SER and OP of a multi-antenna single-relay AF transmission with OSTBC and MRC. In [63], OSTBC based opportunistic relaying scenario is investigated where SER and OP expressions are derived. Recently, employing

MRT, has attracted several interest in the research of cooperative/relay structures since MRT can achieve full available diversity and perform better than the well-known STBCs while requiring low receiver complexity [9]. Although MRT requires feedback of channel state information (CSI) to the transmitter, this may cause negligible overhead when the channel is very slow fading or when the channel is almost reciprocal e.g. indoor wireless mesh networks. In [64], authors investigate a MIMO-MRT network and derives SER and OP for Nakagami-m fading channels. Besides, employing MRT has been investigated in single-relay dual-hop networks in [65]-[70]. Reference [65] considers a network in which multiple-antennas employ MRT at the source and derives OP for Rayleigh fading channels. In [66], DF MRT-based multi-antenna cooperative network is considered and OP is derived. Likewise, in [67], MRT is investigated both at the source and relay where SER is derived. Moreover, [68] and [69] consider a network where source and destination employing MRT/MRC and SER and OP are derived for Nakagami-m and Rayleigh-Rician fading channels. In [70], MRT/MRC scheme is applied at both hops where SER and OP are derived in the presence of feedback delay, channel estimation errors and antenna correlation. In addition, PRS schemes employing MRT is investigated in [71]-[73] and the references therein. In [71]-[72], OP and SER are derived over Nakagami-m and Rayleigh fading channels respectively whereas [73] considers the impact of feedback delay and channel estimation errors on a similar scenario where ergodic capacity and OP are derived.

In this chapter, different from the previous literature, dual-hop AF conventional and opportunistic relay transmissions with MRT is investigated. Note that this low complexity scheme can be useful in wireless mesh or ad-hoc networks especially with massive number of relays and antennas to obtain high reliability in practice. The main contributions of this chapter are outlined as follows:

- A tractable e2e SNR bound is presented and PDF, cumulative distribution function (CDF) and moment generating function (MGF) of the e2e SNR are derived.
- By using CDF and MGF expressions, SER, OP and ergodic capacity for both conventional and opportunistic relaying scenarios are derived and compared.
- Diversity and array gains of conventional and opportunistic networks are obtained by using asymptotic behavior of SER and OP.
- Impact of imperfect channel estimations which is critical for the performance of MRT, is explored.
- By using asymptotic OP, optimal source and relay power factors are obtained.
- To verify the correctness of our analytical study, numerical examples are presented.

The remainder of the chapter is organized as follows. In Section 3.2, system model is presented. Section 3.3 describes performance analysis for conventional

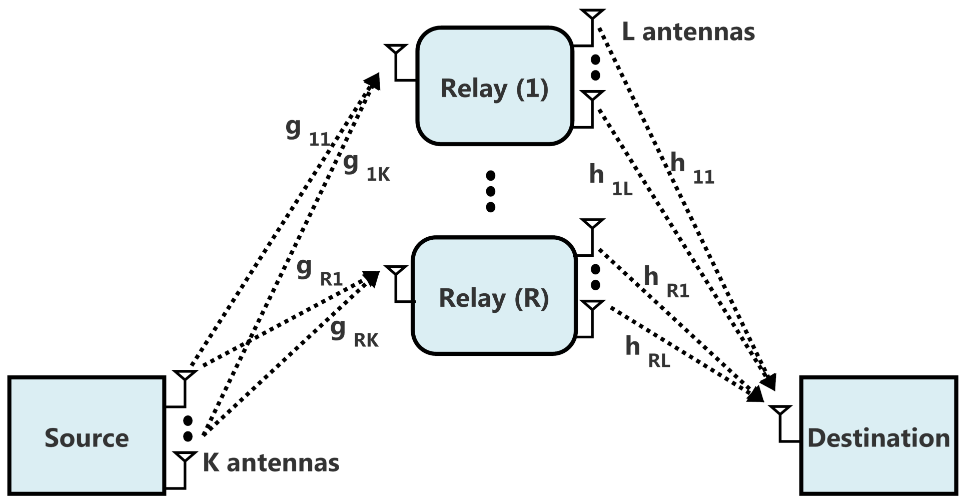

Figure 3.1. Block diagram of dual-hop AF multi-relay system with MRT

and opportunistic networks. Moreover, impact of imperfect channel estimations are investigated. In Section 3.4, optimum source and relay powers that minimize asymptotic OP is studied. Numerical examples are provided in Section 3.5 and finally and Section 3.6 concludes this chapter.

3.2. System Model

The block diagram is depicted in Fig. 3.1. Source node having K antennas transmits to the destination node through R-relays each having L antennas. The communication between source to destination takes place in two phases: In conventional relaying, source transmits signal x to all relays by using MRT in the first phase, in the second phase, each relay amplifies the received signal with an appropriate variable gain and forwards to the destination by using MRT. At the destination, signals coming from R relays are combined by using MRC to obtain maximum diversity gain. In opportunistic relaying, best $S \to R \to D$ path is selected to maximize the e2e SNR at the destination. We assume source, relays and destination know perfect channel state information as needed for optimum MRT. Also, the direct link is assumed to be unavailable due to heavy shadowing.

For the r-th relay $r = \{1, \ldots, R\}$, the channel vectors for $S \to r$ and $r \to D$ paths are given as $\boldsymbol{g}_r$ and $\boldsymbol{h}_r$ respectively. The $\boldsymbol{g}_r$ and $\boldsymbol{h}_r$ vectors are modeled as $\boldsymbol{g}_r \sim \mathcal{CN}(0, \boldsymbol{I}_K)$ and $\boldsymbol{h}_r \sim \mathcal{CN}(0, \boldsymbol{I}_L)$ respectively. The received signal at the r-th relay is written as

$$y_r = \sqrt{\mathcal{P}_s} \boldsymbol{g}_r \boldsymbol{w}_{g_r} x + n_r. \tag{3.1}$$

As mentioned above, each relay uses AF relaying with a variable gain in order to assist the transmission. Assuming that fading coefficients remain almost constant over each frame, the received signal at the destination from r-th relay is given by

$$y_{rd} = \sqrt{\mathcal{P}_r} G_r \boldsymbol{h}_r \boldsymbol{w}_{h_r} y_r + n_{rd}. \tag{3.2}$$

In (3.2), $\mathcal{P}_s$ and $\mathcal{P}_r$ are denoted as transmit powers at the source and relay respectively. MRT based weight vectors for $S \to R$ and $R \to D$ paths are given as $\boldsymbol{w}_{g_r} = (\boldsymbol{g}_r^{\dagger}/\|\boldsymbol{g}_r\|)$ and $\boldsymbol{w}_{h_r} = (\boldsymbol{h}_r^{\dagger}/\|\boldsymbol{h}_r\|)$ respectively. Noise samples (n_r, n_{rd}) are modeled as $n_r, n_{rd} \sim \mathcal{CN}(0, N_0)$ and scaling factor G_r is selected to normalize the power at the relay as shown below

$$G_r^2 = \frac{1}{\mathcal{P}_s|\boldsymbol{g}_r\boldsymbol{w}_{g_r}|^2}. \tag{3.3}$$

The noise at the relay is not considered to simplify the scaling factor G_r above. With the help of (3.1)-(3.3) and after some manipulations, SNR can be written as follows

$$\gamma_d = \begin{cases} \sum\limits_{r=1}^{R}\left(\frac{\gamma_{g_r}\gamma_{h_r}}{\gamma_{g_r}+\gamma_{h_r}}\right), & \text{Conventional relaying} \\ \max\limits_{0\leq r\leq R}\left(\frac{\gamma_{g_r}\gamma_{h_r}}{\gamma_{g_r}+\gamma_{h_r}}\right), & \text{Opportunistic relaying,} \end{cases} \tag{3.4}$$

where $\gamma_{g_r} = \frac{\mathcal{P}_s}{N_0}\|\boldsymbol{g}_r\|^2$ and $\gamma_{h_r} = \frac{\mathcal{P}_r}{N_0}\|\boldsymbol{h}_r\|^2$ represent the received SNRs at $S \rightarrow R$ and $R \rightarrow D$ transmissions.

3.3. Performance Analysis

In this section, the performance analysis of a dual-hop multi-antenna/multi-relay AF MRT transmission scheme is presented. To this end, PDF, CDF and MGF of SNR is obtained, then SER, OP and ergodic capacity for both opportunistic and conventional relaying are derived. In addition, diversity and array gains are found by deriving asymptotic expressions of SER and OP. Finally, the impact of imperfect channel estimations on the proposed scenario are examined.

3.3.1. SNR statistics

As the analysis of SER and OP becomes quite complicated in multi-antenna/multi-relay networks, we resort to compute tight lower bounds on these performance indicators by simplifying the SNR expressions given in (3.4) similar to [15], [76]

$$\sum_{r=1}^{R}\left(\frac{\gamma_{g_r}\gamma_{h_r}}{\gamma_{g_r}+\gamma_{h_r}}\right) \leqslant \gamma_{up}^{cv} = \sum_{r=1}^{R}\min(\gamma_{g_r},\gamma_{h_r}), \tag{3.5}$$

and

$$\max_{0\leq r\leq R}\left(\frac{\gamma_{g_r}\gamma_{h_r}}{\gamma_{g_r}+\gamma_{h_r}}\right) \leqslant \gamma_{up}^{op} = \max_{0\leq r\leq R}\min(\gamma_{g_r},\gamma_{h_r}), \tag{3.6}$$

where superscript *cv* and *op* denotes conventional and opportunistic schemes. To simplify further, we denote $\rho_r = \min(\gamma_{g_r},\gamma_{h_r})$, then the CDF of ρ_r, can be expressed as

$$\begin{aligned}\mathcal{F}_{\rho_r}(\gamma) &= \Pr[\min(\gamma_{g_r},\gamma_{h_r}) < \gamma] \\ &= 1 - \Pr[\gamma_{g_r} > \gamma]\Pr[\gamma_{h_r} > \gamma].\end{aligned} \tag{3.7}$$

PDF expressions of γ_{gr} and γ_{hr} can be obtained as in [9]

$$\begin{aligned}f_{\gamma_{g_r}}(\gamma) &= \frac{\gamma^{K-1}}{\Omega_{g_r}^{K}\Gamma(K)}e^{-\gamma/\Omega_{g_r}} \\ f_{\gamma_{h_r}}(\gamma) &= \frac{\gamma^{L-1}}{\Omega_{h_r}^{L}\Gamma(L)}e^{-\gamma/\Omega_{h_r}},\end{aligned} \tag{3.8}$$

where $\Omega_{g_r} = \frac{\mathcal{P}_s}{N_0}$ and $\Omega_{h_r} = \frac{\mathcal{P}_r}{N_0}$ are the average SNRs per antenna and $\Gamma(\cdot)$ is the gamma function as described in [1, eqn. (8.310.1)]. Integrating (3.8) w.r.t. γ gives us the CDFs of γ_{g_r} and γ_{h_r} which are

$$\begin{aligned} \mathcal{F}_{\gamma_{g_r}}(\gamma) &= \frac{\Gamma\left(K, \frac{\gamma}{\Omega_{g_r}}\right)}{\Gamma(K)} \\ \mathcal{F}_{\gamma_{h_r}}(\gamma) &= \frac{\Gamma\left(L, \frac{\gamma}{\Omega_{h_r}}\right)}{\Gamma(L)}. \end{aligned} \tag{3.9}$$

By substituting (3.9) in (3.7), $\mathcal{F}_{\rho_r}(\gamma)$ can be written as follows

$$\mathcal{F}_{\rho_r}(\gamma) = 1 - \frac{\Gamma(K, \frac{\gamma}{\Omega_{g_r}})\Gamma\left(L, \frac{\gamma}{\Omega_{h_r}}\right)}{\Gamma(K)\Gamma(L)}. \tag{3.10}$$

PDF of ρ_r can be found by taking the derivative of (3.10) w.r.t. γ

$$f_{\rho_r}(\gamma) = \frac{1}{\Gamma(K)\Gamma(L)} \left(\frac{\gamma^{K-1}}{\Omega_{g_r}^K} e^{-\gamma/\Omega_{g_r}} \Gamma\left(L, \frac{\gamma}{\Omega_{h_r}}\right) + \frac{\gamma^{L-1}}{\Omega_{h_r}^L} e^{-\gamma/\Omega_{h_r}} \Gamma\left(K, \frac{\gamma}{\Omega_{g_r}}\right) \right). \tag{3.11}$$

MGF of (3.11) can be obtained by using the definition ($\mathcal{M}_x(s) = \mathbb{E}[e^{-sx}]$) and [1, eqn.(6.455.1)] as shown below

$$\begin{aligned} \mathcal{M}_{\rho_r}(s) = &\frac{\Gamma(K+L)}{\Gamma(K)\Gamma(L)\Omega_{g_r}^K\Omega_{h_r}^L \left(s + (1/\Omega_{g_r}) + (1/\Omega_{h_r})\right)^{K+L}} \\ &\times \left[(1/K)\, {}_2F_1\left(1, K+L; K+1; \frac{s + (1/\Omega_{g_r})}{s + (1/\Omega_{g_r}) + (1/\Omega_{h_r})}\right) \right. \\ &\left. + (1/L)\, {}_2F_1\left(1, K+L; L+1; \frac{s + (1/\Omega_{h_r})}{s + (1/\Omega_{g_r}) + (1/\Omega_{h_r})}\right) \right], \end{aligned} \tag{3.12}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is Gauss' hypergeometric function which is defined in [1, eq.(9.100)]. If we assume $K = L = M$, (3.12) can be simplified as

$$\mathcal{M}_{\rho_r}(s) = \frac{2\Gamma(2M)}{M\Gamma(M)^2\Omega_{\rho_r}^{2M}(s + (2/\Omega_{\rho_r}))^{2M}}\, {}_2F_1\left(1, 2M; M+1; \frac{s\Omega_{\rho_r}+1}{s\Omega_{\rho_r}+2}\right). \tag{3.13}$$

3.3.2. Symbol Error Rate and Outage Probability

3.3.2.1. Conventional Relaying. Having found the MGF of SNR for 1 relay, we can easily extend it to R-relays by using the MGF approach as all channel coefficients between $S \to R$ and $R \to D$ path are independent.

$$\mathcal{M}_{\gamma_{up}^{cv}}(s) = \prod_{r=1}^{R} \mathcal{M}_{\gamma_{\rho_r}}(s). \tag{3.14}$$

With the help of (3.12) and (3.14), SER and OP for conventional relaying can be obtained. For example, for M-PSK modulation, SER can be obtained as

$$P_s^{cv}(e) = \frac{1}{\pi}\int_0^{\phi} \mathcal{M}_{\gamma_{up}^{cv}}\left(\frac{g_{PSK}}{\sin^2(\theta)}\right) d\theta, \tag{3.15}$$

where $\phi = (M-1)\pi/M$, $g_{PSK} = \sin^2(\pi/M)$ i.e., $g_{PSK} = 1$ for BPSK modulation.

Similar to SER, OP (P_{out}^{cv}) is a widely used performance indicator in wireless communication systems. P_{out}^{cv} is defined as the probability of SNR falling below a certain threshold γ_{th} and can be computed by taking the inverse Laplace transform of $\mathcal{M}_{\gamma_{up}^{cv}}(s)$ at γ_{th} as follows

$$P_{out}^{cv} = \left[\mathcal{L}^{-1}\left(\frac{\mathcal{M}_{\gamma_{up}^{cv}}(s)}{s}\right)\right]_{s=\gamma_{th}}, \tag{3.16}$$

where $\mathcal{L}^{-1}(\cdot)$ denotes the inverse Laplace transform.

To the best of our knowledge, closed form expressions of SER and OP are not available in the literature. However, similar to previous studies in cooperative/relay communication systems, SER can be obtained approximately as shown in [74] and OP can be found numerically by using well-known software programs such as MAPLE or MATHEMATICA. For BPSK modulation, approximate SER can be written as shown in [74, eqn.10]

$$P_s(e) = \frac{1}{12}\mathcal{M}_{\gamma_{up}^{cv}}(1) + \frac{1}{4}\mathcal{M}_{\gamma_{up}^{cv}}(1.3) - \frac{1}{12}\mathcal{M}_{\gamma_{up}^{cv}}\left(\frac{1}{\sin^2(\theta)}\right), \tag{3.17}$$

In [74], it is shown that approximate SER expressions are valid and accurate in the whole integral region.

3.3.2.2. Opportunistic Relaying. In opportunistic relaying networks, CDF of received SNR ($\mathcal{F}_{\gamma_{up}^{op}}(\gamma)$) can be written as $\mathcal{F}_{\gamma_{up}^{op}}(\gamma) = \{\mathcal{F}_{\rho_r}(\gamma)\}^R$. With the help of high order statistics [75], equation (3.10) and [1, eqn. 8.352.7], $\mathcal{F}_{\gamma_{up}^{op}}(\gamma)$ can be expressed as

$$\mathcal{F}_{\gamma_{up}^{op}}(\gamma) = \left(1 - e^{-\frac{\gamma}{\Omega_{g_r}}}\sum_{k=0}^{K-1}\left(\frac{\gamma}{\Omega_{g_r}}\right)^k\frac{1}{k!} \times e^{-\frac{\gamma}{\Omega_{h_r}}}\sum_{l=0}^{L-1}\left(\frac{\gamma}{\Omega_{h_r}}\right)^l\frac{1}{l!}\right)^R. \tag{3.18}$$

By applying binomial [1, eqn.(1.111.1)] and multinomial [1, eqn.(0.314)] expansions respectively, $\mathcal{F}_{\gamma_{up}^{op}}(\gamma)$ becomes

$$\mathcal{F}_{\gamma_{up}^{op}}(\gamma)=\sum_{r=0}^{R}\sum_{k=0}^{r(K-1)}\sum_{l=0}^{r(L-1)}\binom{R}{r}(-1)^{r}e^{-r\frac{\gamma}{\Omega_{g_r}}}e^{-r\frac{\gamma}{\Omega_{h_r}}}\mathcal{X}_k(r)\mathcal{X}_l(r)\gamma^{k+l}, \tag{3.19}$$

where combination operation denotes binomial coefficients and multinomial coefficients can be written as $\mathcal{X}_t(r)=\frac{1}{tk_0}\sum_{\rho=1}^{t}(r\rho-t+\rho)k_\rho\mathcal{X}_{t-\rho}(r), t\geqslant 1$ [1, eqn.(0.314)], where $k_\rho=(\frac{1}{\Omega_m})^{\rho}\frac{1}{\rho!}$, $\mathcal{X}_0(r)=k_0^r=1$, $t\in\{k,l\}$ and $m\in\{g_r,h_r\}$.

