2007-417

TÜBİTAK

TÜRKİYE BİLİMSEL VE TEKNOLOJİK ARAŞTIRMA KURUMU

THE SCIENTIFIC AND TECHNOLOGICAL RESEARCH COUNCIL OF TURKEY

Elektrik, Elektronik ve Enformatik Araştırma Grubu

Electrical, Electronical and Informatics Research Group

TÜBİTAK
Proje No: 104E166

Gezgin İletişim Sistemlerinde Alıcı Tasarımı için Yeni İleri Sinyal İşleme Algoritmalarının Geliştirilmesi

Proje Yöneticisi: Prof.Dr. Erdal PANAYIRCI, *Kadir Has Üniversitesi*

Araştırmacı: Prof.Dr. Hakan Ali ÇIRPAN , *İstanbul Üniversitesi*
Araştırmacı: Dr. Hakan DOĞAN, *İstanbul Üniversitesi*

ARALIK 2007
İSTANBUL

1_8(

TÜBİTAK

PROJE ÖZET BİLGİ FORMU

Proje No: 104E166
Proje Başlığı: Gezgin İletişim Sistemlerinde Alıcı Tasarımı için Yeni İleri Sinyal İşleme Algoritmalarının Geliştirilmesi
Proje Yürütücüsü ve Araştırmacılar: Prof.Dr. Erdal Panayırcı Prof.Dr. Hakan Ali Çırpan
Projenin Yürütüldüğü Kuruluş ve Adresi: Kadir Has Üniversitesi, Mühendislik Fakültesi, Elektrik-Elektronik Mühendisliği Bölümü, PK: 34083 Cibali- İstanbul
Destekleyen Kuruluş(ların) Adı ve Adresi: TÜBİTAK Tunus Cad. No:80 06100 Kavaklıdere-ANKARA
Projenin Başlangıç ve Bitiş Tarihleri: 01/01/2005 - 31/12/2007
Öz (en çok 70 kelime): Özellikle radyo kanallarının doğası ve sınırlı bant genişliği kullanımı gibi bazı temel engeller gelecek nesil gezgin ve telsiz iletişim sistemlerinin tasarımı ve geliştirilmesinde karşımıza önemli bir problem olarak çıkmaktadır. Bundan dolayı, tüm iletişim ağ kapasitesini arttırmak için radyo spekturumunu etkin olarak kullanan alıcıların geliştirilmesi günümüzde önemli bir araştırma alanını oluşturmaktadır. Bu alıcıların en kritik bölümünü oluşturan kanal kestirimi, frekans ve faz eş zamanlama, denkleştirme ve veri dizisi sezimi problemlerinin işlem karmaşıklığı en az olacak bir şekilde çözülmesi gerekmektedir. Özellikle yetersiz kalan geleneksel çözümlerin yerine son yıllarda ileri sinyal işleme algoritmalarının uygulama yönünde büyük çabalar sarfedildiği ve bu çabalar sonucunda çok başarılı sonuçlar elde edildiği görülmektedir. Özellikle Beklenti/En Büyükleme (Expectation/Maximization) ve ilgili yöntemlerdeki (SAGE, HMM, Baum-Welch, Monte-Carlo, SMC) son gelişmelerden, gezgin ve telsiz iletişim sistemlerinin alıcı tasarımlarındaki kritik fonksiyonların hızlı ve hesap karmaşıklığı az olarak gerçekleştirilmesini cazip hale getirdiği anlaşılmaktadır. Bundan dolayı bu projede alıcı tasarımlarında karşılaşılan ve

[7] PANAYIRCI E., DOĞAN H., ÇIRPAN H.A and FLEURY B.H, Joint Data Detection and Channel Estimation for Uplink MC-CDMA Systems with EM-Based algorithms, *International Symposium on Communication Theory and Applications, ISCTA'07*, 16th - 20th Temmuz, 2007, Ambleside, Lake District, UK

[8] DOĞAN H., PANAYIRCI E., ÇIRPAN H.A., Iterative Channel Estimation Techniques for Uplink MC-CDMA Systems, IEEE-ISPPIT, The 7th IEEE International Symposium on Signal Processing and Information Technology, December 15-18, 2007, Cairo, Egypt

[9] PANAYIRCI E, DOĞAN H., ÇIRPAN H.A., FLEURY B.H., KOCIAN A, Iterative Joint Data Detection and Channel Estimation for DS-CDMA Systems in the Presence of Time-Varying Channels, IEEE International Conference on Communications (ICC), Beijing, China, 19-23 May 2008.

Ulusal Konferanslar

[1] DOĞAN H , ÇIRPAN H.A, PANAYIRCI E., Turbo/Katlamali Kodlanmis SFBC/STBC-OFDM Sistemlerinde Yinelemeli Kanal Kestirimi -En iyi ögrenci makalesi 2. cilik ödülü- *IEEE-SIU 2006* Antalya, Belek 17-19 Nisan 2006

[2] DOĞAN H , PANAYIRCI E., ÇIRPAN H.A and FLEURY B.H, Aşağı Link MC-CDMA Sistemlerinde Kullanilan PIC Alıcının EM-MAP Tabanlı Olarak İlklendirilmesi- *IEEE-SIU 2007* Eskişehir, 11-13 Haziran 2007, IEEE Öğrenci Makalesi Üçüncülük Ödülü.

[3] DOĞAN H , ÇIRPAN H.A, PANAYIRCI E., Frekans Seçici Kanallarda Çalısan Yukari Link MC-CDMA Sistemleri için EM Tabanlı Birlesik Bilgi Sezim ve Kanal Kestirim Yöntemi- *IEEE-SIU 2007* Eskişehir, 11-13 Haziran 2007

İçindekiler

Önsöz

Gezgin ve telsiz iletişim sistemlerinin alıcı tasarımlarında karşılaşılan ve işlem karmaşıklığı çok yüksek olan özellikle kanal kestirimi, eş zamanlama, denkleştirme gibi tekniklerin etkin bir şekilde gerçekleştirilmesi için çeşitli ileri sayısal sinyal işleme algoritmaları geliştirilmiştir. Bu kapsamda son dönemde önerilen verici çeşitlemeli turbo kodlanmış OFDM ve Aşağı/ Yukarı link MC-CDMA sistemleri için çeşitli alıcı tasarımları yapılmıştır.

Daha sonra yapılacak çalışmalarda faydalı olması dileği ile emeği geçen herkese teşekkürler.

Bu çalışma 104E166 numaralı TÜBİTAK araştırma projesi tarafından desteklenmiştir.

Ocak 2008 **Prof.Dr. Erdal Panayırcı**

işlem karmaşıklığı çok yüksek olan özellikle kanal kestirimi, eş zamanlama, denkleştirme gibi tekniklerin etkin bir şekilde gerçekleştirilmesi için çeşitli ileri sayısal sinyal işleme algoritmaları geliştirilmiştir.

Anahtar Kelimeler: Beklenti/En Büyükleme (BEB) Algoritması, Bayesian Monte-Carlo (BMC), Kanal Kestirim, Denkleştirme, Eş Zamanlama, Gezgin ve Telsiz İletişim

Projeden Yapılan Yayınlar:

SCI Makaleler

[1] ÇIRPAN H.A , PANAYIRCI E., DOĞAN H., Nondata-aided Channel Estimation for OFDM Systems with Space-Frequency Transmit Diversity, *IEEE Transactions on Vehicular Technology*, Vol. 55, Issue 2, pp. 449 – 457, Mart 2006.

[2] PANAYIRCI E., ÇIRPAN H.A, MOENECLAEY M. and NOELS N. , Blind Phase Noise Estimation in OFDM systems by sequential Monte Carlo method, *European Transactions on Telecommunications: Special Issue on Multi Carrier Spread Spectrum,* pp. Vol:17. 685-693, 2006.

[3] DOĞAN H., ÇIRPAN H.A. and PANAYIRCI E., Iterative Channel Estimation and Decoding of Turbo Coded SFBC-OFDM Systems, *IEEE Trans. Wireless Commun.*, vol.6, no.7, Temmuz 2007.

[4] DOĞAN H., PANAYIRCI E, ÇIRPAN H.A., FLEURY B.H., MAP Channel Estimation Based PIC Receiver for Downlink MC-CDMA Systems, *EURASIP Journal on Wireless Communications and Networking, Special Issue on Multicarrier Systems* (EKİM 2007'de yayınlanmak üzere kabul edildi).

[5] DOĞAN H., ÇIRPAN H.A. and PANAYIRCI E., An Efficient Joint Channel Estimation and Decoding Algorithm for Turbo-Coded Space-Time OFDM Receivers, IET Communications (KASIM 2007'de yayınlanmak üzere kabul edildi).

[6] PANAYIRCI E, DOĞAN H., ÇIRPAN H.A., FLEURY B.H., KOCIAN A. , Iterative

Joint Data Detection and Channel Estimation for Uplink MC-CDMA Systems in the Presence of Frequency Selective Channels, IEEE-JSAC dergisine sunuldu.

Uluslararası Konferanslar

[1] DOĞAN H., ÇIRPAN H.A. and PANAYIRCI E., EM based MAP Iterative Channel Estimation for Turbo Coded SF-OFDM systems, *4 th International Symposium on Turbo Codes & Related Topics*, 3-7 Nisan 2006, Munih, Almanya

[2] PANAYIRCI E., ÇIRPAN H.A, MOENECLAEY M. and NOELS N., Blind Phase Noise Estimation and Data Detection Based on SMC Technique and Unscented Filtering, *European Signal Processing Conference (EUSIPCO-2006)*, Eylül. 4-8 2006, Florence Italy.

[3] DOĞAN H., ÇIRPAN H.A. and PANAYIRCI E., Iterative Channel Estimation and Decoding of Turbo/Convolutionally Coded STBC-OFDM Systems, *NEWCOM-AcoRN Joint Worshop 2006*, Eylül. 19-22 2006, Vienna, Austria.

[4] DOĞAN H., PANAYIRCI E., ÇIRPAN H.A., EM-Based MAP Channel Estimation and Data Detection for Downlink MC-CDMA Systems, *IEEE Wireless Communications & Networking Conference (WCNC-2007)*, Mart 11-15 2007, Hong Kong.

[5] PANAYIRCI E., DOĞAN H., ÇIRPAN H.A and FLEURY B.H, "Joint Data Detection and Channel Estimation for Uplink MC-CDMA Systems over Frequency Selective Channels"6th International Workshop on Multi-Carrier Spread Spectrum (MC-SS 2007), Mayıs 07-09 2007, Herrsching, Germany.

[6] PANAYIRCI E., DOĞAN H., ÇIRPAN H.A. and FLEURY B.H., “An Efficient Joint Data Detection and Channel Estimation Technique for Uplink MC-CDMA Systems Based on SAGE Algorithm” 16th IST Mobile & Wireless Communications Summit, Budapest, Hungary, Temmuz 1-5 2007.

ŞEKİL LİSTESİ

TABLO LİSTESİ

ÖZET

Özellikle radyo kanallarının doğası ve sınırlı bant genişliği kullanımı gibi bazı temel engeller gelecek nesil gezgin ve telsiz iletişim sistemlerinin tasarımı ve geliştirilmesinde karşımıza önemli bir problem olarak çıkmaktadır. Bundan dolayı, tüm iletişim ağ kapasitesini arttırmak için radyo spekturumunu etkin olarak kullanan alıcıların geliştirilmesi günümüzde önemli bir araştırma alanını oluşturmaktadır. Bu alıcıların en kritik bölümünü oluşturan kanal kestirimi, frekans ve faz eş zamanlama, denkleştirme ve veri dizisi sezimi problemlerinin işlem karmaşıklığı en az olacak bir şekilde çözülmesi gerekmektedir. Özellikle yetersiz kalan geleneksel çözümlerin yerine son yıllarda ileri sinyal işleme algoritmalarının uygulama yönünde büyük çabalar sarfedildiği ve bu çabalar sonucunda çok başarılı sonuçlar elde edildiği görülmektedir. Özellikle Beklenti/En Büyükleme (Expectation/Maximization) ve ilgili yöntemlerdeki (SAGE, HMM, Baum-Welch, Monte-Carlo, SMC) son gelişmelerden, gezgin ve telsiz iletişim sistemlerinin alıcı tasarımlarındaki kritik fonksiyonların hızlı ve hesap karmaşıklığı az olarak gerçekleştirilmesini cazip hale getirdiği anlaşılmaktadır. Bundan dolayı bu projede alıcı tasarımlarında karşılaşılan ve işlem karmaşıklığı çok yüksek olan özellikle kanal kestirimi, eş zamanlama, denkleştirme gibi tekniklerin etkin bir şekilde gerçekleştirilmesi için çeşitli ileri sayısal sinyal işleme algoritmalarının geliştirilmesi önerilmiştir.

ABSTRACT

Traditional wireless technologies are not well suited to meet the extremely demanding requirements of providing the very high data rates with the ubiquity, mobility and portability characteristic of cellular systems. Some fundamental barriers, related to the nature of the radio channel as well as the limited bandwidth availability at the frequencies of interest, stand in the way. Unique sets of efficient signal processing algorithms and techniques is the one of the primary enablers that will allow lifting these limits. Especially, the development of computationally efficient expectation-maximization (EM) and related methodologies (such as SAGE, MCEM, HMM, Baum-Welch, sequential Monte Carlo) together with the availability of inexpensive and rapid computing power made them very attractive for solving difficult signal processing problem for the design of receivers in wireless communications. Applications of advanced signal processing algorithms mentioned above include, but are not limited to, joint/blind/sequence detection, decoding, synchronization, equalization as well as channel estimation techniques employed in advanced wireless communication systems such as OFDM/OFDMA, Space-Time-Frequency Coding, MIMO, CDMA and with Multi User Detection, Time- and Frequency-Selective MIMO Channels. Although over the past decade such methods have been successfully applied in a variety of communication contexts, many technical challenges remain in emerging applications, whose solutions will provide the bridge between the theoretical potential of such techniques and their practical utility.

1. GİRİŞ

Gezgin ve telsiz iletişim sistemlerinin sunduğu servislere olan yoğun talep her geçen gün hızla artmaktadır. Ancak, geleneksel gezgin ve telsiz iletişim sistemleri, kapasite, evrensel boyutta gezginlik, yüksek hızda veri ve çoğul ortam hizmetleri sunma gereksinimlerini yeterince karşılayamamaktadır. Dolayısıyla, bütün bu eksikliklerin giderilerek geleceğin iletişim sistemlerinin evrensel boyutta gezgin haberleşmeyi destekleyecek şekilde tasarlanması amaçlanmaktadır. Bu amaca yönelik olarak gezgin ve telsiz iletişim standartlarının daha iyi servis kalitesi, daha büyük kapasite ve akıllı servisler sağlayacak biçimde yenilenmesi ve geliştirilmesi için çalışmalar son yıllarda büyük yoğunluk kazanmıştır.

Özellikle radyo kanallarının doğası ve sınırlı bant genişliği kullanımı gibi bazı temel engeller gelecek nesil gezgin ve telsiz iletişim sistemlerinin tasarımı ve geliştirilmesinde karşımıza önemli bir problem olarak çıkmaktadır. Bundan dolayı, tüm iletişim ağ kapasitesini arttırmak için radyo spekturumunu etkin olarak kullanan alıcıların geliştirilmesi günümüzde önemli bir araştırma alanını oluşturmaktadır. Bu alıcıların en kritik bölümünü oluşturan kanal kestirimi, frekans ve faz eş zamanlama, denkleştirme ve veri dizisi sezimi problemlerinin işlem karmaşıklığı enaz olacak bir şekilde çözülmesi gerekmektedir. Özellikle yetersiz kalan geleneksel çözümlerin yerine son yıllarda ileri sinyal işleme algoritmalarının uygulama yönünde büyük çabalar sarfedildiği ve bu çabalar sonucunda çok başarılı sonuçlar elde edildiği görülmektedir. Son literatür çalışmalarından, Beklenti /En Büyükleme ve ilgili yöntemlerdeki (SAGE, HMM, Baum-Welch, Monte-Carlo, SMC) gelişmeler, gezgin ve telsiz iletişim sistemlerinin alıcı tasarımlarındaki kritik fonksiyonların hızlı ve hesap karmaşıklığı az olacak biçimde gerçekleştirilmesini cazip hale getirdiği anlaşılmaktadır [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]

Beklenti en büyükleme (BEB) algoritması gezgin ve telsiz iletişim sistemlerinin fiziksel katmanında kanal kestirimi, denkleştirme ve eş zamanlama gibi çeşitli sinyal işleme problemleri için uyarlanmıştır [3], [4], [5], [6], [7], [8], [9], [10], [11]. BEB algoritması, olası paralel işleme uygulamalarında ayrı ayrı hesaplama yapma olanağı sunması bakımından klasik kestirimcilere göre, özyineli bir alternatif oluşturmaktadır. Böylece çok boyutlu kestirim problemleri daha düşük boyutlu kestirim problemlerine dönüştürülerek ortaya çıkacak işlem yoğunluğundan kurtulabilmek mümkün olmaktadır. Özellikle yeni nesil gezgin ve telsiz iletişim sistemlerinin geliştirilmesinde birçok sinyal işleme tekniği bir alternatif yaklaşım olarak sunulmuştur [4], [5], [6], [7], [8], [9], [10], [11]. Araştırma grubumuz tarafından da ileri sinyal işleme algoritmalarının yeni nesil iletişim sistemleri için çeşitli uygulamar araştırılmaktadır. Ortagonal frekans bölmeli çoğullamalı (OFDM) sistemlerinde frekans ve faz eşzamanlaması için BEB algoritmasına dayalı hızlı ve iteratif yapıda en büyük olabilirlik yöntemi önerilmiş ve bu algoritmaların özellikle frekans-seçici kanallar için de çok iyi sonuçlar verdiği gösterilmiştir [9], [10]. OFDM, uzay-zaman/ uzay-frekans blok kodlamalı OFDM sistemleri için hesaplama karmaşıklığı açısından verimli, veri desteksiz BEB algoritmasına dayalı MAP (maximum a posteriori) kanal kestirim algoritmaları önerilmiş ve başarımı analitik olarak incelenmiştir [12], [13], [16], [17]. Ayrıca, saklı Markov model parametrelerinin kestirimi için kullanılan ve BEB algoritmasının özel bir hali olan Baum-Welch algoritması da uzay-zaman blok kodlamalı sistemlerde gözü kapalı kanal kestirimcisi geliştirilmesi için uyarlanmıştır [14].

Diğer taraftan güçlü bir istatistiksel araç olarak bilinen Bayes Monte Carlo (BMC) yöntemi son yıllarda özellikle iletişim alanında çalışan araçtırıcıların ilgisini çekmiş ve bunun sonucunda pek çok kestirim/sezim problemine yeni alternatif çözüm oluşturmuştur [14], [15]. Bu yöntemin uygulamaları, hesaplama yükü açısından oldukça yoğun olmasına rağmen, hızlı bilgisayarların yaygınlığı ve paralel uygulamalar sonucunda oldukça zor sinyal işleme problemlerinin çözümü için çekici hale gelmiştir. Özellikle BMC yöntemleri çok zor ve dinamik telsiz ortamlarında güvenilir ve hızlı iletişim için algoritmaların geliştirilmesine imkan sağlamıştır [16], [17], [18], [19]. Bu yöntem de yeni nesil gezgin ve telsiz iletişim sistemleri için, OFDM, uzay-zaman OFDM ve turbo kodlanmış OFDM sistemlerinde alıcı tasarımı problemine çeşitli senaryolar için uyarlanmıştır[16], [17], [19]

Günümüzde büyük önem kazanan yeni nesil gezgin–telsiz iletişim sistemlerinin en kritik bölümünü oluşturan alıcıların tasarımlarında şimdiye kadar geliştirilmiş geleneksel yöntem ve teknikler yetersiz kalmaktadır. Dolayısıyla BEB ve ilgili BMC yöntemleri teori ve pratikteki telsiz haberleşmesi ve ileri sinyal işleme araştırmalarını etkilemekte ayrıca haberleşme endüstrisine de büyük teknolojik ve ticari etkiler ve üstünlükler getirmektedir. Bu projenin temel amacı, alıcı tasarımlarında karşılaşılan ve işlem karmaşıklığı çok yüksek olan özellikle kanal kestirimi, eş zamanlama, denkleştirme gibi tekniklerin etkin bir şekilde gerçekleştirilmesi için çeşitli ileri sayısal sinyal işleme algoritmalarının geliştirilmesidir.

2 GENEL BİLGİLER

2.1 MOBİL HABERLEŞME KANALLARI VE KARAKTERİSTİKLERİ

Mobil haberleşme sistemlerinde genellikle alıcı ve verici antenler haberleşme kanalındaki mevcut engellerden dolayı birbirlerini direk olarak göremezler ve vericiden çıkan elektromanyetik dalga, alıcıya birden fazla yol üzerinden gelmektedir. Çoklu yol üzerinden alıcıya gelen sinyaller, farklı gecikmelere maruz kalarak fazında ve genliğinde rasgele değişimlere uğramaktadırlar. Alıcıya gelen bu sinyallerin toplamı, elektromanyetik dalgaların fazlarına bağlı olarak yapıcı veya bozucu olabilir. Bu sebeple gelen sinyallerin toplamı, alınan sinyalin gücünde dalgalanmalara sebep olmaktadır.

Diğer taraftan sinyallerin kanalda çeşitli gecikmelere maruz kalarak alıcıya gelmesi, gecikme yayınımı olarak adlandırılır. Sembol zamanıyla gecikme yayınımı arasındaki ilişki, kanalın frekans cevabını etkilemektedir. Bu ifade açılacak olursa; çoklu yol gecikmesi sembol süresinden küçükse kanal frekans-seçici olmayan (düz sönümlemeli), değilse frekans-seçici olarak adlandırılır. Çoklu yol yayınımının yanında ek olarak Doppler etkisi de mobil haberleşme kanalından iletimi olumsuz etkilemektedir. Alıcıya gelen sinyalin geliş açısı, mobil birimlerin hareketine göre değişmekte ve bu değişim Doppler etkisi olarak isimlendirilmektedir. Alınan sinyalin frekansında kaymaya sebep olan bu etki kanalları zaman seçici olan ve olmayan şeklinde iki gruba ayırmaktadır. Sembol süresinin kanaldaki ilişkili zaman aralığından oldukça düşük olduğu durumda, kanal zamanda seçici olmayan yani düz olarak, diğer durumda ise zaman seçici kanal olarak adlandırılır.

Frekans-seçicilik ve zaman seçicilik, sönümlemeli kanalın iki ayrı özelliğidir. Bu iki kavram, kanallar için ayrı ayrı ele alınırsa, mobil haberleşme kanalları dört çeşide ayrılarak incelenebilir.

Düz sönümlemeli kanallar: Zamanda ve frekansta değişime sahip olmayan kanallardır.

Frekans-seçici kanallar: Kanalın frekansta seçici, zamanda ise düz olduğu kanallardır.

Zaman-seçici sönümlemeli kanallar: Kanalın zamanla değiştiği yani seçici olduğu, frekansta ise düz sönümlemeli olduğu durumlardır.

Çift-seçici sönümlemeli kanallar: Kanalların zamanda ve frekansta seçici olduğu durumlardır.

2.2 İLETİŞİM KANAL MODELİ VE KARHUNEN LOEVE (KL) SERİ AÇILIMININ KULLANIMI

Çift seçici sönümlemeli kanallar genellikle durağan sönümlemeli kanallar olarak modellenmektedir[30]. Bu modele göre, kanalın bir sembol uzunluğu için değişmediği ve art arda gelen sembollerinin aralarında ilişkili veya bağımsız olabileceği varsayılmaktadır. Bu yaklaşım zaman bölmeli çoklu erişim (TDMA), frekans atlamalı geniş spektrumlu (FHSS) ve OFDM gibi sistemlere uygulanabilmektedir. Verici ve alıcı antenler arasındaki kanalın, durağan sönümlemeli bir kanal olduğu, bir OFDM sembolü boyunca frekans-seçici ve art arda gelen OFDM sembolü için Doppler frekansına göre zamanla değiştiği varsayılmıştır. Kanalın ayrık zamanlı temel bant eşdeğer birim basamak yanıtı "L" kanalın uzunluğunu göstermek üzere $\mathbf{h}_\mu = [h_{\mu,0}(n),\cdots,h_{\mu,L}(n)]^T$ olarak tanımlanmaktadır. Bu durumda "μ." verici anten ile alıcı anten arasındaki kanalın frekans yanıtı "n." ayrık zamanı için $\mathbf{H}_\mu(n)$ olarak gösterelim.

Kablosuz iletişimde, kanaldaki değişimler temel işlevlerine açılım yaklaşımıyla modellenebilir[31]. Fourier ve Taylor seri açılımları, polinom açılımı belirgin modellemede sıkça kullanılan önemli açılımlardır. Bunlardan farklı olarak rastlantısal süreçlerin temel açılımı için bilinen diğer etkili bir yöntem ise Karhunen-Loeve (KL) açılımıdır. Ayrıca KL açılımının çok yollu sönümlemeli kanalların benzetimlerinde de kullanılabileceği gösterilmiştir [32]. KL açılım yöntemiyle $\mathbf{H}_\mu(n)$ ifadesini temel birim dik işlevlerin doğrusal kombinasyonları olarak

$$\mathbf{H}_\mu(n) = \mathbf{\Psi}\mathbf{G}_\mu(n) \tag{2.1}$$

yazılabilir. Burada $\mathbf{\Psi} = [\psi_0,\psi_1,\cdots\psi_{N_c-1}]$'ler dik temel vektörlerini ve $\mathbf{G}_\mu(n) = [G_\mu(1,n),\cdots,G_\mu(N_c,n)]^T$'ler açılım ağırlık katsayılarını göstermektedir. Bu yöntemle aynı istatistiksel özelliklere sahip sonsuz sayıda $\mathbf{H}_\mu(n)$'ler üretilebilir. Ayrıca farklı temel işlevleri kullanarak türlü özelliklere sahip başka kanallar da üretilebilmektedir. Kanal ilişki matrisinin $\mathbf{C}_{\mathbf{H}_\mu} = E[\mathbf{H}_\mu\mathbf{H}_\mu^\dagger]$ özdeğer açılımı[44]

$$\mathbf{C}_{\mathbf{H}_\mu} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^\dagger \tag{2.2}$$

şeklinde yazılabilir. Burada, $\mathbf{\Lambda}$ kanal ilişki matrisinin özdeğerlerinin oluşturduğu köşegen matrisi, $\mathbf{\Psi}$ ise öz işlev vektörlerinin oluşturduğu matrisi göstermektedir. Ayrıca özdeğer açılımında elde edilen özdeğerler de ağırlık katsayılarının varyanslarını "$\mathbf{\Lambda} = E\{\mathbf{G}_\mu\mathbf{G}_\mu^\dagger\}$" oluşturmaktadır. Burada özdeğer matrisi Λ'nın köşegen matris biçimde çıkması, karmaşık Gauss rastlantısal değişkenler biçiminde modellenen kanal katsayılarının KL dönüşümü sonucunda birbirinden bağımsız hale dönüştüklerini göstermektedir. Böylece kanal kestiriminde, KL dönüşümü sonucu elde edilen bağımsız $\mathbf{G}_{\mu}$ açılım katsayıları kullanılabilir.

2.3 ÜSSEL ALÇALAN GÜÇ-GECİKME PROFİLİNE SAHİP KANAL MODELİ

Verici ve alıcı anten arasındaki OFDM kanalın frekans cevabı olan $\mathbf{H}_{\mu}(k)$'nın üssel alçalan güç-gecikme profiline (power-delay profile)sahip olduğu $\theta(\tau_{\mu}) = C\exp(-\tau_{\mu}/\tau_{rms})$ ve kanalda meydana gelen gecikmeleri ifade eden τ_{μ}'ların çevrimsel önek üzerinde tekdüze ve bağımsız olarak dağıldığı varsayılmıştır. C ise normalizasyon katsayısı olmak üzere OFDM kanalı için sırasıyla farklı altbantlara ait normalize edilmiş ayrık kanal ilişkileri (k,k') farklı altbantları ifade etmek üzere[33]

$$C_{\mu}(k,k') = \frac{1-e^{-L(1/\tau_{rms}+j2\pi(k-k^{1})/N_c)}}{\tau_{rms}(1-e^{-L/\tau_{rms}})(1/\tau_{rms}+2\pi(k-k^{1})/N_c)} \qquad (2.3)$$

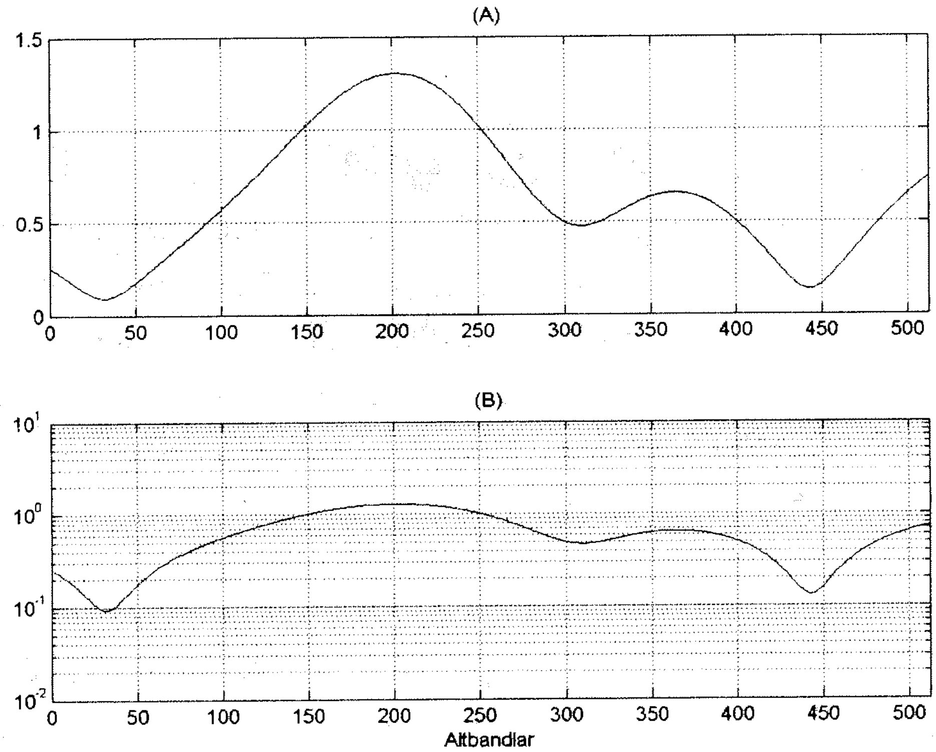
şeklinde tanımlanmıştır. Burada "N_c" kullanılan altbant sayısını, "L" kanalın derecesini, "τ_{rms}" ise güç gecikme profilinin ortalama azalmasını göstermektedir. Bir OFDM sembolü boyunca sabit kanal parametreleri, gelecek diğer OFDM sembolü için Doppler frekansına göre değişime uğrayacaktır. Farklı OFDM sembolleri için ayrık ilişki ifadesi Jake modeline göre

$$J_0(2\pi(n-n')f_dT_s) \qquad (2.4)$$

şeklinde verilmiştir. Burada J_0 sıfırıncı dereceden birinci çeşit Bessel işlevini, f_d Doppler frekansını, T_s ise bir OFDM sembolünün gitmesi için gerekli süreyi ifade etmektedir.