As defined above, OP is the probability of e2e SNR falling below a certain threshold and it can be obtained as $P_{out}^{op}=\mathcal{F}_{\gamma_{up}^{op}}(\gamma_{th})$. In addition, for the systems whose conditional SER expression is in the form of $\mathbb{E}[a\mathcal{Q}(\sqrt{2b\gamma})]$, SER can be computed by using the CDF of SNR as [43]

$$P_s^{op}(e)=\frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^\infty\gamma^{-1/2}e^{-b\gamma}\mathcal{F}_{\gamma_{up}^{op}}(\gamma)d\gamma, \tag{3.20}$$

where a and b denotes modulation coefficients, i.e., $\{a=1,b=0.5\}$ for BFSK modulation, $\{a=1,b=1\}$ for BPSK and $\{a=2(M-1)/M,b=3/(M^2-1)\}$ for M-PAM. Also, $\{a=2,b=\sin^2(\pi/M)\}$ for approximate M-PSK. By substituting (3.19) in (3.20) with the help of [1, eqn. 3.351.3], SER can be obtained as

$$P_s^{op}(e)=\frac{a\sqrt{b}}{2\sqrt{\pi}}\sum_{r=0}^{R}\sum_{k=0}^{r(K-1)}\sum_{l=0}^{r(L-1)}\binom{R}{r}(-1)^{r}\mathcal{X}_k(r)\mathcal{X}_l(r)\Gamma\left(k+l-\frac{3}{2}\right)(b+r\Omega_r)^{-k-l-\frac{1}{2}}, \tag{3.21}$$

where $\Omega_r=\frac{\Omega_{g_r}+\Omega_{h_r}}{\Omega_{g_r}\Omega_{h_r}}$.

3.3.3. Diversity and Array Gains

Here, asymptotic SER and OP expressions are examined to obtain diversity ($\mathcal{G}_d$) and array ($\mathcal{G}_a$) gains.

<u>3.3.3.1. Conventional Relaying.</u> At high SNR, $\mathcal{F}_{\rho_r}(\gamma)$ can be expressed as [77, eqn. 6]

$$\mathcal{F}_{\rho_r}(\gamma) = \frac{\Upsilon\left(K, \frac{\gamma}{\Omega_{g_r}}\right)}{\Gamma(K)} + \frac{\Upsilon\left(L, \frac{\gamma}{\Omega_{h_r}}\right)}{\Gamma(L)}. \tag{3.22}$$

By using the asymptotic behavior of lower incomplete Gamma function given in [78, eqn. 45.9.1], asymptotic $\mathcal{F}^{\infty}_{\rho_r}(\gamma)$ can be expressed as

$$\mathcal{F}^{\infty}_{\rho_r}(\gamma) = \frac{\gamma^K}{\Gamma(K+1)\Omega^K_{g_r}} + \frac{\gamma^L}{\Gamma(L+1)\Omega^L_{h_r}}, \tag{3.23}$$

To obtain asymptotic SER and OP expressions for conventional relaying, we need to obtain $\mathcal{M}_{\gamma^{cv,\infty}_{up}}(s)$. Therefore, by using the relationship between MGF and CDF i.e., $\mathcal{M}^{\infty}_{\rho_r}(s) = s\int_0^{\infty} e^{-s\gamma}\mathcal{F}^{\infty}_{\rho_r}(\gamma)d\gamma$, with the help of [1, eqn. 3.351.3] and then substituting $\mathcal{M}^{\infty}_{\rho_r}(s)$ in (3.14), $\mathcal{M}_{\gamma^{cv,\infty}_{up}}(s)$ can be obtained as

$$\mathcal{M}_{\gamma^{cv,\infty}_{up}}(s) = \prod_{r=1}^{R}\left(\frac{1}{s^K\Omega^K_{g_r}} + \frac{1}{s^L\Omega^L_{h_r}}\right). \tag{3.24}$$

To obtain the inverse Laplace transform of (3.24) is highly complicated.

For this, we assume both hops are balanced i.e., $K = L = M$ and $\Omega_{g_r} = \Omega_{h_r} = \Omega$. Then for large average SNR, $\mathcal{F}_{\gamma_{up}^{cv,\infty}}(\gamma)$ can be expressed as

$$\mathcal{F}_{\gamma_{up}^{cv,\infty}}(\gamma) = \mathcal{A}\left(\frac{\gamma}{\Omega}\right)^{MR}, \tag{3.25}$$

where $\mathcal{A} = \frac{2^R}{\Gamma(MR+1)}$. As $P_{out}^{cv,\infty} = \mathcal{F}_{\gamma_{up}^{cv,\infty}}(\gamma_{th}) = \mathcal{A}\left(\frac{\gamma_{th}}{\Omega}\right)^{MR}$ [44], diversity and array gains can be obtained as $\mathcal{G}_d = MR$ and $\mathcal{G}_a = \left(\frac{2^R \gamma_{th}^{MR}}{\Gamma(MR+1)}\right)^{-1/\mathcal{G}_d}$. By substituting (3.25) in (3.20) and with the help of [44, Prop. 1], asymptotic SER can be obtained as

$$P_s^{cv,\infty}(e) = \frac{a\mathcal{A}\Gamma(MR+1/2)}{2\sqrt{\pi}(b\Omega)^{MR}} + \text{H.O.T.}, \tag{3.26}$$

where a, b are modulation coefficients as described above.

<u>3.3.3.2. Opportunistic Relaying.</u> As mentioned above, in opportunistic networks, $\mathcal{F}_{\gamma_{up}^{op,\infty}}(\gamma)$ can be written as $\mathcal{F}_{\gamma_{up}^{op,\infty}}(\gamma) = \{\mathcal{F}_{\rho_r}^{\infty}(\gamma)\}^R$. By using (3.23) and replacing γ with γ_{th}, $P_{out}^{op,\infty}$ can be obtained as

$$P_{out}^{op,\infty} = \left(\frac{\gamma_{th}^K}{\Gamma(K+1)\Omega_{g_r}^K} + \frac{\gamma_{th}^L}{\Gamma(L+1)\Omega_{h_r}^L}\right)^R. \tag{3.27}$$

By using [44, Prop. 5], $P_{out}^{op,\infty}$ can be expressed as

$$P_{out}^{op,\infty} \approx \mathcal{Z}\left(\frac{\gamma_{th}}{\Omega}\right)^{\mathcal{G}_d} + \text{H.O.T.}, \tag{3.28}$$

where $\Omega \in \{\Omega_{g_r}, \Omega_{h_r}\}$, H.O.T denotes high order terms and $\mathcal{Z}$ is

$$\mathcal{Z} = \begin{cases} \prod_{r=1}^{R} \left(\frac{1}{\Gamma(K+1)}\right), & K < L \\ \prod_{r=1}^{R} \left(\frac{1}{\Gamma(K+1)} + \frac{1}{\Gamma(L+1)}\right), & K = L \\ \prod_{r=1}^{R} \left(\frac{1}{\Gamma(L+1)}\right), & K > L. \end{cases} \tag{3.29}$$

Diversity and array gains can be expressed as

$$\mathcal{G}_d = R \min(K, L)$$
$$\mathcal{G}_a = \mathcal{Z}^{-1/(R \min K, L)}. \tag{3.30}$$

By substituting (3.28) in (3.20) and after γ_{th} is replaced with γ, asymptotic SER can be obtained as follows

$$P_s^{op,\infty}(e) = \frac{2^{\mathcal{G}_d - 1} a \mathcal{Z} \Gamma(\mathcal{G}_d + 1/2)}{\sqrt{\pi}(2b\Omega)^{\mathcal{G}_d}} + \text{H.O.T.} \tag{3.31}$$

When the diversity gain obtained from opportunistic is compared with that of conventional one, it is obserbed that conventional scheme has better array gain but equal diversity with opportunistic.

3.3.4. Ergodic Capacity

Ergodic capacity can be specified as the maximum mutual information (or expectation of information rate) between source and destination. Ergodic capacity for conventional relaying can be expressed as

$$C_{erg}^{cv} = \frac{1}{R}\mathbb{E}\left[\log_2(1+\gamma_{up}^{cv})\right] = \frac{1}{R}\log_2(e)\int_0^\infty \log_2(1+\gamma) f_{\gamma_{up}^{cv}}(\gamma) d\gamma, \tag{3.32}$$

where $f_{\gamma_{up}^{cv}}(\gamma)$ can be find by taking the inverse Laplace transform of $\mathcal{M}_{\gamma_{up}}(s)$ as follows

$$f_{\gamma_{up}^{cv}}(\gamma) = \left[\mathcal{L}^{-1}\left(\mathcal{M}_{\gamma_{up}^{cv}}(s)\right)\right]_{s=\gamma}. \tag{3.33}$$

By substituting (3.33) in (3.32), an upper bound on C_{erg}^{cv} can be computed numerically. For opportunistic relaying, ergodic capacity can be expressed by using the CDF of SNR as follows

$$C_{erg}^{op} = \log_2(e)\int_0^\infty \frac{1}{1+\gamma}\mathcal{F}_{\gamma_{up}^{op}}(\gamma) d\gamma. \tag{3.34}$$

Substituting (3.19) into (3.34) with the help of [1, eqn. 3.353.5], an upper bound on C_{erg}^{op} can be found as

$$\begin{aligned} C_{erg}^{op} = &\log_2(e)\sum_{r=0}^{R}\sum_{k=0}^{r(K-1)}\sum_{l=0}^{r(L-1)}\binom{R}{r}(-1)^r \mathcal{X}_k(r)\mathcal{X}_l(r)\times \\ &\left\{(-1)^{k+l-1}e^{r\Omega_r}\mathrm{Ei}\big(-\Omega_r\big) + \sum_{z=1}^{k+l}(z-1)!(-1)^{k+l-z}(\Omega_r)^{-z}\right\}, \end{aligned} \tag{3.35}$$

where $\mathrm{Ei}(x) = -\int_{-x}^{\infty}\frac{e^{-t}}{t}dt$ denotes exponential integral.

3.3.5. Impact of Imperfect Channel Estimations

In this section, the effects of imperfect channel estimations on the proposed scenarios is investigated. For this, $S \rightarrow R$ and $R \rightarrow D$ paths are assumed to be erroneously estimated as shown below

$$\begin{aligned} \boldsymbol{g}_r &= \tilde{\boldsymbol{g}}_r + \boldsymbol{\xi}_{g_r} \\ \boldsymbol{h}_r &= \tilde{\boldsymbol{h}}_r + \boldsymbol{\xi}_{h_r}, \end{aligned} \tag{3.36}$$

where channel estimates $\tilde{\boldsymbol{g}}_r$ and $\tilde{\boldsymbol{h}}_r$ are modeled as $\tilde{\boldsymbol{g}}_r \sim \mathcal{CN}(0, \boldsymbol{I}_K \sigma^2_{\tilde{g}_r})$ and $\tilde{\boldsymbol{h}}_r \sim \mathcal{CN}(0, \boldsymbol{I}_L \sigma^2_{\tilde{h}_r})$. Estimation errors ($\boldsymbol{\xi}_{g_r}$ and $\boldsymbol{\xi}_{h_r}$) are given as $\boldsymbol{\xi}_{g_r} \sim \mathcal{CN}(0, \boldsymbol{I}_K \sigma^2_{\xi_{g_r}})$ and $\boldsymbol{\xi}_{g_r} \sim \mathcal{CN}(0, \boldsymbol{I}_L \sigma^2_{\xi_{h_r}})$ [79]-[81]. MRT based weight vectors can be specified as $\boldsymbol{w}_{\tilde{g}_r} = (\tilde{\boldsymbol{g}}_r^\dagger / \|\tilde{\boldsymbol{g}}_r\|)$, $\boldsymbol{w}_{\tilde{h}_r} = (\tilde{\boldsymbol{h}}_r^\dagger / \|\tilde{\boldsymbol{h}}_r\|)$ respectively. The scaling factor becomes

$$\tilde{G}_r^2 = \frac{1}{\mathcal{P}_s |\tilde{\boldsymbol{g}}_r \boldsymbol{w}_{\tilde{g}_r}|^2}. \tag{3.37}$$

By substituting (3.36), (3.37) in (3.1) and (3.2) and after some manipulations, the received signal at the destination can be expressed as

$$\begin{aligned} y_{r_d} &= \sqrt{P_r} G_r \boldsymbol{w}_{h_r} (\tilde{\boldsymbol{h}}_r + \boldsymbol{\xi}_{h_r})(\sqrt{P_s} \boldsymbol{w}_{g_r} (\tilde{\boldsymbol{g}}_r + \boldsymbol{\xi}_{g_r}) x + n_r) + n_{rd} \\ &= \sqrt{P_s}\sqrt{P_r} G_r \|\tilde{\boldsymbol{g}}_r\| \|\tilde{\boldsymbol{h}}_r\| + \sqrt{P_s}\sqrt{P_r} G_r \|\tilde{\boldsymbol{h}}_r\| \boldsymbol{w}_{g_r} \boldsymbol{\xi}_{g_r} + \sqrt{P_s}\sqrt{P_r} G_r \|\tilde{\boldsymbol{g}}_r\| \boldsymbol{w}_{h_r} \boldsymbol{\xi}_{h_r} \\ &+ \sqrt{P_s}\sqrt{P_r} G_r \boldsymbol{w}_{g_r} \boldsymbol{\xi}_{g_r} \boldsymbol{w}_{h_r} \boldsymbol{\xi}_{h_r} + \sqrt{P_r} G_r \|\tilde{\boldsymbol{h}}_r\| n_r + \sqrt{P_r} G_r \boldsymbol{w}_{h_r} \boldsymbol{\xi}_{h_r} n_r + n_{rd}. \end{aligned} \tag{3.38}$$

We can divide y_{rd} into 3 parts; The signal part is written as

$$\sqrt{P_s}\sqrt{P_r}G_r\|\tilde{\boldsymbol{g}}_r\|\|\tilde{\boldsymbol{h}}_r\|, \tag{3.39}$$

the noise part can be expressed as

$$\sqrt{P_r}G_r\|\tilde{\boldsymbol{h}}_r\|n_r + n_{rd}, \tag{3.40}$$

and the estimation error part can be specified as

$$\sqrt{P_s}\sqrt{P_r}G_r\|\tilde{\boldsymbol{h}}_r\|\boldsymbol{w}_{g_r}^T\boldsymbol{\xi}_{g_r} + \sqrt{P_s}\sqrt{P_r}G_r\|\tilde{\boldsymbol{g}}_r\|\boldsymbol{w}_{h_r}^T\boldsymbol{\xi}_{h_r} + \sqrt{P_s}\sqrt{P_r}G_r\boldsymbol{w}_{g_r}^T\boldsymbol{\xi}_{g_r}\boldsymbol{w}_{h_r}^T\boldsymbol{\xi}_{h_r} + \sqrt{P_r}G_r\boldsymbol{w}_{h_r}^T\boldsymbol{\xi}_{h_r}n_r. \tag{3.41}$$

Note that $\tilde{\boldsymbol{g}}_r$, $\tilde{\boldsymbol{h}}_r$, $\boldsymbol{\xi}_{g_r}$, $\boldsymbol{\xi}_{h_r}$, n_{sr} and n_{rd} are mutually independent from each other. With the help of (3.38) and (3.37) and after some manipulations, end-to-end effective SNR can be written as follows

$$\gamma_d^{ef} = \begin{cases} \sum_{r=1}^{R}\left(\frac{\gamma_{g_r}^{ef}\gamma_{h_r}^{ef}}{\mathcal{A}_r\gamma_{g_r}^{ef}+\mathcal{B}_r\gamma_{h_r}^{ef}+\mathcal{C}_r}\right), & \text{Conventional relaying} \\ \max\limits_{0\le r\le R}\left(\frac{\gamma_{g_r}^{ef}\gamma_{h_r}^{ef}}{\mathcal{A}_r\gamma_{g_r}^{ef}+\mathcal{B}_r\gamma_{h_r}^{ef}+\mathcal{C}_r}\right), & \text{Opportunistic relaying} \end{cases} \tag{3.42}$$

where $\gamma_{g_r}^{ef} = \frac{\mathcal{P}_s}{N_0}\|\tilde{\boldsymbol{g}}_r\|^2$ and $\gamma_{h_r}^{ef} = \frac{\mathcal{P}_r}{N_0}\|\tilde{\boldsymbol{h}}_r\|^2$. Also, $\mathcal{A}_r = 1+\frac{\mathcal{P}_r}{N_0}\sigma^2_{\xi_{h_r}}$, $\mathcal{B}_r = 1+\frac{\mathcal{P}_s}{N_0}\sigma^2_{\xi_{g_r}}$ and $\mathcal{C}_r = \frac{\mathcal{P}_r}{N_0}\sigma^2_{\xi_{g_r}} + \frac{\mathcal{P}_s}{N_0}\frac{\mathcal{P}_r}{N_0}\sigma^2_{\xi_{g_r}}\sigma^2_{\xi_{h_r}}$. After effective SNRs are approximately written as in (3.5) and (3.6), $\mathcal{F}_{\rho_r}^{ef}(\gamma)$ can be obtained as

$$\mathcal{F}_{\rho_r}^{ef}(\gamma) = 1 - \frac{\Gamma(K, \mathcal{B}_r \frac{\gamma}{\Omega_{g_r}})\Gamma\left(L, \mathcal{A}_r \frac{\gamma}{\Omega_{h_r}}\right)}{\Gamma(K)\Gamma(L)}. \tag{3.43}$$

From (3.43), it can be observed that the CDF of SNR deteriorates from the negative effects of imperfect channel estimations. By applying the same theoretical steps to (3.39), SER and OP in the presence of channel estimation errors can be obtained for both conventional and opportunistic relay networks.

3.4. Optimal Power Allocation

In this section, our aim is to improve the performance of the dual-hop single-relay multi-antenna network by obtaining optimum $\mathcal{P}_s$ and $\mathcal{P}_r$ values to minimize the OP under a power fraction α. For this, we first express P_{out}^{∞} by using (3.23) and replacing γ with γ_{th} as can be seen below

Table 3.1. Optimum power values for $\mathcal{P}_t = 10$ and $\gamma_{th} = 7$ dB.

K, L	Optimum α values
$2, 1$	$\alpha = 0.2925$
$1, 2$	$\alpha = 0.7074$
$3, 1$	$\alpha = 0.1711$
$4, 1$	$\alpha = 0.1104$

$$P_{out}^{\infty} = \frac{A}{\mathcal{P}_s^K} + \frac{B}{\mathcal{P}_r^L}, \tag{3.44}$$

where $A = \frac{\left(\gamma_{th} \times N_0\right)^K}{\Gamma(K+1)}$ and $B = \frac{\left(\gamma_{th} \times N_0\right)^L}{\Gamma(L+1)}$. We assume $\mathcal{P}_s = \alpha \mathcal{P}_t$ and $\mathcal{P}_r = (1-\alpha)\mathcal{P}_t$, where $\mathcal{P}_t$ the total transmit power available in the network. Hence, the power allocation problem can be formed as follows

$$\min_{\alpha} P_{out}^{\infty}, \text{ subject to} : 0 < \alpha < 1. \tag{3.45}$$

By substituting $\mathcal{P}_s = \alpha \mathcal{P}_t$ and $\mathcal{P}_r = (1-\alpha)\mathcal{P}_t$ in (3.44), then taking the second derivative of P_{out}^{∞} w.r.t α, we recognize that P_{out}^{∞} is a strictly convex function of α. Therefore, taking the first derivative of (3.44) and equating to zero, we can obtain optimal value of α as follows

$$\begin{gathered} \frac{\alpha^{K+1}}{(1-\alpha)^{L+1}} = \frac{KA}{LB} \mathcal{P}_t^{L-K}, \text{ when } K \neq L \\ \alpha = \frac{1}{2}, \text{ when } K = L. \end{gathered} \tag{3.46}$$

The closed form solution of (3.46) is difficult to obtain, but numerical results can be obtained by using root-finding algorithms such as Bisection or Newton. Table 3.1 gives some examples for $\mathcal{P}_t = 10$ dB. From the table, we understand that when $K > L$, source power decreases and relay power increases, or when $L > K$, source power increases and relay power decreases to minimize OP.

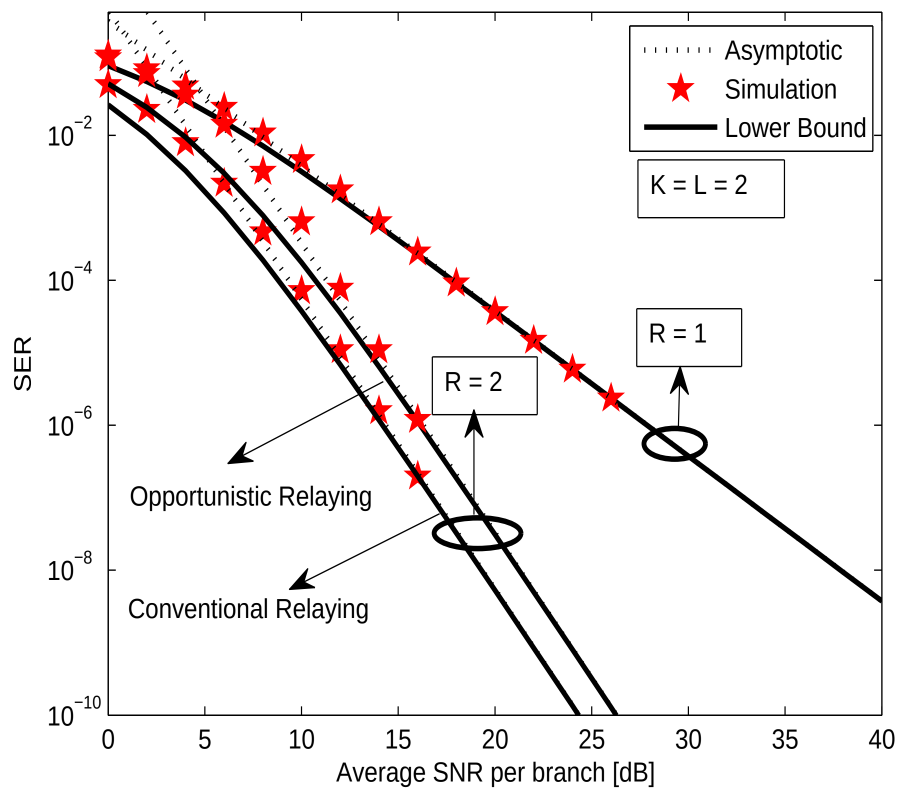

Figure 3.2. SER comparison of theoretical bounds with exact simulations

3.5. Numerical Examples

In this section, several numerical examples are provided to verify and demonstrate our analytical study to gain further insight about the usefulness of the proposed system. SER and OPs are obtained via Monte-Carlo simulations where BPSK signalling and Rayleigh fading channel model are used. For simplicity, both transmit powers between $S \rightarrow R$ and $R \rightarrow D$ links are assumed to be equal i.e., $(\mathcal{P}_s = \mathcal{P}_r = \mathcal{P}_t/2)$ and horizontal axes of all figures represent

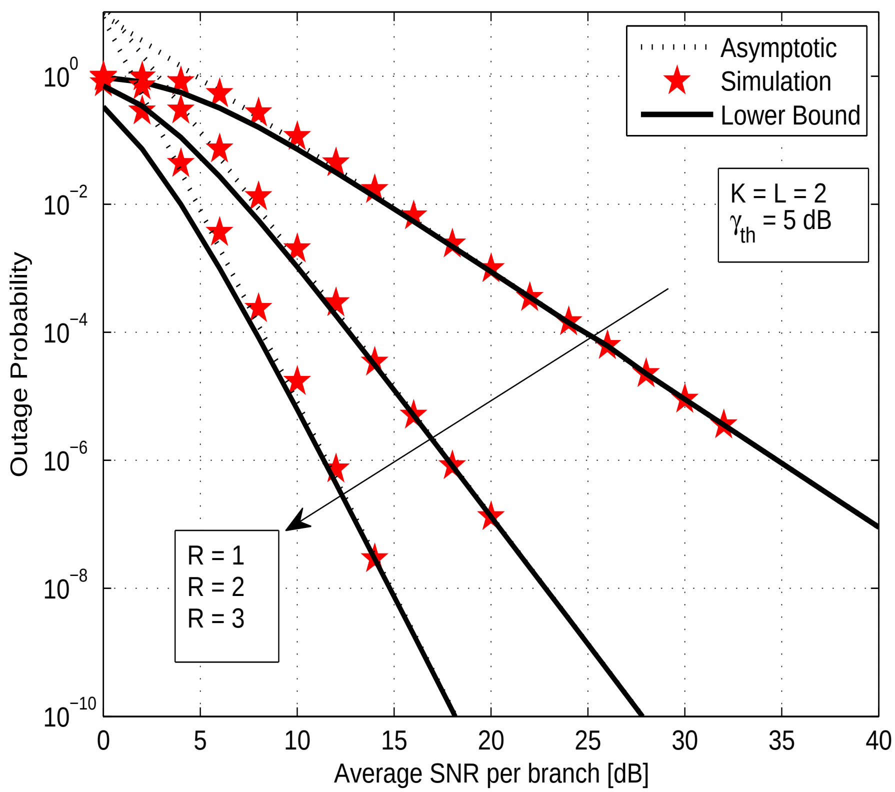

Figure 3.3. Outage probability of conventional relaying

the average SNR per branch unless otherwise stated.

Figure 3.2 depicts the SER of opportunistic and conventional relaying schemes for $K = L = 2$ and $R = 1, 2$. Comparing derived lower bound and asymptotic results with the simulation, it can be observed that the theoretical results match almost perfectly with the simulation at especially medium to high SNRs. In addition, we understand that conventional relaying achieves average 2 dB better SER then opportunistic relaying despite the diversity orders are

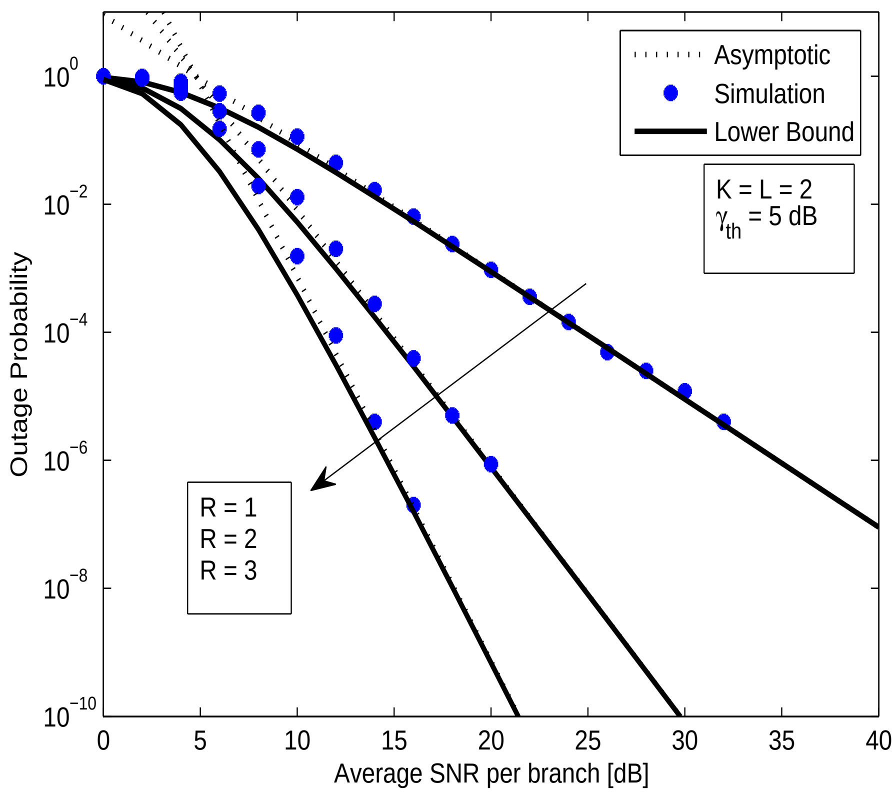

Figure 3.4. Outage performance of opportunistic relaying

identical e.g., 2 and 4 for $R = 1, 2$ respectively. Interestingly, due to simple structure of MRT technique, one can satisfy error performance requirements by exploiting few of the available users as relays without the overhead of changing receiver structure and executing channel coding/decoding algorithms.

In Fig. 3.3 and 3.4, OPs of conventional and opportunistic relaying is drawn for $R = 1, 2, 3$ and when $K = L = 2$. From both figures, it is observed that as the number of relays increase, the performance significantly improves

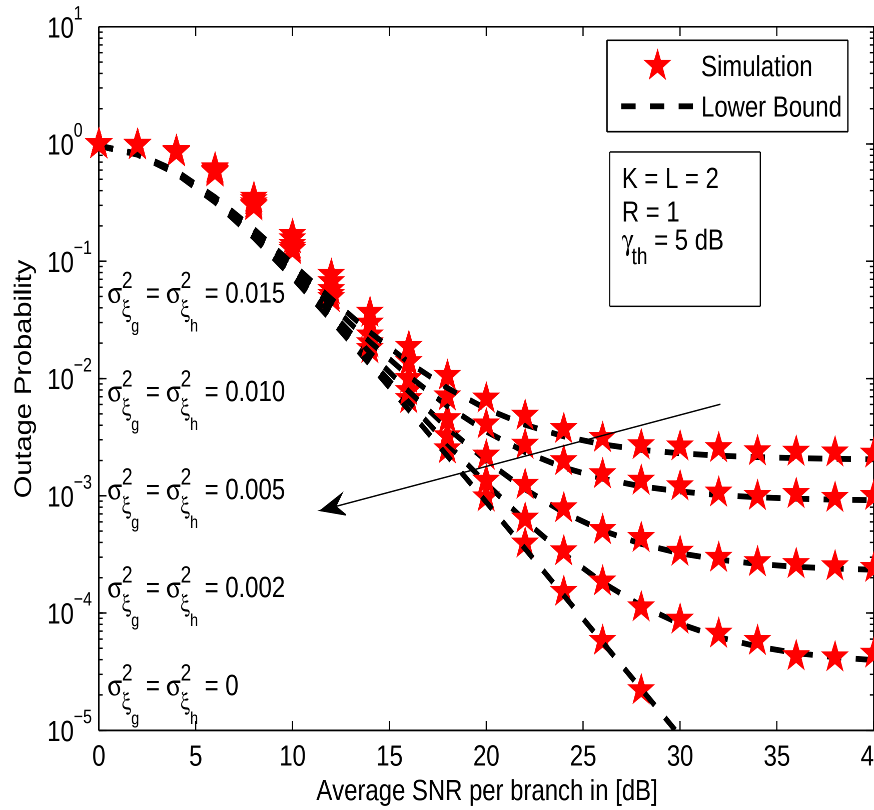

Figure 3.5. Impact of imperfect channel estimations on the proposed network when $R = 1$

e.g., the difference between $R = 2$ and $R = 3$ is about 9 dB at 10^{-10} P_{out}. Similar to Fig. 3.2, asymptotic and approximate results of both figures matches perfectly with the simulation at all cases especially at medium to high SNRs. In addition, conventional scheme is complex but average $2-3$ dB superior than opportunistic case, despite the diversity orders are exactly the same e.g., $2, 4, 6$ for $R = 1, 2, 3$.

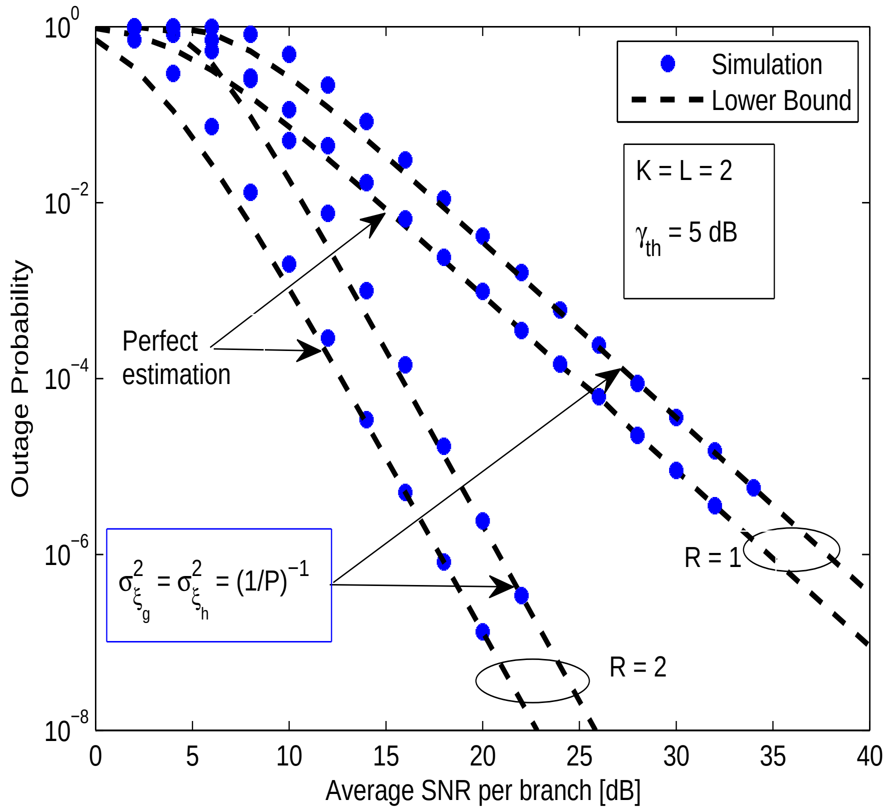

Figure 3.6. Outage probability performance of conventional relaying for SNR dependent variances of estimation errors

In Fig. 3.5, the impact of imperfect channel estimations on the OP is demonstrated for different values of fixed estimation error variances. From this figure, we can clearly observe error floors due to channel estimation errors when the error variances cannot be improved with increased SNR. After especially 15 dB, error floors results in huge performance loses as no diversity can be obtained. Furthermore, it is observed that the lower bound is in an excellent agreement with the simulation results in all cases especially at medium to high SNRs.

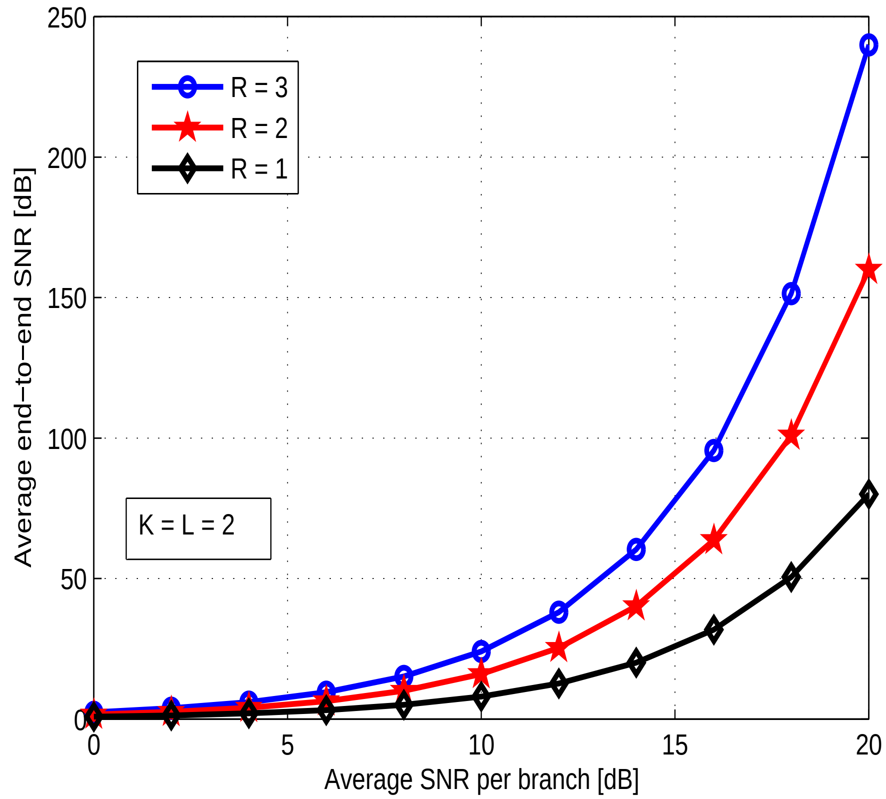

Figure 3.7. Average end-to-end SNR performance of conventional relaying

Fig. 3.6 illustrates OP performance of exact simulation with lower bound when the estimation error variances are SNR dependent (variable). As seen from the figure, as the SNR increases, estimation error variances decrease and consequently error floors vanish so the diversity orders remain same with that of the perfect estimation case i.e., 2 and 4 at $R = 1$ and $R = 2$ with an average 2 dB performance loss. Furthermore, as it is observed in the previous figures, lower bound provides an excellent match with the exact simulation result at especially after 8 dB.