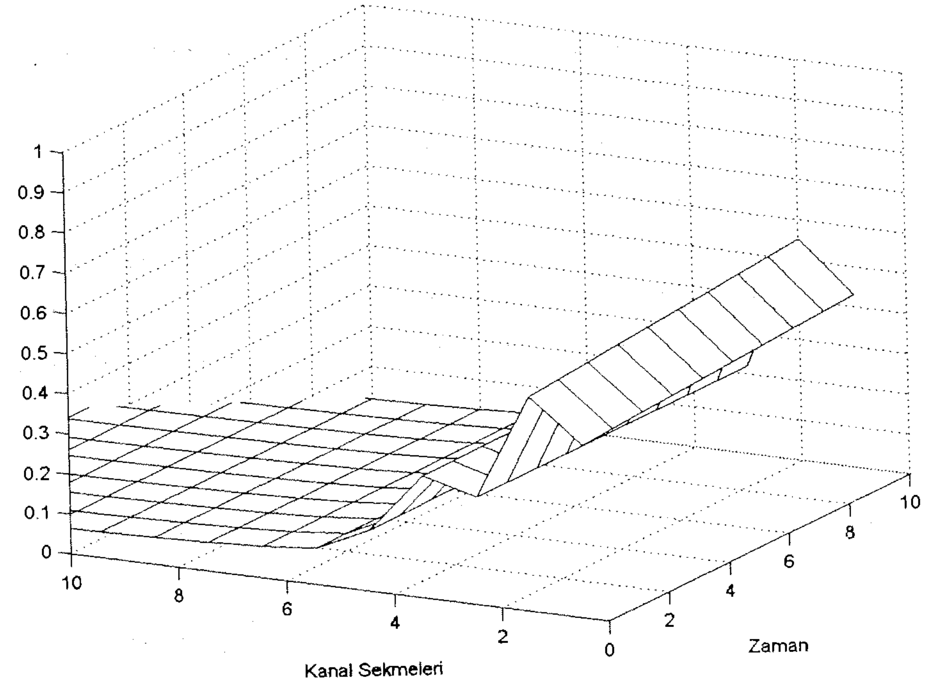
2.3.1 Zayıflayan Güç Gecikme Karakterine Sahip Kanal Modeli Bilgisayar Benzetimleri

Verici antenden sinyallerin düşüş katsayıları ($\delta = 0.2$) olan yükseltilmiş kosinüs biçimindeki verici süzgeçleri ile biçimlendirilerek, 2.28 MHz'lik bant genişliğinden BPSK modülasyon tekniği ile iletildikleri varsayılmaktadır. Toplam iletim bandı 512 altbanta bölünerek iletim süresi T_s, 1.052 μsn'lik bölümü çevirimli önek olmak üzere toplam 136 μsn'dir. Kanalın ortalama gecikme süresi ise $\tau_{rms} = 0.263\mu$sn'dir. OFDM kanalın frekans izgesinde karakteristiğini daha iyi anlamak için aşağıda örnek bir OFDM kanalının frekans ve zaman izgesinin genlik yanıtları incelenmiştir.

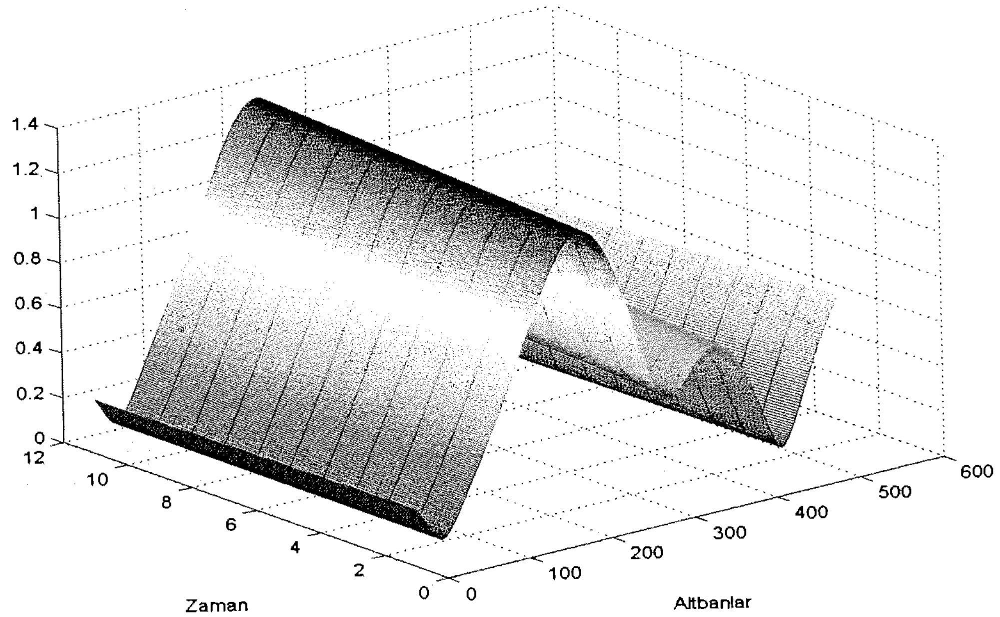


Şekil 2.1: Zayıflayan güç gecikme karakterine sahip bir OFDM kanalının frekans izgesinin genlik cevabı (A) : Doğrusal (B): Logaritmik

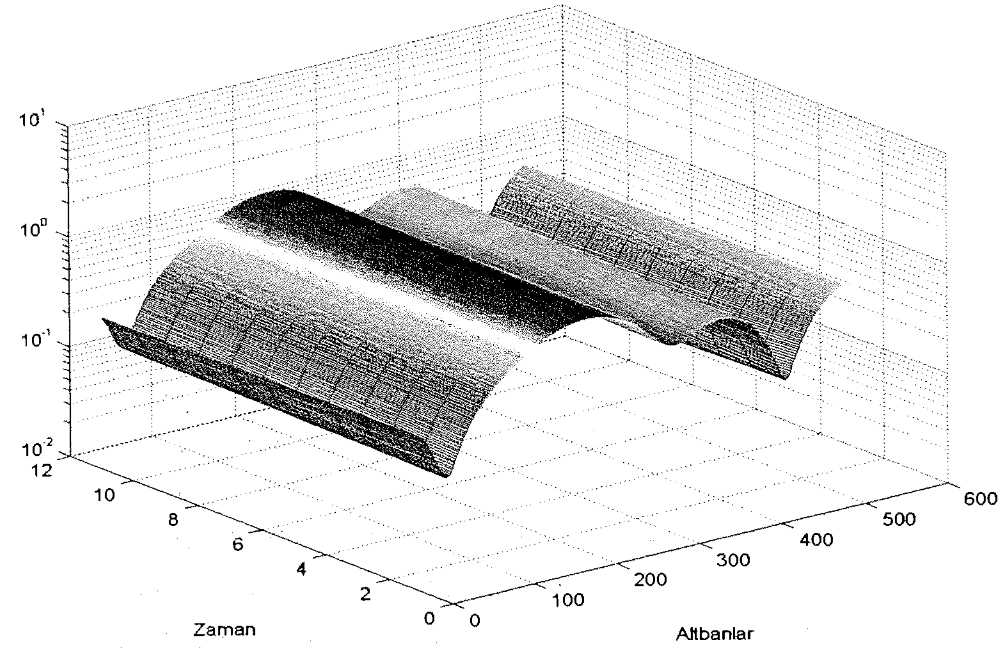
Şekil 2.1'de görüleceği üzere kanal frekans-seçici bir özellik taşımaktadır. Şekil 2.2'de kanala ait sekmelerin değişimi zamana göre incelenmiş ve kanalın yaklaşık 8 sekmeden oluştuğu görülmüştür. Maksimum Doppler frekansının $f_{Dm} = 10Hz$ olmasından dolayı, bu benzetimde kanal sekmelerinin (taps) zamanla değişiminin oldukça yavaş olduğu gözlemlenmiştir. Kanalın frekans cevabının zamanla değişimi ise, doğrusal ve logaritmik olarak sırasıyla Şekil 2.3 ve Şekil 2.4'te gösterilmiştir. Kanalın zamanla değişimi, bir sonraki OFDM sembolü için (2.4) denkleminde ifade edildiği üzere Doppler frekansına ve iletilen OFDM sembolünün süresine bağlı olarak değiştiği bilinmektedir. Artan maksimum Doppler frekans değerlerine göre kanalın değişiminin gözlenmesi için benzer bilgisayar benzetimleri 100Hz için Şekil 2.5, Şekil 2.6 ve Şekil 2.7'de verilmiştir.



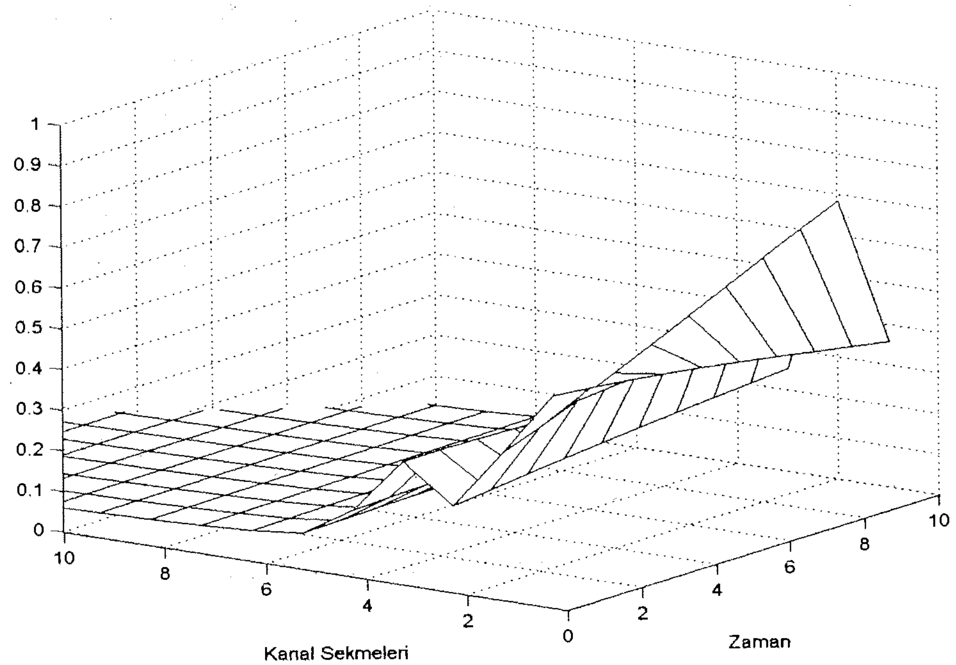
Şekil 2.2: Zayıflayan güç gecikme karakterine sahip bir OFDM kanalının zaman sekmelerinin genlik cevabı $f_{Dm} = 10Hz$



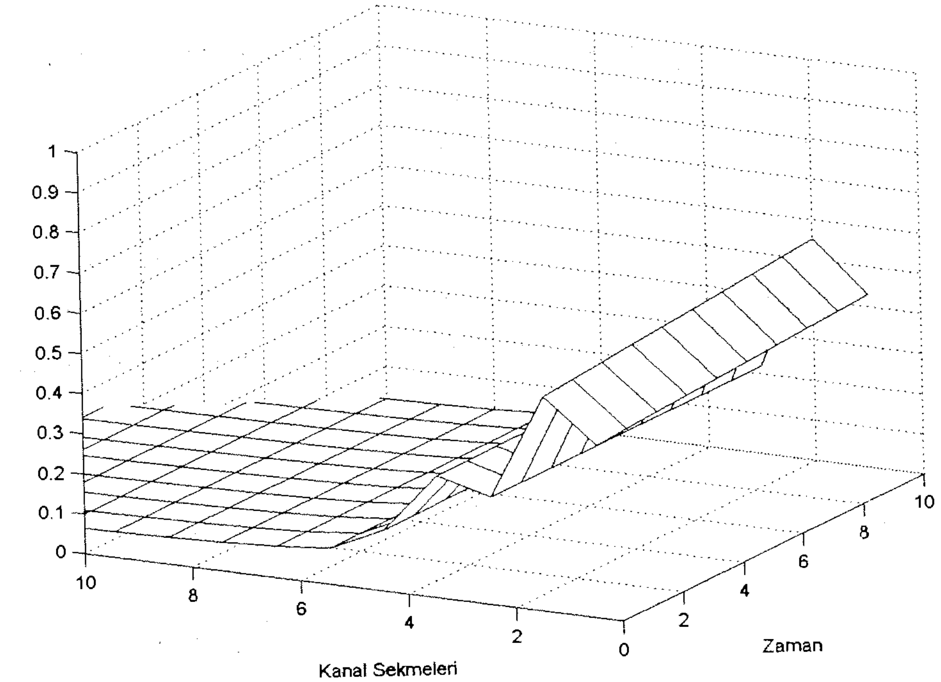
Şekil 2.3: Örnek bir OFDM kanalının frekans cevabının zamanla değişen genlik cevabı $f_{Dm} = 10Hz$



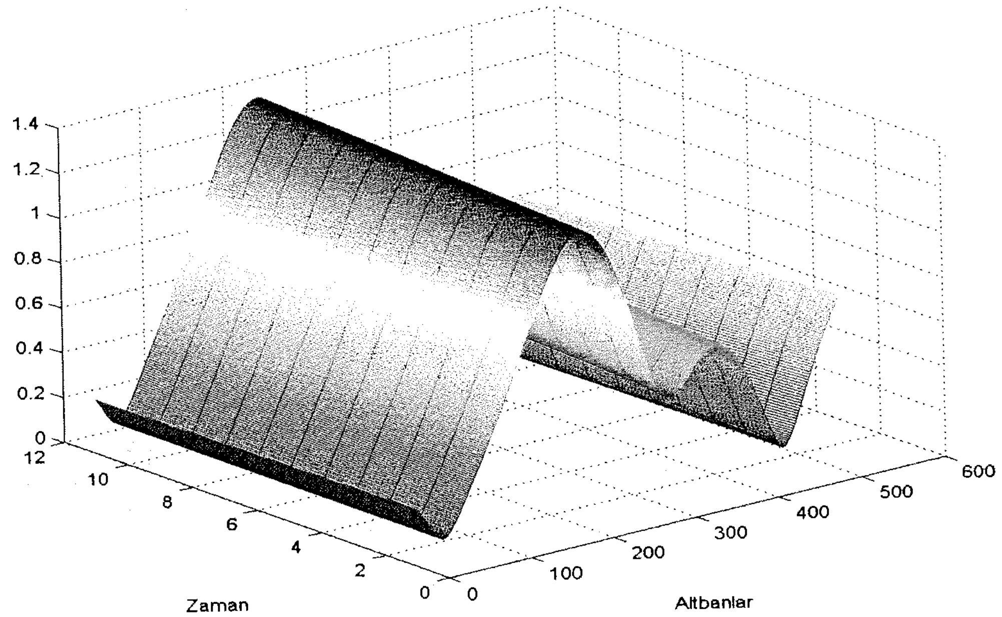
Şekil 2.4: Örnek bir OFDM kanalının frekans cevabının zamanla değişen logaritmik genlik cevabı $f_{Dm} = 10Hz$



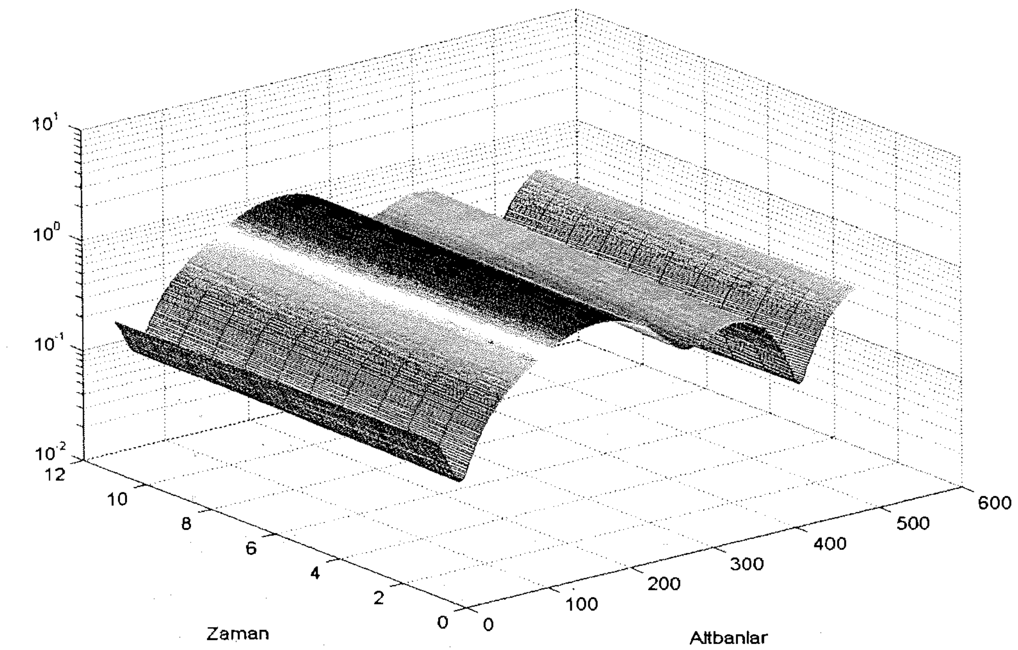
Şekil 2.5: Zayıflayan güç gecikme karakterine sahip bir OFDM kanalının zaman sekmelerinin genlik cevabı $f_{Dm} = 100Hz$



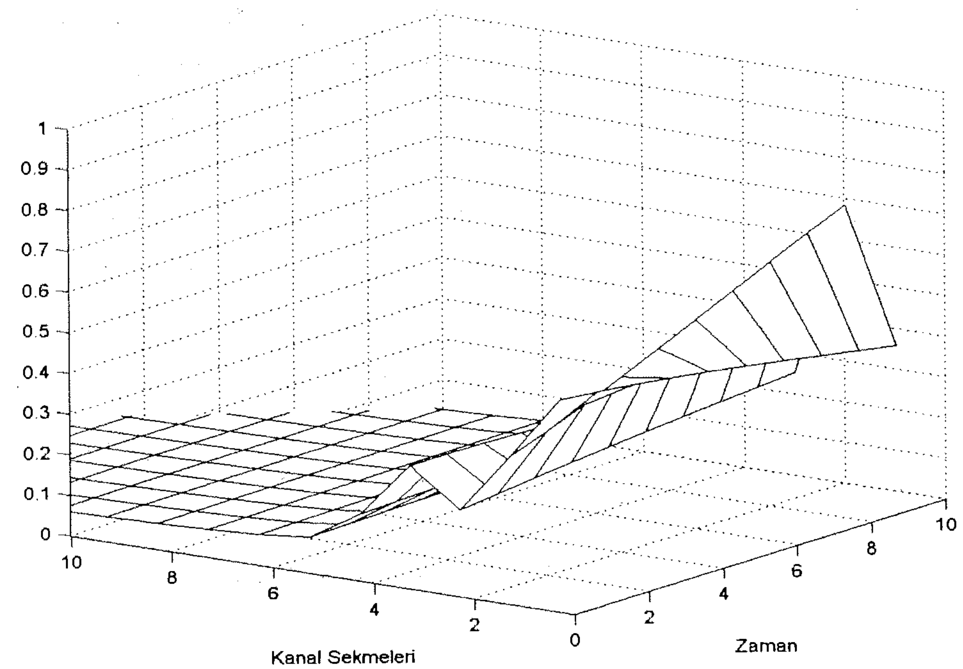
Şekil 2.2: Zayıflayan güç gecikme karakterine sahip bir OFDM kanalının zaman sekmelerinin genlik cevabı $f_{Dm} = 10Hz$



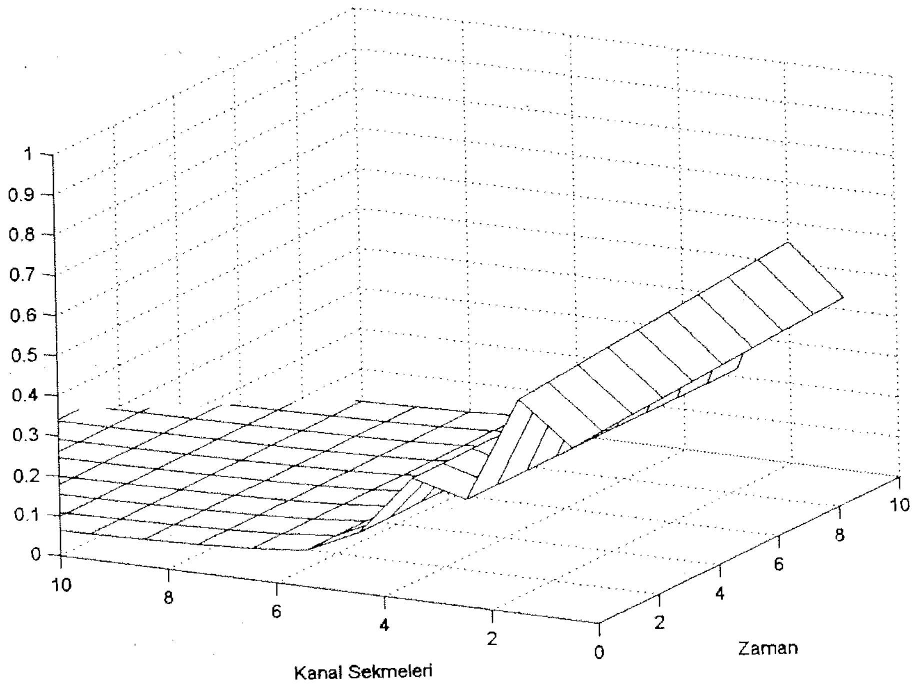
Şekil 2.3: Örnek bir OFDM kanalının frekans cevabının zamanla değişen genlik cevabı $f_{Dm} = 10Hz$



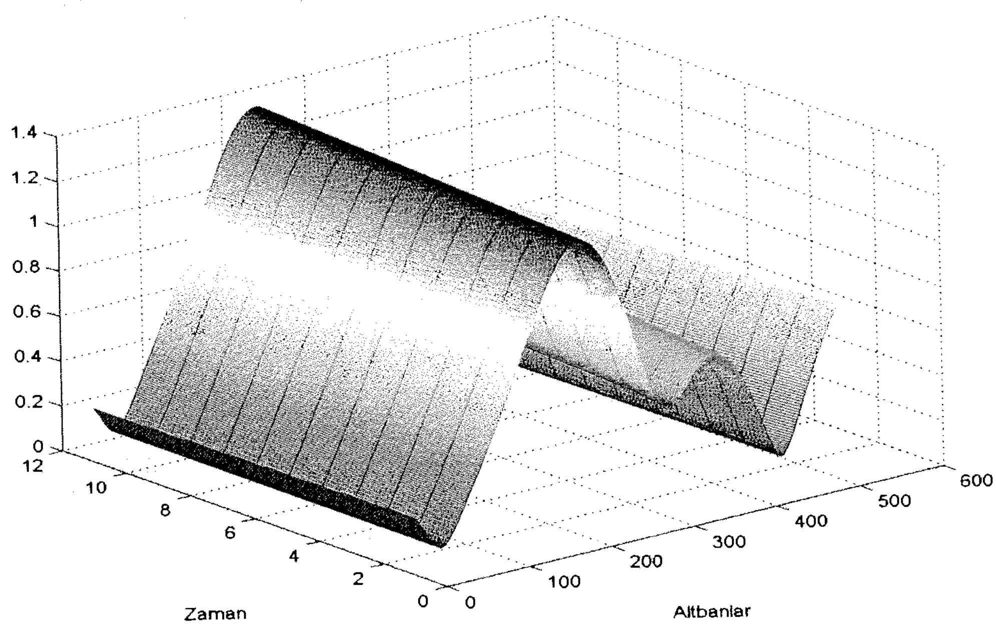
Şekil 2.4: Örnek bir OFDM kanalının frekans cevabının zamanla değişen logaritmik genlik cevabı $f_{Dm} = 10Hz$



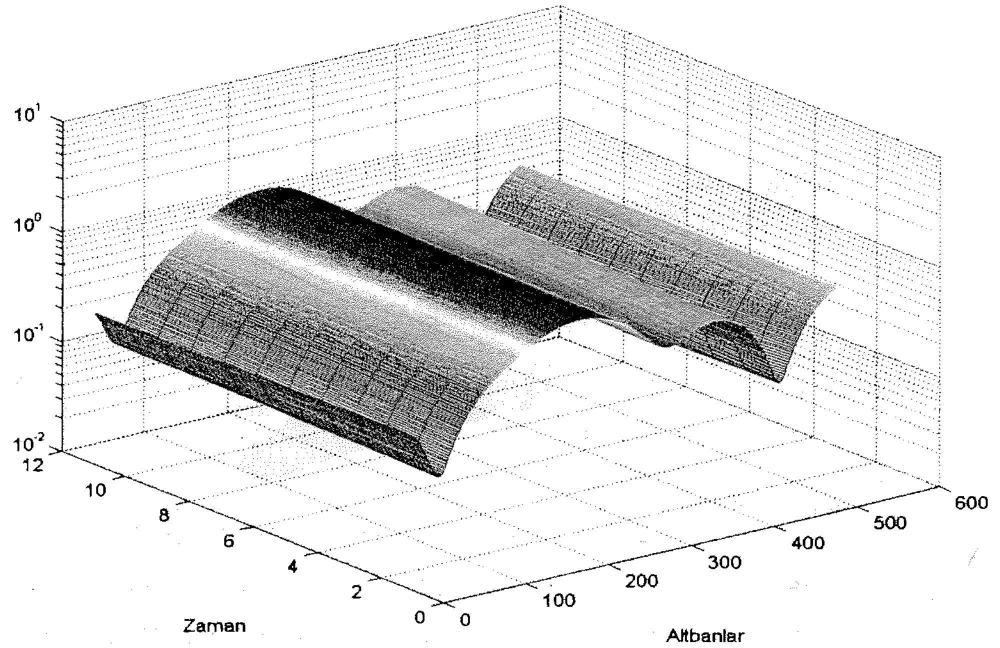
Şekil 2.5: Zayıflayan güç gecikme karakterine sahip bir OFDM kanalının zaman sekmelerinin genlik cevabı $f_{Dm} = 100Hz$



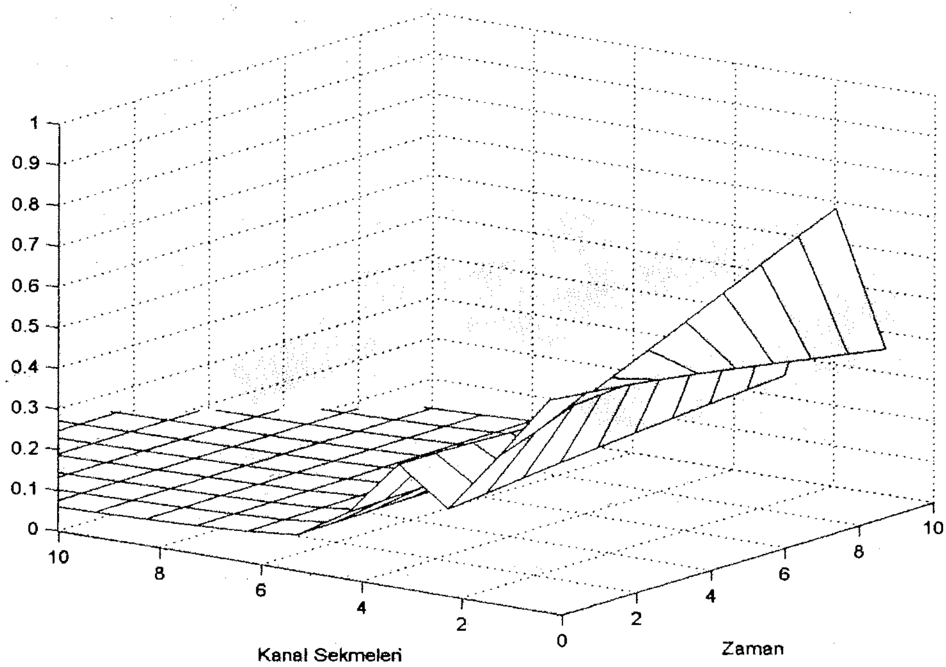
Şekil 2.2: Zayıflayan güç gecikme karakterine sahip bir OFDM kanalının zaman sekmelerinin genlik cevabı $f_{Dm} = 10Hz$