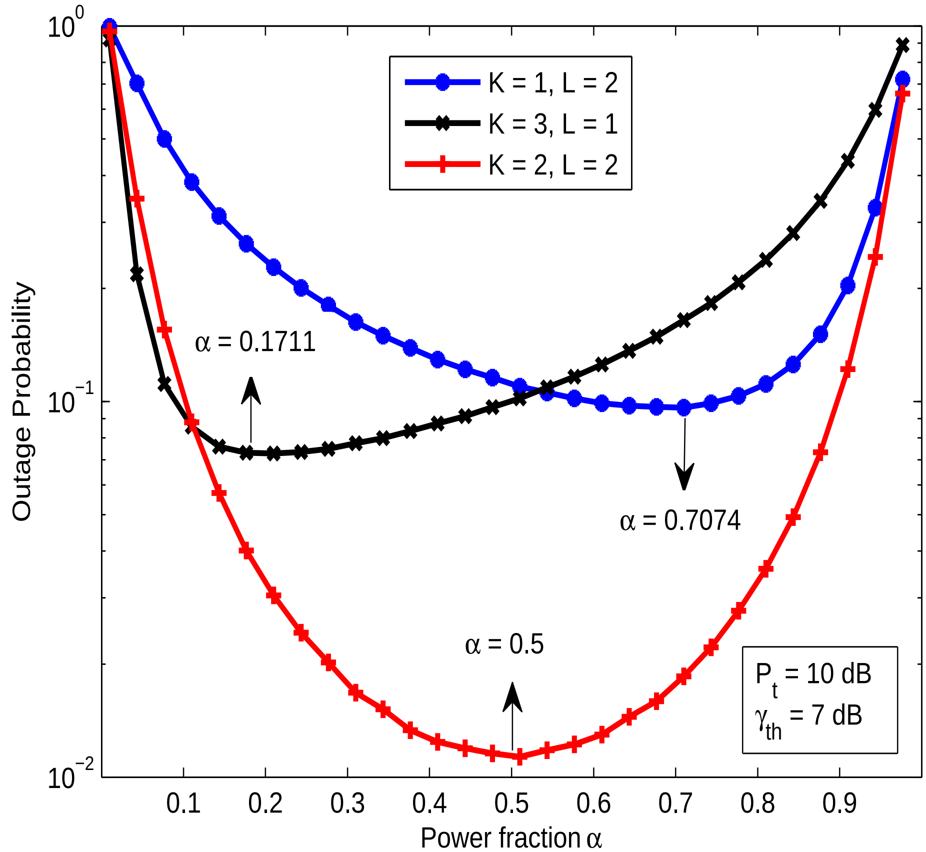

Figure 3.8. Optimal power allocation under power fraction α

In 3.7, average end-to-end SNR is depicted for different scenarios. As seen from the figure, average end-to-end SNR gain increases significantly with the number of relays, e.g., when $P/N0$ is 15 dB, adding each relay will help to increase end-to-end SNR by 25dB which is quite appealing from system design perspective.

Fig. 3.8 shows the usefulness of power allocation which obtains optimum

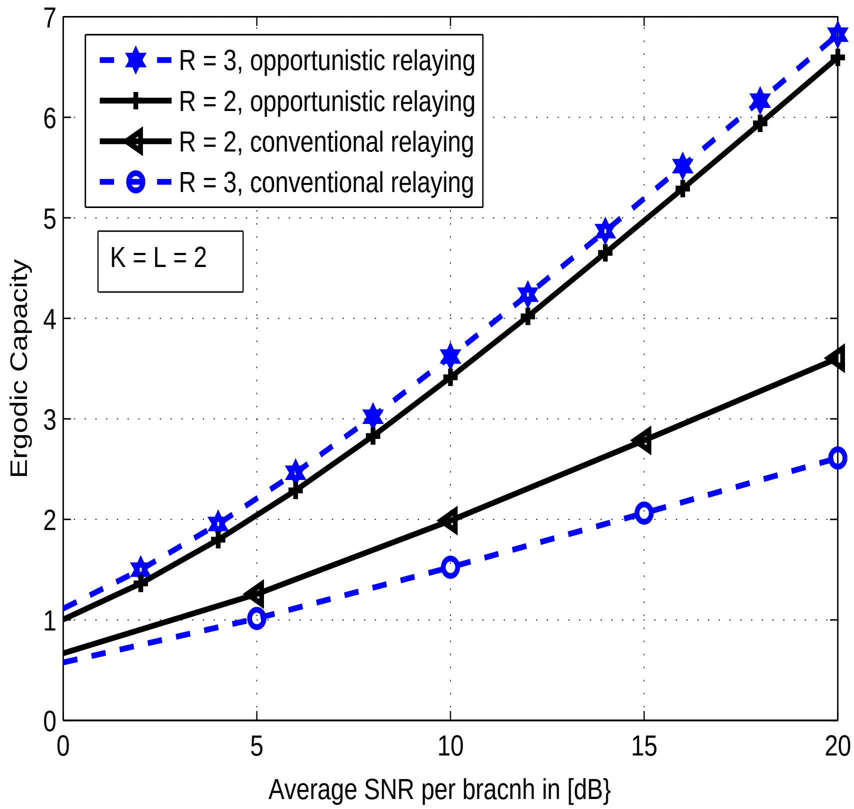

Figure 3.9. Ergodic capacity comparison of conventional and opportunistic relaying

power fraction values to minimize OP. In this figure, total power is set to 10 dB and 3 different cases are drawn. From all cases, we infer that optimum power allocation yields a much better performance then $\alpha = 1/2$. For example, when $K = 3, L = 1$, OP is lower than $10^{-1}P_{out}$ at $\alpha = 0.1711$ or when $K = 1, L = 2$, source power must be increased to 7.074 dB to obtain a much better outage performance. However, when $K = L$, source and relay powers are equal (5 dB). All these values shown in Fig. 3.8, can be verified from Table 3.1.

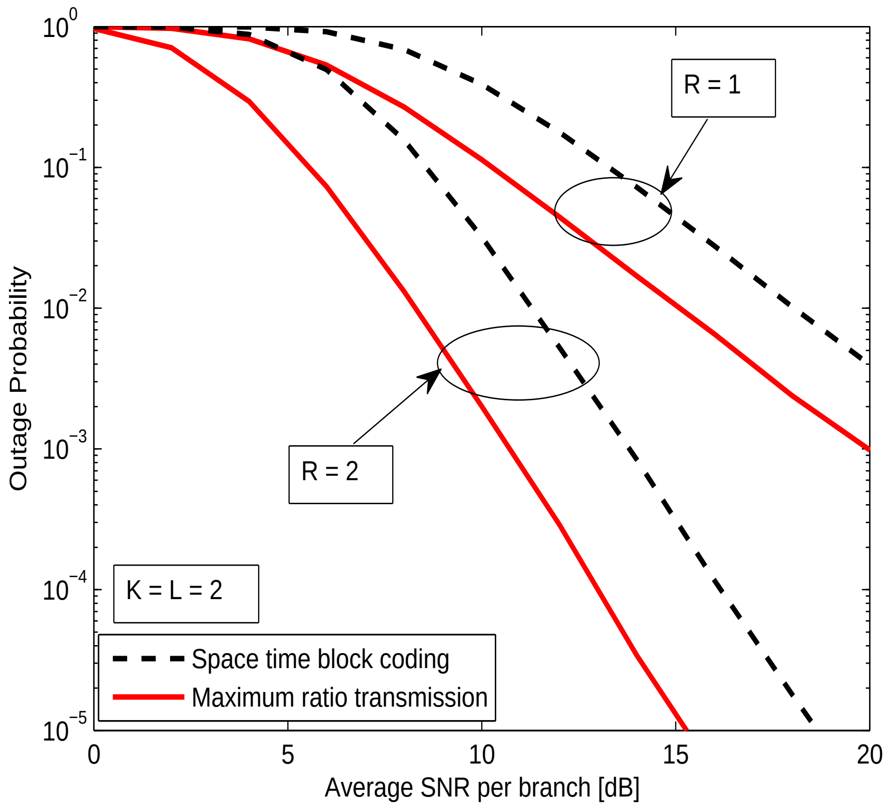

Figure 3.10. Outage probability comparison of OSTBC with MRT for conventional networks

In, Fig. 3.9, ergodic capacity of conventional and opportunistic relaying schemes, is illustrated. As can be seen, in opportunistic relaying, increasing R increases ergodic capacity. However, as conventional relaying uses R time slots in the transmission, ergodic capacity decreases by a factor of R which can be observed from (3.31). Therefore, opportunistic relaying is much superior than conventional in terms of ergodic capacity. It should be noted that, to improve the ergodic capacity of conventional scheme, number of antennas at the source and relay can be increased.

Fig. 3.10 presents the performance comparison of well known STBC with MRT within conventional transmission for $R = 1$ and $R = 2$ when $K = L = 2$ transmit antennas are used at the source and relays. As can be seen, MRT performs 3 dB better than STBC while both can achieve the same diversity orders. We can deduce that using MRT instead of OSTBC can be preferable due to performance gain and low receiver complexity advantage especially when the wireless network experiences slow fading.

3.6. Chapter Summary

In this chapter, multi-antenna/multi-relay AF MRT with both conventional and opportunistic networks are investigated. In conventional relaying, source and all relays employing MRT participate the transmission to obtain considerable diversity gain. In contrast, opportunistic relaying selects the best path to maximize the e2e SNR at the destination and obtain identical diversity gains with low computational complexity. For both models, PDF, CDF and MGF are derived. Approximate and asymptotic SER and OP expressions are obtained, ergodic capacity is derived and diversity and array gains are computed. In addition, optimum source and relay powers are obtained and the theoretical derivations are verified by numerical examples. The proposed multi-relay MRT can be a promising option in practical wireless communication networks as they can provide high diversity gains while requiring low receiver complexity.

4. MAXIMUM RATIO TRANSMISSION WITH RELAY SELECTION IN TWO-WAY RELAY NETWORKS

In this chapter, we investigate the performance of an AF MIMO TWRN where two sources are equipped with multiple antennas employing MRT and the communication is carried through the selected relay resulting in the largest received power. Assuming the fading channel coefficients are Nakagami-m distributed, the SSER and OPs are derived for each user and the overall system. In addition, diversity and array gains are obtained using the derived asymptotic SSER and system OP expressions. With the help of asymptotic system OP, the optimum location of relay is found by solving the convex optimization problem. Furthermore, the impact of limited feedback and imperfect channel estimations are investigated on the performance of the proposed structure. Finally, theoretical findings are validated by simulation results.

4.1. Introduction

To enhance the communication reliability in TWRNs, multiple antennas and relays have been studied to explore the improved performance [82]-[92]. In [82]-[83], [84] and [85], performance of an AF opportunistic TWRN is analyzed for Rayleigh, Nakagami-m and Rician fading channels respectively. Moreover in [86]-[87], power optimization is studied in opportunistic TWRNs where [86] obtains system outage for Nakagami-m fading channels and [87] derives both OP and SER for Rayleigh fading channels. Like multi-relay transmission scenarios, multiple antennas at the sources with MRT becomes popular in TWRNs. User OP of an AF MIMO TWRN with MRT is investigated over Nakagami-m fading channels in [88] whereas in [89], joint optimization of power allocation and relay location are examined to minimize overall SER and ergodic sum rate over Nakagami-m fading channels. In addition, a comparison of antenna selection and MRT is considered in [90], where SSER expression is obtained for Nakagami-m fading channels. In [91], an AF MIMO TWRN is analyzed where joint MRT and antenna selection is compared with joint transmit-receive antenna selection and system OP is derived for Nakagami-m fading channels. Besides, optimal beamformer design for TWRNs is presented in [92] where capacity limits are obtained.

In the wide body of literature, there is no previous work which investigates the performance of joint MRT and relay selection in TWRNs even for Rayleigh fading channels. In this chapter, the performance of an AF MIMO TWRN is examined where multiple-antenna sources exchange information through R-relays having single antenna and derive user, system OPs and SSER over flat Nakagami-m fading channels. The main contributions can be listed as follows:

- User, system OPs and SSER over flat Nakagami-m fading channels are derived to quantify the performance of the proposed network.
- Asymptotic SSER and system outage expressions are derived to obtain diversity and coding gains.

- The impact of imperfect channel estimations and limited feedback are investigated on the proposed network.
- To improve overall system performance, the problem of relay location optimization is examined to minimize system OP.
- To verify the correctness of analytical results, numerical examples are presented and compared with theoretical results.

The remainder of the chapter is organized as follows. Channel and network models are presented in Section 4.2. In Section 4.3, PDF, CDF and MGF of the e2e SNR are derived. In Section 4.4, user, system outages and SSER expressions are derived. Then, to obtain diversity and array gains, asymptotic system OP and SSER are obtained. In addition, impact of imperfect channel estimations and limited feedback are presented. In Section 4.5, optimum relay location to minimize system OP is obtained and numerical examples are given in Section 4.6. Finally conclusions are drawn in Section 4.7.

4.2. System Model

This chapter focuses on an AF MIMO TWRN consisting of two source terminals (S_1 and S_2) having N_1 and N_2 antennas respectively, communicates via R-relays each having single antenna as depicted in Fig. 1. The direct link between two source terminals is assumed to be not available due to large path loss effect, distance or heavy shadowing. Therefore, the transmission between S_1

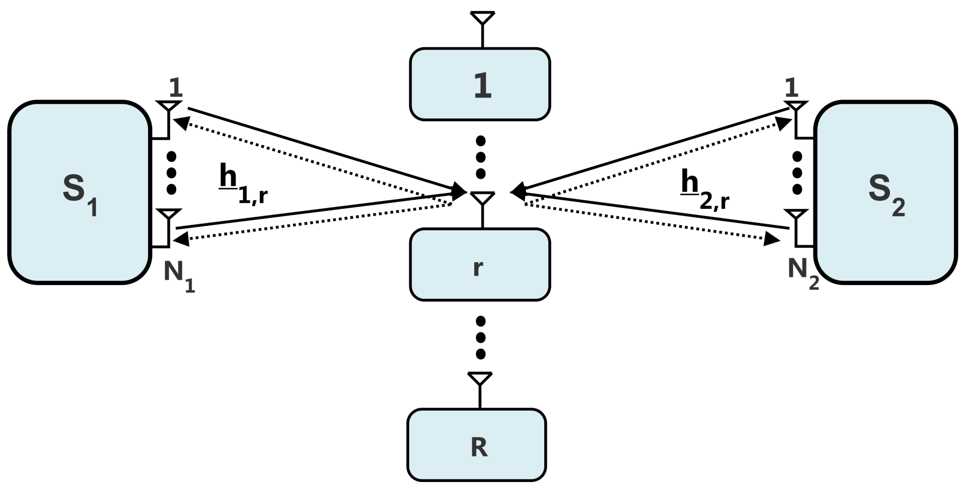

Figure 4.1. Block diagram of MIMO TWRN with relay selection

and S_2 can be done with the help of selected relay r, $\{1 \leq r \leq R\}$. All channel coefficients between $S_1 \rightarrow r$ and $S_2 \rightarrow r$ hops are modeled as independent and identically distributed (i.i.d) flat Nakagami-m fading with m_1 and m_2 severity parameters respectively, whereas both hops are assumed to be i.n.i.d Nakagami-m fading channel i.e., $m_1 \neq m_2$, and $N_1 \neq N_2$. The communication between two source terminals is divided into two time slots. In the first time slot, S_1 and S_2 simultaneously transmit their signals x_1 and x_2 respectively by using MRT technique [9]. As we assume equal power at all nodes, the received signal at the r-th relay can be written as follows

$$y_r = \sqrt{Pd_{1,r}^{-\alpha}}\mathbf{h}_{1,r}\mathbf{w}_{1,r}x_1 + \sqrt{Pd_{2,r}^{-\alpha}}\mathbf{h}_{2,r}\mathbf{w}_{2,r}x_2 + n_r. \tag{4.1}$$

In the second time slot, relay amplifies the received signal with a scaling factor G_r and forwards to S_1 and S_2 by using MRC. The received signal at both source nodes can be expressed as

$$y_{s_1} = \mathbf{w}_{1,r}^T \left(\sqrt{P d_{1,r}^{-\alpha}} G_r \mathbf{h}_{1,r}^T y_r + \mathbf{n}_1 \right)$$
$$y_{s_2} = \mathbf{w}_{2,r}^T \left(\sqrt{P d_{2,r}^{-\alpha}} G_r \mathbf{h}_{2,r}^T y_r + \mathbf{n}_2 \right), \tag{4.2}$$

where $d_{1,r}$ and $d_{2,r}$ are distances between $S_1 \to r$ and $S_2 \to r$ respectively, α is the path loss component, $\mathbf{h}_{1,r}$ and $\mathbf{h}_{2,r}$ are $N_1 \times 1$ and $N_2 \times 1$ channel vectors between $S_1 \to r$ and $S_2 \to r$ respectively. MRT based weight vectors $\mathbf{w}_{1,r}$ and $\mathbf{w}_{2,r}$ are specified as $\mathbf{w}_{1,r} = (\mathbf{h}_{1,r}^{\dagger}/\|\mathbf{h}_{1,r}\|)$ and $\mathbf{w}_{2,r} = (\mathbf{h}_{2,r}^{\dagger}/\|\mathbf{h}_{2,r}\|)$. Noise sample n_r and vectors $\mathbf{n}_1$, $\mathbf{n}_2$ are modeled as complex additive white Gaussian noise (AWGN) with zero mean and N_0 noise power. Scaling factor G_r is given as

$$G_r = \frac{1}{\sqrt{P d_{1,r}^{-\alpha} \|\mathbf{h}_{1,r}\|^2 + P d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2}}. \tag{4.3}$$

Substituting (4.1) in (4.2) with the help of (4.3) and after the self interference term drops due to perfect channel reciprocity, the following e2e SNRs can be obtained as below.

$$\gamma_{S_1 \to r \to S_2} = \frac{\frac{P}{N_0} d_{1,r}^{-\alpha} \|\mathbf{h}_{1,r}\|^2 \frac{P}{N_0} d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2}{2 \frac{P}{N_0} d_{1,r}^{-\alpha} \|\mathbf{h}_{1,r}\|^2 + \frac{P}{N_0} d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2} = \frac{\gamma_{S_1} \gamma_{S_2}}{2\gamma_{S_1} + \gamma_{S_2}}$$
$$\gamma_{S_2 \to r \to S_1} = \frac{\frac{P}{N_0} d_{1,r}^{-\alpha} \|\mathbf{h}_{1,r}\|^2 \frac{P}{N_0} d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2}{\frac{P}{N_0} d_{1,r}^{-\alpha} \|\mathbf{h}_{1,r}\|^2 + 2 \frac{P}{N_0} d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2} = \frac{\gamma_{S_1} \gamma_{S_2}}{\gamma_{S_1} + 2\gamma_{S_2}}, \tag{4.4}$$

where $\gamma_{S_1} = \frac{P}{N_0} d_{1,r}^{-\alpha} \|\mathbf{h}_{1,r}\|^2$ and $\gamma_{S_2} = \frac{P}{N_0} d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2$ are the instantaneous SNRs at $S_1 \to r$ and $S_2 \to r$ hops. As the exact system OP and SSER becomes quite complicated in MIMO TWRNs, we resort computing tight lower bounds on these performance indicators by simplifying the e2e SNRs given in (4.4) as

$$\gamma_{S_i \to r \to S_j} \leqslant \gamma^{up}_{S_i \to r \to S_j} = \min(\gamma_{S_i}, \gamma_{S_j}/2), \tag{4.5}$$

where $i, j \in \{1, 2\}$ and $i \neq j$. Since user outage is defined as the probability of e2e SNR being lower than a certain threshold γ_{th}, optimal relay selection (for user 1 or 2) can be shown as follows

$$R^{us}_{*} = \arg \max_{1 \leqslant r \leqslant R} \min(\gamma_{S_i}, \gamma_{S_j}/2). \tag{4.6}$$

System outage on the other hand can be defined as if at least one of the source nodes is in outage and relay selection can be expressed as [93, eqn. 11]

$$R^{sys}_{*} = \arg \max_{1 \leqslant r \leqslant R} \min(\gamma_{S_1 \to r \to S_2}, \gamma_{S_2 \to r \to S_1}). \tag{4.7}$$

Equations (4.6) and (4.7) show the selection policy of the best relay for each user or overall system respectively.

4.3. SNR Statistics

In this section, we derive PDF, CDF and MGF of the SNR for any user $i, j = \{1, 2\}$, $i \neq j$. With the help of (4.5), CDF of $S_i \to r \to S_j$ can be expressed as

$$\mathcal{F}_{\gamma^{up}_{S_i\to r\to S_j}}(\gamma) = \Pr\left[\min(\gamma_{S_i}, \gamma_{S_j}/2) \leqslant \gamma\right]$$
$$= 1 - \Pr[\gamma_{S_i} > \gamma]\Pr[\gamma_{S_j} > 2\gamma], \tag{4.8}$$

and the PDFs of γ_{S_i} and γ_{S_j} can be written as

$$f_{\gamma_{S_i}}(\gamma) = \frac{m_i^{m_iN_i}\gamma^{m_iN_i-1}e^{-m_i\gamma/\Omega_i}}{\Omega_i^{m_iN_i}\Gamma(m_iN_i)}$$
$$f_{\gamma_{S_j}}(\gamma) = \frac{m_j^{m_jN_j}\gamma^{m_jN_j-1}e^{-m_j\gamma/\Omega_j}}{\Omega_j^{m_jN_j}\Gamma(m_jN_j)}. \tag{4.9}$$

Therefore, integrating (4.9) w.r.t γ, with the help of [1, eqn.(8.350.2)] and then substituting in (4.8), $\mathcal{F}_{\gamma^{up}_{S_i\to r\to S_j}}$ can be obtained as

$$\mathcal{F}_{\gamma^{up}_{S_i\to r\to S_j}}(\gamma) = \left(1 - \frac{\Gamma\left(m_iN_i, m_i\frac{\gamma}{\Omega_i}\right)}{\Gamma(m_iN_i)} \times \frac{\Gamma\left(m_jN_j, 2m_j\frac{\gamma}{\Omega_j}\right)}{\Gamma(m_jN_j)}\right). \tag{4.10}$$

We denote $\Omega_i = d_{i,r}^{-\alpha}\bar{\gamma}$, $\Omega_j = d_{j,r}^{-\alpha}\bar{\gamma}$ as average SNRs and $\bar{\gamma} = P/N_0$. With the help of [1, eqn.(8.352.7)], (4.10) can be expressed as

$$\mathcal{F}_{\gamma^{up}_{S_i\to r\to S_j}}(\gamma) = \left(1 - e^{-m_i\frac{\gamma}{\Omega_i}}\sum_{z=0}^{m_iN_i-1}\left(m_i\frac{\gamma}{\Omega_i}\right)^z\frac{1}{z!}\right.$$
$$\left.\times e^{-2m_j\frac{\gamma}{\Omega_j}}\sum_{v=0}^{m_jN_j-1}\left(2m_j\frac{\gamma}{\Omega_j}\right)^v\frac{1}{v!}\right). \tag{4.11}$$

With the help of high order statistics [75] and (4.6), $\mathcal{F}_{\gamma^{up}_{S_i\to R_*^{us}\to S_j}}(\gamma)$ can be obtained as $\mathcal{F}_{\gamma^{up}_{S_i\to R_*^{us}\to S_j}}(\gamma) = \left\{\mathcal{F}_{\gamma^{up}_{S_i\to r\to S_j}}(\gamma)\right\}^R$. By applying binomial [1, eqn.(1.111.1)] and multinomial expansions [1, eqn.(0.314)] respectively, $\mathcal{F}_{\gamma^{up}_{S_i\to R_*^{us}\to S_j}}(\gamma)$ can be obtained as

$$\mathcal{F}_{\gamma^{up}_{S_i\to R_*^{us}\to S_j}}(\gamma) = \sum_{r=0}^{R}\sum_{z=0}^{r(m_iN_i-1)}\sum_{v=0}^{r(m_jN_j-1)}\binom{R}{r}(-1)^r e^{-\gamma\left(\frac{rm_i\Omega_j+2rm_j\Omega_i}{\Omega_i\Omega_j}\right)} \times\mathcal{X}_z(r)\mathcal{X}_v(r)\gamma^{v+z}, \quad (4.12)$$

where combination operation denotes binomial coefficients and $\mathcal{X}_t(r)$ shows multinomial coefficients which can be found as

$$\mathcal{X}_t(r) = \frac{1}{tk_0}\sum_{\rho=1}^{t}(r\rho - t + \rho)k_\rho\mathcal{X}_{t-\rho}(r), t \geqslant 1. \quad (4.13)$$

Multinomial coefficients can be obtained by using [1, eqn.(0.314)]; $k_\rho = (Am_l\frac{1}{\Omega_l})^\rho\frac{1}{\rho!}$, $\mathcal{X}_0(r) = k_0^r = 1$, $t \in \{v, z\}$, $A = \{1, 2\}$ and $l \in \{i, j\}$. As $\mathcal{F}_{\gamma^{up}_{S_i\to R_*^{us}\to S_j}}(\gamma)$ is obtained, the PDF of $\gamma^{up}_{S_i\to R_*^{us}\to S_j}$ can be found by taking the derivative of (4.12) w.r.t. γ as

$$f_{\gamma^{up}_{S_i\to R_*^{us}\to S_j}}(\gamma) = \sum_{r=0}^{R}\sum_{z=0}^{r(m_iN_i-1)}\sum_{v=0}^{r(m_jN_j-1)}\binom{R}{r}(-1)^r\mathcal{X}_z(r)\mathcal{X}_v(r) \times e^{-\gamma\left(\frac{rm_i}{\Omega_i}+\frac{2rm_j}{\Omega_j}\right)}\gamma^{v+z}\left(\gamma(v+z) - \frac{rm_i}{\Omega_i} - \frac{2rm_j}{\Omega_j}\right), \quad (4.14)$$

and the MGF of $\gamma^{up}_{S_i \to R^{us}_* \to S_j}$ can be obtained as

$$\mathcal{M}_{\gamma^{up}_{S_i \to R^{us}_* \to S_j}}(s) = s \int_0^\infty e^{-s\gamma} \mathcal{F}_{\gamma^{up}_{S_i \to R^{us}_* \to S_j}}(\gamma) d\gamma. \tag{4.15}$$

By substituting (4.12) in (4.15) and with the help of [1, eqn. 3.351.3], $\mathcal{M}_{\gamma^{up}_{S_i \to R^{us}_* \to S_j}}(s)$ can be expressed as follows

$$\mathcal{M}_{\gamma^{up}_{S_i \to R^{us}_* \to S_j}}(s) = \sum_{r=0}^{R} \sum_{z=0}^{r(m_i N_i - 1)} \sum_{v=0}^{r(m_j N_j - 1)} \binom{R}{r} (-1)^r s \mathcal{X}_z(r) \mathcal{X}_v(r) \Gamma(v + z - 1) \times \left(s + \frac{r m_i}{\Omega_i} + \frac{2 r m_j}{\Omega_j} \right)^{-v-z-1}. \tag{4.16}$$

MGF of SNR (4.16) or CDF of SNR (4.12) can be used to obtain SSER and OPs.

4.4. Performance Analysis

In this section, we first derive OPs and SSER for flat Nakagami-m fading channels. Then, by deriving asymptotic expressions of SSER and system OP, diversity and array gains are obtained. Finally, the impact of practical transmission impairments such as limited feedback (of channel coefficients) and channel estimation errors are investigated.

4.4.1. User and System Outage Probabilities

User OP is defined as the probability of e2e SNR ($\gamma^{up}_{S_i \to r \to S_j}$) falling below a certain threshold γ_{th} and it can be computed as $P^{us}_{out} = \mathcal{F}_{\gamma^{up}_{S_i \to R^{us}_{*} \to S_j}}(\gamma_{th})$. System OP on the other hand means if $S_1 \to r \to S_2$ or $S_2 \to r \to S_1$ path is in outage. With the help of (4.7), system OP can be expressed as

$$\begin{aligned} P^{sys}_{out} &= \mathcal{F}_{\gamma}(\gamma_{th}), \\ \mathcal{F}_{\gamma}(\gamma_{th}) &= \textstyle\prod_{r=1}^{R} \Pr\Big(\min\left(\gamma_{S_1 \to r \to S_2}, \gamma_{S_2 \to r \to S_1}\right) \leqslant \gamma_{th} \Big), \\ &= \textstyle\prod_{r=1}^{R} \Big(1 - \Pr\left[\gamma_{S_1 \to r \to S_2} > \gamma_{th}\right] \times \Pr\left[\gamma_{S_2 \to r \to S_1} > \gamma_{th}\right]\Big). \end{aligned} \tag{4.17}$$

Substituting (4.5) in (4.17) gives

$$\begin{aligned} \mathcal{F}_{\gamma}(\gamma_{th}) = \prod_{r=1}^{R} \Bigg(1 - \Pr\left[\min\left(\gamma_{S_1}, \frac{\gamma_{S_2}}{2}\right) > \gamma_{th}\right] \\ \times \Pr\left[\min\left(\frac{\gamma_{S_1}}{2}, \gamma_{S_2}\right) > \gamma_{th}\right] \Bigg). \end{aligned} \tag{4.18}$$

As (4.18) is highly complicated, simple lower bounds on system OP is investigated. For this, the following Lemma is proposed.

Lemma: E2e SNRs can be upper bounded by dividing (4.4) to $\gamma_{S_1} = \frac{P}{N_0} d_{1,r}^{-\alpha} \|\mathbf{h}_{1,r}\|^2$ and $\gamma_{S_2} = \frac{P}{N_0} d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2$ which is

$$\gamma_{S_1 \to r \to S_2} = \frac{\frac{P}{N_0} d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2}{2 + \frac{d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2}{d_{1,r}^{-\alpha} \|\mathbf{h}_{1,r}\|^2}} \leqslant \frac{P}{2N_0} d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2 = \frac{1}{2}\gamma_{S_2}, \tag{4.19}$$

and similarly $\gamma_{S_2 \to r \to S_1} \leqslant \frac{1}{2}\gamma_{S_1}$. As $\frac{P}{N_0} d_{1,r}^{-\alpha} \|\mathbf{h}_{1,r}\|^2 > 0$ and $\frac{P}{N_0} d_{2,r}^{-\alpha} \|\mathbf{h}_{2,r}\|^2 > 0$, these approximate results are valid. By using these simple bounds, (4.18) can be approximate written as

$$\begin{aligned} P_{out}^{sys} &= \mathcal{F}_{\gamma}^{up}(\gamma_{th}), \\ \mathcal{F}_{\gamma}^{up}(\gamma_{th}) &= \prod_{r=1}^{R} \Pr\Big[\min\left(\gamma_{S_1}/2, \gamma_{S_2}/2\right) \leqslant \gamma_{th}\Big]. \end{aligned} \tag{4.20}$$

By using similar theoretical steps as given in (4.8)-(4.12), system OP (lower bound) can be obtained as follows

$$\begin{aligned} P_{out}^{sys} = \sum_{r=0}^{R} \sum_{z=0}^{r(m_1 N_1 - 1)} \sum_{v=0}^{r(m_2 N_2 - 1)} \binom{R}{r} (-1)^r e^{-\gamma_{th}\left(\frac{2rm_1\Omega_2 + 2rm_2\Omega_1}{\Omega_1\Omega_2}\right)} \\ \times \mathcal{X}_z(r) \mathcal{X}_v(r) \gamma_{th}^{v+z}, \end{aligned} \tag{4.21}$$

where multinomial coefficients are as given in (4.13), only difference is $k_\rho = (2m_l \frac{1}{\Omega_l})^\rho \frac{1}{\rho!}$, $l = \{1, 2\}$.

4.4.2. Sum Symbol Error Rate

SSER which can be defined as the summation of SER at S_1 and S_2 nodes, is one of the most important performance criterion in TWRNs. Mathematically, it can be expressed as

$$P_s(e) = P_{s_1}(e) + P_{s_2}(e). \tag{4.22}$$

For M-PSK modulation, by using the MGF of SNR (4.16), we can write SSER as follows [43]

$$P_{s,PSK}(e) = \frac{1}{\pi}\int_0^{\phi}\left[\mathcal{M}^{up}_{\gamma_{S_1\to R_*^{us}\to S_2}}\left(\frac{g_{PSK}}{\sin^2(\theta)}\right) + \mathcal{M}^{up}_{\gamma_{S_2\to R_*^{us}\to S_1}}\left(\frac{g_{PSK}}{\sin^2(\theta)}\right)\right]d\theta, \tag{4.23}$$

where $g_{PSK} = \sin^2(\pi/M)$ and $\phi = (M-1)\pi/M$. For M-QAM modulation, SSER can be written as [43]

$$\begin{aligned} P_{s,QAM}(e) = B\bigg[\int_0^{\pi/2}\mathcal{M}^{up}_{\gamma_{S_1\to R_*^{us}\to S_2}}\left(\frac{g_{QAM}}{\sin^2(\theta)}\right) + \mathcal{M}^{up}_{\gamma_{S_2\to R_*^{us}\to S_1}}\left(\frac{g_{QAM}}{\sin^2(\theta)}\right)d\theta\bigg] \\ - B(1-1/\sqrt{M})\bigg[\int_0^{\pi/4}\mathcal{M}^{up}_{\gamma_{S_1\to R_*^{us}\to S_2}}\left(\frac{g_{QAM}}{\sin^2(\theta)}\right) + \mathcal{M}^{up}_{\gamma_{S_2\to R_*^{us}\to S_1}}\left(\frac{g_{QAM}}{\sin^2(\theta)}\right)\bigg], \end{aligned} \tag{4.24}$$

where $B = \frac{4}{\pi}(1-1/\sqrt{M})$. By substituting (4.16) in (4.23) and (4.24), SSER can be obtained for M-ary modulations. In addition, for the systems whose conditional error probability is in the form of $a\mathbb{E}[Q(\sqrt{2b\gamma})]$, SSER can also be obtained by using the CDF of SNR as follows

$$P_s(e) = \frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^{\infty}\gamma^{-1/2}e^{-b\gamma}\left(\mathcal{F}_{\gamma^{up}_{S_1\to R_*^{us}\to S_2}}(\gamma) + \mathcal{F}_{\gamma^{up}_{S_2\to R_*^{us}\to S_1}}(\gamma)\right)d\gamma, \tag{4.25}$$

where a and b denotes modulation coefficients as described above. Furthermore, we can obtain approximate results for M-ary modulation types [43]. Substituting (4.12) into (4.25), with the help of [1, eqn. 3.351.3], $P_s(e)$ can be obtained as follows

$$P_s(e) = \frac{a\sqrt{b}}{2\sqrt{\pi}} \sum_{r=0}^{R} \binom{R}{r} (-1)^r \times$$
$$\Bigg[\sum_{z=0}^{r(m_1N_1-1)} \sum_{v=0}^{r(m_2N_2-1)} \mathcal{X}_z(r)\mathcal{X}_v(r)\Gamma(\mathcal{V}) \Bigg(b + \frac{rm_1}{\Omega_1} + \frac{2rm_2}{\Omega_2} \Bigg)^{\mathcal{N}}$$
$$+ \sum_{z=0}^{r(m_2N_2-1)} \sum_{v=0}^{r(m_1N_1-1)} \mathcal{X}_z(r)\mathcal{X}_v(r)\Gamma(\mathcal{V}) \Bigg(b + \frac{rm_2}{\Omega_2} + \frac{2rm_1}{\Omega_1} \Bigg)^{\mathcal{N}} \Bigg], \quad (4.26)$$

where $\mathcal{N} = -v - z - 1/2$ and $\mathcal{V} = v + z - 3/2$.