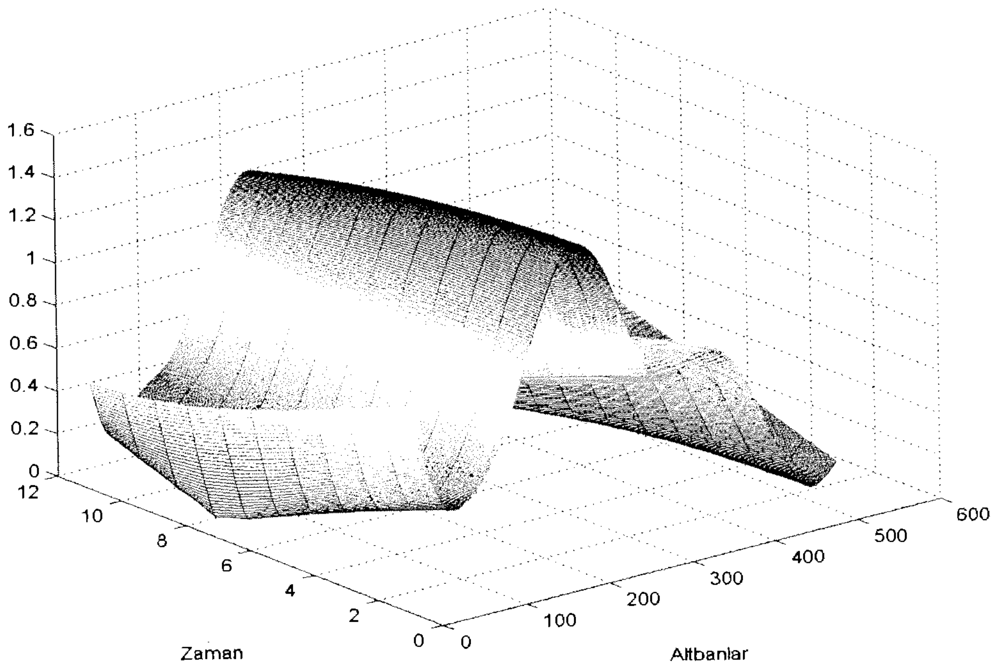
Şekil 2.3: Örnek bir OFDM kanalının frekans cevabının zamanla değişen genlik cevabı $f_{Dm} = 10Hz$



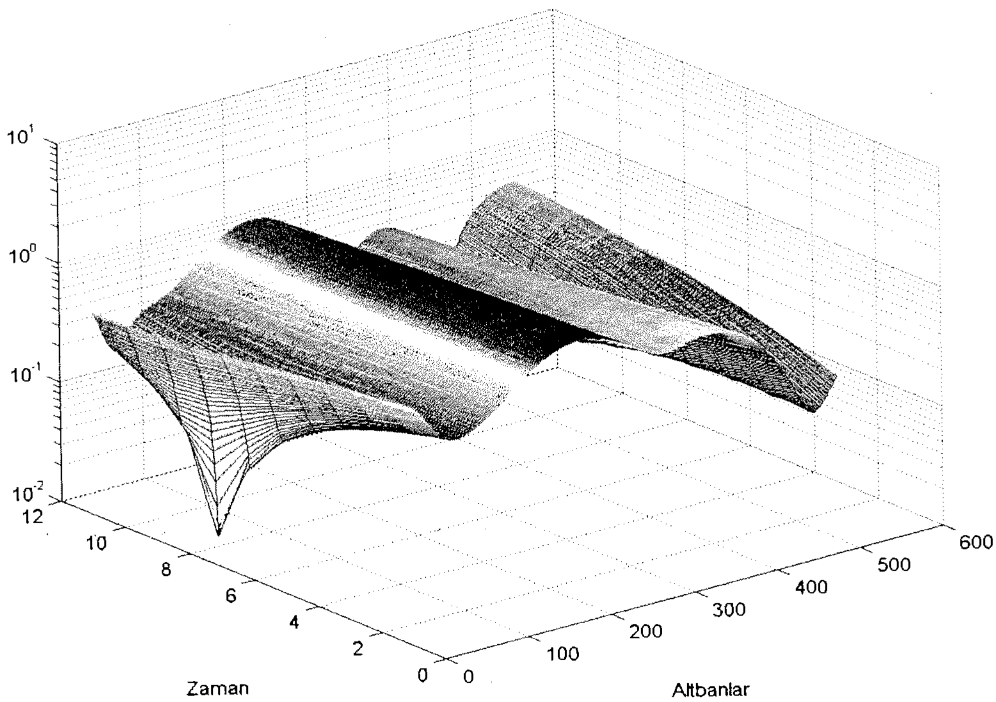
Şekil 2.4: Örnek bir OFDM kanalının frekans cevabının zamanla değişen logaritmik genlik cevabı $f_{Dm} = 10Hz$



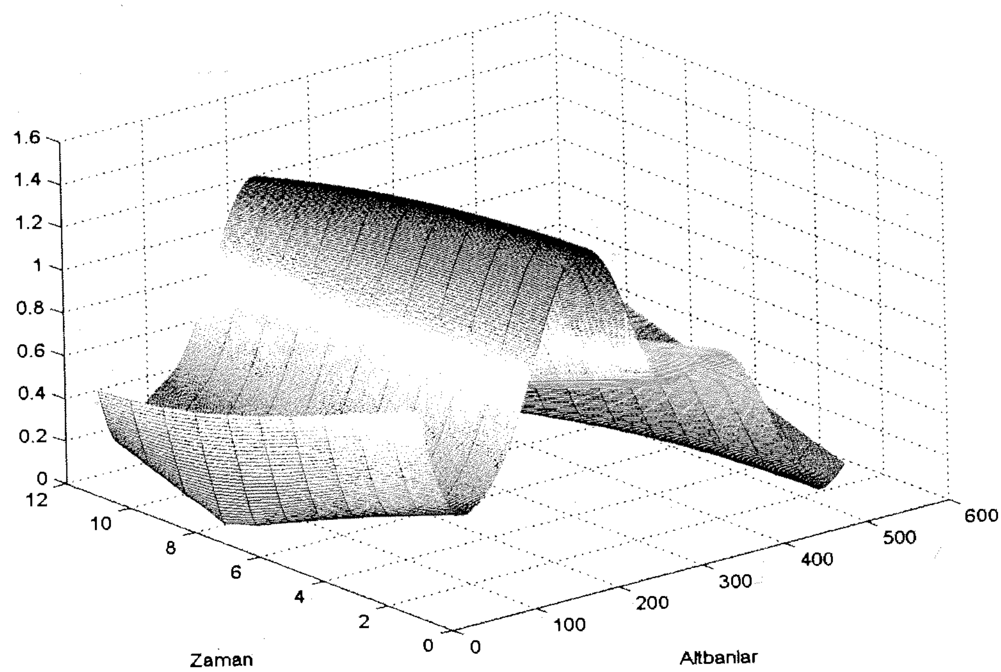
Şekil 2.5: Zayıflayan güç gecikme karakterine sahip bir OFDM kanalının zaman sekmelerinin genlik cevabı $f_{Dm} = 100Hz$



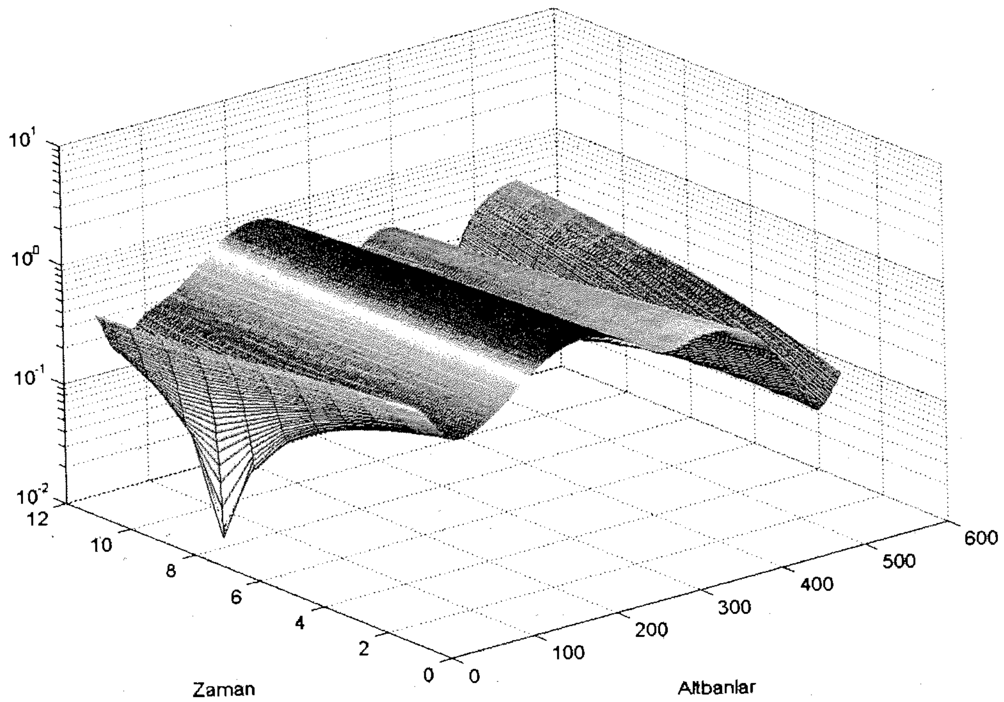
Şekil 2.6: Örnek bir OFDM kanalının frekans izgesinin zamanla değişen genlik cevabı $f_{Dm} = 100Hz$



Şekil 2.7: Örnek bir OFDM kanalının frekans cevabının zamanla değişen logaritmik genlik cevabı $f_{Dm} = 100Hz$



Şekil 2.6: Örnek bir OFDM kanalının frekans izgesinin zamanla değişen genlik cevabı $f_{Dm} = 100Hz$



Şekil 2.7: Örnek bir OFDM kanalının frekans cevabının zamanla değişen logaritmik genlik cevabı $f_{Dm} = 100Hz$

2.4 COST-207 KANAL MODELİ

Bilimsel araştırma ve kalkınma konularında ülkelerin ulusal düzeyde gerçekleştirdikleri projeler arasında koordinasyonun sağlanması amacıyla Ekim 1971'de oluşturulan COST (Bilimsel ve Teknik Araştırma Alanında Avrupa İşbirliği) birimi çeşitli projelere imza atmıştır. Bu projelerin en önemlilerinden biri elektromanyetik dalga yayınımı hakkında yapılmış, sayısal yerel mobil radyo haberleşmesi isimli COST-207 projesidir[34]. COST-207 projesi dahilinde çoklu yola sahip mobil haberleşme kanalları, uygulamadaki kanal modellerinin geniş anlamda durağan (WSSUS) oldukları varsayımına dayanarak modellenmiştir. COST-207 kapsamında elektromanyetik dalgaların yayınımını etkileyen ortamlar tipik olarak

i) Kırsal bölgeler (RA),
ii) Orta ölçekli şehirler (TU),
iii) Kötü yayınım koşullarına sahip bölgeler (BU),
iv) Dağlık alanlar (HT)

olmak üzere 4 farklı gruba ayrılmıştır. Önerilen mobil iletişim kanal grupları için Doppler ve gecikme güç spektrumları Tablo.2.1 ve Tablo2.2'de verilmiştir.

Tablo 2.1: COST-207 kanal modeline göre çeşitli kanallara ait güç gecikmesinin spektrum yoğunlukları $S_h(\tau)$

Yayılım Ortamı	$S_h(\tau)$	Gecikme Yayılımı, T_m
RA	$c_{RA}e^{-9.2\tau/\mu s}$, $0 \le \tau \le 0.7\mu sn$ 0 , *diğer durumlar*	$0.7\mu sn$
TU	$c_{TU}e^{-\tau/\mu s}$, $0 \le \tau \le 7\mu sn$ 0 , *diğer durumlar*	$7\mu sn$
BU	$c_{BU}e^{-\tau/\mu s}$, $0 \le \tau \le 5\mu sn$ $c_{BU}e^{5-\tau/\mu s}$, $5 \le \tau \le 10\mu sn$ 0 , *diğer durumlar*	$10\mu sn$
HT	$c_{HT}e^{-3.5\tau/\mu s}$, $0 \le \tau \le 2\mu sn$ $0.1c_{HT}e^{15-\tau/\mu s}$, $15 \le \tau \le 20\mu sn$ 0 , *diğer durumlar*	$20\mu sn$

Tablo 2.1'de verilen c_{RA}, c_{TU} c_{BU} c_{HT} katsayıları $\int_0^\infty S_h(\tau)d\tau = 1$ olacak şekilde hesaplanıp bu proje dahilinde

$$c_{RA} = \frac{9.2}{1-e^{-6.44}} \tag{2.5}$$

$$c_{TU} = \frac{1}{1-e^{-7}} \tag{2.6}$$

$$c_{BU} = \frac{2}{3(1-e^{-5})} \tag{2.7}$$

$$c_{TU} = \frac{1}{(1-e^{-7})/3.5+(1-e^{-5})/10} \tag{2.8}$$

olarak verilmiştir. GSM sistemleri için sembol süresinin $3.7\mu sn$ olduğu düşünülürse Tablo 2.1'e göre kırsal bölge (RA) kanal modeli, sinyal üzerine düz sönümlemeli kanal olarak etki ederken, diğer kanal modelleri ise frekans-seçici kanal olarak etki etmektedir. COST-207 çalışmasında Doppler güç spektrum yoğunlukları Tablo 2.2'de verilmiştir.

Tablo 2.2: COST-207 Kanal modeline göre Doppler güç spektrum yoğunlukları $S_h(\lambda)$

Çeşiti	$S_h(\lambda)$	Yayılım Gecikmesi	Doppler Yayınımı
Jakes	$\frac{1}{\pi f_{\max}}\sqrt{1-(f/f_{\max})}$	$0 \le \tau \le 0.5\mu sn$	$f_{\max}/\sqrt{2}$
Gauss-I	$A_1 \exp(-\frac{(f+0.8f_{\max})^2}{2(0.05f_{\max})^2})+0.1\,A_1 \exp(-\frac{(f-0.4f_{\max})^2}{2(0.1f_{\max})^2})$	$0.5 \le \tau \le 2\mu sn$	$0.45f_{\max}$
Gauss-II	$A_2 \exp(-\frac{(f-0.7f_{\max})^2}{2(0.1f_{\max})^2})+10^{-1.5}$ $A_2 \exp(-\frac{(f+0.4f_{\max})^2}{2(0.15f_{\max})^2})$	$\tau \ge 2\mu sn$	$0.25f_{\max}$
Rice	$\frac{0.41^2}{\pi f_{\max}}\sqrt{1-(f/f_{\max})}+0.91^2\delta(f-0.7f_{\max})$	$\tau = 0\mu sn$	$0.39f_{\max}$

Tablo 2.2'de verilen A_1, A_2 katsayıları $\int_0^{\infty} S_h(\lambda)d\lambda = 1$ olacak şekilde hesaplanıp COST 207 projesi dahilinde

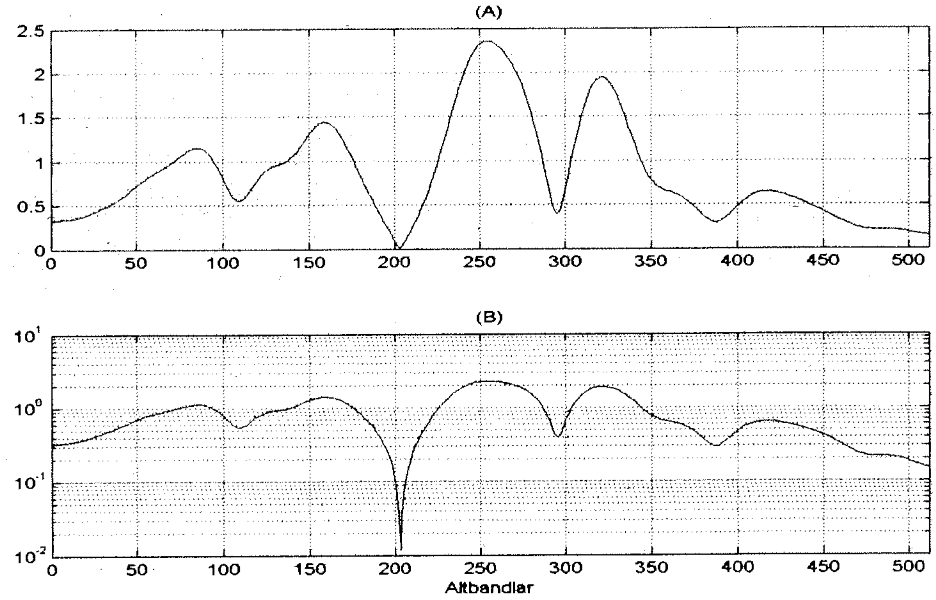
$$A_1 = \frac{50}{\sqrt{18\pi}f_{max}}, \quad A_2 = \frac{10^{1.5}}{\sqrt{2\pi}(\sqrt{10}+0.15)f_{max}} \qquad (2.9)$$

olarak verilmiştir.

2.4.1 COST-207 Kanal Modeli için yapılan Bilgisayar Benzetimleri

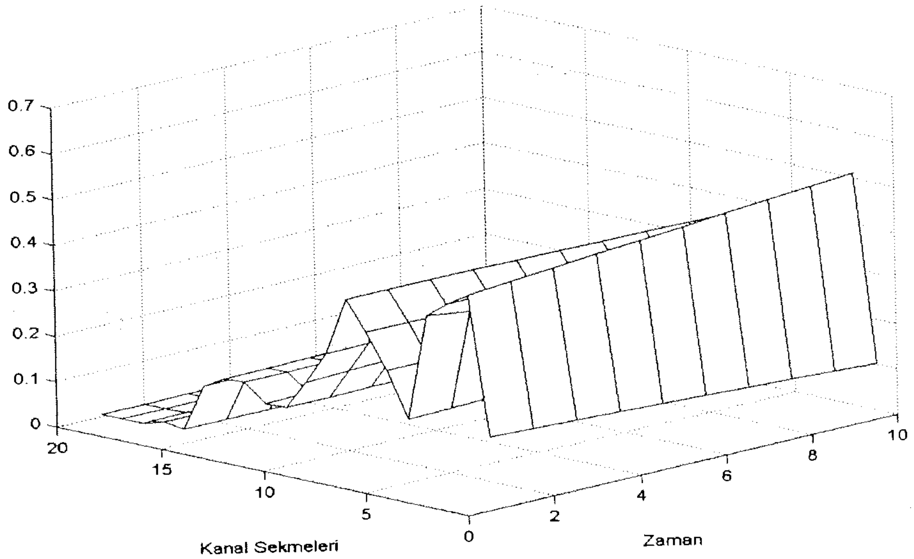
Verici antenden iletilen sinyal, düşüş katsayısı (rolloff factor) $\vartheta = 0.2$ olan yükseltilmiş kosinüs biçimindeki verici süzgeçi ile biçimlendirilmiştir. Toplam iletişim bandı, 512 altbanda bölünerek, iletim süresi *Ts*, 40 μsn'lik kısmı çevirimli önek (cyclic prefix) olmak üzere, toplam 328 μsn'dir. Bilgisayar benzetimlerde kullanılan TU ve BU kanalların karakteristikleri Bölüm 2.4'de verilmiştir.

Şekil 2.8 ve Şekil 2.9'da, sırasıyla kanalın frekans cevabının ve zamandaki sekmelerinin değişimleri TU kanallar için incelenmiştir. Yapılan bu benzetimlerde, kanalın *Beek* kanal modeline göre frekans-seçiciliğinin daha fazla olduğu gözlenmiştir. Şekil 2.9'da kanala ait sekmelerin çiziminde TU kanalların zamanda yaklaşık 10-15 katsayıya sahip olduğu görülmüştür. Ayrıca, Şekil 2.10 ve Şekil 2.11'de yavaş sönümlemeli olarak modellenen kanalın frekans cevabı, sırasıyla doğrusal ve logaritmik olarak incelenmiştir.

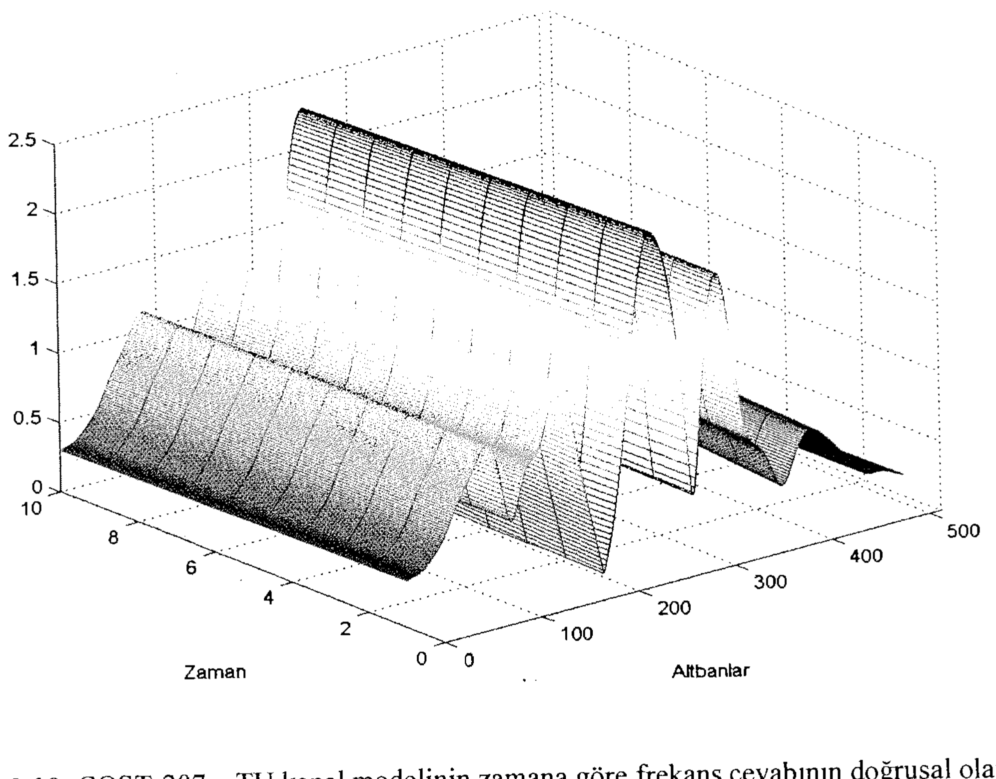


Şekil 2.8:COST-207-TU kanalının frekans izgesinin genlik cevabı

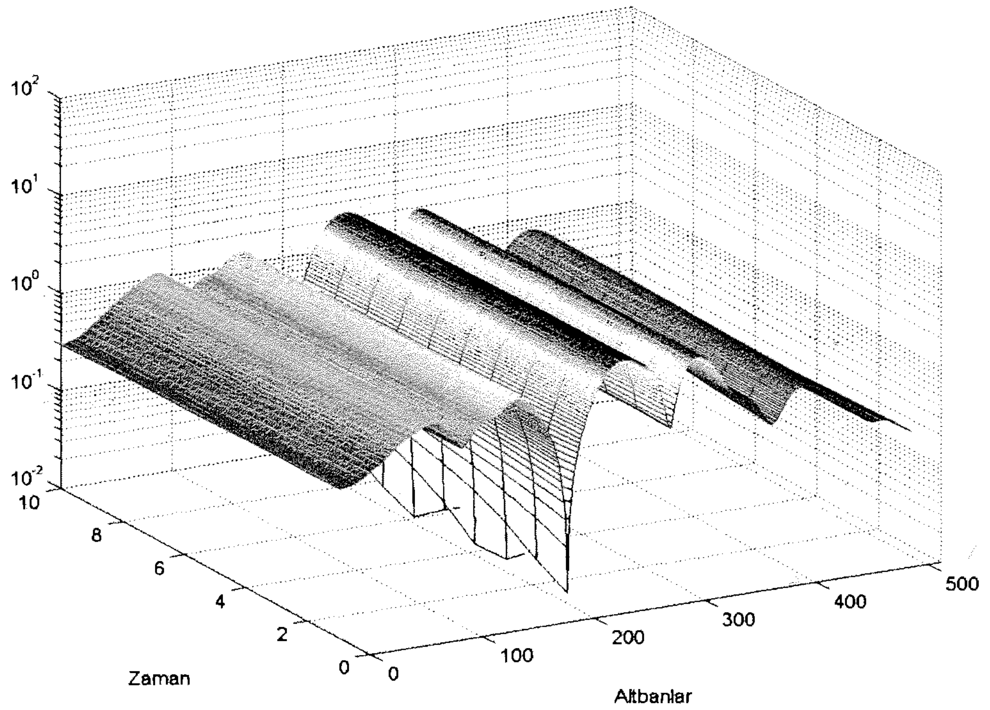
(A) : Doğrusal (B): Logaritmik



Şekil 2.9: COST-207 - TU kanal modelinin zamana göre kanal sekmelerinin genlik değişimi (fdmax=10 Hz, altband sayısı=512)



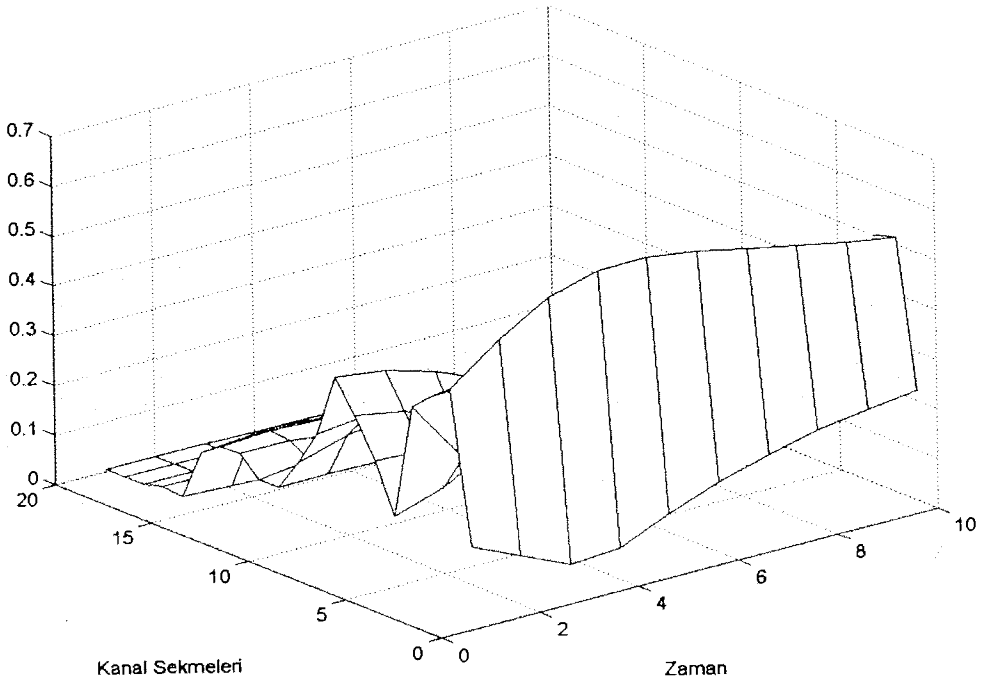
Şekil 2.10: COST-207 – TU kanal modelinin zamana göre frekans cevabının doğrusal olarak genlik değişimi (fdmax=10 Hz, altband sayısı=512)



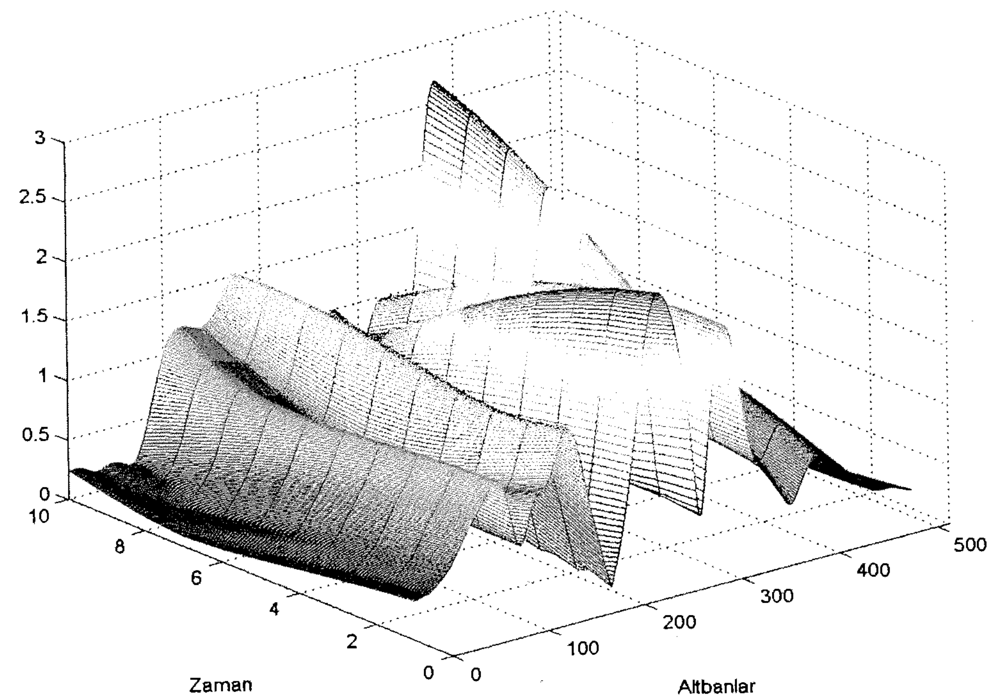
Şekil 2.11: COST-207 – TU Kanal modelinin zamana göre frekans cevabının logaritmik olarak genlik değişimi (fdmax=10 Hz, altband sayısı=512)

TU kanal modeli için artan maksimum Doppler frekans değerlerine göre, kanalın frekans cevabının ve zamandaki sekmelerin değişimlerinin artması beklenmektedir. Şekil 2.12'de TU kanal modelinin zamandaki sekmelerinin değişimi incelenmiş ve Şekil 2.9'a göre değişimin oldukça hızlı olduğu gözlemlenmiştir. Ayrıca, artan maksimum Doppler frekans etkisinin, kanalın frekans cevabına olan etkisi, sırasıyla doğrusal ve logaritmik olarak Şekil 2.13 ve Şekil 2.14'te incelenmiştir.