4.4.3. Asymptotic Analysis

In this section, asymptotic system OP and SSER expressions are derived to obtain diversity ($\mathcal{G}_d$) and array gains ($\mathcal{G}_a$) [44].
At high SNR, by using [77, eqn. 6], (4.20) can be expressed as

$$P_{out}^{sys} = \prod_{r=1}^{R} \Big(\mathcal{F}_{\gamma_{S_1}}(2\gamma_{th}) + \mathcal{F}_{\gamma_{S_2}}(2\gamma_{th}) \Big)$$
$$= \prod_{r=1}^{R} \left(\frac{\Upsilon\Big(m_1N_1, 2m_1\frac{\gamma_{th}}{\Omega_1}\Big)}{\Gamma(m_1N_1)} + \frac{\Upsilon\Big(m_2N_2, 2m_2\frac{\gamma_{th}}{\Omega_2}\Big)}{\Gamma(m_2N_2)} \right). \quad (4.27)$$

By using the asymptotic behavior of lower incomplete gamma function given in [78, eqn.(45.9.1)], i.e., $\Upsilon(k, v \to 0) \to v^k/k$, (4.27) can be expressed as

$$P_{out}^{sys,\infty} = \prod_{r=1}^{R} \left(\frac{(2m_1\gamma_{th})^{m_1N_1}}{\Gamma(m_1N_1+1)\Omega_1^{m_1N_1}} + \frac{(2m_2\gamma_{th})^{m_2N_2}}{\Gamma(m_2N_2+1)\Omega_2^{m_2N_2}} \right), \quad (4.28)$$

For large enough $\bar{\gamma}$ and with the help of [44], asymptotic system OP can be obtained as

$$P_{out}^{sys,\infty} \approx \mathcal{J}\left(\bar{\gamma}\right)^{-\sum_{r=1}^{R}\min(m_1N_1,m_2N_2)} + \text{H.O.T.}, \tag{4.29}$$

where H.O.T denotes high order terms and $\mathcal{J}$ is given as

$$\mathcal{J} = \begin{cases} \prod_{r=1}^{R}\left(\frac{(2m_1\gamma_{th})^{m_1N_1}}{\Gamma(m_1N_1+1)d_1^{-\alpha m_1N_1}}\right), & m_1N_1 < m_2N_2 \\ \prod_{r=1}^{R}\left(\frac{(2m_1\gamma_{th})^{m_1N_1}}{\Gamma(m_1N_1+1)d_1^{-\alpha m_1N_1}} + \frac{(2m_2\gamma_{th})^{m_2N_2}}{\Gamma(m_2N_2+1)d_2^{-\alpha m_2N_2}}\right), & m_1N_1 = m_2N_2 \\ \prod_{r=1}^{R}\left(\frac{(2m_2\gamma_{th})^{m_2N_2}}{\Gamma(m_2N_2+1)d_2^{-\alpha m_1N_1}}\right), & m_1N_1 > m_2N_2. \end{cases} \tag{4.30}$$

As described in [44], $P_{out}^{sys,\infty} \approx (\mathcal{G}_a\bar{\gamma})^{-\mathcal{G}_d}$, so diversity and array gains become

$$\begin{aligned} \mathcal{G}_d &= \textstyle\sum_{r=1}^{R}\min(m_1N_1,m_2N_2) \\ \mathcal{G}_a &= \mathcal{J}^{-1/\sum_{r=1}^{R}\min(m_1N_1,m_2N_2)}. \end{aligned} \tag{4.31}$$

From (4.31), we can understand that, the number of relays have a direct impact on the diversity order. On the other hand, minimum number of severity parameters and antennas at both sources are more important. To obtain asymptotic SSER, the asymptotic property of lower incomplete Gamma function is used as described above. Similar to (4.28), $\mathcal{F}_{\gamma_{S_i}\to R_*^{us}\to S_j}^{up,\infty}$ can be expressed as

$$\mathcal{F}_{\gamma_{S_i}\to R_*^{us}\to S_j}^{up,\infty}(\gamma) = \prod_{r=1}^{R}\left(\frac{(2m_i\gamma)^{m_iN_i}}{\Gamma(m_iN_i+1)\Omega_i^{m_iN_i}} + \frac{(m_j\gamma)^{m_jN_j}}{\Gamma(m_jN_j+1)\Omega_j^{m_jN_j}}\right). \tag{4.32}$$

To simplify the theoretical complexity, $S_1 \rightarrow r$ and $S_2 \rightarrow r$ hops are assumed to be balanced i.e., $m_i = m_j = m$, $N_i = N_j = N$ and $\Omega_i = \Omega_j = \Omega$, then (4.32) can be expressed in a simple form

$$\mathcal{F}^{up,\infty}_{\gamma_{S_i} \rightarrow R_*^{us} \rightarrow S_j}(\gamma) = \mathcal{A}\left(\frac{\gamma}{\Omega}\right)^{\mathcal{B}}, \tag{4.33}$$

where $\mathcal{A} = \left[(2^{mN}+1)(m)^{mN}/\Gamma(mN+1)\right]^R$ and $\mathcal{B} = mNR$. For balanced hops, (4.25) becomes

$$P_s(e) = \frac{a\sqrt{b}}{\sqrt{\pi}} \int_0^\infty \gamma^{-1/2} e^{-b\gamma} \mathcal{F}_{\gamma^{up,\infty}_{S_i \rightarrow R_*^{us} \rightarrow S_j}}(\gamma) d\gamma \tag{4.34}$$

By substituting (4.33) in (4.34) with the help of [44, Proposition 1.], asymptotic SSER can be expressed as

$$P_s^\infty(e) = \frac{\mathcal{A} a \Gamma(\mathcal{B}+1/2)}{\sqrt{\pi}(b\Omega)^{\mathcal{B}}} + \text{H.O.T.}, \tag{4.35}$$

where a, b are modulation coefficients as described above and $\mathcal{G}_d = \mathcal{B} = mNR$. Therefore, the diversity order of the asymptotic system OP derived in (4.31) verifies the diversity order obtained from (4.35) when the hops are balanced.

4.4.4. Impact of practical transmission impairments

To maximize the effects of MRT, we mainly assume a full-rate perfect feedback of channel coefficients at the relay node. However, if a wireless network suffers from power or bandwidth constraints, the feedback rate becomes insufficient which causes huge loses on the MRT performance. As shown in [94], the effects of limited feedback on the PDF of SNR can be expressed as

$$f_{\gamma_{S_i}}(\gamma) = \frac{m_i^{m_i N_i} \gamma^{m_i N_i - 1} e^{-m_i \gamma / \Omega_i (1-\xi)}}{(\Omega_i (1-\xi))^{m_i N_i} \Gamma(m_i N_i)}, \tag{4.36}$$

where $f_{\gamma_{S_i}}(\gamma)$ can be obtained similarly. In (4.36), ξ denotes the rate of feedback, i.e., $\xi = 0$ shows full-rate feedback. Substituting (4.36) in (4.9) and applying same theoretical steps as shown above, system OP and SSER in the presence of limited feedback can be obtained.

In addition, to examine the impact of imperfect channel estimations, we derive effective e2e SNRs and obtain CDF of SNR. For this, both hops are assumed to be erroneously estimated and show the relationship between channel vectors and estimation errors as [95]-[96]

$$\begin{aligned} \mathbf{h}_{1,r} &= \tilde{\mathbf{h}}_{1,r} + \boldsymbol{\xi}_{1,r} \\ \mathbf{h}_{2,r} &= \tilde{\mathbf{h}}_{2,r} + \boldsymbol{\xi}_{2,r} \end{aligned} \tag{4.37}$$

where $\tilde{\mathbf{h}}_{1,r}$ and $\tilde{\mathbf{h}}_{2,r}$ are channel estimates and $\boldsymbol{\xi}_{1,r}$, $\boldsymbol{\xi}_{2,r}$ are estimation error vectors. Note that MRT based weight vectors become $\mathbf{w}_{1,r} = (\tilde{\mathbf{h}}_{1,r}^{\dagger} / \|\mathbf{h}_{1,r}\|)$ and $\mathbf{w}_{2,r} = (\tilde{\mathbf{h}}_{2,r}^{\dagger} / \|\mathbf{h}_{2,r}\|)$. Substituting (4.37) into (4.1), (4.2) and substituting $\tilde{\mathbf{h}}_{1,r}$ and $\tilde{\mathbf{h}}_{2,r}$ into (4.3) and after removing the self-interference term, e2e SNRs can be written as follows

$$\gamma_{S_i \to r \to S_j} = \frac{\gamma_{S_i} \gamma_{S_j}}{\varphi \gamma_{S_i} + \beta \gamma_{S_j} + \lambda}, \tag{4.38}$$

where, $\varphi = 2+4\frac{P}{N_0}\sigma^2_{\xi_{i,r}} + \frac{P}{N_0}\sigma^2_{\xi_{j,r}}$, $\beta = 1+\frac{P}{N_0}\sigma^2_{\xi_{i,r}}$, $\lambda = \frac{P^2}{N_0^2}\sigma^2_{\xi_i}\sigma^2_{\xi_j} + \frac{P}{N_0}\sigma^2_{\xi_i} + \frac{P^2}{N_0^2}\sigma^2_{\xi_i}\sigma^2_{\xi_j}$ and $i, j = \{1, 2\}$, $i \neq j$. The upper bound given in (4.5) becomes $\gamma_{S_i \to r \to S_j} \leqslant \min\left(\frac{\gamma_{S_i}}{\beta}, \frac{\gamma_{S_j}}{\varphi}\right)$ and (4.8) is written as

$$\begin{aligned}\mathcal{F}_{\gamma^{up}_{S_i \to r \to S_j}}(\gamma) &= \Big(1 - \Pr[\gamma_{S_i} > \beta\gamma] \Pr[\gamma_{S_j} > \varphi\gamma]\Big) \\ &= \left(1 - \frac{\Gamma\Big(m_i N_i, \beta m_i \frac{\gamma}{\Omega_i}\Big)}{\Gamma(m_i N_i)} \times \frac{\Gamma\Big(m_j N_j, \varphi m_j \frac{\gamma}{\Omega_j}\Big)}{\Gamma(m_j N_j)}\right),\end{aligned} \tag{4.39}$$

From (4.39), it can be observed that the CDF of SNR deteriorates from the negative effects of imperfect channel estimations e.g., φ and β. Applying same theoretical steps to (4.39), SSER and system OP in the presence of channel estimation errors can be easily derived.

4.5. Relay Location Optimization

Relay location optimization is an important design problem in relay networks to improve overall system performance and to combat the effects of co-channel interference. We assume normalized distance between S_1 and S_2 i.e., $d_1 + d_2 = 1$, $d_1 = d, d_2 = 1 - d$ and $R = 1$. Under these assumptions, relay location is optimized to minimize asymptotic system OP as shown below

$$\min_d P^{sys,\infty}_{out} \text{ subject to: } 0 < d < 1. \tag{4.40}$$

We first take the second derivative of $P_{out}^{sys,\infty}$, which is

$$\frac{\partial^2 P_{out}^{sys,\infty}}{\partial d^2} = \mathcal{Z}_1 m_1 N_1 \alpha (m_1 N_1 \alpha - 1) d^{m_1 N_1 \alpha - 2} + \mathcal{Z}_2 m_2 N_2 \alpha (m_2 N_2 \alpha - 1)(1-d)^{m_2 N_2 \alpha - 2}, \tag{4.41}$$

where $\mathcal{Z}_1 = \frac{(2m_1\gamma_{th})^{m_1N_1}}{\Gamma(m_1N_1+1)\bar{\gamma}^{m_1N_1}}$ and $\mathcal{Z}_2 = \frac{(2m_2\gamma_{th})^{m_2N_2}}{\Gamma(m_2N_2+1)\bar{\gamma}^{m_2N_2}}$. Since $m_1N_1\alpha > 1$ and $m_2N_2\alpha > 1$, it is understood that the proposed problem is convex i.e., $\frac{\partial^2 P_{out}^{sys,\infty}}{\partial d^2} > 0$. Hence, if we take the first derivative of $P_{out}^{sys,\infty}$ w.r.t d and equalize to zero, optimum relay location can be found

$$\frac{\partial P_{out}^{sys,\infty}}{\partial d} = \mathcal{Z}_1 m_1 N_1 \alpha d^{m_1 N_1 \alpha - 1} - \mathcal{Z}_2 m_2 N_2 \alpha (1-d)^{m_2 N_2 \alpha - 1} = 0. \tag{4.42}$$

After some manipulations

$$d^{m_1 N_1 \alpha - 1} = \frac{\mathcal{Z}_2 m_2 N_2}{\mathcal{Z}_1 m_1 N_1}(1-d)^{m_2 N_2 \alpha - 1}. \tag{4.43}$$

To obtain optimum relay distance, root finding algorithms can be applied. For different channel conditions and number of antennas, Table 4.1 shows optimum relay distances when $\bar{\gamma} = 10$ dB, $\alpha = 2$ and $\gamma_{th} = 3$ dB. It can be understood the table that when $m_1N_1 > m_2N_2$, optimum relay location must be close to S_2 and when $m_2N_2 > m_1N_1$, optimum position of relay must be near S_1 to minimize system OP. Besides, when $m_1 = N_1 = m_2 = N_2$, optimum relay location is in the middle of both sources, i.e., $d = 1/2$.

Table 4.1. Optimum relay distance for $\alpha = 2$, $\gamma_{th} = 3$ dB and $\bar{\gamma} = 10$ dB.

$(m_1, N_1), (m_2, N_2)$	Optimum relay distance
$(1,1),(1,1)$	$d_1 = d_2 = 1/2$
$(1,2),(1,1)$	$d_1 = 0.7975$, $d_2 = 0.2025$
$(1,2),(2,2)$	$d_1 = 0.3130$, $d_2 = 0.6870$

4.6. Numerical Examples

In this section, various numerical examples for different number of antennas, relays and fading severity are presented to verify the analytical results and demonstrate the usefulness of the proposed system. SSER and system OP curves are obtained via Monte-Carlo simulations where BPSK signalling is used. In the simulations, path-loss component is chosen as $\alpha = 1.6$ to represent factory or office environment [5] and distances are given as $d_1 = d_2 = d = 0.5$ unless otherwise stated.

Fig. 4.2 depicts the SSER vs Ω performance of the proposed network for balanced hops $m_1 = m_2 = 1$, $N_1 = N_2 = 2$ and different number of relays. It is observed from the figure that using more relays both yield a much better performance and enhanced diversity orders (slope of the curves), i.e., 16 dB SNR

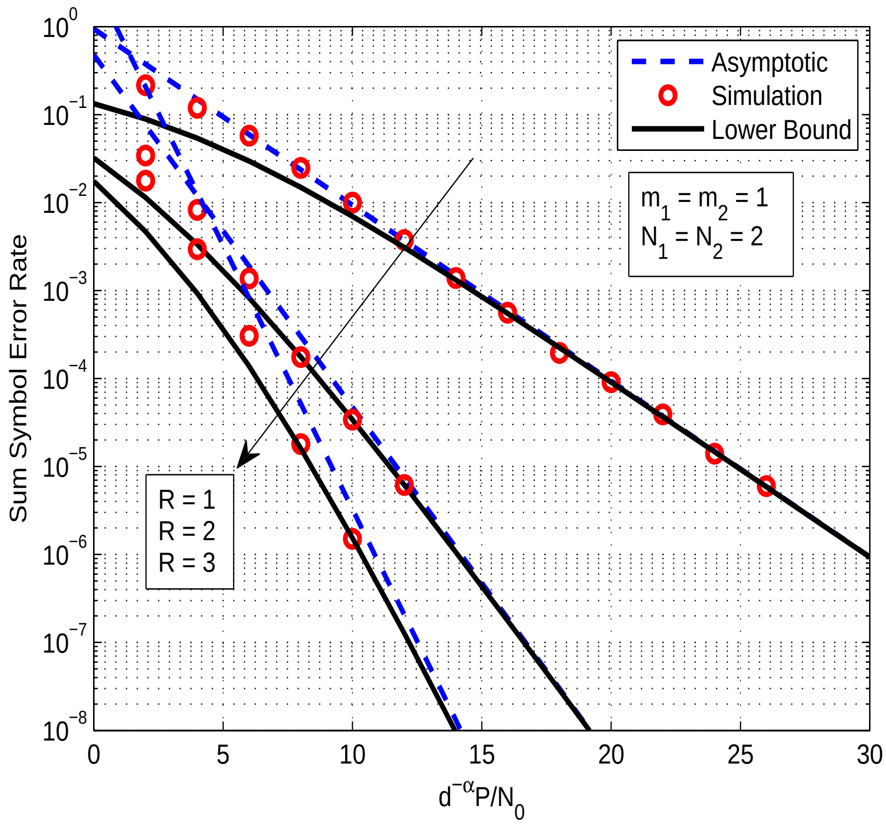

Figure 4.2. Sum SER performance of MIMO AF TWRN for $m_1 = m_2 = 1$ i.e., Rayleigh fading channel and $R = 1, 2, 3$.

gain can be obtained and the diversity order becomes 2 to 4 if $R = 1$ is compared with $R = 2$. Besides, the theoretical results precisely match with the simulation at all cases and the slopes of the curves verify the diversity gains obtained e.g., 2, 4 and 6. In addition, from the system design perspective, it is observed that using 2 relays and 2 antennas at both sources can achieve 10^{-8} SSER at 19 dB SNR which is quite appealing.

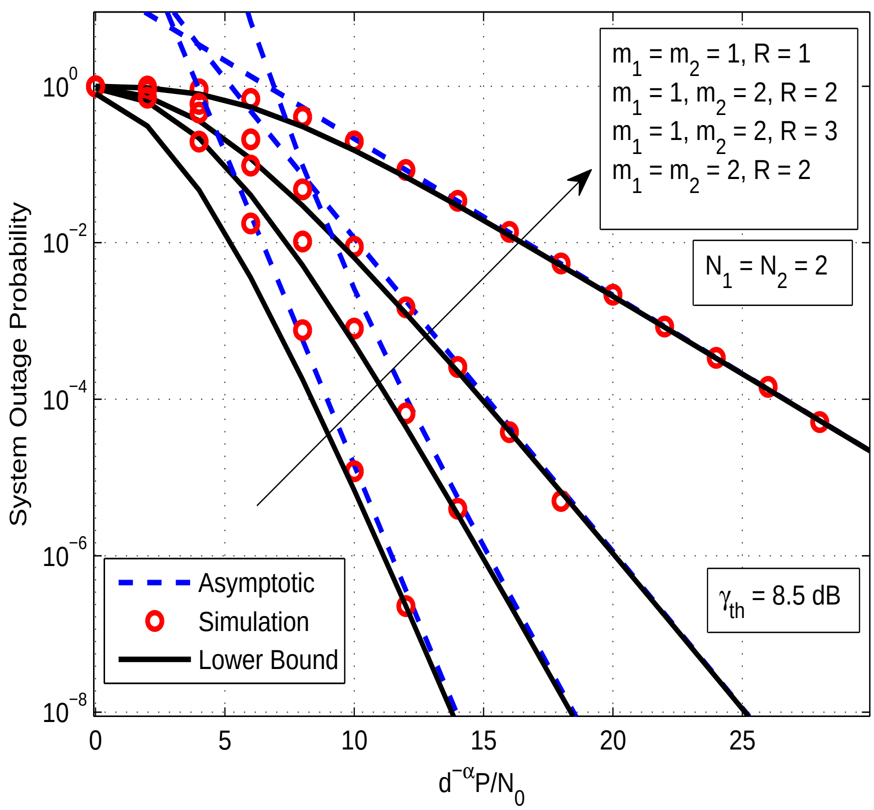

Figure 4.3. System OP performance for different number of relays and fading severity.