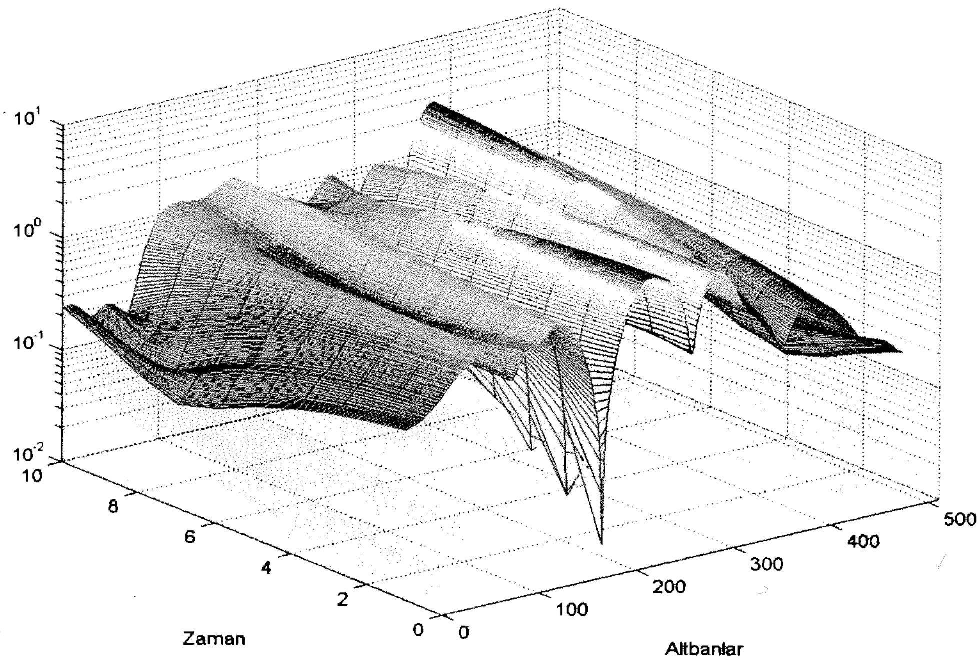
BU kanallarda, iletişimin TU kanallara göre daha zor olduğunu, yani kanalın daha frekans-seçici olduğunu göstermek için Şekil 2.15 ve Şekil 2.16'da, sırasıyla kanalın frekans yanıtının ve zamandaki sekmelerinin değişimleri incelenmiştir. Frekans-seçiciliği fazla olan bu kanalların zamandaki sekme sayısının yaklaşık 20-25 olduğu görülmüştür. Benzer şekilde, BU kanalların zamana göre kanal frekans cevabının değişimi doğrusal ve logaritmik olarak Şekil 2.17 ve Şekil 2.18'de verilmiştir.



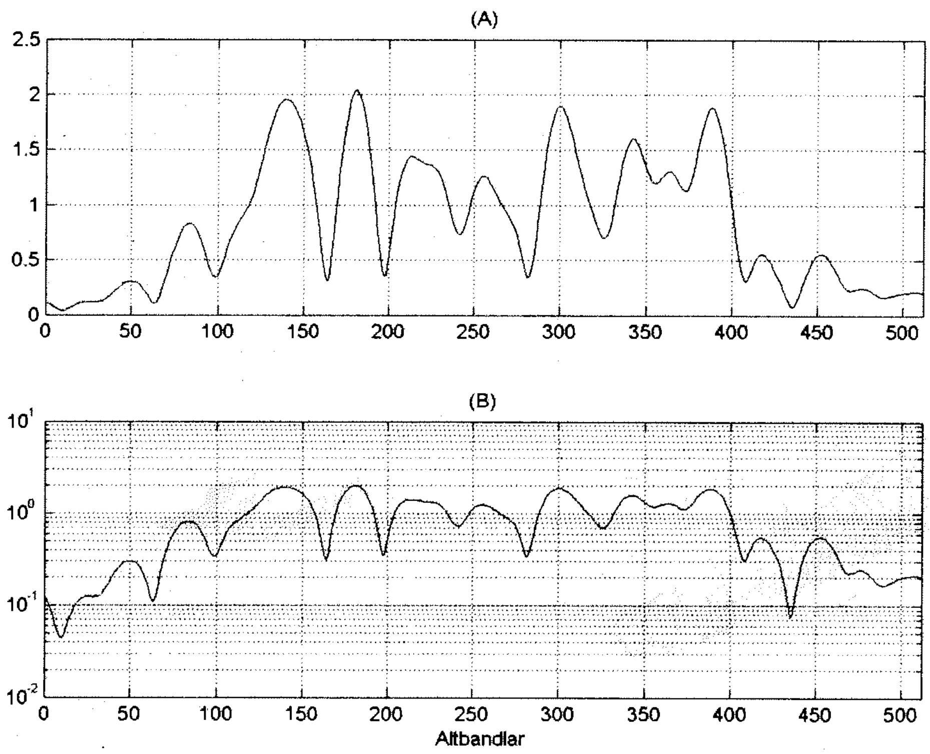
Şekil 2.12: COST-207 – TU kanal modelinin zamana göre kanal sekmelerinin genlik değişimi (fdmax=100 Hz, altband sayısı=512)



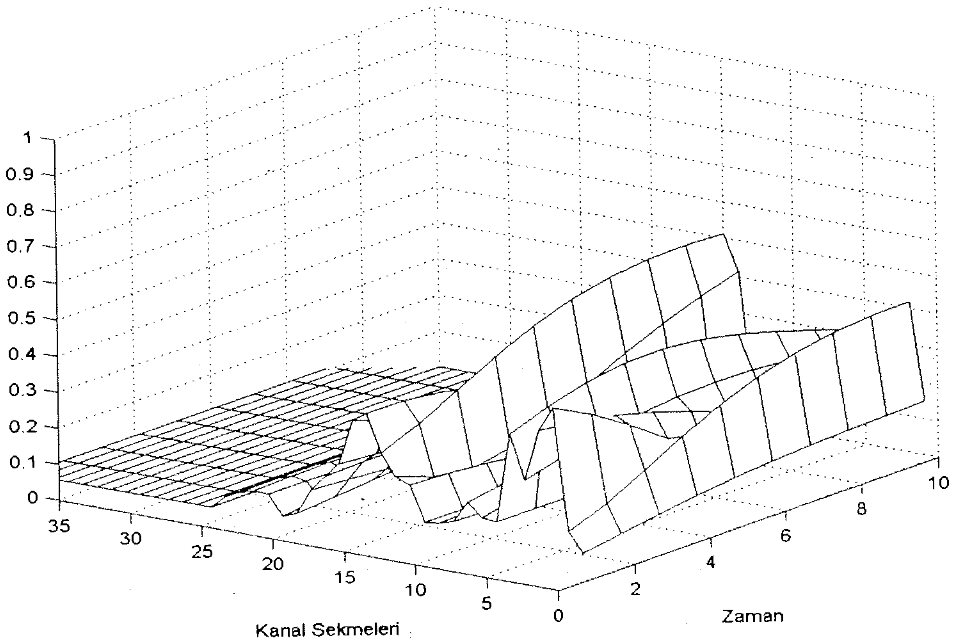
Şekil 2.13: COST-207 – TU kanal modelinin zamana göre frekans cevabının doğrusal olarak genlik değişimi (fdmax=100 Hz, altband sayısı=512)



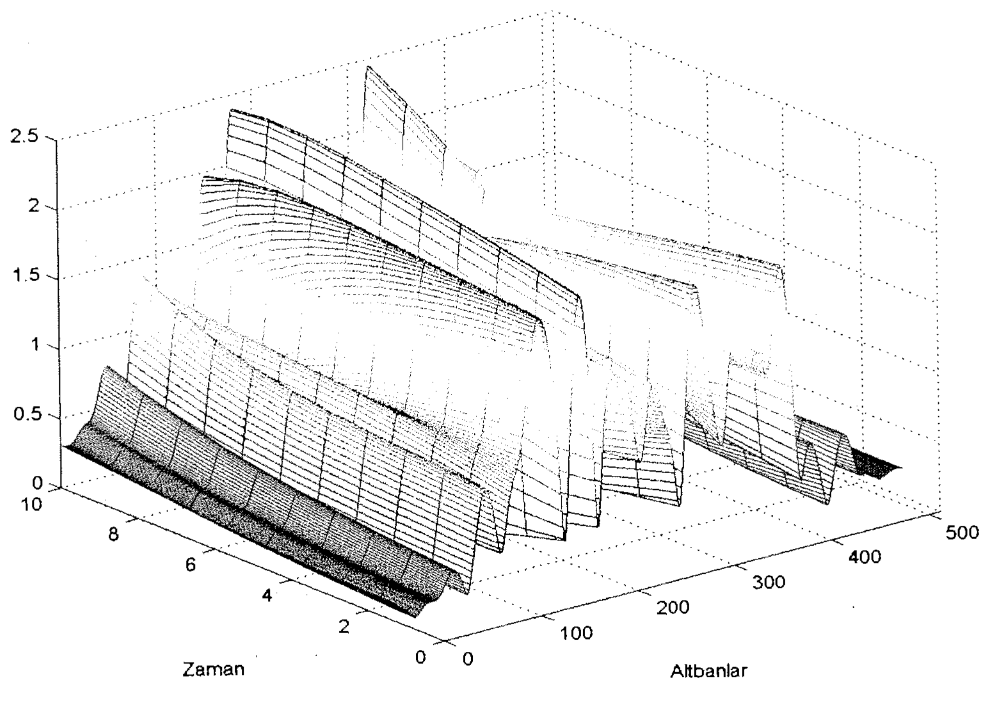
Şekil 2.14: COST-207 – TU kanal modelinin zamana göre frekans cevabının logaritmik olarak genlik değişimi (fdmax=100 Hz, altband sayısı=512)



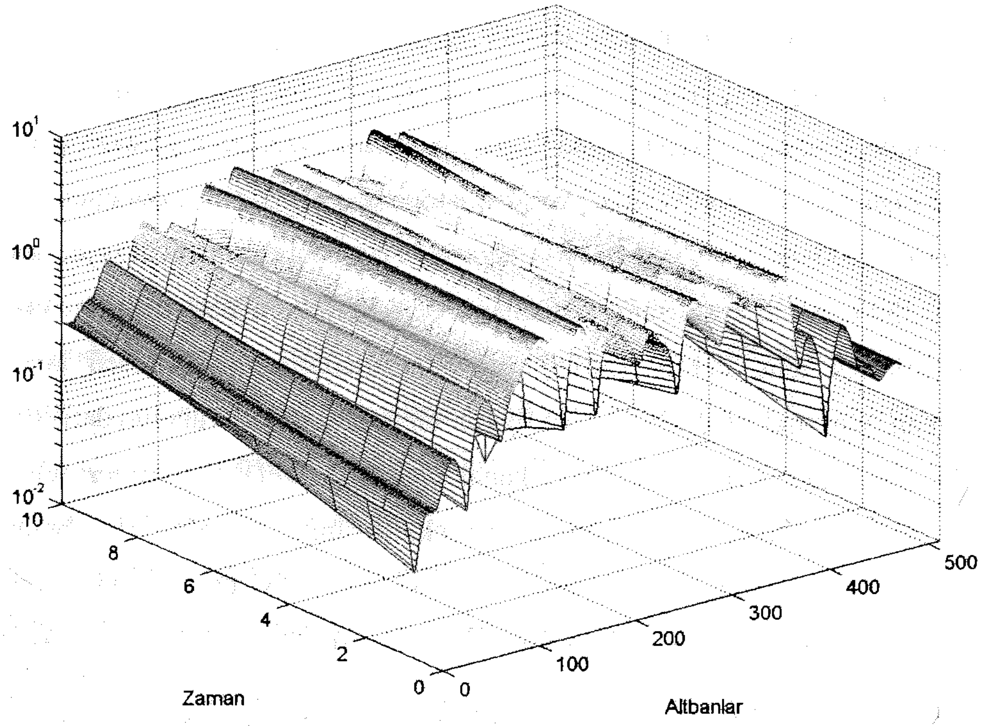
Şekil 2.15:COST-207 BU kanalının frekans izgesinin genlik cevabı (A) : Doğrusal (B): Logaritmik



Şekil 2.16: COST-207 – BU kanal modelinin zamana göre kanal sekmelerinin genlik değişimi (fdmax=100 Hz, altband sayısı=512)



Şekil 2.17: COST-207 – BU kanal modelinin zamana göre frekans cevabının doğrusal olarak genlik değişimi (fdmax=100 Hz, altband sayısı=512)



Şekil 2.18: COST-207 – BU kanal modelinin zamana göre frekans cevabının logaritmik olarak genlik değişimi (fdmax=100 Hz, altband sayısı=512)

3. GEREÇ VE YÖNTEM

Özellikle son yıllarda, geleneksel tekniklerin dışında çok hızlı, işlem karmaşıklığı az, Beklenti En Büyükleme (BEB) ve Bayes Monte Carlo (BMC) tekniklerine dayalı başarımı yüksek alıcı yapıları üzerinde çalışmalar yapılmıştadır. Bu çalışmalar, araştırma grubumuzun daha önceki yaptığı çalışmalar doğrultusunda, verici-çeşitlemeli ve kanal kodlamalı OFDM sistemlerinde kanal kestirimi ve veri sezimi üzerinde yoğunlaşılmıştır. Proje amaçları doğrultusunda, özellikle, kanal kestirimi/eşzamanlama/denkleştirme işlemlerini daha etkin bir biçimde gerçekleştirmek için, iteratif (turbo) yapıdaki alıcıların, kod çözücü tarafından kendilerine sağlanan, yumuşak (soft) bilgiyi en uygun bir şekilde nasıl kullanmaları gerektiği araştırılmıştır. Bu doğrultuda verici çeşitlemeli kanal kodlamalı OFDM sistemleri için MAP kanal kestirimi, EM algoritması yardımıyla gerçekleştirilmiş ve sistemin başarımını belirlemek amacıyla yoğun bilgisayar benzetimleri yapılmıştır. Ayrıca çalışmada, önerilen yapının işlemsel karmaşıklığı geleneksel yöntemlerle karşılaştırılmış ve kanal kestirimi sonucunda elde edilebilecek enaz karesel hata sınırını veren "Değiştirilmiş Cramer-Rao Alt Sınırı" (MCRLB) analitik olarak çıkarılmıştır. Yapılan bilgisayar benzetimlerinde, önerilen kanal kestirimcinin başarımının kuramsal alt sınıra yaklaştığı ve daha önce önerilen kanal kestirimcilerden daha iyi olduğu gözlemlenmiştir. Araştırma grubumuz bu çalışmadan bağımsız olarak siztemde oluşan faz gürültüsünün kestirimi için de bu dönemde iterative Monte-Carlo (SMC) tekniğine dayalı çeşitli çalışmalar gerçekleştirmiştir. Yapılan bu çalışmalarının ayrıntıları ve elde edilen sonuçlar Bölüm 3.1'de sunulmuştur. Ayrıca proje önerisinde belirtildiği üzere, tek kullanıcılı alıcılar için proje kapsamında önerilen alıcı yapıları çok kullanıcılı alıcılar için de gözönüne alınmış, bu durumda her kullanıcıya ilişkin parametrelerin birleşik kestirimi (joint estimation) için yeni algoritmalar geliştirilmiştir. Bu çerçevede çok taşıyıcılı (Multi carrier) sistemlerin, Kod Bölmeli Çoklu Erişim (CDMA) sistemleriyle birleştirilmesi ile oluşturulan MC-CDMA sistemleri üzerinde çalışmalar yapılmıştır. Frekans-seçici kanallarda yüksek hızda çok kullanıcıyı destekleyen bu sistemler, baz istasyonundan mobil alıcıya iletişimi öngören aşağı link ve mobil kullanıcıdan baz istasyonuna iletişimi öngören yukarı link olarak iki ayrı durum için incelenmiştir. Yapılan bu çalışmalarının ayrıntıları ve elde edilen sonuçlar Bölüm 3.2'de sunulmuştur.

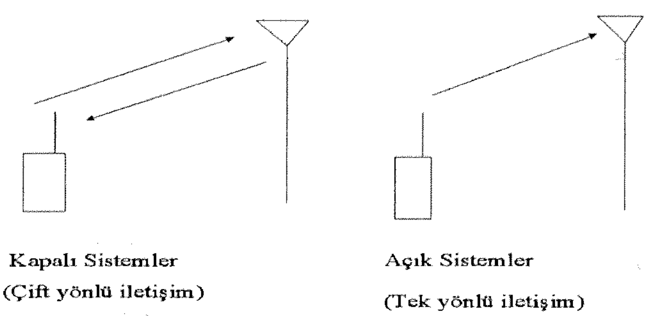
3.1 STBC/SFBC- OFDM SİSTEMLERİ İÇİN KANAL KESTİRİM TABANLI TURBO ALICI TASARIMLARI

Yeni nesil geniş bandlı kablosuz mobil iletişim sistemleri üzerinde çalışan araştırmacıların hedefi kablolu iletişim sistemleriyle rekabet edecek yüksek hızda ve kalitede iletişim sağlayabilen teknolojiler geliştirmektir. Mümkün olabilecek en iyi sistem verimliliğini sağlayacak teknikler üzerindeki çalışmalar yoğun olarak sürüdürülmektedir. Kablosuz iletişimde güvenilir bir iletişimin yapılmasını engelleyen ve kablolu sistemlerle rekabetini zorlaştıran en önemli neden, çoklu yol yayınımlarından kaynaklanan sönümlemelerdir. Bu kanallarda hata oranın azaltılması bir başka deyişle iletişim kalitesinin artırılması oldukça zordur. Örneğin, toplamsal Gauss gürültülü bir kanalda efektif bit hata oranını (BER) uygun modülasyon ve kodlama teknikleri ile 10^{-2} den 10^{-3} düzeyine indirebilmek için Sinyal Gürültü Oranında (SGO) 1-2 dB'lik bir arttırım yeterliyken bu başarımın sönümlemeli bir kanalda sağlanabilmesi için SGO'nın 10 dB'le kadar yükseltilmesi gerekmektedir [20]. SGO oranın yükseltilmesi mobil cihazın kullanım süresinin artmasına sebeb olacak ve maliyeti yüksek olan güçlendiriciler kullanılmasını gerektirecektir. İletim gücünü artırarak yeterli bir başarım sağlanabilse de bu yeni nesil haberleşme sistemleri için uygun bir çözüm değildir.

Çoklu yol etkisini azaltmak için kullanılan geleneksel yöntem verici güç kontrolüdür. Vericinin alıcıyla iletişim kurarak kanal durumunu belirlemesi halinde, göndereceği sinyalin gücünü ayarlayarak yapılan iletişimin kanaldan en az biçimde etkilenmesi sağlanabilir. Ancak bu yaklaşımda iki temel tasarım problemi ile karşılaşılmaktadır. Birinci problem, vericinin kanaldan değişik düzeylerde sinyal gücü göndermesi nedeniyle, vericinin güç dinamik aralığının büyük ölçekte artmasıdır. Diğer bir problem ise vericinin kanal bilgisine sahip olması için çift yönlü bir iletişimin kurulması gerektiğidir. Verici ve alıcı arasında çift yönlü (kapalı) iletişim kurulabilse de, bu sistemin karmaşıklığını ve maliyetini artırması nedeniyle uygulamada pek tercih edilen bir durum değildir.

Son dönemde önerilen diğer etkili bir çözüm ise zaman, frekans ya da uzayda çeşitlilik sağlayan teknik ve yöntemlerdir. Kablosuz iletişimde hata başarımını iyileştirmek için birçok alıcı ve verici anten kullanımına dayanan bu kodlama teknikleri, farklı antenlerden iletilen sinyallere zaman ve uzaya ilişkin geçici ilintiler eklenmesiyle gerçekleştirilmektedir. Böylece alıcıda çeşitlilik sağlanarak bant genişliğinden ödün vermeksizin sistemin kodlama kazancı artırılmış olur. Uygulama açısından gerçeklenmesi kolay ve etkili bu uzay ve zamanda çeşitleme teknikleri basit bir alıcı yapısıyla kablosuz iletişim sisteminin kapasitesini büyük ölçüde artıracaktır.

Çoklu anten kullanımı, alıcıda ve vericide kullanıldığı durumlara göre ise *alıcı çeşitlemesi* ve *verici çeşitlemesi* olarak adlandırılırlar. Alıcı çeşitlemesi, alıcı anten çıkışlarından elde edilen sinyallerin kanaldaki sönümlemeyi yok edecek şekilde uygun bir biçimde birleştirilmesine dayanmaktadır. Bu yöntem, GSM ve IS-136 sistemlerinde mobil cihazdan baz istasyonu yönündeki iletişim (uplink) için kullanılmaktadır. Ancak alıcı çeşitleme yöntemi, mobil cihazdaki güç ve fiziksel boyutlarda kısıtlamalardan dolayı baz istasyonundan mobil cihaz yönündeki iletişimler(downlink) için kullanılması pek uygun değildir. Baz istasyonunda ise bu türden kısıtlamaların olmaması nedeniyle genel olarak 'downlink' iletişimde verici anten çeşitlemesi kavramı ortaya çıkmış ve geliştirilmiştir. Diğer taraftan verici anten çeşitlemesinin kullanılması durumunda ortaya iki temel problem çıkmaktadır. Bu problemler, alıcının sönümlemeli kanalın anlık bilgisine sahip olmaması ve iletilen sinyallerin alıcıya gelmeden kanalda karışmasıdır. Bu problemler, alıcı çeşitlemesi yöntemine kıyasla bir takım ek işlemlerin yapılması gerektiğini göstermektedir. Verici çeşitleme teknikleri açık ve kapalı çevirim olarak sınıflandırılmaktadır (Bkz. Şekil 3.1).



Şekil 3.1: Açık ve Kapalı Sistemler

Kapalı sistemler; verici ile alıcı arasında bir geri dönüşümün olduğu ve bu geri dönüşüm yardımıyla vericinin kanal bilgisine sahip olduğu sistemlerdir. Bir çok kablosuz sistem (güç kontrol amaçlı) geri dönüşüm yoluna sahip kapalı sistemler olmasına karşın, mobil cihazların hareketliliğinin kanaldaki değişim hızını artırdığı ve mobil cihazın baz istasyonundan uzak mesafede olabileceği de göz önüne alındığında açık çevirim sistemlerin yeni nesil kablosuz iletişim sistemler için daha uygun olduğu görülmektedir.

Son zamanlarda düz sönümlemeli kanallar için yüksek başarımlı uzay-zaman/frekans kodlayıcılar ve ML (En büyük olabilirlikli) kod çözücüleri içeren verici çeşitleme yöntemleri üzerinde yoğun çalışmalar yapılmıştır [20],[21],[22]. Ancak gerçek uygulamalarda, düz sönümlemeli kanallar için tasarlanan bu kodlayıcıların özelikle frekans-seçici kanallarda çalışması durumunda, kanal parametrelerinin kestirilmesi ve denkleştirme/kod çözme işlemlerinin alıcılarda işlemsel olarak oldukça büyük yük getirdiği görülmektedir. Geniş bandlı kanallarda ise uzun süreli gecikme yayınımından dolayı kestirilmesi gerekli kanal parametreleri ve kod çözümünde kullanılacak durum sayısı büyük oranda artmaktadır. Bu durumda karmaşık sinyal işleme algoritmalarına gereksinim duyulmaktadır.

Birbirine dik birçok alttaşıyıcı üzerinden sinyalleri paralel olarak göndererek uzun kanal birim basamak yanıtının etkisinden kurtulmayı sağlayan Dikgen-Frekans-Bölüşümlü-Çoğullama (OFDM) sistemleri üzerinde son yıllarda yoğun araştırma ve geliştirme çalışmaları yapılmaktadır. Özellikle, OFDM'in verici çeşitleme yöntemleriyle birleştirilmesi durumunda, verici çeşitleme işlemlerinin frekans-seçici kanallarda daha düşük karmaşıkla gerçekleştirilebilmesine olanak sağladığı ortaya konulmuştur [23]. Ancak iletişim sistemlerinde kullanılan servislerin artmasıyla birlikte, bilgi iletim hızında ve kalitesindeki artış talebi, kablosuz iletişimde sistemleri verimli bir şekilde kullanan kanal kodlama yöntemlerinin kullanılmasını kesinlikle gerekli hale getirmiştir. Genel olarak kodlama kazancı sağlamayan verici çeşitleme yöntemleri için dış-kanal kodlayıcılarının eklenmesiyle gerekli hata başarımına erişmek mümkün olmaktadır. Son dönemde kanal kodlama alanında yönelim, sağladıkları yüksek başarımdan dolayı, vericide paralel ve/veya seri bağlı kodlayıcıların, alıcıda ise yumuşak-kararlı, tekrarlı-kod çözücülerin kullanıldığı, turbo kodlamalı sistemlerdir. Bu sistemlerin Shannon-limitine yakın başarımlarda çalışırlarken bile, kabul edilebilir bir karmaşıklıkla gerçekleştirilebilecekleri anlaşılmaktadır. Verici çeşitleme tekniklerinin turbo yapısı ile birleştirilmeleri durumunda, paralel ve/veya seri bağlı kodlayıcılar vericilere dış kodlayıcı olarak eklenmektedir.

Turbo kodlar genel olarak serpiştirici (interleaver) tarafından ayrılmış paralel veya seri bağlı kodların birleştirilmesiyle oluşturulmuştur ve genellikle yüksek hızda iletişim için gerekli olan bandgenişliğini arttıran düşük iletim hızlı kodlardır. Turbo kodlama kullanımıyla düşen iletim hızını arttırmak için iletim hızı birden büyük olan verici çeşitleme yöntemleri kullanılabilir. Ancak literatürden bilindiği gibi Alamouti verici anten çeşitleme tasarımı [20] karmaşık değerli bilgi için diklik ve tam hızı sağlayan tek yöntemdir. Diklik özelliği ise, ileriki bölümlerde açıklanacağı gibi, kanal kestiriminde çok önemli bir yeri vardır. Ayrıca Alamouti kodlamasının diklik yapısı iletim kanallarının ayrımını sağlayarak denkleştiricinin karmaşıklığını azaltmaktadır. Bunun sonucu olarak, Alamouti tekniği WCDMA, CDMA2000 gibi kablosuz haberleşme standartlarına girmiş bulunmaktadır. Bu nedenle, turbo kodlanmış verici çeşitlemeli sistemlerin OFDM sistemleriyle bütünleştirmesi ile oluşacak yeni yapıların gelecek kuşak iletişim sistemleri işin mükemmel bir çözüm olacağı anlaşılmaktadır. İletilen sembolleri uzay-zaman'da dik hale getiren Alamouti yöntemi tam uzamsal çeşitlemeyi sağlamaktadır. Uzamsal yapıya ek olarak, frekans-seçici kanallar üzerinden çok yollu çeşitleme yapmak için Alamouti blok kodlama yöntemi frekans domeninde de gerçekleştirilebilir. Bu sebeble verici çeşitleme sistemleriyle OFDM sistemlerinin kullanılması durumda uzay-frekans kodlamalı sistem yapıları ortaya atılmıştır.

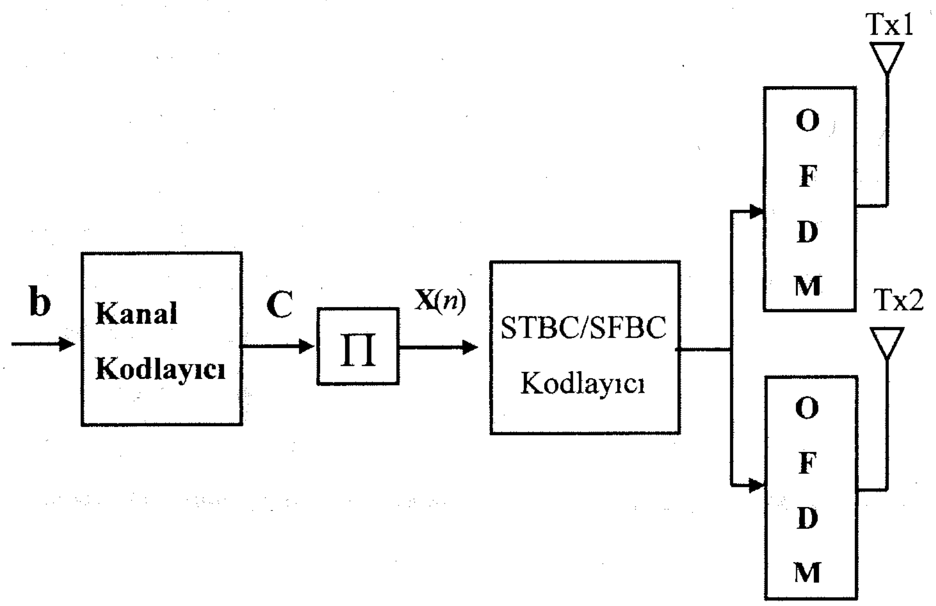
OFDM sistemlerinde verici çeşitliliğinin kullanılması uzay-zaman blok kodlama (STBC) yanında uzay-frekans blok kodlama tekniğinin de (SFBC) kullanılmasına imkan vermektedir. Kanal bilgisinin alıcıda bilindiği veya kestirildiği varsayımı altında, yavaş sönümlemeli kanallar için SFBC-OFDM sistemlerinin STBC-OFDM sistemleriyle aynı başarımı sağladığı, hızlı sönümlemeli kanallarda ise daha iyi bir başarıma sahip oldukları gösterilmiştir[24], [42]. Ayrıca SFBC-OFDM tekniği işlem sürecini bir blok üzerinde tamamladığı için STBC-OFDM sistemlerine kıyasla gerekli belleğin yarısını kullanmaktadır. Benzer şekilde, kod çözme işleminde gecikme süresi STBC-OFDM'de oluşan gecikmenin yarısı kadardır. SFBC-OFDM tekniğinde tüm alt kanallar altbandlardan oluşan gruplara ayrılır. Altband gruplama işlemi uygun sistem parametreleri yardımıyla kod yapısı ve çözümünü basitleştirerek çeşitleme kazancı sağlamaktadır.

Verici çeşitlemeli OFDM sistemleri için kanal kestirimi Li'nin [25],[26] çalışmalarıyla yoğunluk kazanmıştır. Ancak verici çeşitlemeli OFDM yapılarına yönelik ilk çalışmalar kanal kodlamasının kullanılmadığı sistemlerdir. Özellikle OFDM tekniğinin mutlaka bir kanal kodlayıcısı ile birlikte kullanılması gerekliliği, son zamanlarda yapılan çalışmaların kodlu sistemler üzerinde yoğunlaşmasına neden olmuştur. Bunun doğal bir sonucu olarak da, bir takım etkili ve hızlı istatistiksel sinyal işlem algoritmalarının özellikle karmaşık yapıda iletişim sistemlerinin kanal, taşıyıcı frekans ve faz kayma parametrelerinin kestirimlerinde geniş bir uygulama alanı bulduğu gözlenmektedir. Örneğin, Beklenti Enbüyükleme (EM) algoritması, uzay zaman blok kodları için önerilmiş [5],[27] daha sonra OFDM sistemleri ile bütünleştirilmiş dış kodlayıcıların kullanıldığı ve kullanılmadığı sistemler için genelleştirilmiştir [8],[28],[29]. Bu kapsamda, [8] de, dış kodlayıcı olarak turbo kodlamanın kullanıldığı, uzay-zaman blok kodlamalı OFDM sistemleri için EM tabanlı bir enbüyük sonsal (MAP) kestirimci önerilmiştir. Yapılan bu çalışmada, iletilen sembollerin kanal kestiriminde direkt olarak (pilot sembollerin) kullanıldığı ve kanalın birkaç OFDM sembolü boyunca değişmediği varsayılmıştır. Ayrıca karıştırılmış sinyallerin genel kestirimi için teklif edilen EM algoritmasının değişik bir versiyonu konvolüsyonel kodlayıcıların kullanıldığı iletim çeşitlemeli OFDM sistemlerine uygulanmış ve elde edilen sonuçlar, EM algoritmasının hızlandırılmış bir versiyonu olan, SAGE algoritması ile karşılaştırılmıştır [28]. Bu çalışmada kanal en büyük olabilirlikli olarak zaman izgesinde kestirilmiştir. Ayrıca, [28] deki çalışmalar STBC-OFDM ve SFBC-OFDM sistemleri için genişletilmiştir [29].

Bu bölümde, litaratürde önerilen EM algoritlarından farklı olarak [5], [27], [8], [28], [29] dış kodlayıcılı STBC/SFBC-OFDM sistemleri için MAP-EM kanal kestirim tabanlı bir turbo alıcı yapısı önerilmiştir. Daha önce teklif edilen, iletilen veri dizisinin alıcıda bilindiğini varsayan *eğitim dizili* yaklaşımlardan farklı olarak, bu çalışmada *bilgi-desteksiz* (non-data-aided) çalışan EM tabanlı yeni bir kanal kestirim algoritması önerilmiştir. Ayrıca Karhuenen Loeve (KL) açılımı yardımıyla önerilen kanal kestirimcinin karmaşıklığının azaltılabileceği gösterilmiştir.

3.1.1 Kanal Kodlayıcılı SFBC-OFDM ve SFBC-OFDM Sistemleri İçin Birleştirilmiş Sinyal Modeli

Şekil 3.2'de gösterildiği gibi ikili bilgi bitlerinden oluşan **b** vektörü dış-kanal kodlayıcı tarafından kodlanmaktadır. Kodlanmış bilgi dizisi serpiştirici tarafından rasgele serpiştirilmekte ve modüle edilerek $\mathbf{X}(n)$ dizisine dönüştürülmektedir. Şekil 3.2'deki serpiştiricinin temel fonksiyonu, patlamalı (burst) hataların olasılığının azaltılması ve kodlanmış bitler arasındaki ilişkinin ortadan kaldırılmasıdır. Bu durum kullanılacak kanal kestirim algoritmasının çalışması için büyük önem taşımaktadır. Son olarak, modüle edilmiş sinyal STBC/SFBC kodlayıcı tarafından 2 anten ile OFDM altbandlarından alıcıya iletilmektedir.



Şekil3.2: Verici Çeşitlemeli-Turbo Kodlanmış-OFDM Sistemleri için Verici yapısı

$N_c \times 1$ boyutlu, kanal kodlanmış, serpiştirilmiş ve modüle edilmiş "$n.$" zaman aralığında iletilen sinyal $\mathbf{X}(n) = [X(nN_c), X(nN_c+1), \cdots, X(nN_c+N_c)]^T$ ile, "$k.$" veri sembolü

$$X_k(n) = X(nN_c + k) \quad k = 1, \ldots, N_c \tag{2.10}$$

İle gösterilirse, bir OFDM bloğunu oluşturan veri sembol dizisi de $\mathbf{X}(n)=\left[X_1(n),X_2(n),\cdots,X_{N_c}(n)\right]^T$ biçiminde ifade edilebilir. Verici çeşitlemeyi sağlayacak kodlayıcı, alt kanal gruplandırma işlemiyle $\mathbf{X}(n)$ dizisini tek ve çift elemanları $\mathbf{X}_e(n)$ ve $\mathbf{X}_o(n)$ olmak üzere Şekil 3.2'deki gibi iki gruba ayırır.

$$\begin{aligned}\mathbf{X}_e(n)&=\left[X_2(n),X_4(n),\cdots,X_{N_c-2}(n),X_{N_c}(n)\right]^T\\ \mathbf{X}_o(n)&=\left[X_1(n),X_3(n),\cdots,X_{N_c-3}(n),X_{N_c-1}(n)\right]^T.\end{aligned}\qquad(2.11)$$

Tasarlanacak verici çeşitlemesinin SFBC olması durumunda; kodlayıcı 1. antenin çift numaralı altbandlarından $\mathbf{X}_e(n)$ dizisini, tek numaralı altbandlarından $-\mathbf{X}_o^*(n)$ dizisini göndermektedir. Benzer şekilde, $\mathbf{X}_o(n)$ dizisi 2. antenin çift numaralı altbandlarından, $\mathbf{X}_e^*(n)$ dizisi ise tek numaralı altbandlarından gönderilmektedir. Buna göre 1. ve 2. antenden iletilen bilgi dizileri sırasıyla $\mathbf{X}_1(n)$, $\mathbf{X}_2(n)$ olmak üzere,

$$\begin{aligned}\mathbf{X}_1(n)&=\left[X_2(n),-X_1^*(n),\cdots,X_{N_c}(n),-X_{N_c-1}^*(n)\right]^T\\ \mathbf{X}_2(n)&=\left[X_1(n),X_2^*(n),\cdots,X_{N_c-1}(n),X_{N_c}^*(n)\right]^T\end{aligned}\qquad(2.12)$$

şeklinde gösterilebilir. N_c adet alt taşıyıcı için alınan sinyal $\mathbf{R}_e=\left[R_2(n),R_4(n),\cdots,R_{N_c}(n)\right]^T$ ve $\mathbf{R}_o=\left[R_1(n),R_3(n),\cdots,R_{N_c-1}(n)\right]^T$ olmak üzere tek ve çift kısımlarına bölünürse alıcıdaki işaret

$$\begin{aligned}\mathbf{R}_e(n)&=X_e(n)\mathbf{H}_{1,e}+X_o(n)\mathbf{H}_{2,e}+\mathbf{W}_e(n)\\ \mathbf{R}_o(n)&=-X_o^*(n)\mathbf{H}_{1,o}+X_e^*(n)\mathbf{H}_{2,o}+\mathbf{W}_o(n)\end{aligned}\qquad(2.13)$$

şeklinde ifade edilir. Burada $X_e(n)$ ve $X_o(n)$ köşegen elemanları sırasıyla $\mathbf{X}_e(n)$ ve $\mathbf{X}_o(n)$ vektörünün elemanları olmak üzere $(N_c/2)\ x\,(N_c/2)$ boyutunda köşegen matrislerdir. μ. verici anten ile alıcı anten arasındaki kanalın N_c sayıda altbandı için frekans yanıtı $N_c\times 1$ boyutlu bir $\mathbf{H}_\mu(n)=\left[H_{\mu,1}(n),H_{\mu,2}(n),\cdots,H_{\mu,N_c}(n)\right]^T$ vektörü ile gösterilirse, $\mathbf{H}_\mu(n)$, (2.14)'deki gibi $N_c/2\times 1$ boyutlu tek ve çift elemanlarından oluşan iki vektörle ifade edilebilir.

$$\begin{aligned}\mathbf{H}_{\mu,e}(n) &= \left[H_{\mu,2}(n), H_{\mu,4}(n), \cdots, H_{\mu,N_c}(n)\right]^T \\ \mathbf{H}_{\mu,o}(n) &= \left[H_{\mu,1}(n), H_{\mu,3}(n), \cdots, H_{\mu,N_{c-1}}(n)\right]^T\end{aligned} \tag{2.14}$$

$\mathbf{W}_e(n)$ ve $\mathbf{W}_o(n)$ vektörleri ise, OFDM alt bantları için $(N_c/2)x1$ boyutunda ortalaması sıfır ve varyansı σ^2 olan bağımsız Gauss dağılımlı gürültü vektörleridir. (2.13) denklemi bilgi sembolleri $\mathbf{X}_e(n)$ ve $\mathbf{X}_o(n)$'ların farklı kanalların bitişik alt bant gruplarından iletildiğini göstermektedir. Kanal parametrelerinin kestirimi ve bilgi dizisinin çözümü için bitişik altbandlar için karmaşık kanal kazançlarının yaklaşık olarak eşit olduğu kabul edilirse $\mathbf{H}_{1,e}(n) \approx \mathbf{H}_{1,o}(n)$ $\mathbf{H}_{2,e}(n) \approx \mathbf{H}_{2,o}(n)$, (2.13) denklemi

$$\begin{bmatrix}\mathbf{R}_e(\mathrm{n}) \\ \mathbf{R}_o(\mathrm{n})\end{bmatrix} = \begin{bmatrix} X_e(n) & X_o(n) \\ -X_o^*(n) & X_e^*(n)\end{bmatrix}\begin{bmatrix}\mathbf{H}_{1,e}(n) \\ \mathbf{H}_{2,e}(n)\end{bmatrix} + \begin{bmatrix}\mathbf{W}_e(\mathrm{n}) \\ \mathbf{W}_o(\mathrm{n})\end{bmatrix} \tag{2.15}$$

şeklinde yazılabilir. Benzer şekilde STBC-OFDM sistemleri için kodlayıcı $X_e(n)$ ve $X_o(n)$ bilgi vektörlerini "n" ve "$n+1$" olmak üzere iki ayrı zaman aralığında gönderilmesi durumunda STBC-OFDM sistemi için alınan sinyal denklemleri

$$\begin{aligned}\mathbf{R}(n) &= X_e(n)\mathbf{H}_1(n) + X_o(n)\mathbf{H}_2(n) + \mathbf{W}(n) \\ \mathbf{R}(n+1) &= -X_o^*(n)\mathbf{H}_1(n+1) + X_e^*(n)\mathbf{H}_2(n+1) + \mathbf{W}(n+1)\end{aligned} \tag{2.16}$$

şeklinde ifade edilebilir. Buna göre "n" anında $X_e(n)$ 1. antenden $X_o(n)$ ise 2.antenden iletilmektedir. Benzer şekilde "$n+1$"anında ise $-X_o^*(n)$ 1. antenden, $X_e^*(n)$ ise ikinci antenden iletilmektedir. SFBC'de yapılan varsayımdan farklı olarak kanalın n. ve $n+1$ anında değişmediğini kabul ederek

$$\begin{bmatrix}\mathbf{R}(\mathrm{n}) \\ \mathbf{R}(\mathrm{n}+1)\end{bmatrix} = \begin{bmatrix} X_e(n) & X_o(n) \\ -X_o^*(n) & X_e^*(n)\end{bmatrix}\begin{bmatrix}\mathbf{H}_1(\mathrm{n}) \\ \mathbf{H}_2(\mathrm{n})\end{bmatrix} + \begin{bmatrix}\mathbf{W}\ (\mathrm{n}) \\ \mathbf{W}\ (\mathrm{n}+1)\end{bmatrix} \tag{2.17}$$

şeklinde yazabiliriz. Burada iletilen dizlerin boyutu $(N_c/2)\,x\,(N_c/2)$ olduğu için kullanılan kanalların alt band sayısının $(N_c/2)$ boyutunda olduğu unutulmamalıdır. Sırasıyla SFBC-OFDM ve STBC-OFDM için alınan sinyal modelini ifade eden (2.15) ve (2.17) denklemi genel olarak

$$\mathbf{R} = X\mathbf{H} + \mathbf{W} \tag{2.18}$$

biçiminde ifade edilebilir.

3.1.2 İletişim Kanal Modeli ve Karhunen Loeve (KL) Seri Açılımının Kullanımı

Çift-seçici sönümlemeli kanallar genellikle blok sönümlemeli kanallar olarak modellenmektedir[30]. Bu modele göre kanalın belli bir blok uzunluğu için değişmediği ve ard arda gelen bloklar arasında ilişkili veya bağımsız olabileceği varsayılmaktadır. Bu yaklaşım zaman-bölmeli-çoklu-erişim (TDMA), frekans-atlamalı-geniş-spektrum (FHSS) ve OFDM gibi sistemlere uygulanabilmektedir. Verici ve alıcı anten arasındaki kanalın blok sönümlemeli bir kanal olduğu, bir OFDM sembolü boyunca frekans-seçici ve ard arda gelen OFDM sembolü için Doppler frekansına göre zamanla değiştiği varsayılmıştır. Kanalın ayrık zamanlı temel band eşdeğer birim basamak yanıtı $h_\mu = [h_{\mu,0}(n), \cdots, h_{\mu,L}(n)]^T$ olarak tanımlanmaktadır. Burada "L" kanalın uzunluğunu göstermektedir. Buna göre "μ." verici anten ile alıcı anten arasındaki kanalın frekans yanıtı "n." ayrık zamanı için $\mathbf{H}_\mu(n)$ olarak gösterilmektedir.

Kablosuz iletişimde kanaldaki değişimler temel taban işlevlerine açılım yaklaşımıyla modellenebilir [31]. Fourier ve Taylor seri açılımları, polinom açılımı belirgin modellemede kullanılan bilinen önemli açılımlardır. Bundan farklı olarak rastlantısal süreçlerin temel açılımı için bilinen diğer bir etkili yöntem ise Karhunen-Loeve (KL) açılımıdır. Ayrıca KL açılımının çok yollu sönümlemeli kanalların benzetimlerindede kullanılabileceği gösterilmiştir [32]. KL açılım yöntemiyle $\mathbf{H}_\mu(n)$'i temel birimdik işlevlerin doğrusal kombinasyonları olarak

$$\mathbf{H}_\mu(n) = \Psi\, \mathbf{G}_\mu(n) \tag{2.19}$$

biçiminde yazılabilir. Burada $\Psi = [\psi_0, \psi_1, \cdots \psi_{N_c-1}]$'ler dik temel vektörlerini ve $G_\mu(n) = [G_{\mu,1}(n), \cdots, G_{\mu,N_c-1}(n)]^T$'ler açılım ağırlık katsayılarını göstermektedir. Bu yöntemle aynı istatiksel özelliklere sahip sonsuz sayıda $\mathbf{H}_\mu(n)$'ler üretilebilir. Ayrıca farklı temel işlevleri kullanarak türlü özelliklere sahip başka kanallar da üretilebilmektedir. Kanal ilişki matrisinin $\mathbf{C}_{\mathbf{H}_\mu} = E[\mathbf{H}_\mu \mathbf{H}_\mu^\dagger]$ öz değer açılımı

$$\mathbf{C}_{\mathbf{H}_\mu} = \Psi \Lambda \Psi^\dagger \tag{2.20}$$

şeklinde yazılabilir. Burada Λ, kanal ilişki matrisinin özdeğerlerin oluşturduğu köşegen matrisi, Ψ ise öz işlev vektörlerinin oluşturduğu matrisi göstermektedir. Ayrıca özdeğer açılımında elde edilen özdeğerlerde ağırlık katsayılarının varyanslarını "$\Lambda = E\{\mathbf{G}_\mu \mathbf{G}_\mu^\dagger\}$" oluşturmaktadır.

Burada özdeğer matrisi Λ'nın köşegen matris biçimde çıkması, karmaşık Gauss rastlantısal değişkenler biçiminde modellenen kanal katsayılarının KL dönüşümü sonucunda birbirinden bağımsız hale dönüştüklerini göstermektedir. Böylece kanal kestiriminde, KL dönüşümü sonucu elde edilen bağımsız G_μ açılım katsayıları kullanılabilir.

3.1.3 Kanal Kestirimi/Denkleştirme

Turbo kodlamanın gelişimiyle, literatürde yeni sinyal sezimi, yinelemeli kanal kestirimi ve kod çözümü teknikleri önerilmiş ve yapılan araştırmalar, yinelemesiz yöntemlerin yinelemeli yöntemlere göre bit hata başarımlarının daha iyi olduklarını göstermiştir. Bu nedenle, bu projede verici çeşitlenmiş-turbo kodlamalı-OFDM sistemleri için Beklenti en büyükleme (EM) tabanlı MAP kanal kestirim yöntemi önerilmektedir. Ayrık frekans dönüşümü (DFT) tabanlı zaman izgesinde yapılan kanal kestirimcilerin örneklenmeyen uzaylı kanallar için izge sızıntısına sebeb olduğu ve hata düzlemi oluşturduğu için önerilen algoritma frekans izgesinde çalışmaktadır [38][39]. İletilen sembollerin kanal karıştırıcı yardımıyla ilişkisiz hale dönüştürülmesi algoritmanın uygulamada kullanılmasını mümkün kılmaktadır.

Sezim ve kestirim işlemlerini birleştirmenin en genel yolu, gönderilen bilgi dizisi ve kanal katsayılarının birleşik olasılığını en büyükleme işlemidir. Bilinmeyen belirgin kanal modeli genel olarak, iletim ortamının modellenmesinde kullanılmaktadır. Bu çalışmada ise, kanala ait istatiksel bilginin (Kanalın altbandları arasındaki ilişki) alıcıda bilindiği, *Bayesian* temelli bir yaklaşım benimsenmiştir. Yapılacak kestirim işleminde $\mathbf{H}$'ın öncül (a priori) PDF'i verilmiş raslantısal bir değişken olduğu varsayılmakta ve böylece kanala ait var olan öncül bilgi, kestirimcinin *Bayesian* yaklaşımıyla daha doğru işlem yapmasını sağlamaktadır. Bulunacak MAP kestirimci ifadesi gözlemlenen $\mathbf{R}$ vektörüne göre

$$\hat{\mathbf{H}}_{MAP} = \arg\max_{\mathbf{H}} \left[p(\mathbf{H}|\mathbf{R}) \right] \tag{2.21}$$

şeklinde yazılabilir. KL seri açılımı yardımıyla kanala ait raslantısal değişkenin ilişkisiz taban işlevleri bulunabilir. Bu sebeble, KL açılımı ilişkili olan çok yollu kanal parametreleri kestirimi problemini, ilişkisiz katsayıların kestirimi problemine dönüştürmektedir. Buna göre MAP kanal kestirimci ifadesi sonsal (posterior) PDF'i en büyükleyecek şekilde

$$\tilde{\mathbf{G}}_{MAP} = \arg\max_{G} \left[p(\mathbf{G}|\mathbf{R}) \right] \tag{2.22}$$

yazılabilir. (2.22) denkleminden MAP kestirimini hesaplamak işlem karmaşıklığı açısından çok yüksek bir eniyileme probleminin çözümünü gerektirmektedir ve genelde analitik bir çözümü yoktur. Sayısal yöntemlerle elde edilecek çözümlerin ise karmaşıklığının oldukça fazla olması nedeniyle uygulamada pek gerçekçi değildir. Ancak problem itaratif EM algoritmasıyla düşük karmaşıklı bir şekilde çözülebilir. Bu algoritma sonsal olasılık dağılım fonksiyonunun tekdüze artışını garanti altına alarak tümevarımsal bir yöntemle $\mathbf{G}$ değerlerini yinelemeli olarak bulmaktadır. $\mathbf{G}$'nin q. iterasyon adımında kestirimini $\mathbf{G}^{(q)}$ ile göstererek, yardımcı işlev olan

$$Q(\mathbf{G}|\mathbf{G}^{(q)}) = \sum_{X} p(\mathbf{R}, X, \mathbf{G}^{(q)}) \log p(\mathbf{R}, X, \mathbf{G}^{(q)}) \tag{2.23}$$

ifadesinin enbüyüklenmesiyle elde edilir. İletilen kodlanmış sembollerin kanal serpiştirici sayesinde ilişkisiz hale getirildiği ve eş dağılımlı olarak $\mathbf{G}$' den bağımsız oldukları göz önüne alınırsa $p(\mathbf{R}, X, \mathbf{G})$,

$$\log p(\mathbf{R}, X, \mathbf{G}) = \log(X|\mathbf{G}) + \log p(\mathbf{R}|X, \mathbf{G}) + \log p(\mathbf{G}) \tag{2.24}$$

şeklinde yazılabilir. Bu denklemdeki ilk terim, bilgi sembolleri eşit olasılıklı ve $\mathbf{G}$'den bağımsız olduğu için sabittir ve sonuçta kanal kestirim ifadesini etkilememektedir. Ayrıca gürültü bileşenlerinin bunlardan bağımsız olduğu göz önüne alınırsa, $\mathbf{R}$ sinyali için logaritması alınmış koşullu olasılık fonksiyonu,

$$\log p(\mathbf{R}|X, \mathbf{G}) \propto \left[-(\mathbf{R} - X\tilde{\mathbf{\Psi}}\mathbf{G})^{\dagger} \mathbf{\Sigma}^{-1} (\mathbf{R} - X\tilde{\mathbf{\Psi}}\mathbf{G}) \right] \tag{2.25}$$

şeklinde yazılabilir. Burada "$\mathbf{\Sigma}$", gürültünün varyanslarını gösteren ve elemanları $\mathbf{\Sigma}[k,k]=\sigma^2$ olan köşegen matrisi göstermektedir. (2.23), (2.24) ve (2.25) denklemlerini göz önüne alarak uzay zaman blok kodlanmış sistemler için "n" anındaki kanalların bütün altbandlarını ifade eden $\mathbf{H}$ vektörü $\mathbf{G}=\left[\mathbf{G}_1^T\mathbf{G}_2^T\right]^T$, $\tilde{\mathbf{\Psi}}=diag(\mathbf{\Psi}\ \mathbf{\Psi})$ olmak üzere ifade edilirken, Uzay frekans blok kodlanmış sistemler için ise kanalların çift altbandlarının frekans yanıtını ifade eden $\mathbf{H}$ vektörü $\mathbf{G}=\left[\mathbf{G}_{1,e}^T\mathbf{G}_{2,e}^T\right]^T$, $\tilde{\mathbf{\Psi}}=diag(\mathbf{\Psi}_e\ \mathbf{\Psi}_e)$ şeklinde ifade edilmiştir. (2.23) denkleminin çözümünü kanallara ait öncül olasılık yoğunluk işlevini SFBC-OFDM ve STBC-OFDM sitemleri için tanımlayarak gerçekleştirilebilir.

SFBC-OFDM durumu:

Sönümlemeli kanalın öncül olasılık yoğunluk işlevi $\mathbf{G}=\left[\mathbf{G}_{1,e}^T\mathbf{G}_{2,e}^T\right]^T$, $\tilde{\mathbf{\Lambda}}=diag(\mathbf{\Lambda}_e\ \mathbf{\Lambda}_e)$ göstermek üzere

$$p(\mathbf{G})\,\square\,\exp(-\mathbf{G}\tilde{\mathbf{\Lambda}}^{-1}\mathbf{G}) \qquad (2.26)$$

olarak ifade edilebilir. Ayrıca $\tilde{\mathbf{\Psi}}=diag(\tilde{\mathbf{\Psi}}_e\tilde{\mathbf{\Psi}}_e)$ ve $\mathbf{\Sigma}=diag(\mathbf{\Sigma}\,\mathbf{\Sigma})$ ifadeleri SFBC için yazılırsa, alınan sinyal modeli (2.13), koşullu olasılık dağılım işlevini (2.25) ve öncül olasılığı gösteren (2.26) denklemleri göz önüne alınarak

$$\begin{aligned} p(\mathbf{R}|X,\mathbf{G}) \approx & -\left[\mathbf{R}_\mathbf{e}(n)-X_e(n)\mathbf{H}_{1,e}-X_o(n)\mathbf{H}_{2,e}\right]^\dagger\mathbf{\Sigma}\left[\mathbf{R}_\mathbf{e}(n)-X_e(n)\mathbf{H}_{1,e}-X_o(n)\mathbf{H}_{2,e}\right] \\ & -\left[\mathbf{R}_o(n)+X_o^\dagger(n)\mathbf{H}_{1,e}-X_e^\dagger(n)\mathbf{H}_{2,e}\right]^\dagger\mathbf{\Sigma}\left[\mathbf{R}_o(n)+X_o^\dagger(n)\mathbf{H}_{1,e}-X_e^\dagger(n)\mathbf{H}_{2,e}\right], \end{aligned} \qquad (2.27)$$

$$p(\mathbf{G})\approx-\mathbf{G}_{1,e}^\dagger\Lambda_e\mathbf{G}_{1,e}-\mathbf{G}_{2,e}^\dagger\Lambda_e\mathbf{G}_{2,e} \qquad (2.28)$$

ilişkileri yazılabilir. (2.23) denklemini (2.27), (2.28) denklemleri ve $\|X_e(n)\|^2=\|X_o(n)\|^2=\frac{1}{2}\mathbf{I}$ eşitliğini kullandıktan sonra $\mathbf{G}_{1,e}$ ve $\mathbf{G}_{2,e}$'ye göre türev alınıp sıfıra eşitlenirse

$$
\begin{aligned}
\frac{\partial Q}{\partial \mathbf{G}_{1,e}} &= \sum_{X} p(\mathbf{R}, X, \mathbf{G}^{(q)}) \left[\boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi}_e \left(X_e^{\dagger}(n) \mathbf{R}_e(n) - X_o(n) \mathbf{R}_o(n) - \mathbf{H}_{1,e} \right) - \boldsymbol{\Lambda}_e^{-1} \mathbf{G}_{1,e} \right] = 0 \\
\frac{\partial Q}{\partial \mathbf{G}_{2,e}} & \sum_{X} p(\mathbf{R}, X, \mathbf{G}^{(q)}) \left[\boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi}_e \left(X_o^{\dagger}(n) \mathbf{R}_e(n) + X_e(n) \mathbf{R}_o(n) - \mathbf{H}_{2,e} \right) - \boldsymbol{\Lambda}_e^{-1} \mathbf{G}_{2,e} \right] = 0
\end{aligned}
\tag{2.29}
$$

denklemleri elde edilir. Burada $p(\mathbf{R}, X, \mathbf{G}^{(q)})$ ifadesi $p(\mathbf{R} | X, \mathbf{G}^{(q)})$ ifadesi ile yer değiştirilerek X 'ler üzerinden ortalaması alınırsa (2.29) denklemi

$$
\begin{aligned}
\boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi}_e \left(\hat{X}_e^{\dagger}(n) \mathbf{R}_e(n) - \hat{X}_o(n) \mathbf{R}_o(n) - \mathbf{H}_{1,e} \right) &= \boldsymbol{\Lambda}_e^{-1} \mathbf{G}_{1,e} \\
\boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi}_e \left(\hat{X}_o^{\dagger}(n) \mathbf{R}_e(n) + \hat{X}_e(n) \mathbf{R}_o(n) - \mathbf{H}_{2,e} \right) &= \boldsymbol{\Lambda}_e^{-1} \mathbf{G}_{2,e}
\end{aligned}
\tag{2.30}
$$

şeklinde yazılabilir. Burada $\hat{X}_o(n)$ ve $\hat{X}_e(n)$ gönderilen sembollere ait sonsal olasılıkları göstermek üzere q. iterasyonda MAP kod çözücü tarafından hesaplanacaktır. (2.30) denkleminden $\mathbf{G}_1$ ve $\mathbf{G}_2$ ifadeleri çekilerek ve KL taban açılımı yardımıyla verici antenler ve alıcı anten arasındaki kanalların kestirimini ifade eden $\hat{\mathbf{H}}_{1,e}^{(q+1)}$ ve $\hat{\mathbf{H}}_{2,e}^{(q+1)}$, $(\mathbf{I} + \boldsymbol{\Sigma}\boldsymbol{\Lambda}_e^{-1})^{-1} = diag\left(\frac{\lambda_2}{\sigma^2 + \lambda_2}, \frac{\lambda_4}{\sigma^2 + \lambda_4} \cdots, \frac{\lambda_{N_c}}{\sigma^2 + \lambda_{N_c}} \right)$ olmak üzere

$$
\mathbf{H}_{1,e}^{(i+1)} = \boldsymbol{\Psi}_e (\mathbf{I} + \boldsymbol{\Sigma}\boldsymbol{\Lambda}_e^{-1})^{-1} \boldsymbol{\Psi}_e^{\dagger} \left[\hat{X}_e^{\dagger(q)} \mathbf{R}_e(n) - \hat{X}_o^{(q)} \mathbf{R}_o(n) \right] \tag{2.31}
$$

$$
\mathbf{H}_{2,e}^{(i+1)} = \boldsymbol{\Psi}_e (\mathbf{I} + \boldsymbol{\Sigma}\boldsymbol{\Lambda}_e^{-1})^{-1} \boldsymbol{\Psi}_e^{-1} \left[\hat{X}_o^{\dagger(q)} \mathbf{R}_e(n) + \hat{X}_e^{(q)} \mathbf{R}_o(n) \right] \tag{2.32}
$$

şeklinde bulunabilir.