Fig. 4.3 illustrates the system OP for unbalanced links and different number of relays when $\gamma_{th} = 8.5$ dB. As can be seen, the proposed lower bound for system OP is in an excellent agreement with the simulation results in all cases especially at medium to high SNRs. In addition, the asymptotic curves of the system OP verifies the diversity order derived in (4.31) i.e., $\mathcal{G}_d = \sum_{r=1}^{R} \min(m_1 N_1, m_2 N_2)$. As we observed in the previous figure, increasing the total number of relays have a direct impact both on system performance and

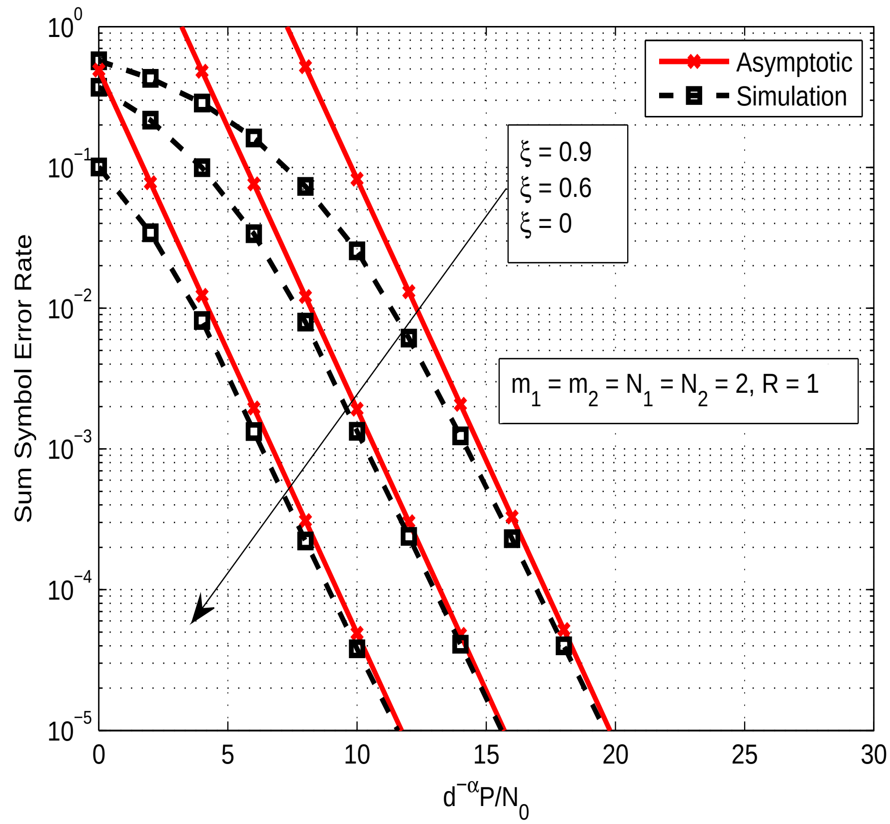

Figure 4.4. Impact of limited feedback on the SSER performance.

diversity orders.

In Fig. 4.4 and 4.5, the impact of limited feedback and imperfect channel estimations on SSER and system OP are demonstrated respectively. From Fig. 4.4, we can clearly observe that, although the limited feedback deteriorates the SSER performance, there is no change on the diversity gains. In contrast, imperfect channel estimations in Fig. 4.5 not only have an adverse effect on the performance of system OP but also error floors result in huge performance

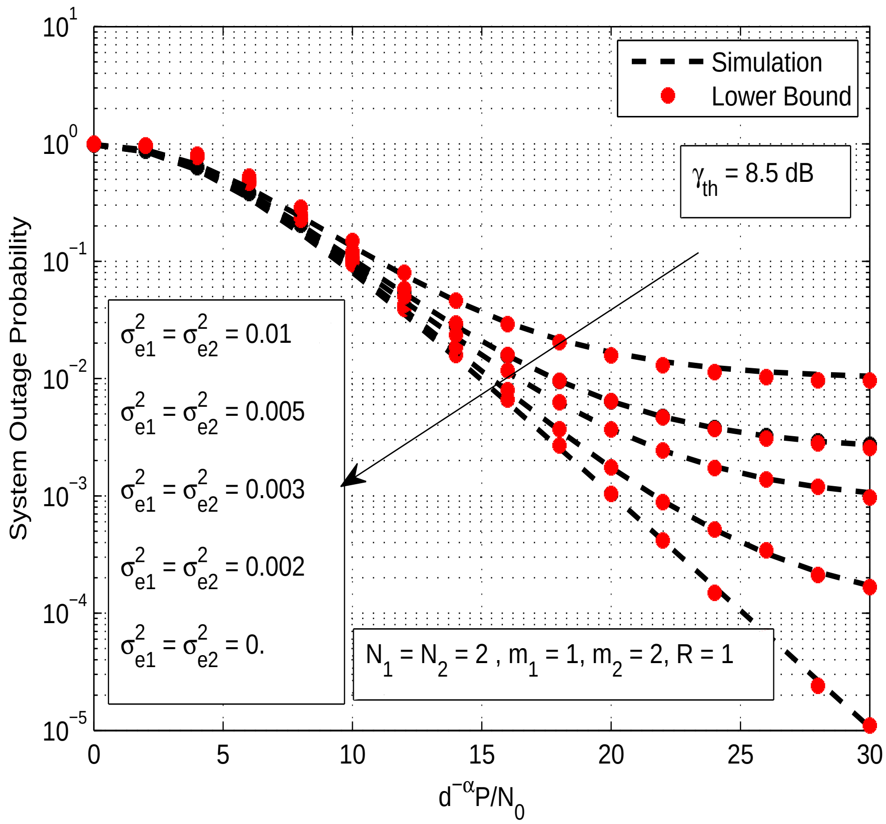

Figure 4.5. Impact of imperfect channel estimations on the system OP.

loses as no diversity can be obtained. Besides, as observed in the previous figures, asymptotic SSER matches quite good with simulation in Fig. 4.4 and the proposed lower bound for system OP provides an excellent match with the simulations in Fig. 4.5. In addition, if the impact of imperfect channel estimations on OWRNs and TWRNs are compared, it is understood that the erroneously estimated channel vectors have more adverse effect on TWRNs as both source nodes are being affected from both channel vectors (4.38).

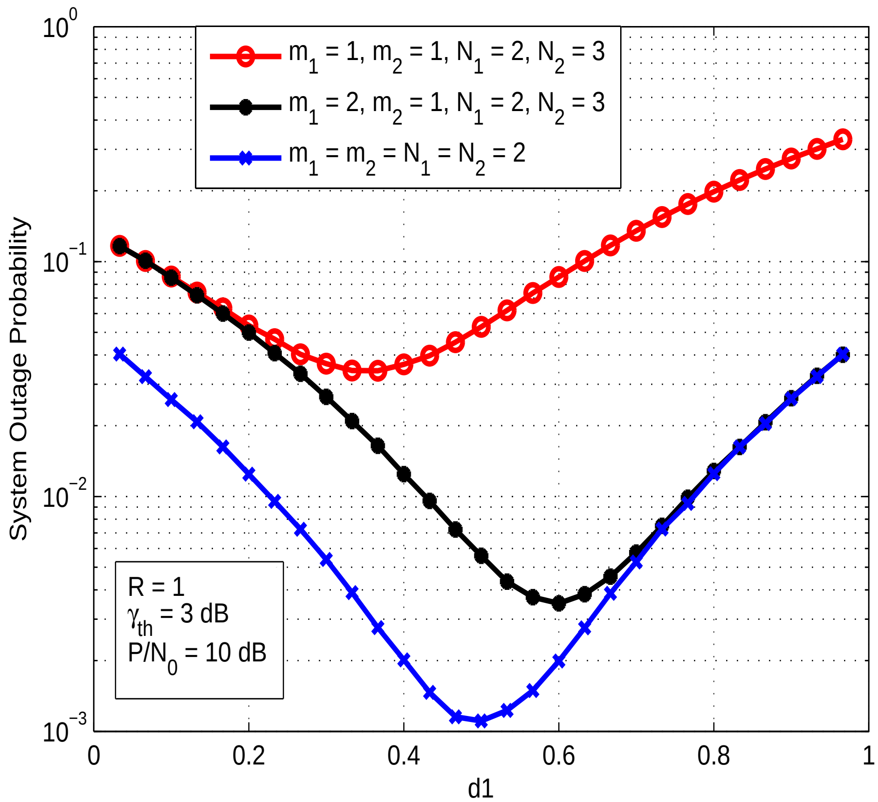

Figure 4.6. System OP vs d_1 for different number of antennas and severity parameters.

Fig. 4.6 plots optimum relay locations for different number of antennas and fading severity when $P/N_0 = 10$ dB and $R = 1$. As can be seen, this figure can be easily used to obtain optimum relay locations. For example, when $m_2N_2 > m_1N_1$, $d_2 > d_1$. In contrast, $d_1 > d_2$, when $m_1N_1 > m_2N_2$. Likewise, when $m_1N_1 = m_2N_2$ optimum distance becomes $d_1 = d_2 = 1/2$. All these results can be justified by using eqn. (4.43). In addition, Fig. 4.7 presents the effect of optimum relay location on the system OP for different number of antennas,

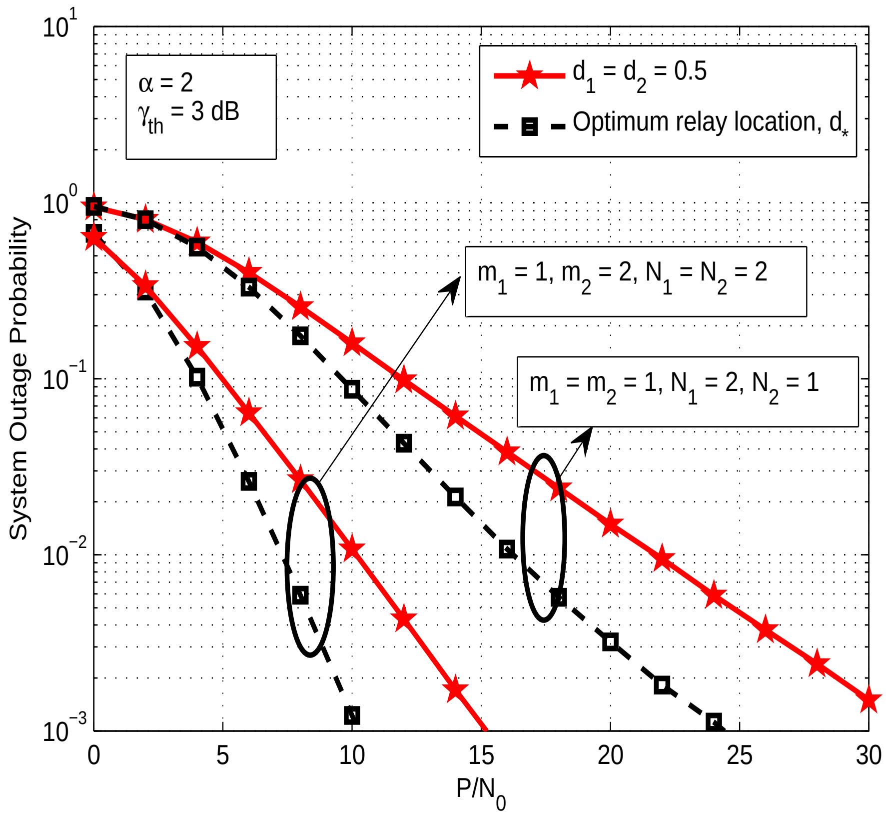

Figure 4.7. System OP performance for different number of antennas, relays and severity parameter for optimum and suboptimum ($d_1 = d_2 = 1/2$) relay location.

relays and fading severity. Especially in this figure, the optimum values obtained in Table 3.1 are used and the effect of optimum relay location is investigated on the system OP. As can be seen from both cases that optimum relay position can bring up to 6 dB performance gain and can enhance the diversity orders which is quite important from the system design perspective.

4.7. Chapter Summary

In this chapter, the performance of joint MRT and relay selection in AF MIMO TWRNs are proposed and analyzed. For the proposed network, we derive approximate and asymptotic user, system OPs and SSER for flat i.n.i.d Nakagami-m fading channels and obtain diversity and array gains. In addition, important performance indicators such as limited feedback and imperfect channel estimations are investigated which are critical on the performance of MRT. At the end, it is observed that the optimum location of relay can both improve system performance and diversity gains which are quite important for the system designer.

5. TWO-WAY RELAY NETWORKS WITH JOINT ANTENNA AND RELAY SELECTION

In this chapter, the performance of a MIMO AF TWRN with joint antenna and relay selection is analyzed over Nakagami-m fading channels. We derive both approximate and asymptotic system OP expressions and present diversity and coding gains for arbitrary number of antennas, relays and fading severity. Finally, our analytical findings are verified by numerical examples

5.1. Introduction

To leverage the advantages of TWRNs, multiple antennas employing MRT have previously studied in [88]-[92] and the references therein. To lower the cost and complexity with a minimal loss in performance gains, antenna selection is investigated in TWRNs recently [91], [97]-[100]. In [97], antenna selection is studied in AF TWRNs and system OP is derived for Rayleigh fading channels. In [98]-[99], antenna selection is analyzed for DF TWRNs and SER is derived. Reference [100] considers two new joint transmit-receive antenna selection strategies and derives system OP for Rayleigh fading channels. For a similar system model, [91] derives closed form and approximate system OPs for Nakagami-m fading channels.

In the literature, the outage expressions in recent TWRN studies (e.g. [91], [100]) are quite complicated in general which makes it difficult to gain insights about the system behavior. In this paper, an AF MIMO TWRN with joint antenna and relay selection is considered where approximate and asymptotic system OPs are derived for Nakagami-m fading channels. The main contributions of this chapter can be listed as

- Approximate system OP is derived by using simple bounds of e2e SNRs.
- Diversity and array gains are obtained by using the high SNR analysis.
- All theoretical results are conformed with Monte-Carlo simulation.

The remainder of this chapter is organized as follows. Channel and network model is presented in Section 5.2. In Section 5.3, CDF of SNR is derived and approximate system OP is obtained. In section 5.4, diversity and array gains are obtained by using the high SNR analysis. Section 5.5 presents numerical examples and Section 5.6 concludes this chapter.

5.2. System model

We consider an AF MIMO TWRN consisting of two source terminals having N_K and N_L antennas communicating via R-relays having N_r antennas $\{r = 1 \ldots R\}$. System block diagram is shown in Fig. 5.1. The direct link between two source terminals are assumed to be unavailable e.g. due to heavy shadowing. All transmit-receive antenna pairs between $S_1 \rightarrow r$ and $S_2 \rightarrow r$ hops are assumed to be modeled as i.i.d Nakagami-m with fading severity parameters m_1 and m_2 respectively. The communication between two terminals

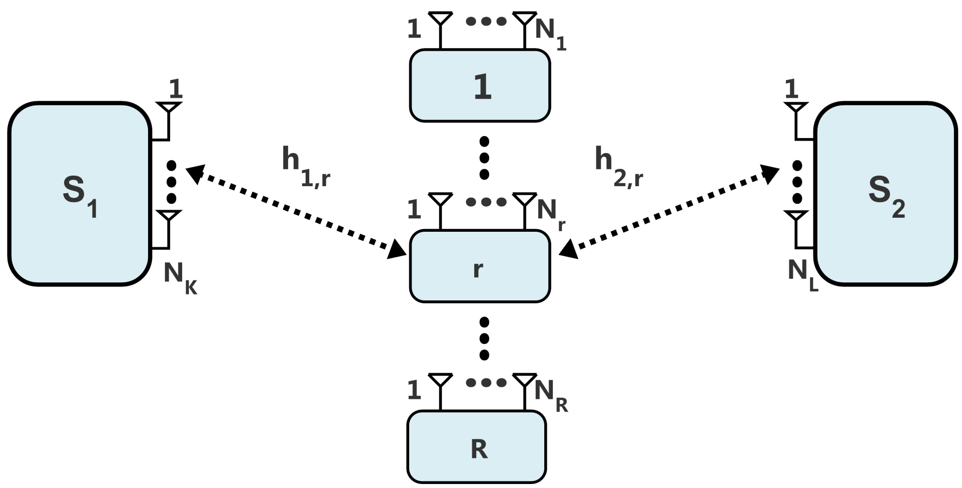

Figure 5.1. Block diagram of MIMO AF TWRN with multiple antennas and relays

takes place in two time slots. In the first time slot, both sources transmit their signals x_1 and x_2 concurrently through their selected k-th and l-th antennas. As we assume equal power at S_1, S_2 and r i.e., $P_1 = P_2 = P_r = P$, the received signal at the selected r-th relay and j-th antenna (best pairs) can be written as,

$$y_r = \sqrt{P}h_{1,r}^{(k,j)}x_1 + \sqrt{P}h_{2,r}^{(l,j)}x_2 + n_r, \tag{5.1}$$

where $h_{1,r}^{(k,j)}$, $h_{2,r}^{(l,j)}$ are the selected channel coefficients between $S_1 \rightarrow r$ and $S_2 \rightarrow r$ paths respectively. n_r is the complex additive white Gaussian noise (AWGN) with zero-mean and N_0 variance. Note that, antennas and relays are selected to minimize the system OP which can be achieved by maximizing the e2e SNR of the weakest source. In the second time slot, r-th relay amplifies the received signal with gain G_r and forwards to both source terminals. As

$S \rightarrow r$ and $r \rightarrow S$ paths are assumed to be reciprocal in general TWRNs, same antennas can be used. Hence, the received signal at S_1 and S_2 can be expressed as

$$
\begin{aligned}
y_{S_1} &= \sqrt{P} G_r h_{1,r}^{(k,j)} y_r + n_1, \\
y_{S_2} &= \sqrt{P} G_r h_{2,r}^{(l,j)} y_r + n_2,
\end{aligned}
\tag{5.2}
$$

where n_1,n_2 are the AWGN noises at S_1 and S_2 with zero-mean and N_0 noise power. The amplifying gain is given as

$$
G_r = \frac{1}{\sqrt{P|h_{1,r}^{(k,j)}|^2 + P|h_{2,r}^{(l,j)}|^2}}. \tag{5.3}
$$

Substituting (5.3) in (5.2) and after removing the self interference term, the e2e SNR for both terminals can be written as follows

$$
\begin{aligned}
\gamma_{S_1 \rightarrow r \rightarrow S_2}^{(k,l,j)} &= \frac{\frac{P}{N_0}|h_{1,r}^{(k,j)}|^2 \frac{P}{N_0}|h_{2,r}^{(l,j)}|^2}{2\frac{P}{N_0}|h_{1,r}^{(k,j)}|^2 + \frac{P}{N_0}|h_{2,r}^{(l,j)}|^2} = \frac{\gamma_{S_1}^{(k,j)}\gamma_{S_2}^{(l,j)}}{2\gamma_{S_1}^{(k,j)} + \gamma_{S_2}^{(l,j)}} \\
\gamma_{S_2 \rightarrow r \rightarrow S_1}^{(k,l,j)} &= \frac{\frac{P}{N_0}|h_{1,r}^{(k,j)}|^2 \frac{P}{N_0}|h_{2,r}^{(l,j)}|^2}{\frac{P}{N_0}|h_{1,r}^{(k,j)}|^2 + 2\frac{P}{N_0}|h_{2,r}^{(l,j)}|^2} = \frac{\gamma_{S_1}^{(k,j)}\gamma_{S_2}^{(l,j)}}{\gamma_{S_1}^{(k,j)} + 2\gamma_{S_2}^{(l,j)}},
\end{aligned}
\tag{5.4}
$$

where $\gamma_{S_1}^{(k,j)} = \frac{P}{N_0}|h_{1,r}^{(k,j)}|^2$ and $\gamma_{S_2}^{(l,j)} = \frac{P}{N_0}|h_{2,r}^{(l,j)}|^2$.