STBC-OFDM durumu:

Sönümlemeli kanalın öncül olasılık yoğunluk işlevi $\mathbf{G} = \left[\mathbf{G}_1^T \mathbf{G}_2^T \right]^T$, $\tilde{\boldsymbol{\Lambda}} = diag(\boldsymbol{\Lambda}\,\boldsymbol{\Lambda})$ olmak üzere

$$
p(\mathbf{G}) \propto \exp(-\mathbf{G}\tilde{\boldsymbol{\Lambda}}^{-1}\mathbf{G}) \tag{2.33}
$$

olarak ifade edilebilir. Ayrıca $\tilde{\boldsymbol{\Psi}} = diag(\tilde{\boldsymbol{\Psi}}\tilde{\boldsymbol{\Psi}})$ ve $\boldsymbol{\Sigma} = diag(\boldsymbol{\Sigma}\boldsymbol{\Sigma})$ ifadeleri STBC için yazılırsa, alınan sinyal modeli (2.17), koşullu olasılık dağılım işlevini (2.25) ve öncül olasılığı gösteren (2.33) denklemlerini göz önüne alarak

$$\begin{aligned} p(\mathbf{R}|X,\mathbf{G}) \approx & -\left[\mathbf{R}(n) - X_e(n)\mathbf{H}_1(n) - X_o(n)\mathbf{H}_2(n)\right]^{\dagger} \boldsymbol{\Sigma}\left[\mathbf{R}(n) - X_e(n)\mathbf{H}_1(n) - X_o(n)\mathbf{H}_2(n)\right] \\ & -\left[\mathbf{R}(n+1) + X_o^{\dagger}(n)\mathbf{H}_1(n) - X_e^{\dagger}(n)\mathbf{H}_2(n)\right]^{\dagger} \boldsymbol{\Sigma}\left[\mathbf{R}(n+1) + X_o^{\dagger}(n)\mathbf{H}_1(n) - X_e^{\dagger}(n)\mathbf{H}_2(n)\right] \end{aligned} \tag{2.34}$$

$$p(\mathbf{G}) \approx -\mathbf{G}_1^{\dagger}\Lambda\mathbf{G}_1 - \mathbf{G}_2^{\dagger}\Lambda\mathbf{G}_2 \tag{2.35}$$

ilişkileri yazılabilir. (2.23) denklemini, (2.34), (2.35) denklemleri ve $\|X_e(n)\|^2 = \|X_o(n)\|^2 = \frac{1}{2}\mathbf{I}$ eşitliği kullanılarak yazıldıktan sonra $\mathbf{G}_{1,e}$ ve $\mathbf{G}_{2,e}$ 'ye göre türev alınıp sıfıra eşitlenirse

$$\begin{aligned} \frac{\partial Q}{\partial \mathbf{G}_1} &= \sum_X p(\mathbf{R}, X, \mathbf{G}^{(q)})\left[\boldsymbol{\Sigma}^{-1}\boldsymbol{\Psi}\left(X_e^{\dagger}(n)\mathbf{R}(n) - X_o(n)\mathbf{R}(n) - \mathbf{H}_1\right) - \boldsymbol{\Lambda}^{-1}\mathbf{G}_1\right] = 0 \\ \frac{\partial Q}{\partial \mathbf{G}_1} &= \sum_X p(\mathbf{R}, X, \mathbf{G}^{(q)})\left[\boldsymbol{\Sigma}^{-1}\boldsymbol{\Psi}\left(X_o^{\dagger}(n)\mathbf{R}(n+1) + X_e(n)\mathbf{R}(n+1) - \mathbf{H}_2\right) - \boldsymbol{\Lambda}^{-1}\mathbf{G}_2\right] = 0 \end{aligned} \tag{2.36}$$

ifadeleri elde edilir. Burada $p(\mathbf{R}, X, \mathbf{G}^{(q)})$ ifadesi $p(\mathbf{R}|X, \mathbf{G}^{(q)})$ ifadesi ile yer değiştirilip X 'ler üzerinden ortalaması alınırsa (2.36) denklemi

$$\begin{aligned} \boldsymbol{\Sigma}^{-1}\boldsymbol{\Psi}\left(\hat{X}_e^{\dagger}(n)\mathbf{R}(n) - \hat{X}_o(n)\mathbf{R}(n) - \mathbf{H}_1\right) &= \boldsymbol{\Lambda}^{-1}\mathbf{G}_1 \\ \boldsymbol{\Sigma}^{-1}\boldsymbol{\Psi}\left(\hat{X}_o^{\dagger}(n)\mathbf{R}(n+1) + \hat{X}_e(n)\mathbf{R}(n+1) - \mathbf{H}_2\right) &= \boldsymbol{\Lambda}^{-1}\mathbf{G}_2 \end{aligned} \tag{2.37}$$

şeklinde yazılabilir Burada $\hat{X}_o(n)$ ve $\hat{X}_e(n)$ gönderilen sembollere ait sonsal olasılıkları göstermek üzere q. yineleme adımında MAP kod çözücü tarafından hesaplanacaktır. (2.37) denkleminden $\mathbf{G}_1$ ve $\mathbf{G}_2$ ifadeleri çekilerek ve KL taban açılımı yardımıyla verici antenler ve alıcı anten arasındaki kanalların kestirimini ifade eden $\hat{\mathbf{H}}_{1,e}^{(q+1)}$ ve $\hat{\mathbf{H}}_{2,e}^{(q+1)}$'ler,

$(\mathbf{I} + \boldsymbol{\Sigma}\boldsymbol{\Lambda}_{\varepsilon}^{-1})^{-1} = diag\left(\frac{\lambda_1}{\sigma^2 + \lambda_1}, \frac{\lambda_2}{\sigma^2 + \lambda_2}, \ldots, \frac{\lambda_{N_c/2}}{\sigma^2 + \lambda_{N_c/2}}\right)$ olmak üzere

$$\hat{\mathbf{H}}_1^{(i+1)} = \mathbf{\Psi}(\mathbf{I}+\Sigma\Lambda^{-1})^{-1}\mathbf{\Psi}^H\left[\hat{X}_e^{\dagger(q)}\mathbf{R}(n) - \hat{X}_o^{(q)}\mathbf{R}(n+1)\right] \quad (2.38)$$

$$\hat{\mathbf{H}}_2^{(i+1)} = \mathbf{\Psi}(\mathbf{I}+\Sigma\Lambda^{-1})^{-1}\mathbf{\Psi}^H\left[\hat{X}_o^{\dagger(q)}\mathbf{R}(n) + \hat{X}_e^{(q)}\mathbf{R}(n+1)\right] \quad (2.39)$$

şeklinde bulunabilir.

İlklendirme:

Algoritmanın çalışmasıda gerekli olan başlangıç değerlerinin belirlenmesi için vericide her OFDM sembolü içine N_{PS} sayıda pilot sembol yerleştirilir. Vericiden iletilen pilot semboller yardımıyla alt-örneklenmiş bir sinyal modeli elde edilir ve $\hat{\mathbf{H}}_i^{(0)}$'ler için doğrusal-enaz-karesel hata kestirimi (LMMSE) yapılır[35]. Kanaldaki bütün altbandlara ait cevabın elde edilmesi için interpolasyon yapılır. (Lagrange interpolasyon tekniği). Sonuç olarak elde edilen kanal başlangıç değerleri EM algoritmasının yakınsaması için kullanılır. Alınan **R** sinyaline göre EM algoritması kanal kestirimi ve data kestirimini sırasıyla yinelemeli olarak gerçekleştirmektedir. Buna göre kanal kestiriminin başlangıç değeri $\mathbf{G}^{(0)}$ şeklinde pilot sembollerinden elde edilmektedir. İteratif enbüyükleme işlemi, $(q+1)$ iterasyon adımında $\mathbf{G}^{(q+1)}$ şeklinde ifade edilmektedir ve $\mathbf{G}^{(q+1)} = \arg\max_{\mathbf{G}} Q(\mathbf{G}|\mathbf{G}^{(q)})$ olarak hesaplanmaktadır.

3.1.4 Yinelemeli Kanal Denkleştirme ve Kod Çözümü İşlemi:

EM algoritmasından elde edilen kanal katsayıları yardımıyla, alınan **R** sinyalinden iletilen veri dizisi basit bir matris çarpımlarıyla bulunabilir. Bu bölümde önceden verilen kanal kodlanmış STBC-OFDM ve SFBC-OFDM sistemlerine ilişkin sinyal modellerini kanal denkleştirme işlemleri için tekrardan aşağıdaki gibi yazılsın.

SFBC OFDM Durumu:

$H_{1,e}(n) = diag(\mathbf{H}_{1,e}(n))$, $H_{1,o}(n) = diag(\mathbf{H}_{1,o}(n))$ olmak üzere $N_c \times N_c$ boyutlu kanal matrisi

$$H = \begin{bmatrix} H_{1,e}(n) & H_{2,e}(n) \\ H_{2,0}^{\dagger}(n) & -H_{1,o}^{\dagger}(n) \end{bmatrix} \quad (2.40)$$

ve $\mathbf{R}=\left[\mathbf{R}_e^T(n),\mathbf{R}_o^\dagger(n)\right]^T$, $\tilde{\mathbf{X}}=\left[\mathbf{X}_e^T(n),\mathbf{X}_o^\dagger(n)\right]^T$, $\tilde{\mathbf{W}}=\left[\mathbf{W}_e^T(n),\mathbf{W}_o^\dagger(n)\right]^T$ şeklinde tanımlanırsa (2.13) denklemi kanal denkleştirme için

$$\tilde{\mathbf{R}}=H\tilde{\mathbf{X}}+\tilde{\mathbf{W}} \tag{2.41}$$

şeklinde tekrar düzenlenebilir.

STBC OFDM Durumu:

$H_1(n)=diag(\mathbf{H}_1(n))$, $H_1(n)=diag(\mathbf{H}_1(n))$ olmak üzere $N_c\times N_c$ boyutlu kanal matrisi

$$H=\begin{bmatrix} H_1(n) & H_2(n) \\ H_2^\dagger(n+1) & -H_1^\dagger(n+1) \end{bmatrix} \tag{2.42}$$

olarak ve $\mathbf{R}=\left[\mathbf{R}^T(n),\mathbf{R}^\dagger(n+1)\right]^T$, $\tilde{\mathbf{X}}=\left[\mathbf{X}_e^T(n),\mathbf{X}_o^\dagger(n)\right]^T$, $\tilde{\mathbf{W}}=\left[\mathbf{W}_e^T(n),\mathbf{W}_o^\dagger(n)\right]^T$ şeklinde tanımlanırsa (2.16) denklemi SFBC modeline benzer şekilde kanal denkleştirme için

$$\tilde{\mathbf{R}}=H\tilde{\mathbf{X}}+\tilde{\mathbf{W}} \tag{2.43}$$

şeklinde yazılabilir. Kanal frekans yanıtının bilinmesi durumunda (2.38), (2.39) ve (2.43) denklemlerinden $\tilde{\mathbf{X}}$ ifadesini elde etmek için bir çok doğrusal denkleştirici önerilebilir. Bu çalışmada En-küçük-ortalamalı-karesel hataya sahip (MMSE) kestirimciden yola çıkılarak, basitleştirmeler yardımıyla Sıfır-Zorlamalı denkleştirici (ZF) elde edilecektir. Burada $\tilde{\mathbf{X}}$ vektörünün ortalamasını $\bar{\tilde{\mathbf{X}}}$, ortak değişinti matrisini $\mathbf{C}_{\tilde{\mathbf{X}}}$ ve gürültünün ortak değişinti matrisini $\mathbf{C}_{\tilde{\mathbf{W}}}$ göstermek üzere $\tilde{\mathbf{X}}$ ifadesi için MMSE kestirimciyi ifade eden $\mathbf{D}$ vektörü

$$\mathbf{D}=\bar{\tilde{\mathbf{X}}}+\mathbf{C}_{\tilde{\mathbf{X}}}H^\dagger(H^\dagger\mathbf{C}_{\tilde{\mathbf{X}}}H+\mathbf{C}_{\tilde{\mathbf{W}}})^{-1}(\tilde{\mathbf{R}}-H\bar{\tilde{\mathbf{X}}}) \tag{2.44}$$

şeklinde hesaplanabilir. Kanal matrisi H'nin ölçeklenmiş birim matris olduğunu, SFBC kodlama durumunda altbandlara ait kanal katsayıları için $H_{1,e,k}^2+H_{2,e,k}^2\approx 1$ yaklaşımı, STBC kodlama durumunda ise $H_{1,k}^2+H_{2,k}^2\approx 1$ yaklaşımı yapılırsa, MMSE kestirimci ifadesi basitleştirmek için "$H^\dagger H$" ifadesi

$$H^{\dagger}H \approx \mathbf{I}_{N_c \times N_c} \tag{2.45}$$

olarak yazılabilir. Ayrıca Tücker'in [36] çalışmasındaki $\bar{\tilde{\mathbf{X}}} = 0, \mathbf{C}_{\tilde{\mathbf{X}}} = \frac{1}{2}\mathbf{I}$ varsayımlar benimsenirse (2.44) denklemi

$$\mathbf{D} = \left(\mathbf{I} + 2\sigma_n^2\mathbf{I}\right)^{-1} H^{\dagger}\tilde{\mathbf{R}} \tag{2.46}$$

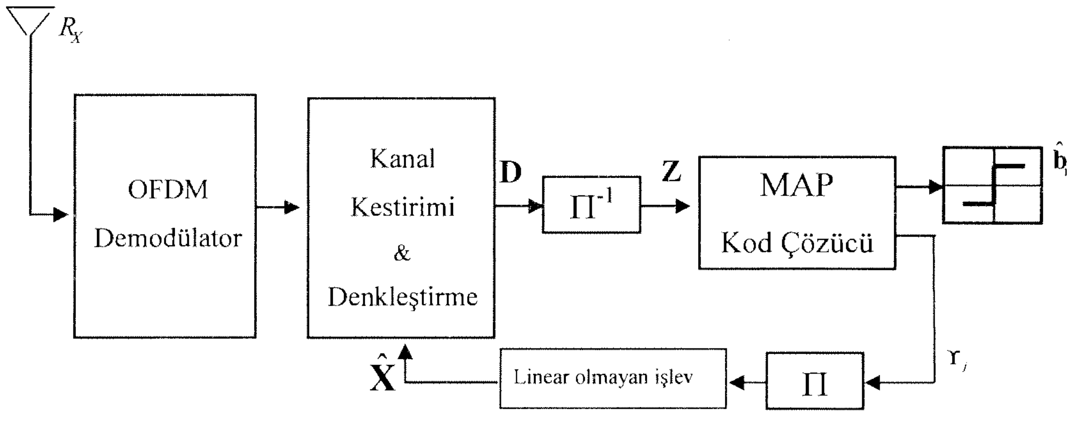
şekline dönüştürülebilir. Burada gürültüye ait ortak değişinti matriside $\mathbf{C}_{\tilde{\mathbf{W}}} = 0$ olarak alırsak doğrusal denkleştiricimiz sıfır zorlamalı denkleştirici haline gelirki $\boldsymbol{\eta}(n) = H^{\dagger}\tilde{\mathbf{W}}$ olmak üzere

$$\mathbf{D} = H^{\dagger}\tilde{\mathbf{R}} = H^{\dagger}H\tilde{\mathbf{X}} + \boldsymbol{\eta} \tag{2.47}$$

olacaktır.

3.1.5 Alıcı Yapısı

Verici çeşitlemeli kanal kodlanmış OFDM sistemleri için önerilen alıcı yapısı Şekil 3.3'te verilmiştir. Alıcıya gelen sinyal OFDM demodülatöründen çıktıktan sonra pilot semboller yardımıyla kanal kestirimi ve denkleştirme işlemi yapılır. Denkleştirilmiş sinyal $\{\mathbf{D}\}$ ters serpiştiriciden geçirilir ve $\{\mathbf{Z}\}$ sinyali elde edilir. MAP kod çözücüye giren $\{\mathbf{Z}\}$ sinyali için kanaldan iletilen kodlanmış sembollere ve bilgi bitlerine ait Logaritma-Olasılık-Oranı (Log Likelihood Ratio, "LLR") değerleri hesaplanır. Bilgi bitlerine ait LLR değerlerinden $\{\hat{\mathbf{b}}_i\}$ bilgi bitleri kestirilir. Kodlanmış sembollere ait LLR değerleri $\{\Upsilon_j\}$ ise serpiştiriciden geçirilerek linear olmayan bir işleve uygulanır. Linear olmayan işlev bloğu $\{\Upsilon_j\}$ dizisine ait yumuşak değerleri hesaplayarak $\{\hat{\mathbf{X}}\}$ dizisini bulur. İletilen sinyale ilişkin kestirimi gösteren $\{\hat{\mathbf{X}}\}$ dizisi önceden yapılan kestirim işlemini iyileştirmek için $\hat{X}_e^{(q)}$ ve $\hat{X}_o^{(q)}$ olarak kanal kestirimi işleminde kullanılır. İyileştirilen kanal kestirim değerleri ile tekrar denkleştirme işlemi yapılır. Önerilen yineleme işlemi belirlenen döngü sayısı kadar devam etmektedir.



Şekil 3.3 : Verici çeşitlemeli-Turbo Kodlanmış-OFDM Sistemleri için Önerilen Alıcı Yapısı

3.1.6 Lineer Olmayan İşlev

Turbo alıcı yapısında MAP kod çözücünün hesapladığı kodlanmış sembollere ait LLR değerlerinin yumuşak değerleri linear olmayan işlev yardımıyla hesaplanmaktadır. Buna göre kodlanmış sembollere ait LLR hesaplamasından $a = 1/\sqrt{2}$ için $P(X_k = a)$ ve $P(X_k = -a)$ değerlerini bulalım. Kodlanmış sembollere ait LLR

$$L(X_k) = \log_e \left[\frac{P(X_k = -a)}{P(X_k = +a)} \right] = \log_e \left[\frac{P(X_k = -a)}{1 - P(X_k = +a)} \right] \quad (2.48)$$

şeklinde yazılırsa, (2.48) denkleminden $P(X_k = -a)$ ifadesi

$$P(X_k = -a) = \frac{e^{L(X_k)}}{1 + e^{L(X_k)}} \quad (2.49)$$

olarak elde edilir. Benzer şekilde, $P(X_k = -a)$ ifadeside

$$P(X_k = a) = 1 - P(X_k = -a) = 1 - \frac{e^{L(X_k)}}{1 + e^{L(X_k)}} = \frac{1}{1 + e^{L(X_k)}} \quad (2.50)$$

şeklinde ifade edilebilir. BPSK modüleli sinyaller için X_k ifadesinin beklenen değeri (istatiksel-ortalaması)

$$\bar{X}_k = \sum_{x_b \in \{+a,-a\}} x_b . P(X_k = x_b) = a(P(X_k = +a) - P(X_k = -a)) \tag{2.51}$$

şeklinde hesaplanabilmektedir. Bu denklemde $P(X_k = +a)$ ve $P(X_k = -a)$ değerleri (2.51)'de yerlerine koyulursa, X_k ifadesinin ortalaması

$$\bar{X}_k = a\left(\frac{e^{L(X_k)}}{1+e^{L(X_k)}} - \frac{1}{1+e^{L(X_k)}}\right) = \frac{e^{L(X_k)}-1}{e^{L(X_k)}+1} = a\tanh(L(X_k)/2) \tag{2.52}$$

olarak bulunur.

3.1.7 Kanal Kestirimcinin Karmaşıklığının İncelenmesi:

Frekans domeninde kanal kestirimi için klasik linear-en-küçük-ortalama-karesel kestirimci ifadesi SFBC sistemleri için $\mathbf{P}_1 = X_e^{\dagger(q)}\mathbf{R}_e(n) - X_o^{(q)}\mathbf{R}_o(n)$, $\mathbf{P}_2 = X_o^{\dagger(q)}\mathbf{R}_e(n) + X_e^{(q)}\mathbf{R}_o(n)$ olmak üzere aşağıdaki biçimde verilmiştir[19].

$$\mathbf{H}_{\mu,e} = \underbrace{\mathbf{C}_{\mathbf{H}_{\mu,e}}(\mathbf{\Sigma}+\mathbf{C}_{\mathbf{H}_{\mu,e}})^{-1}}_{\text{Önden hesaplama}}\mathbf{P}_{\mu}. \tag{2.53}$$

Benzer ifade STBC sistemleri için $\mathbf{P}_1 = \hat{X}^{\dagger(q)}(n)\mathbf{R}(n) - \hat{X}^{(q)}(n+1)\mathbf{R}_o(n+1)$, $\mathbf{P}_2 = \hat{X}^{\dagger(q)}(n+1)\mathbf{R}(n) - \hat{X}^{(q)}(n)\mathbf{R}(n+1)$ olmak üzere

$$\mathbf{H}_{\mu} = \underbrace{\mathbf{C}_{\mathbf{H}_{\mu}}(\mathbf{\Sigma}+\mathbf{C}_{\mathbf{H}_{\mu}})^{-1}}_{\text{Önden hesaplama}}\mathbf{P}_{\mu} \tag{2.54}$$

şeklinde yazılabilir (2.53) ve (2.54) denklemlerindeki $\mathbf{C}_{\mathbf{H}_{\mu}}(\mathbf{\Sigma}+\mathbf{C}_{\mathbf{H}_{\mu}})^{-1}$ ve $\mathbf{C}_{\mathbf{H}_{\mu,e}}(\mathbf{\Sigma}_e+\mathbf{C}_{\mathbf{H}_{\mu,e}})^{-1}$ ifadeleri iletilen bilgi sembollerinden bağımsız olduğu için önceden matris tersinin alınmasıyla her bir kestirim ifadesi için kullanılabilir. OFDM Kanalına ait altbandlara ait ilişki işlevi ve kanaldaki gürültünün gücünün bilindiği varsayılırsa, SFBC ve STBC sistemleri için önceden hesaplanacak kısım $N_c/2 \times N_c/2$ boyutlu bir matris tersi alma işlemi gerektirmektedir. Bu matrisin $N_c/2\times1$'lik $\mathbf{P}_{\mu}$ vektörü ile çarpımı işlemi $N_c^2/4$ adet karmaşık çarpma işlemi gerektirmektedir.

Ancak önden hesaplama işlemi büyük boyutlu matris ters alma işlemi gerektirdiğinden işlem karmaşıklığı oldukça fazladır. Ayrıca OFDM Kanalına ait altbandlara ait ilişki işlevi ve kanaldaki gürültünün gücünün hakkındaki bilgide yapılan ufak hatalar matris ters alma işleminde katlamalı olarak artmaktadır[37]. Diğer taraftan, tersi alınan matrisin boyutunun artmasıyla yapılan hatada artmaktadır. Bu yüzden gerekli matris tersi alma işleminden kurtulmak için KL tabanlı bir yaklaşım önerilmiştir. KL açılımının kullanılmasıyla $\mathbf{H}$'a ait yinelemeli kanal kestirim ifadesi STBC sistemleri için

$$\hat{\mathbf{H}}_{\mu} = \mathbf{\Psi}(\mathbf{I} + \mathbf{\Sigma\Lambda}^{-1})^{-1}\mathbf{\Psi}^{\dagger}\mathbf{P}_{\mu} \tag{2.55}$$

şeklinde yazılabilir. Oluşan işlem karmaşıklığını daha çok azaltmak için $\mathbf{\Lambda}$ köşegen matrisinin en büyük r elemanına karşılık gelen $\mathbf{\Psi}$ matrisinin sadece r sütününu olarak düşük rank yaklaşımı gerçekleştirilebilir. $\mathbf{\Psi}$ matrisinin $(N_c/2)-r$ sütününu iptal ederek $(N_c/2)\times r$ boyutlu $\mathbf{\Psi}_r$ matrisini ve $r\times r$ boyutlu $\mathbf{\Sigma}_r$ matrisi kullanılarak kestirimcinin ifadesi

$$\hat{\mathbf{H}}_{\mu} = \mathbf{\Psi}_r \underbrace{(\mathbf{I} + \mathbf{\Sigma\Lambda}_r^{-1})^{-1}\mathbf{\Psi}_r^{\dagger}}_{\text{Önden hesaplama}}\mathbf{P}_{\mu} \tag{2.56}$$

şeklinde yazılabilir. Burada $(\mathbf{I} + \mathbf{\Sigma}_r\mathbf{\Lambda}_r^{-1})^{-1} = \left\{\frac{\lambda_o}{\lambda_o + \sigma^2}, \ldots, \frac{\lambda_{r-1}}{\lambda_{r-1} + \sigma^2}\right\}$ olarak ifade edilmektedir.

Önden hesaplama işlemi yapılması durumunda düşük ranklı kestirimci ilk olarak önden hesaplanmış $r\times(N_c/2)$ matrisi ile $(N_c/2)\times 1$ boyutlu $\mathbf{P}_{\mu}$ vektörünü çarpar. Daha sonra ise $(N_c/2)\times r$ boyutlu $\mathbf{\Psi}_r$ matrisi ile ilk işlemde hesaplanan $r\times 1$ matrisini çarpmaktadır. Sonuçta yapılan toplam karmaşık çarpma işlemi $N_c r$'dir. Ton başına düşen karmaşıklık olarak ifade edilirse önceden belirttiğimiz ters alma işleminde meydana gelen olumsuzlardan kurtulmanın yanında ton başına düşen karmaşık çarpma işlemi sayısı $N_c/4$'ten r'ye indirilmiştir. Benzer şekilde SFBC sistemleri içinde KL dönüşümü yapılıp rank düşürme işlemi uygulandığında $\mathbf{\Psi}_e$ matrisinin sadece "r" sütününu alarak elde edilen $\mathbf{\Psi}_{e,r}$ matrisi yardımıyla

$$\hat{\mathbf{H}}_{\mu} = \mathbf{\Psi}_{e,r} \underbrace{(\mathbf{I} + \mathbf{\Sigma\Lambda}_{e,r}^{-1})^{-1}\mathbf{\Psi}_{e,r}^{\dagger}}_{\text{Önden hesaplama}}\mathbf{P}_{\mu} \tag{2.57}$$

yazılabilir. Burada $\Lambda_{e,r}$, Λ_e köşegen matrisinin en büyük r elemanı içeren $r \times r$ 'lik köşegen matristir.

3.1.8 Değiştirilmiş Cramer-Rao Sınırı

Bir parametre kestirim algoritmasının erişibileceği en iyi başarım, Cramer-Rao alt sınırı (CRLB) belirlenir. CRLB'nın hesaplaması Fisher bilgi matrisinin (FIM) tersinin alınmasıyla gerçekleştirilebilir. Elde edilen kestirim ifadesinde öncül *(a priori)* olasılığının kullanılması durumda değiştirilmiş FIM ifadesi, $\mathbf{J}_P(\mathbf{G})$ öncül bilgiyi göstermek üzere,

$$\mathbf{J}_M(\mathbf{G}) \square -\underbrace{E\left[\frac{\partial^2 In\, p(\mathbf{R}|\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T}\right]}_{\mathbf{J}(\mathbf{G})} - \underbrace{E\left[\frac{\partial^2 In\, p(\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T}\right]}_{\mathbf{J}_P(\mathbf{G})} \quad (2.58)$$

şeklinde ifade edilebilir[15]. Kanal taban işlevlerinin katsayılarını ifade eden "$\mathbf{G}$" ve iletim anında sisteme eklenen gürültüyü ifade eden "$\mathbf{W}$"'nin birbirinden bağımsız olduğunu, gürültünün sıfır ortalamalı ve iletilen sembollerin kanal serpiştirici tarafından birbirinden ilişkisiz hale dönüştürüldüğü göz önüne alınırsa, $\mathbf{R}$ 'nin alınması durumunda $\mathbf{G}$ 'ye ait koşullu olasılık işlevi

$$p(\mathbf{R}|\mathbf{G}) = E_X\left\{p(\mathbf{R}|X,\mathbf{G})\right\} \square \frac{1}{\sigma^2} E_X\left\{(\mathbf{R} - X\tilde{\mathbf{\Psi}}\mathbf{G})^{\dagger}(\mathbf{R} - X\tilde{\mathbf{\Psi}}\mathbf{G})\right\} \quad (2.59)$$

şeklinde yazılabilir. Uzay-zaman blok kodlanmış sistemler $\mathbf{G} = \left[\mathbf{G}_1^T \mathbf{G}_2^T\right]^T$, $\tilde{\mathbf{\Psi}} = diag(\mathbf{\Psi}\,\mathbf{\Psi})$ biçiminde ifade edilirken, Uzay-frekans blok kodlanmış sistemler için ise $\mathbf{G}$ vektörü, $\mathbf{G} = \left[\mathbf{G}_{1,e}^T \mathbf{G}_{2,e}^T\right]^T$, $\tilde{\mathbf{\Psi}} = diag(\mathbf{\Psi}_e\,\mathbf{\Psi}_e)$ şeklinde ifade edilmektedir. Genel ifadeden ayrılmaksızın (2.58) denklemindeki $\mathbf{J}(\mathbf{G})$ 'ye ait türevleme işlemleri (2.59) denklemine göre yapılırsa

$$\frac{\partial In\, p(\mathbf{R}|\mathbf{G})}{\partial \mathbf{G}^T} = -\frac{1}{\sigma^2}(\mathbf{R} - X\tilde{\mathbf{\Psi}}\mathbf{G})^{\dagger} X\tilde{\mathbf{\Psi}} \quad (2.60)$$

$$\frac{\partial^2 In\, p(\mathbf{R}|\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T} = -\frac{1}{\sigma^2}\tilde{\mathbf{\Psi}}^{\dagger} X^{\dagger} X\tilde{\mathbf{\Psi}} \quad (2.61)$$

denklemleri bulunabilir. Ayrıca verici çeşitleme yöntemleri olarak kullandığımız STBC ve SFBC kodlarının ($X^{\dagger}X = \mathbf{I}$) ve kanalın taban işlevlerinin ($\tilde{\mathbf{\Psi}}^{\dagger}\tilde{\mathbf{\Psi}} = \mathbf{I}$) diklik özelliği sayesinde $\mathbf{J}(\mathbf{G})$ ifadesini basitleştirilmiş olarak

$$J(G) = -\frac{1}{\sigma^2}\mathbf{I} \tag{2.62}$$

olarak yazabiliriz. (2.58) denklemindeki ikinci terim olan $\mathbf{J}_P(\mathbf{G})$ ifadesinin türevleri ise $\mathbf{G}$ 'ye ait öncül olasılık işlevi $p(\mathbf{G}) \propto \exp(-\mathbf{G}^{\dagger}\Lambda\mathbf{G})$ yardımıyla

$$\frac{\partial In\, p(\mathbf{R}|\mathbf{G})}{\partial \mathbf{G}^T} = -\mathbf{G}^{\dagger}\tilde{\Lambda}^{-1}, \tag{2.63}$$

$$\frac{\partial^2 In\, p(\mathbf{R}|\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T} = -\tilde{\Lambda}^{-1} \tag{2.64}$$

olarak bulunabilir. Değiştirilmiş FIM'e ait hesaplamayı ifade eden (2.58) ilişkisine göre bulunan (2.62) ve (2.64) denklemlerinin negatif beklenti ifadeleri alınarak toplanırsa, değiştirilmiş FIM

$$\mathbf{J}_M(\mathbf{G}) = \frac{1}{\sigma^2}\mathbf{I} + \tilde{\Lambda}^{-1} \tag{2.65}$$

şeklinde bulunur. $\mathbf{J}_M(\mathbf{G})$ matrisinin tersi $\mathbf{G}$ 'ye ait MCRB ifadesini vermektedir

$$MCRB(\hat{\mathbf{G}}) = \mathbf{J}_M^{-1}(\mathbf{G}). \tag{2.66}$$

Buna göre elde edilen $MCRB(\hat{\mathbf{G}})$ ifadesi elemanları $\mathbf{J}_M(\mathbf{G})$ matrisinin elemanlarının tersine eşit olan köşegen matris olarak bulunur.

3.2 MC-CDMA SİSTEMLERİ

OFDM ve CDMA sistemlerinin kombinasyonunu öngören MC-CDMA sistemleri her iki sistemin avatajlarını kullanabilmek için 1993 yılında önerilmiştir [46],[47]. İlk dönemde bu sistemlerin başarımı kanal parametrelerinin alıcıda tamamen bilinmesi durumunda incelenmiştir. Yapılan çalışmalarda, MC-CDMA sistemlerinde bilgi sezimi için en küçük karesel kestirimci ve PIC (Paralel Karışım Engelleyici) sistemlerinin birleştirilmesini öngören (Tümleşik MMSE-PIC) iteratif yöntemlerin iteratif olmayan yöntemlerden üstünlüğü gösterilmiştir[48].

Kanal bilgisinin alıcıda bilinmemesi durumunda ise, mevcut önerilen bilgi sezim yöntemlerinin başarımları *Iraji* tarafından incelenmiştir[49]. Pilot tonlar yardımıyla yapılan en küçük karesel kanal kestiriminin (LS) başarımı, kanalın tamamiyle bilinmesi durumundaki başarımıyla karşılaştırılmış ve PIC alıcının başarımının sistemin ilklendirilmesinde kullanılan kanal kestirim ve bilgi sezimi işlemiyle oldukça ilgili olduğu görülmüştür. *Kuhn ve Iraji*'nin yaptığı bu çalışmalardan kanal kestirim işleminin iteratif bir biçimde alıcı yapısına katılması gerektiği görülmüştür.

Tüm kullanıcılara ait bilgi sezimi ve kanal kestirimi ne kadar iyi yapılırsa PIC alıcınında o kadar iyi çalışması, PIC yapısının ilklendirilebilmesi için genel sinyal modeline ait birleşik bilgi ve kanal kestirim işleminin gerekliliğini ortaya çıkarmıştır. EM algoritması yüksek karmaşıklı ve çözümü oldukça zor olan olasılık fonksyonlarının en büyüklemesi için önerilen iteratif bir yöntemdir. Karıştırılmış sinyallerin genel kestirimi için önerilen yapı [2] OFDM sistemlerinde yapılacak kanal kestirimi için önerilmiş SAGE biçimi ile karşılaştırılmıştır[28]. Ayrıca daha sonra uzay-frekans/uzay zaman blok kodlamalı OFDM sistemleri için MAP kanal kestirimide bu yöntemle gerçekleştirilmiştir [15].

EM algoritması CDMA sistemleri için *Nelson ve Poor* tarafından sürekli zaman kanal parametrelerinin kestiriminden ziyade bilgi sezimi işleminin gerçekleştirilmesi için önerilmiş ve genelleştirilmiş yapısı olan SAGE biçimi ile karşılaştırılmıştır[10]. Daha sonraları EM tabanlı alıcı yapıları yukarı link DS-CDMA sistemleri için *Kocian* tarafından incelenmiştir[51].

3.3 AŞAĞI LİNK MC-CDMA SİSTEMLERİ İÇİN ALICI YAPISI ÇALIŞMALARI

Bu çalışmada, frekans-seçici kanallar üzerinden çalışan aşağı link MC-CDMA sistemleri için alıcıda kanal bilgisinin olmaması durumunda, alıcı tasarımı problemine yaklaşımda bulunulmuştur. Alıcıdaki paralel karışım engelleyicinin (PIC) düzgün çalışması için gerekli birleşik MAP kanal kestirimi ve bilgi sezimi işleminin "Beklenti-Enbüyükleme (EM)" algoritması tabanlı olarak yapılması öngörülmüştür. Ayrıca, algoritmanın karmaşıklığının azaltılması için kanal değişimleri rastgele bir süreç olarak modellenip Karhunen-Loeve (KL) dik seri açılımı uygulanmıştır. Yapılan bilgisayar benzetimlerinde önerilen algoritmanın başarımı daha önce önerilen algoritmaların başarımlarıyla karşılaştırılmış ve KL açılımı sayesinde algoritmanın karmaşıklığının azaltılabileceği gösterilmiştir.

Mobil hücre içerisinde k. kullanıcıya ait gönderilecek MPSK modüleli sembol b^k ile gösterilmek üzere toplam aktif kullanıcı sayısının K adet olduğu varsayılmaktadır. Baz istasyonu N_c boyunda birimdik yayıcı seri yardımıyla b^k sembolünü yaymaktadır. Her bir kullanıcıya ait birimdik yayıcı seri $\mathbf{c}^k = (c_1^k, c_2^k, ..., c_{N_c}^k)^T$ vektörü ile gösterilmek üzere, vektörün her bir elemanı c_i^k, $\{-\frac{1}{\sqrt{N_c}}, \frac{1}{\sqrt{N_c}}\}$ değerlerini almaktadır. Bütün kullanıcılara ait yayılmış semboller toplanarak ters ayrık Fourier dönüşümü (IDFT) alınır ve daha sonra gönderilecek sinyale kanalda meydana gelebilecek gecikmeden fazla olacak şekilde çevirimli önek eklenmektedir. Yapılacak kanal kestiriminin başlangıç değerlerinin elde edilmesi için OFDM bloğu içine eşit aralıklarla pilot tonlar eklenir. Bu çalışmada, notasyon ve sinyal modelini basitleştirmek için kullanıcılar ait yayma miktarlarının aynı olduğu ve OFDM sisteminde kullanılan alt band sayısına eşit olduğu varsayılmıştır.

Gönderilen işaret frekans-seçici kanal üzerinden alıcıya geldiğinde, işarete ait önek kaldırılarak ayrık Fourier dönüşümü (DFT) uygulanır. Yapılan bu işlemlerden sonra, yayma kod matrisi $N_c \times K$ boyutlu olmak üzere $\mathbf{C} = [\mathbf{c}^1, ..., \mathbf{c}^K]$ ile, K adet kullanıcıya ait iletilecek sembolleri gösteren $\mathbf{b} = [b^1, ..., b^K]^T$ vektörü $K \times 1$ boyutlu olmak üzere elde edilen sinyal modeli

$$\mathbf{R} = H\mathbf{C}\mathbf{b} + \mathbf{W} \tag{2.67}$$

şeklinde yazılabilir. Burada H elemanları her bir altbanda ait karmaşık sönümleme katsayılarını ifade eden $N_c \times N_c$ boyutlu köşegen kanal matrisi, $\mathbf{W}$ ise kanalda eklenen toplamsal gürültüyü ifade eden $N_c \times 1$ boyutlu sıfır ortalamalı ve boyut başına $\sigma^2/2$ varyansa sahip Gauss dağılımlı vektörü ifade etmektedir. Yayma dizilerinin dikliği ise $\mathbf{C}^T\mathbf{C} = \mathbf{I}$ olarak yazılabilir. Bu çalışmada, gözlemlenen sinyal modeli (3.58)'e göre MAP-EM kanal kestirimi önerilecektir. MAP-EM kanal kestirimi aynı zamanda bilgi sezimi işleminin iteratif olrak iyileşmesini sağlamaktadır [41], [53]. Bir sonraki bölümde alıcı için önerilecek kanal kestirimcinin çıkarımından önce altbandları arasında ilişkiye sahip kanalın, KL açılımı yardımıyla ilişkisiz kanal parametleri şeklinde ifade edilebiceği gösterilecektir.

3.3.1 Kanal Frekans Cevabının KL Açılımı

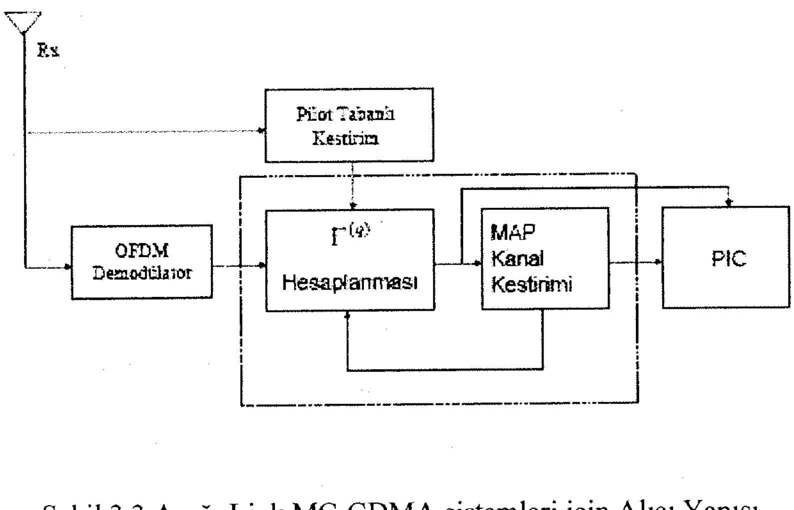
Alıcı ve verici anten arasındaki sönümlemeli kanalın her bir OFDM sembolü için değişmediği, diğer zaman aralığında ise değiştiği kabul edilmiştir. Kanalın frekans cevabı frekans-seçici olarak modellenmiş ve ilişkili altbandlara ait frekans cevabını gösteren $\mathbf{H} = [H_1, H_2, ..., H_{N_c}]^T$ vektörü KL açılımı yardımıyla modellenerek birimdik taban fonksyonlarının doğrusal kombinasyonu sayesinde ifade edilebilmektedir. Birim dik taban fonksyonları ψ_i ile ve bu açılıma ait ağırlık katsayılarıda G_i ile gösterilmek üzere, $\Psi = [\psi_1, \psi_2, \cdots, \psi_{N_c}]$ ve $\mathbf{G} = [G_1, \ \cdots, G_{N_c}]^T$ vektörleri tanımlanarak alıcı ve verici arasındaki mobil iletişim kanalı

$$\mathbf{H} = \Psi \mathbf{G} \tag{2.68}$$

şeklinde yazılabilir. Farklı Ψ 'ların (taban fonksyonlarının) kullanılması sonucu farklı özelliklere sahip kanallar modellenebilir. Kanala ait özilinti matrisi $\mathbf{C_H} = E\left[\mathbf{HH}^\dagger\right]$ olmak üzere

$$\mathbf{C_H} = \Psi \Lambda \Psi^\dagger \tag{2.69}$$

şeklinde öz değer ve öz vektörlerine ayrıştırılabilir. Burada Λ ağırlık katsayılarının varyanslarını gösteren köşegen matristir ve $\Lambda = E\{\mathbf{GG}^\dagger\}$ şeklinde ifade edilmektedir.



Şekil 3.3 Aşağı Link MC-CDMA sistemleri için Alıcı Yapısı.

3.3.2 EM tabanlı MAP kanal kestirimi

MC-CDMA sistemlerinde frekans domeninde gerçekleştirilebilen kanal denkleştirme işlemi kanalın frekans cevabına ihtiyaç duymaktadır. Zaman izgesinde kanal kestirim işlemi gerçekleştirilebilmesede, ayrık Fourier dönüşümü (DFT) tabanlı zaman izgesinde yapılan kanal kestirimcilerin örneklenmeyen uzaylı kanallar için izge sızıntısına sebeb olduğu ve hata düzlemi oluşturduğu bilinmektedir[38]. Diğer taraftan kanalın frekans cevabının denkleştirme işleminde direk kullanılması zaman domeninde yapılan kestirimin tekrardan frekans domenine geçirilmesini zorunlu kılmaktadır. KL açılımı yardımıyla (2.67) denklemi

$$\mathbf{R} = diag(\mathbf{Cb})\Psi\mathbf{G} + \mathbf{W} \tag{2.70}$$

şeklinde tekrar yazılabilir. (2.70) denkleminde $\mathbf{G}$ 'ye ait MAP kestirim işlemi

$$\hat{\mathbf{G}} = \arg\max_{\mathbf{G}} p(\mathbf{G} \mid \mathbf{R}) \tag{2.71}$$

şeklinde yazılabilir. (2.71) denkleminin enbüyükleme işleminin matematiksel olarak çözümlenmesi oldukça zordur. EM algoritması yardımıyla çözüm iteratif olarak gerçekleştirilebilir. Çözümlenmesi gereken bu problemde *tam* olan bilgi $\chi = \{\mathbf{R}, \mathbf{b}\}$, kestirilecek vektör $\mathbf{G}$ ve *eksik* (tam olmayan) bilgi ise $\mathbf{R}$ şeklinde tanımlanmaktadır. Bu durumda $\mathbf{G}$ 'ye ait kestirim işlemi

$$Q(\mathbf{G} \mid \mathbf{G}^{(q)}) = \sum_{\mathbf{b}} p(\mathbf{R}, \mathbf{b}, \mathbf{G}^{(q)}) \log p(\mathbf{R}, \mathbf{b}, \mathbf{G}) \tag{2.72}$$

Kullback-Leibler denkleminin iteratif olarak en büyükleme problemine dönüşmektedir. Burada $\mathbf{G}^{(q)}$ ifadesi $\mathbf{G}$ 'nin q. adımdaki kestirim işlemini göstermektedir. (2.71)denkleminde verilen koşullu olasılık fonksyonunun en büyükleme işlemindeki monotik artış sağlanabilir. (2.72) denklemindeki $\log p(\mathbf{R}, \mathbf{b}, \mathbf{G})$ ifadesi *Bayes* kuralı yardımıyla,

$$\log p(\mathbf{R},\mathbf{b},\mathbf{G}) = \log p(\mathbf{b}|\mathbf{G}) + \log p(\mathbf{R}|\mathbf{b},\mathbf{G}) + \log p(\mathbf{G}) \tag{2.73}$$

şeklinde ifade edilebilir. (2.73) denklemindeki soldan ilk terim $\mathbf{b}$ ve $\mathbf{G}$ vektörlerinin bağımsız olması ve $\mathbf{b}$ 'lerin eşit olasılıklı olmasından dolayı en büyükleme işlemini etkilememektedir. Son terim $\mathbf{G}$ 'nin alıcıdaki birleşik olasılık yoğunluk işlevi

$$p(\mathbf{G}) \propto \exp(-\mathbf{G}^{\dagger} \Lambda^{-1} \mathbf{G}) \tag{2.74}$$

şeklinde ifade edilebilir. Denklemdeki ikinci terim ise iletilen semboller $\mathbf{b}$, ayrık kanal tanımlanması $\mathbf{G}$ 'ler ve bağımsız gürültü bileşenleri göz önüne alınırsa, alıcıda gözlemlenen $\mathbf{R}$ sinyaline ait koşullu olasılık yoğunluk işlevi

$$p(\mathbf{R} \mid \mathbf{b},\mathbf{G}) \propto \exp\left[-(\mathbf{R}-diag(\mathbf{C}\mathbf{b})\Psi\mathbf{G})^{\dagger}\Sigma^{-1}(\mathbf{R}-diag(\mathbf{C}\mathbf{b})\Psi\mathbf{G})\right] \quad (2.75)$$

şeklinde yazılabilir. Σ burada $N_c \times N_c$ boyutlu ve her bir elemanı $\Sigma[k,k]=\sigma^2 \quad k=1,2,\cdots,N_c$ olmak üzere kanalda eklenen gürültünün varyanslarını ifade etmektedir. (2.74) ve (2.75) denklemleri yardımıyla (2.72) denkleminin $\mathbf{G}$ 'ye göre türev alınarak sıfıra eşitlenmesi sonucu

$$\sum_{b} p(\mathbf{R},\mathbf{b},\mathbf{G}^{(q)})(\Psi^{\dagger}diag(\mathbf{b}^{\dagger}\mathbf{C}^{T})\Sigma^{-1}(\mathbf{R}-diag(\mathbf{C}\mathbf{b})\Psi\mathbf{G})-\Lambda^{-1}\mathbf{G})=0 \quad (2.76)$$

denklemi elde edilir. (2.76) denkleminde, denklem eşitliği bozulmayacak şekilde $p(\mathbf{R},\mathbf{b},\mathbf{G}^{(q)})$ ifadesi yerine $p(\mathbf{b}|\mathbf{R},\mathbf{G}^{(q)})$ ifadesi konulabilir. $\mathbf{b}$ 'ler üzerinden ortalaması alınan (2.76) denkleminin $(q+1)$. iterasyon için $\mathbf{G}$ 'ye ait çözümünü ifade eden $\hat{\mathbf{G}}^{(q+1)}$ vektörü, q. adımda bilgi sembollerine ait sonsal olasılıkları $\Gamma^{(q)}=[\Gamma^{(q)}(1),\Gamma^{(q)}(2),...,\Gamma^{(q)}(K)]$ vektörü göstermek üzere

$$\hat{\mathbf{G}}^{(q+1)}=(\mathbf{T}^{\dagger(q)}\mathbf{T}^{(q)}+\Sigma\Lambda^{-1})^{-1}\mathbf{T}^{\dagger(q)}\mathbf{R} \quad (2.77)$$

$$\mathbf{T}^{(q)}=diag(\mathbf{C}\Gamma^{(q)})\Psi, \quad (2.78)$$

şeklinde ifade edilebilir. $\Gamma^{(q)}$ vektörünün elemanları olan $\Gamma^{(q)}(k)$

$$\Gamma^{(q)}(k)=\sum_{b\in S_k} bP(b^k=b \mid \mathbf{R},\mathbf{G}^{(q)}) \quad (2.79)$$

şeklinde ifade edilebilir. Gönderilen sembol dizisi için $\hat{\mathbf{b}}=0 \quad \mathbf{C}_b=\mathbf{I}$ varsayımları yapılarak $\Gamma^{(q)}$ ifadesi QPSK modüleli sinyaller için

$$\hat{\mathbf{Z}}^{(q)}=\mathbf{C}^{T}(\hat{H}^{\dagger(q)}\hat{H}^{(q)}+\sigma^2\mathbf{I})^{-1}\hat{H}^{\dagger(q)}\mathbf{R}$$

olmak üzere

$$\Gamma^{(q)} = \frac{1}{\sqrt{2}} \tanh\left[\frac{\sqrt{2}}{\sigma^2} Re(\hat{\mathbf{Z}}^{(q)})\right] + \frac{j}{\sqrt{2}} \tanh\left[\frac{\sqrt{2}}{\sigma^2} Im(\hat{\mathbf{Z}}^{(q)})\right] \quad (2.80)$$

hesaplanabilir. Son olarak q. adımda herbir kullanıcılara ait giden sembol dizisini ifade eden $\mathbf{b}$'vektörü, "csign" ifadesi $\text{csign}(a + jb) = \text{sign}(a) + j\text{sign}(b)$ şeklinde tanımlanmak üzere

$$\hat{\mathbf{b}}^{(q)} = \frac{1}{\sqrt{2}} csign(\Gamma^{(q)}) \quad (2.81)$$

şeklinde hesaplanabilir.

3.3.3 Belli sayıda KL katsayısı kullanabilme:

Taban fonksyonları içinden $\mathbf{C_H}\Psi = \Psi\Lambda$ denklemini sağlayacak şekilde "r" tane taban fonksiyonu şeçilmesiyle $\mathbf{G}$ hesaplama işlemi yapılabilirki bu işleme katsayı kesimi işlemi denir. Oluşacak hata "$\varepsilon_r = \mathbf{G} - \mathbf{G}_r$" şeklinde tanımlanırsa açılan taban fonksiyonlara ait ilk en büyük "r" adet öz değer alınmasıyla ortalama karesel en küçük katsayı kesme hatasını ifade eden $\frac{1}{N_c} E[\varepsilon_r^{\dagger}\varepsilon_r]$ hesaplama işlemi

$$\frac{1}{N_c - r} E[\varepsilon_r^{\dagger}\varepsilon_r] = \frac{1}{N_c - r} \sum_{i=r}^{N_c} \lambda_i \quad (2.82)$$

şeklinde bulunabilir. KL açılımındaki katsayı kesme işlemi aslında düşük ranklı yaklaşımı oluşturmaktadır. Böylece Λ matrisinin "r" ranklı yaklaşımınında $N_c - r$ adet eleman ihmal edilerek $\Lambda_r = diag\{\lambda_1, \lambda_2, \cdots, \lambda_r\}$ şeklinde tanımlanabilir. İhmal edilen $N_c - r$ adet özdeğer, geri kalan $\{\lambda_l\}_{l=1}^{r}$ öz değerlere kıyasla oldukça küçüktür. Aslında yapılan işlem Λ matrisini açılımda etkin olan ve olmayan şekilde ikiye ayırarak etkin kısımla $\mathbf{G}$'nin hesaplamasına ait iyi bir yaklaşım yapmaktır.

İşlemsel Karmaşıklık : Kanal kestirimi $\mathbf{H}$ için önerilen geleneksel LMMSE kestirimi

$$\hat{\mathbf{H}} = \mathbf{C_H} \underbrace{\left[\mathbf{C_H} + \Sigma(\text{diag}(\mathbf{Cb})^{\dagger}\text{diag}(\mathbf{Cb}))^{-1}\right]^{-1}}_{\text{'}O(N_c^3)\text{' islemsel karmasiklik}} [\text{diag}(\mathbf{Cb})]^{-1}\mathbf{R} \quad (2.83)$$

olarak hesaplanabilir. (2.83) denklemindeki $\left[\mathbf{C}_\mathbf{H} + \Sigma(\mathrm{diag}(\mathbf{Cb})^{\dagger}\mathrm{diag}(\mathbf{Cb}))^{-1}\right]^{-1}$ ifadesi, gönderilen bilgi sembollerine bağlı olduğundan önceden hesaplanamaz ve alıcıdaki hesaplama işlemi matris tersi alınması gerekliliğinden dolayı, işlemsel karmaşıklığı oldukça fazladır. Ayrıca $\mathbf{C}_\mathbf{H}$ ve Σ değerlerinde yapılabilecek küçük hatalar, matris ters alma işleminden dolayı kanal kestiriminde büyük değişimlere sebep olabilmektedir. Dahası bu etki, tersi alınan matrisin boyutuyla orantılı olarak artmaktadır [37]. Bu açıdan KL açılımlı yaklaşım, büyük ölçekli matris alma işleminden kurtulmak için önerilmiştir. (2.68) ve (2.77) denklemleri ile $\mathbf{H}$ vektörüne ait iteratif kestirim işlemi

$$\hat{\mathbf{H}}^{(q+1)} = \Psi(\mathbf{T}^{\dagger(q)}\mathbf{T}^{(q)} + \Sigma\Lambda^{-1})^{-1}\mathbf{T}^{\dagger(q)}\mathbf{R} \tag{2.84}$$

şeklinde ifade edilebilir. Ancak elde edilen bu form, OFDM sistemlerindeki gibi direk ters alma işleminden kurtulabilmeyi sağlayamamakta ve mevcut LMMSE kestiriminde elde edilecek işlemsel karmaşıklıktan da az değildir. Bu açıdan elde edilen (2.84) denklemi tekrardan

$$\hat{\mathbf{H}}^{(q+1)} = \Psi\Lambda(\Lambda\mathbf{T}^{\dagger(q)}\mathbf{T}^{(q)}\Lambda + \Sigma\Lambda)^{-1}\Lambda\mathbf{T}^{\dagger(q)}\mathbf{R} \tag{2.85}$$

şeklinde yazılıp Λ matrisinin en büyük r adet elemanına ($\Lambda_r = diag\{\lambda_1, \lambda_2, \cdots, \lambda_r\}$) karşılık gelen Ψ ve $\mathbf{T}$ matrislerinin sütun elemanları alınarak oluşturulmuş, $N_c \times r$ boyutlu Ψ_r and $\mathbf{T}_r$ matrisleri yardımıyla düşük ranklı yaklaşım

$$\hat{\mathbf{H}}^{(q+1)} = \Psi_r\Lambda_r\underbrace{(\Lambda_r\mathbf{T}_r^{\dagger(q)}\mathbf{T}_r^{(q)}\Lambda_r + \Sigma_r\Lambda_r)^{-1}}_{\text{"}O(r^3)\text{" islemsel karmasiklik}}\Lambda_r\mathbf{T}_r^{\dagger(q)}\mathbf{R} \tag{2.86}$$

şeklinde hesaplanabilir. Burada Σ_r elemanları σ^2 eşit olan $r \times r$ boyutlu köşegen matristir. Bu durumda (2.86) denklemi

$$\hat{\mathbf{H}}^{(q+1)} = \Psi_r(\mathbf{T}_r^{\dagger(q)}\mathbf{T}_r^{(q)} + \Sigma_r\Lambda_r^{-1})^{-1}\mathbf{T}_r^{\dagger(q)}\mathbf{R} \tag{2.87}$$

şeklinde yazılabilir. Açılımdaki etkin öz değerlerin kullanımıyla elde edilen (2.84) denklemine göre, kanal kestiriminde tersi alınması gerekli olan matrisin boyutu oldukça küçülmektedir. Ayrıca gönderilen her bir OFDM sembolünde pilot tonlarlarla birlikte bozulan OFDM örnekleri sayesinde algoritma kanaldaki değişimlere çabuk yanıt verebilme özelliğine sahiptir.

3.3.4 Paralel Karışım Engelleyici (PIC)

Gönderilen sembollerin kestirimini ifade eden $\hat{\mathbf{b}}$ vektörü PIC alıcı bloğuna girer. Bu blokta k. kullanıcı hariç diğer kullanıcılara ait sinyallerin toplamı

$$\mathbf{R}_{int}^{k} = \hat{H}\mathbf{C}\hat{\mathbf{b}} \quad \text{for} \quad \hat{b}^{k} = 0 \tag{2.88}$$

şeklinde yazılabilir. k. kullanıcıya göre karışım sinyallerini ifade eden $\mathbf{R}_{int}^{k}$ vektörü, gözlemlenen $\mathbf{R}$ sinyalinden çıkartılarak tek kullanıcı sezicisine girerse, k. kullanıcıya ait PIC alıcısı, b_{pic}^{k} değerlerini

$$b_{pic}^{k} = (\mathbf{c}^{k})^{T}[\hat{H}^{\dagger}(\mathbf{R} - \mathbf{R}_{int}^{k})] \quad \text{for} \quad k = 1,\ldots,K \tag{2.89}$$

şeklinde hesaplayabilir. Son iterasyonda QPSK modülasyonu için sezilen semboller

$$\hat{b}_{pic}^{k} = \frac{1}{\sqrt{2}}\text{csign}(b_{pic}^{k}) \quad \text{for} \quad k = 1,\ldots,K \tag{2.90}$$

şeklinde bulunabilir.

3.4 YUKARI LİNK MC-CDMA SİSTEMLERİ İÇİN ALICI YAPISI ÇALIŞMALARI

Bu çalışmada, frekans-seçici kanallarda çalışan yukarı link MC-CDMA sistemleri için birleşik kanal kestirimi ve bilgi sezimi işlemini gerçekleştirebilecek alıcı tasarımı problemi üzerinde çalışılmıştır. Beklenti-Enbüyükleme (EM) algoritması tabanlı alıcı yapıları, her bir kullanıcıya ait bilgi sezimi ve kanal kestirim işleminin ortak olasılık fonksyonunu en büyükleme işlemi için kullanılmış ve problemin kapalı formda çözümü gerçekleştirilmiştir[50]. Önerilen EM tabanlı alıcıların BER başarımı literatürde daha önce önerilen alıcı yapılarıyla karşılaştırılmış ve önerilen alıcı yapısının başarım üstünlüğü gösterilmiştir.