5.3. System Outage Probability

In TWRNs, system OP can be defined as the weakest e2e SNR falling below a certain threshold (γ_{th}), i.e., $S_1 \to r \to S_2$ or $S_2 \to r \to S_1$ path is in outage. Mathematically, it can be expressed as follows

$$P_{out} = \Pr\left[\max_{\substack{1 \leqslant k \leqslant N_K, 1 \leqslant l \leqslant N_L, \\ 1 \leqslant j \leqslant N_r, 1 \leqslant r \leqslant R}} \min\left(\gamma_{S_1 \to r \to S_2}^{(k,l,j)}, \gamma_{S_2 \to r \to S_1}^{(k,l,j)}\right) \leqslant \gamma_{th}\right], \tag{5.5}$$

where $\Pr[\cdot]$ denotes probability of an event. It can be seen from [91], [97] that the analysis of (5.5) is difficult especially for MIMO TWRNs with antenna/relay selection in Nakagami-m fading channels. With the motivation of simplifying the analytical complexity and obtaining a simple outage expression, the *Lemma* shown in (4.19) is used and (5.5) is simplified as follows

$$P_{out} = \Pr\left[\max_{\substack{1 \leqslant k \leqslant N_K, 1 \leqslant l \leqslant N_L, \\ 1 \leqslant j \leqslant N_r, 1 \leqslant r \leqslant R}} \min\left(\frac{\gamma_{S_1}^{(k,j)}}{2}, \frac{\gamma_{S_2}^{(l,j)}}{2}\right) \leqslant \gamma_{th}\right]. \tag{5.6}$$

Obviously this approximation simplifies the theoretical complexity in the derivation of system OP in TWRNs and also performs quite well as can be seen in the Numerical Examples section. By using [77, eqn. 6] and after some manipulations, (5.6) can be expressed as

$$\begin{aligned}
P_{out} &= \mathcal{F}_{\gamma}(\gamma_{th}), \\
\mathcal{F}_{\gamma}(\gamma) &= \prod_{r=1}^{R}\left[\mathcal{F}_{\gamma_{S_1}}(2\gamma) + \mathcal{F}_{\gamma_{S_2}}(2\gamma) - \mathcal{F}_{\gamma_{S_1}}(2\gamma)\mathcal{F}_{\gamma_{S_2}}(2\gamma)\right]^{N_r} \\
&\approx \prod_{r=1}^{R}\left[\mathcal{F}_{\gamma_{S_1}}(2\gamma) + \mathcal{F}_{\gamma_{S_2}}(2\gamma)\right]^{N_r},
\end{aligned} \tag{5.7}$$

where $\mathcal{F}_{\gamma_{S_1}}(2\gamma) = \Pr\left[\gamma_{S_1} \leqslant 2\gamma\right]$ and $\mathcal{F}_{\gamma_{S_2}}(2\gamma) = \Pr\left[\gamma_{S_2} \leqslant 2\gamma\right]$. With the help of order statistics [75] and [1, eqn.(8.352.6)], we can specify $\mathcal{F}_{\gamma_{S_i}}(2\gamma)$, $i = \left\{1, 2\right\}$ as follows

$$\begin{aligned}
\mathcal{F}_{\gamma_{S_i}}(2\gamma) &= \left(\frac{\Upsilon\left(m_i, 2m_i\frac{\gamma}{\Omega}\right)}{\Gamma(m_i)}\right)^{\mathcal{N}} \\
&= \left(1 - e^{-2m_i\frac{\gamma}{\Omega_i}}\sum_{t=0}^{m_i-1}(2m_i\frac{\gamma}{\Omega})^t\frac{1}{t!}\right)^{\mathcal{N}}.
\end{aligned} \tag{5.8}$$

We denote $\Omega = P/N_0$ as average SNR and $\mathcal{N} \in \left\{N_K, N_L\right\}$. By applying binomial [1, eqn.(1.111)] and multinomial expansions [1, eqn.(0.314)] respectively, $\mathcal{F}_{\gamma_{S_i}}(2\gamma)$ can be expressed as

$$\mathcal{F}_{\gamma_{S_i}}(2\gamma) = \sum_{a=0}^{\mathcal{N}}\sum_{t=0}^{a(m_i-1)}\binom{\mathcal{N}}{a}(-1)^a\gamma^t e^{-2m_i a\frac{\gamma}{\Omega}}\mathcal{X}_t(a), \tag{5.9}$$

where combination operation gives binomial coefficients and $\mathcal{X}_t(a)$ stands for multinomial coefficients which is written as

$$\mathcal{X}_t(a) = \frac{1}{tz_0}\sum_{\rho=1}^{t}(a\rho - t + \rho)z_\rho \mathcal{X}_{t-\rho}(a), t \geqslant 1, \tag{5.10}$$

where $z_\rho = (2m_i\frac{1}{\Omega})^\rho \frac{1}{\rho!}$ and $\mathcal{X}_0(a) = z_0^a = 1$. By substituting (5.8) in (5.7), system OP can be obtained as

$$\begin{aligned} P_{out} = \prod_{r=1}^{R}\Bigg[&\sum_{a=0}^{N_K}\sum_{t=0}^{a(m_1-1)}\binom{N_K}{a}(-1)^a\gamma_{th}^t e^{-2m_1 a\frac{\gamma}{\Omega}}\mathcal{X}_t(a) \\ &+\sum_{a=0}^{N_L}\sum_{t=0}^{a(m_2-1)}\binom{N_L}{a}(-1)^a\gamma_{th}^t e^{-2m_2 a\frac{\gamma}{\Omega}}\mathcal{X}_t(a)\Bigg]^{N_r} \end{aligned} \tag{5.11}$$

If both hops are balanced i.e., $N_K = N_L = N_T$ and $m_1 = m_2 = m$, (5.11) can be written as

$$P_{out} = \prod_{r=1}^{R}\Bigg[2\sum_{a=0}^{N_T}\sum_{t=0}^{a(m-1)}\binom{N_T}{a}(-1)^a\gamma_{th}^t e^{-2ma\frac{\gamma}{\Omega}}\mathcal{X}_t(a)\Bigg]^{N_r}. \tag{5.12}$$

5.4. Diversity order and coding gain

Here diversity ($\mathcal{G}_d$) and array gains ($\mathcal{G}_a$) are presented by deriving system OP asymptotically as described in [44]. At high SNR, when $\Omega \to \infty$, the lower incomplete Gamma function can be asymptotically written as $\Upsilon(k, v \to 0) \to$

v^k/k. Therefore, asymptotic system OP can be expressed as

$$P_{out}^{\infty} = \prod_{r=1}^{R} \left(\left(\frac{(2m_1\gamma_{th})^{m_1}}{\Gamma(m_1+1)\Omega^{m_1}} \right)^{N_K} + \left(\frac{(2m_2\gamma_{th})^{m_2}}{\Gamma(m_2+1)\Omega^{m_2}} \right)^{N_L} \right)^{N_r}. \quad (5.13)$$

By using [44, Prop. 5], P_{out}^{∞} can be obtained as

$$P_{out}^{\infty} = \prod_{r=1}^{R} (\mathcal{K})^{N_r} \left(\frac{\gamma_{th}}{\Omega} \right)^{\sum_{r=1}^{R} \left(N_r \times \min(m_1 N_K, m_2 N_L) \right)} + \text{H.O.T.}, \quad (5.14)$$

where H.O.T. denotes high order terms and $\mathcal{K}$ is given as

$$\mathcal{K} = \begin{cases} \left(\frac{(2m_1)^{m_1}}{\Gamma(m_1+1)} \right)^{N_K}, & m_1 N_K < m_2 N_L \\ \left(\frac{(2m_1)^{m_1}}{\Gamma(m_1+1)} \right)^{N_K} + \left(\frac{(2m_2)^{m_2}}{\Gamma(m_2+1)} \right)^{N_L}, & m_1 N_K = m_2 N_L \\ \left(\frac{(2m_2)^{m_2}}{\Gamma(m_2+1)} \right)^{N_L}, & m_1 N_K > m_2 N_L. \end{cases} \quad (5.15)$$

As $P_{out} \approx (\mathcal{G}_c \Omega)^{-\mathcal{G}_d}$, the diversity order becomes $\mathcal{G}_d = \sum_{r=1}^{R} \left(N_r \times \min(m_1 N_K, m_2 N_L) \right)$ and the array gain is $\mathcal{G}_a = (\mathcal{K}\gamma_{th})^{-1/G_d}$.

5.5. Numerical Examples

In this section, numerical examples are presented for different number of antennas and relays to show the usefulness of the proposed system. Monte-Carlo simulations are used where vertical and horizontal axis represents system OP and P/N_0 unless otherwise stated.

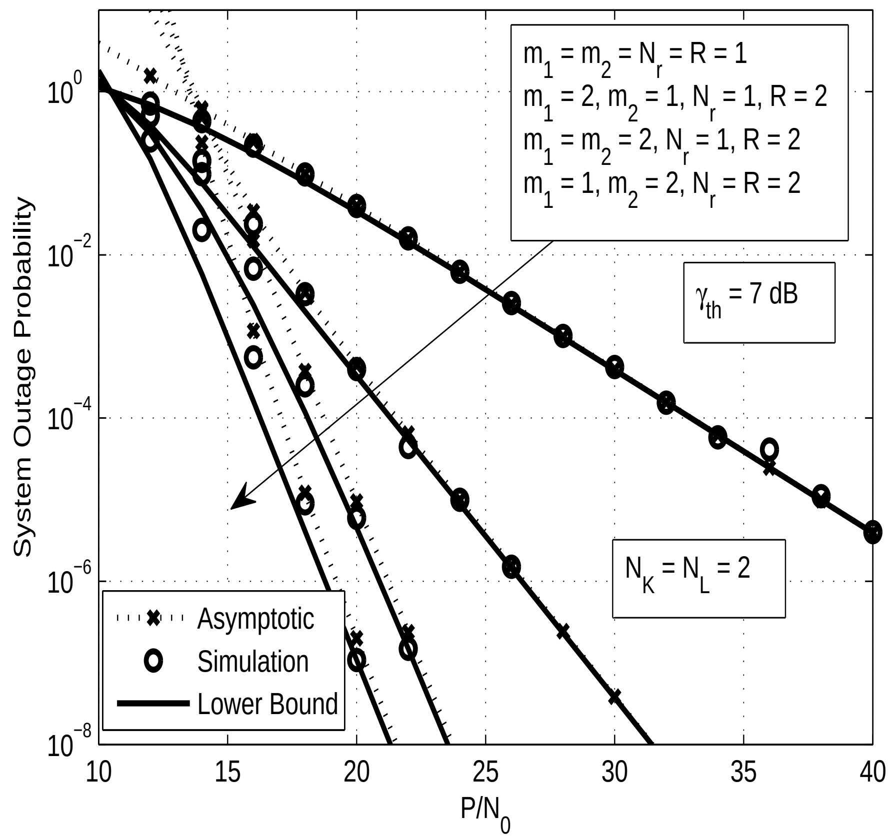

Figure 5.2. System OP performance of MIMO AF TWRN for different channel, antenna and relay configurations.

Fig. 5.2 depicts system OP vs P/N_0 for various system parameters. As can be seen, the proposed lower bound matches almost perfectly with the simulations at especially medium to high SNRs for all cases. In addition, slopes of the curves 2, 4, 8, 8 conforms with the derived diversity orders. From the figure, it is obvious that R and N_r improve outage performance much more than N_K, N_L or severity parameters.

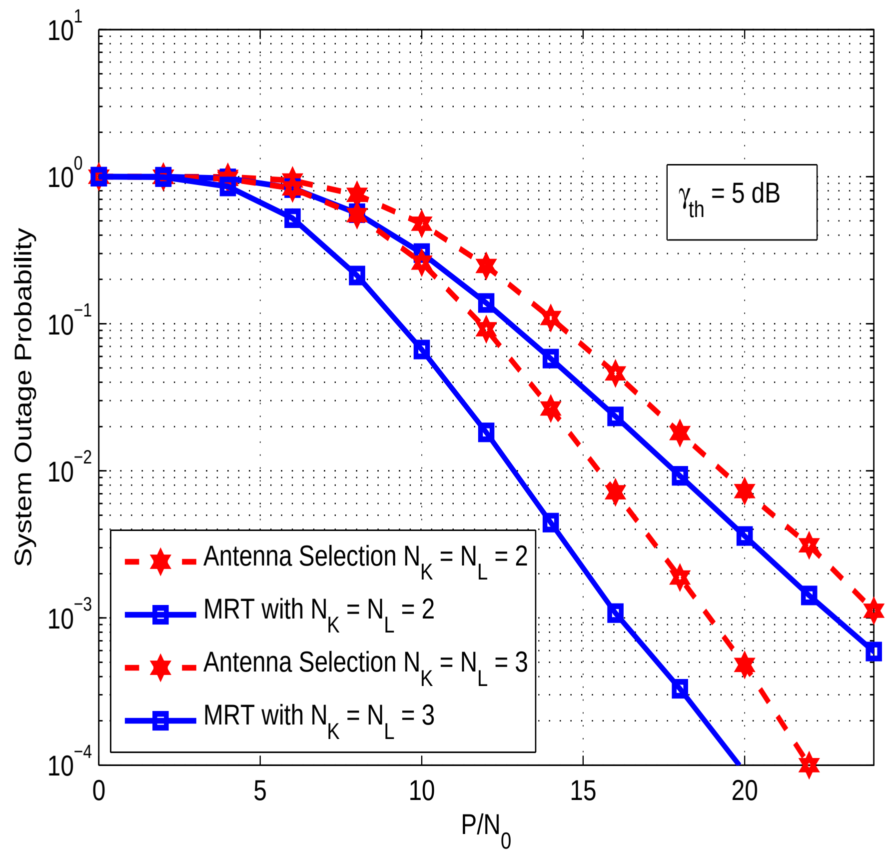

Figure 5.3. Comparison of MRT with antenna selection in terms of system OP for TWRNs.

In Fig. 5.3 MRT is compared with antenna selection in TWRNs. We assume $R = N_r = 1$ and observe that MRT outperforms in both cases. However, it should be noted that implementation complexity of MRT is higher then antenna selection.

5.6. Chapter Summary

In this chapter, system OP of an AF MIMO TWRN with joint antenna and relay selection over i.n.i.d Nakagami-m fading channels is presented. Approximate and asymptotic outage expressions are obtained by simplifying e2e SNRs. Compared to previous studies, the derived outage expression is simpler, can be useful in the design of practical networks e.g. wireless mesh or sensor networks and system designer can get a quick idea about the performance without the need for simulations or prototyping.

6. CONCLUSIONS AND FUTURE RESEARCH

This thesis focuses on the employment and analysis multi-antenna techniques in one-way or two-way relay networks to improve reliability and diversity. The major contributions of this thesis can be listed below.

- In Chapter 3, multi-antenna/multi-relay one-way network is analyzed for opportunistic and conventional relay techniques. To this end, PDF, CDF and MGF of SNR are derived and SER, OP, ergodic capacity are computed. Moreover, diversity and coding gains are obtained and optimal power factors (for single relay) are calculated. The proposed multi-relay MRT can be a promising option in WiFi Mesh networks or wireless sensor networks as they can provide high diversity gains while requiring low receiver complexity.
- In Chapter 4, the performance of MIMO two-way network is investigated where sources employing MRT and communication is carried through the best relay. For this network, we first obtain PDF, CDF and MGF of SNR (for 1 user). Then, we derive user, system OPs and sum SER for i.n.i.d Nakagami-m fading channels. In addition, diversity and array gains are obtained and the impact of practical transmission impairments such as imperfect channel estimations and limited feedback are investigated. Finally, by solving the convex optimization problem, optimum relay location is obtained. The proposed model can be widely used in indoor wireless WiFi networks with massive number of antennas and relays (as routers)

to improve reliability while requiring low receiver complexity

- In Chapter 5, an AF MIMO TWRN is proposed where both sources and all relays are equipped with multiple antennas and the communication is carried through best antenna pairs resulting in the largest received power. For this structure, approximate and asymptotic system OPs are derived for i.n.i.d Nakagami-m fading channels and diversity and array gains are obtained. Compared to other studies, the proposed network can achieve high diversity gains and the system designer can get a quick idea about the performance as the derived OP expression is simpler.

The future research directions can be summarized as follows;

- We aim to analyze a mobile two-way multi-relay network in which both source nodes are in motion. To model this structure, cascaded Rayleigh or Nakagami-m fading channels can be used.
- Multi-way relay networks with single-relay and single-antenna are recently investigated in the literature. Therefore, to design and analyze multi-relay or multi-antenna multi-way networks with MRT or antenna selection can be promising.
- Employing MRT or antenna selection in two-way networks are well-investigated. However, zero forcing which can perform better than MRT can be investigated.

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Curriculum Vitae

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