3.4.1 Yukarı Link MC-CDMA Sinyal Modeli

Temel bandda çalışan P adet taşıyıcıya sahip yukarı link MC-CDMA sistemi için K adet aktif kullanıcının olduğu varsayılmıştır. Sistemdeki k. kullanıcının ileteceği sembol frekans domeninde

$P \times 1$ boyutlu yayıcı dizi olan $\mathbf{c}_k$ ile yayılmaktadır. Yayıcı kodla yayılan iletilecek sembol, P noktalı IDFT işleminden sonra paralelden seriye çevrilir ve yayılmış diziye kanalda meydana gelebilecek gecikmeden daha fazla olacak şekilde çevirimli önek eklenir. Bu çalışmada, notasyon ve sinyal modelini basitleştirmek için her bir kullanıcının göndereceği sembolleri yayan yayıcı kodun uzunluklarının aynı ve sistemde kullanılan alt band sayısına eşit olduğu varsayılmıştır. Vericiden çıkan k. kullanıcıya ait sinyal L adet çoklu yola sahip kanal üzerinden iletilmektedir. Kanal katsayılarının bir sembol boyunca değişmediği ancak sembolden sembole değişmekte ve alıcıda kanala ait ikinci dereceden istatistiksel özelliklerin bilindiği varsayılmaktadır.

Alıcıda alınan sinyal, ilk olarak seriden paralele çevirilir ve önek kaldırılarak DFT işlemi uygulanır. $b_k(m)$, k. kullanıcıya ait m. sembolü, $\mathbf{F} \in C^{P \times L}$ olmak üzere (k,l). elemanı $\frac{1}{\sqrt{P}} e^{-j2\pi kl/P}$ şeklinde tanımlanan DFT matrisini göstermek üzere, sinyal modeli

$$\mathbf{y}(m) = \sum_{k=1}^{K} b_k(m)\mathbf{C}_k \mathbf{F}\mathbf{h}_k + \mathbf{w}(m) \quad m = 1,2,\ldots,M \tag{2.91}$$

şeklinde yazılabilir. Burada $\mathbf{w}(m)$ kanalda eklenen toplamsal gürültüyü ifade eden sıfır ortalamalı ve boyut başına $\sigma^2/2$ varyansa sahip gauss dağılımlı $P \times 1$ boyutlu vektör, $\mathbf{C}_k = \mathrm{diag}(\mathbf{c}_k)$ ise k. kullanıcıya ait yayma kodunu gösteren elemanları $\{-\frac{1}{\sqrt{P}}, \frac{1}{\sqrt{P}}\}$ değerlerini alan $\mathbf{c}_k = [c_{k1}, c_{k1}, \ldots, c_{kP}]^T$ şeklinde tanımlanmıştır. Toplamda K adet kullanıcının M adet sembolü göndermek üzere gözlemlenen $\mathbf{y}(m)$ sinyalini vektör formunda $\mathbf{y} = [\mathbf{y}^T(1), \ldots, \mathbf{y}^T(M)]^T$ olacak şekilde yazılırsa alınan sinyal modeli

$$\mathbf{y} = \begin{bmatrix} b_1(1)\mathbf{C}_1\mathbf{F} & \cdots & b_K(1)\mathbf{C}_K\mathbf{F} \\ \vdots & \ddots & \vdots \\ b_1(M)\mathbf{C}_1\mathbf{F} & \cdots & b_K(M)\mathbf{C}_K\mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix} + \begin{bmatrix} \mathbf{w}(1) \\ \vdots \\ \mathbf{w}(M) \end{bmatrix} \tag{2.92}$$

şeklinde elde edilebilir. Bu denklem daha basit formda

$$\mathbf{y} = \mathbf{A}\mathbf{h} + \mathbf{w} \tag{2.93}$$

olarak yazılabilir. Burada $\Sigma_{\mathbf{h}_k} = E[\mathbf{h}_k \mathbf{h}_k^{\dagger}]$ olmak üzere her bir kanalın dağılımı $\mathbf{h}_k \sim \mathcal{N}(0, \Sigma_{\mathbf{h}_k})$ şeklinde modellenmiş ve (2.93) denklemindeki bütün kanalları ifade eden $\mathbf{h}$ vektörüne ait dağılım ise $\Sigma_{\mathbf{h}} = \text{diag}[\Sigma_1, \cdots, \Sigma_K]$ olmak üzere $\mathbf{h} \sim \mathcal{N}(0, \Sigma_{\mathbf{h}})$ şeklinde yazılabilmektedir.

3.4.2 Tümleşik Bilgi Sezimi ve Kanal Kestirimi

Bu çalışmada, (2.91) denkleminde verilen sinyal modeli için tümleşik bilgi sezimi ve kanal kestirimini gerçekleştirecek EM tabanlı iteratif algoritmanın çıkarımı yapılmıştır. (2.91) denklemindeki karıştırılmış sinyallerin toplamını ifade eden sinyal modelininin çözümlenmesi EM algoritmasıyla mümkündür[6]. Toplamsal olarak elde edilen $\mathbf{y}(m)$ sinyalinin çözümlenmesi

$$\mathbf{y}(m) = \sum_{k=1}^{K} \mathbf{x}_k(m), m = 1, 2 \cdots, M \quad (2.94)$$

$$\mathbf{x}_k(m) = b_k(m)\mathbf{C}_k \mathbf{F} \mathbf{h}_k + \mathbf{w}_k(m). \quad (2.95)$$

şeklinde yazılabilir. Burada $\mathbf{x}_k(m)$, gözlemlenen sinyalde k. kullanıcının birim basamak cevabı $\mathbf{h}_k$ ile ifade edilmiş kanal üzerinden gönderdiği sembole ait parçasını göstermektedir. $\mathbf{w}_k(m)$, sisteme etki eden $\mathbf{w}(m)$ gürültüsünün k. kullanıcıya ait kısmını göstermekte olup herbir kullanıcıya düşen gürültülerin toplam varyansları $N_0\beta_k$ olmak üzere $\sum_{k=1}^{K} \mathbf{w}_k(m) = \mathbf{w}(m)$ şeklinde tanımlanmıştır. $N_0\beta_k$ değerleri, $\sum_{k=1}^{K} \beta_k = 1$, $0 \le \beta_k \le 1$ koşullarını sağlamak üzere $\mathbf{x}_k(m)$ sinyali için belirlenmiş gürültü gücünün parçasını belirlemektedir.

Bu durumda çözümlenmesi gereken problem, gözlemlenen $\mathbf{y}$ bilgisinden her bir kullanıcının sinyalini gönderdiği $\mathbf{h}_k$ kanalının ve iletilen sembol dizisinin $\mathbf{b} = \{b_k(m)\}_{k=1,m=1}^{K,M}$ bulunmasıdır. EM algoritmasında gözlemlenen $\mathbf{y}$ dizisi "*eksik*" bilgi olarak, $\mathbf{x}_k = [\mathbf{x}_k(1), ..., \mathbf{x}_k(M)]^T$ $k = 1, 2, \cdots, K$ ise "*tüm*" bilgi olmak üzere, $\chi = \{(\mathbf{x}_1, \mathbf{h}_1), (\mathbf{x}_1, \mathbf{h}_1), ..., (\mathbf{x}_K, \mathbf{h}_K)\}$ şeklinde tanımlanmıştır.

3.4.3 EM algoritmasının uygulanması :

Verilen *tüm* bilgiden $\chi = \{(\mathbf{x}_1, \mathbf{h}_1), (\mathbf{x}_1, \mathbf{h}_1), ..., (\mathbf{x}_K, \mathbf{h}_K)\}$, $\mathbf{b}$'nin kestirimi için koşullu olasılık fonksiyonu

$$\log p(\chi \mid \mathbf{b}) = \sum_{k=1}^{K} \log p(\mathbf{x}_k, \mathbf{h}_k \mid \mathbf{b}_k) \tag{2.96}$$

şeklinde tanımlanmıştır. Burada $\mathbf{b}_k = [b_k(1), b_k(2), \cdots, b_k(M)]$ olmak üzere $p(\mathbf{x}_k, \mathbf{h}_k \mid \mathbf{b}_k)$ ifadesi Bayes kuralından

$$\log p(\mathbf{x}_k, \mathbf{h}_k \mid \mathbf{b}_k) = \log p(\mathbf{x}_k \mid \mathbf{b}_k, \mathbf{h}_k) + \log p(\mathbf{h}_k \mid \mathbf{b}_k) \tag{2.97}$$

olarak yazılabilir. Gönderilen bilgi dizisi $\mathbf{b}_k$ ve kanal cevabı $\mathbf{h}_k$ birbirinden bağımsız olduğu için (2.97) denklemindeki $\log p(\mathbf{h}_k \mid \mathbf{b}_k)$ ihmal edilebilir. Bu durumda EM algoritmasında yapılması gereken ilk işlem "*beklenen değer bulma*" işlemi olup logaritmik-olasılık fonksiyonun ortalamasının alınmasıdır. Gözlemlenmiş $\mathbf{y}$ 'den χ 'e ait koşullu beklenti hesabı yapılırsa i. iterasyon için $\mathbf{b}$ 'nin değeri

$$Q\left(\mathbf{b} \mid \mathbf{b}^{(i)}\right) = E\left\{\log p(\chi \mid \mathbf{b} \mid \mathbf{y}, \mathbf{b}^{(i)}\right\} \tag{2.98}$$

şeklinde yazılabilir. (2.96) denklemindeki $\log p(\chi \mid \mathbf{b})$ ifadesinin özel formunu dikkate alarak (2.98) denklemi

$$Q\left(\mathbf{b} \mid \mathbf{b}^{(i)}\right) = \sum_{k=1}^{K} Q_k(\mathbf{b}_k \mid \mathbf{b}^{(i)}) \tag{2.99}$$

şeklinde ayrıştırılabilmektedir. Buradaki $Q_k(\mathbf{b}_k \mid \mathbf{b}^{(i)})$ ifadesi Bayes kuralı ve (2.97) denklemi göz önüne alınarak

$$Q_k(\mathbf{b}_k \mid \mathbf{b}^{(i)}) = E\left\{\log p(\mathbf{x}_k \mid \mathbf{b}_k, \mathbf{h}_k) \mid \mathbf{y}, \mathbf{b}^{(i)}\right\} \tag{2.100}$$

şeklinde tekrar yazılabilir. (2.95) denkleminde mevcut $\mathbf{b}_k$ dan bağımsız terimler ihmal edilerek $\log p(\mathbf{x}_k \mid \mathbf{b}_k, \mathbf{h}_k)$ ifadesi

$$\log p(\mathbf{x}_k \mid \mathbf{b}_k, \mathbf{h}_k) \square \sum_{m=1}^{M} \Re\{b_k(m)\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{x}_k(m)\} \tag{2.101}$$

olarak yazılabilir. (2.101) denklemindeki ifade, (2.100) denkleminde yerine koyulursa

$$(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{x}_k(m))^{[i]} E\left\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{x}_k(m) \mid \mathbf{y}, \mathbf{b}^{(i)}\right\} \tag{2.102}$$

olmak üzere, $Q_k(\mathbf{b}_k \mid \mathbf{b}^{(i)})$ ifadesi

$$Q_k(\mathbf{b}_k \mid \mathbf{b}^{(i)}) = \sum_{m=1}^{M} \Re\{b_k(m)(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{x}_k(m))^{[i]}\} \tag{2.103}$$

şeklinde yazılabilir. (2.102) denklemindeki şartlı beklenti kuralının uygulanması durumunda

$$\begin{aligned}(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{x}_k(m))^{[i]} &= E\{\mathbf{h}_k^{\dagger}\ E(\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{x}_k(m) \mid \mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h}) \mid \mathbf{y}, \mathbf{b}^{(i)}\} \\ &= E\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T} E(\mathbf{x}_k(m) \mid \mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h}) \mid \mathbf{y}, \mathbf{b}^{(i)}\}\end{aligned} \tag{2.104}$$

denklemi elde edilir. Verilen $\mathbf{y}$, $\mathbf{h}$ ve $\mathbf{b} = \mathbf{b}^{(i)}$ için $\mathbf{x}_k(m)$'e ait koşullu dağılım Gauss olmakta ve $(b_k(m))^{[i]} \square E(b_k(m) \mid \mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h})$ olmak üzere

$$E(\mathbf{x}_k(m) \mid \mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h}) = (b_k(m))^{[i]}\mathbf{C}_k\mathbf{F}\mathbf{h}_k + \beta_k\left(\mathbf{y}(m) - \sum_{j=1}^{K}(b_j(m))^{[i]}\mathbf{C}_j\mathbf{F}\mathbf{h}_j\right) \tag{2.105}$$

şeklinde hesaplanabilmektedir. (2.105) denklemindeki ifadeler, (2.104) denkleminde yerine koyularak $\mathbf{F}^{\dagger}\mathbf{F} = \mathbf{I}$ ve $\mathbf{C}_k^{T}\mathbf{C}_k = \frac{1}{P}\mathbf{I}$ olmak üzere, (2.104) denklemi tekrardan

$$(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{x}_k(m))^{[i]} = \frac{1}{P}(b_k(m))^{[i]} E\{\mathbf{h}_k^{\dagger}\mathbf{h}_k \mid \mathbf{y}, \mathbf{b}^{(i)}\} + \beta_k E\{\mathbf{h}_k^{\dagger} \mid \mathbf{y}, \mathbf{b}^{(i)}\}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{y}(m)$$

$$-\beta_k \sum_{j=1,j\neq k}^{K} (b_j(m))^{[i]} E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{F}\mathbf{h}_j \mid \mathbf{y},\mathbf{b}^{(i)}\} \tag{2.106}$$

olarak yazılabilir. Sistemdeki gürültünün dağılımının $\mathbf{w} \sim N(0,\Sigma^2\mathbf{I})$, $\mathbf{h}$'ın öncül pdf'i ise $\mathbf{h} \sim N(0,\Sigma_{\mathbf{h}})$ olduğu için, verilmiş $\mathbf{y}$ ve $\mathbf{b}^{(i)}$'ye göre $\mathbf{h}$'a ait koşullu pdf

$$p(\mathbf{h} \mid \mathbf{y},\mathbf{b}^{(i)}) \propto p(\mathbf{y} \mid \mathbf{h},\mathbf{b}^{(i)})\,p(\mathbf{h}) \propto \exp\left[-\frac{1}{\sigma^2}(\mathbf{y}-\mathbf{A}\mathbf{h})^\dagger(\mathbf{y}-\mathbf{A}\mathbf{h}) - \mathbf{h}^\dagger\Sigma_{\mathbf{h}}^{-1}\mathbf{h}\right] \tag{2.107}$$

şeklinde yazılabilir. Koşullu pdf olan $p(\mathbf{h} \mid \mathbf{y},\mathbf{b}^{(i)})$

$$\mu_{\mathbf{h}}^{(i)} = \frac{1}{\sigma^2}\Sigma_{\mathbf{h}}^{(i)}\mathbf{A}^{(i)\dagger}\mathbf{y}, \quad \Sigma_{\mathbf{h}}^{(i)} = \left[\Sigma_{\mathbf{h}}^{-1} + \frac{1}{\sigma^2}(\mathbf{A}^{(i)})^\dagger\mathbf{A}^{(i)}\right]^{-1} \tag{2.108}$$

olmak üzere Gauss dağılımına sahip olduğu gösterilebilir. (2.106) denklemindeki sağdan ilk terimdeki $E\{\mathbf{h}_k^\dagger\mathbf{h}_k \mid \mathbf{y},\mathbf{b}^{(i)}\}$ ifadesi, $\Sigma_{\mathbf{h}}[i,j]$, $\Sigma_{\mathbf{h}}$ matrisinin (i,j). elemanını göstermek üzere

$$(\|\mathbf{h}_k\|^2)^{[i]} \triangleq E\{\mathbf{h}_k^\dagger\mathbf{h}_k \mid \mathbf{y},\mathbf{b}^{(i)}\} = \mathrm{tr}\left[\Sigma_{\mathbf{h}}^{(i)}[k,k] + \mu_h^{(i)}[k]\mu_h^{(i)}[k]^\dagger\right] \tag{2.109}$$

şeklinde hesaplanabilir. (2.106) denklemindeki ikinci beklenti hesabı

$$(\mathbf{h}_k)^{[i]} \triangleq E\{\mathbf{h}_k \mid \mathbf{y},\mathbf{b}^{(i)}\} = \mu_h^{(i)}[k] \tag{2.110}$$

şeklinde bulunabilir. (2.106) denklemindeki son terim olan $E\{\mathbf{h}_k^\dagger\mathbf{F}^\dagger\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j \mid \mathbf{y},\mathbf{b}^{(i)}\}$ ifadesinin hesaplaması için $\Psi_j \triangleq \mathbf{C}_j\mathbf{F}$, $\mathbf{s}_j \triangleq \Psi_j\mathbf{h}_j$ tanımlamaları yapılırsa

$$\Sigma_s^{(i)} \triangleq E[\mathbf{s}\mathbf{s}^\dagger \mid \mathbf{y},\mathbf{b}^{(i)}] = E[\Psi\mathbf{h}\mathbf{h}^\dagger\Psi^\dagger \mid \mathbf{y},\mathbf{b}^{(i)}] = \Psi\Sigma_{\mathbf{h}}^{(i)}\Psi^\dagger \tag{2.111}$$

şeklinde ifade edilebilir. Buradan (2.111) denklemi, $\mu_s^{(i)} \triangleq \Psi\mu_h^{(i)}$ olmak üzere

$$E\{\mathbf{h}_k^\dagger\mathbf{F}^\dagger\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j \mid \mathbf{y},\mathbf{b}^{(i)}\} = E[\mathbf{s}^\dagger\mathbf{s} \mid \mathbf{y},\mathbf{b}^{(i)}] = \mathrm{tr}\left[\Sigma_s^{(i)}[k,j] + \mu_s^{(i)}[k]\mu_s^{(i)}[j]^\dagger\right] \tag{2.112}$$

olarak bulununur.

En Büyükleme Adımı (M-Adımı): EM algoritmasını gerçekleştirebilmek için ikinci adımı oluşturan *M-Adımı*'na göre $\mathbf{b}$ ifadesi $(i+1)$. iterasyonda

$$\mathbf{b}^{(i+1)} = \arg\max_{\mathbf{b}} Q(\mathbf{b} \mid \mathbf{b}_i) = \sum_{k=1}^{K} Q_k(\mathbf{b}_k \mid \mathbf{b}^{(i)}). \tag{2.113}$$

olarak yazılabilir. (2.113) denklemindeki her bir $Q_k(\mathbf{b}_k \mid \mathbf{b}^{(i)})$ ifadesi için en büyüklenme işlemi sistemde kodlama kullanılmamışsa ayrı ayrı

$$b_k^{(i+1)}(l) = \operatorname{sgn}\left[\Re\{(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{x}_k(m))^{[i]}\}\right] \tag{2.114}$$

şeklinde gerçeklenebilir. EM algoritması sonucunda kullanıcılara ait gönderilen bilgi bit dizisi kestirimi

$$b_k^{i+1}(l) = \operatorname{sgn}\left[\Re\left\{\frac{1}{P} b_k^{[i]}(m) \left\|\mu_h^{(i)}[k]\right\|^2 (1-\beta_k) + \right.\right.$$

$$\left.\left.\beta_k(\mathbf{h}_k^{(i)\dagger}\Psi_k^T\left[\mathbf{y}(m) - \sum_{j=1, j\neq k}^{K} b_j^{[i]}(m)\Psi_j\mu_h^{(i)}[j]\right]\right\}\right] \tag{2.115}$$

şeklinde hesaplanabilir. Sonuçta (2.115) denklemi, birleşik kanal kestirimi ve parçalı karışım yoketmeli bilgi sezimi olarak düşünülebilir. Alıcı yapısındaki her bir bilgi sezimi için gelen sinyalden, ilgili kullanıcıdan hariç, diğer kullanıcılara ait sinyallerin karışımı azaltılarak tek kullanıcı için önerilmiş alıcı yapısına uygulanmaktadır. Sonuçta K kullanıcılı optimizasyon problemi, çözüm işlemi daha kolay olan K adet birbirinden bağımsız optimizasyon problemine dönüşmektedir.

3.4.5 SAGE algoritmasının uygulanması:

SAGE algoritması Fessler tarafından EM algoritmasının genelleştirilmesi olarak sunulmuştur [52]. Algoritma her adımda bütün parametrelerin aynı anda yenilenmesi yerine, her bir iterasyon için tek bir parametreyi yenilemekte ve bu yineleme esnasında diğer parametreleri eski değerleri olarak korumaktadır. Bizim ele aldığımız sinyal modelinde, EM algoritmasında iletilen sembol dizisi olan $\mathbf{b}$ yerine, SAGE algoritması her bir iterasyonda k. kullanıcıya ait $\mathbf{b}_k = [b_k(1), b_k(2), \cdots, b_k(M)]$

vektörünü yenilemektedir. SAGE algoritması anlaşılacağı üzere seri sıralı bir algoritmadır. EM algoritması gibi monotik artışı garanti altına alırken yakınsama özelliğide genellikle EM algoritmasından daha iyidir. SAGE algoritması, EM algoritması gibi beklenti ve enbüyükleme adımlarından oluşmaktadır. Buna göre i. iterasyon işlemi için Beklenti hesabı

$$Q_k\left(\mathbf{b}_k \mid \mathbf{b}^{(i)}\right) = E\left\{\log p(\chi \mid \mathbf{b}_k, \mathbf{b}_{\bar{k}}^{(i)} \mid \mathbf{y}, \mathbf{b}^{(i)}\right\} \tag{2.116}$$

şeklinde yapılmaktadır. En büyükleme adımında ise $\mathbf{b}_k$ ifadesi

$$\mathbf{b}_k^{(i+1)} = \arg\max_{\mathbf{b}} Q_k(\mathbf{b}_k \mid \mathbf{b}^{(i)}), \qquad \mathbf{b}_{\bar{k}}^{(i+1)} = \mathbf{b}_{\bar{k}}^{(i)}. \tag{2.117}$$

şeklinde hesaplanmaktadır. Verilen tüm bilgi için $\mathbf{b}$ matrisine ait logaritmik olabilirlik işlevi

$$\begin{aligned} \log p(\chi \mid \mathbf{b}) &= \log p(\mathbf{y}, \mathbf{h} \mid \mathbf{b}) \\ &= \log p(\mathbf{y} \mid \mathbf{h}, \mathbf{b}) + \log p(\mathbf{h} \mid \mathbf{b}) \end{aligned} \tag{2.118}$$

olarak ifade edilebilir. (2.118) denklemindeki son terim gönderilen semboller ve kanal birbirinden bağımsız olduğundan enbüyükleme işlemine tabi tutulmamaktadır. İlk terim olan $\log p(\mathbf{y} \mid \mathbf{b}, \mathbf{h})$ ifadesi

$$\log p(\mathbf{y} \mid \mathbf{b}, \mathbf{h}) \square \sum_{m=1}^{M} 2\Re\left\{\left(\sum_{j=1}^{K} b_j(m)\mathbf{C}_j\mathbf{F}\mathbf{h}_j\right)^{\dagger} \mathbf{y}(m)\right\} - \left\|\sum_{j=1}^{K} b_j(m)\mathbf{C}_j\mathbf{F}\mathbf{h}_j\right\|^2. \tag{2.119}$$

olarak yazılabilir. (2.119) denklemini (2.116)'da yerine koyarsak $Q_k(\mathbf{b}_k \mid \mathbf{b}^{(i)})$ ifadesi

$$Q_k(\mathbf{b}_k \mid \mathbf{b}^{(i)}) = \sum_{m=1}^{M} \Re\left\{b_k(m)(\mathbf{h}_k^{(i)})^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{y}(m) - \sum_{j=1, j\neq k}^{K} b_j(m)(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{C}_j\mathbf{F}\mathbf{h}_j)^{(i)}\right\} \tag{2.120}$$

şekline bulunmaktadır. Burada $\mathbf{h}_k^{(i)}$ ve $(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{C}_j\mathbf{F}\mathbf{h}_j)^{(i)}$ ifadeleri

$$\begin{aligned} \mathbf{h}_k^{(i)} &\square E(\mathbf{h}_k \mid \mathbf{y}, \mathbf{b}^{i}) \\ (\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{C}_j\mathbf{F}\mathbf{h}_j)^{(i)} &\square E\left\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^{T}\mathbf{C}_j\mathbf{F}\mathbf{h}_j \mid \mathbf{y}, \mathbf{b}^{i}\right\} \end{aligned}$$

şeklinde tanımlanmış olup sırasıyla (2.110) ve (2.112) denklemleri yardımıyla hesaplanabilir. (2.120) denklemindeki toplam herbir kullanıcı için ayrı ayrı enbüyüklenirse bazı matematiksel işlemlerden sonra

$$b_k^{(i+1)}(m) = \operatorname{sgn}\left[\Re\left\{\left((\mathbf{h}_k^{\dagger})^{(i)}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{y}(m) - \sum_{j=1, j\neq k}^{K} b_j^{(i)}(m)(\mathbf{h}_k\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j)^{(i)}\right)\right\}\right] \tag{2.121}$$

olarak hesaplanabilir. Ayrıca gönderilen çerçeve uzunluğunun M boyutunda yeteri kadar uzun olduğu varsayılırsa (2.112) denklemindeki ikinci terim ihmal edilerek herbir kullanıcıya ait kestirim ifadesi

$$b_k^{i+1}(m) = \operatorname{sgn}\left[\Re\left\{\mu_h^{(i)}[j]\Psi_k^T\left[\mathbf{y}(m) - \sum_{j=1, j\neq k}^{K} b_j^{(i)}(m)\Psi_j\mu_h^{(i)}[j]\right]\right\}\right] \tag{2.122}$$

şeklinde yazılabilir. (2.122) denklemine göre her bir iterasyonda kullanıcılara ait karışım yok edilelerek elde edilen kestirim işlemi daha güvenilir hale gelmektedir. Ancak EM algoirtmasından farklı olarak seri sıralı olduğu için iterasyon işleminde diğer kullanıcıların hesaplamasını beklemesi gerektiğinden zaman problemi oluşturmaktadır. Ayrıca denklemden görüleceği üzere yapılan işlemlerde SAGE-JDE için EM-JDE'ye göre K kat daha fazla kanal kestirim işlemi yapılması gerekmektedir.

4.BULGULAR VE TARTIŞMA/SONUÇ

4.1 VERİCİ ÇEŞİTLEMELİ TURBO KODLANMIŞ OFDM SİSTEMLERİNDE DÜŞÜK KARMAŞIKLI EM TEKNİĞİNE DAYALI KANAL KESTİRİMİ ÇALIŞMALARININ BİLGİSAYAR BENZETİM SONUÇLARI

Önerilen alıcı yapılarının başarımları çeşitli senoryalar için bilgisayar benzetimi yardımıyla MSE ve BER olarak verilmiştir. Benzetimde SFBC/STBC-OFDM sistemlerinin 1/2 kodlama oranınan sahip turbo veya konvolüsyonel olarak kodlandığı durumlar düşünülmüştür. Turbo kodlama durumunda $(1,5_8/7_8)$ üreteç matrisine sahip iki özdeş özyineli sistematik konvolüsyonel kodlayıcının arada rasgele serpiştirici olacak şekilde paralel bağlandığı durum seçilmiştir. Katlamalı kodlayıcı durumda ise $(5_8,7_8)$ üreteç matrisine sahip kodlayıcı kullanılmıştır[40]. Bilgisayar benzetimlerinde iki ayrı kanal modeli gözönüne alınmıştır.

i) Beek Kanal Modeli [33]: Verici antenden iletilen sinyaller, düşüş katsayıları ($\alpha = 0.2$) olan yükseltilmiş-kosinüs verici süzgeçleri ile biçimlendirilip, 2.28 MHz'lik band genişliğinden BPSK modülasyon tekniği ile iletildikleri varsayılmaktadır. Toplam iletişim bandı 512 altbanta bölünerek bir OFDM sembolünün iletim süresi T_s, 1.052 μsn'lik kısmı çevirimli önek olmak üzere, toplam 136 μsn'dir. Kanalın efektif gecikme süresi $\tau_{rms} = 0.263\mu$sn olarak seçilmiştir.

ii) COST-207 Modeli [34]: Yapılacak bilgisayar benzetim çalışmalarında Bölüm 2'de açıklanan genel COST 207 kanal modeline ait TU ve BU senaryoları şeçilmiştir. Verici antenden iletilen sinyaller, düşüş katsayıları $\alpha = 0.2$ olan yükseltilmiş-kosinüs verici süzgeçleri ile biçimlendirilip, 1.07 MHz'lik band genişliğinden BPSK modülasyon tekniği ile iletildikleri varsayılmaktadır. Toplam iletişim bandı 512 altbanta bölünerek bir OFDM sembolünün iletim süresi T_s, 40 μsn'lik kısmı çevirimli önek olmak üzere, toplam 328 μsn olarak seçilmiştir.

Beek kanal modeli için bilgisayar benzetim sonuçları Şekil 4.1-4.8'de sunulmaktadır. Şekil 4.1'de, Turbo kodlanmış SFBC-OFDM sistemleri için EM-MAP kanal kestirimci kullanılması durumunda oluşacak ortalama karesel hata (MSE) başarımı, kanal kestiriminde yapılabilecek en az hatayı gösteren MCRLB alt sınırı, EM-ML ve doğrusal MMSE algoritmalarıyla karşılaştırılmıştır. Bilgisayar benzetim çalışmalarında 8 bilgi sembolüne karşılık bir pilot ton koyulmasıyla algoritmanın başlangıç değerleri saptanmıştır. (PIR=1:8). Şekil 4.1'de EM-MAP algoritmasının EM-ML ve LMMSE algoritmandan üstün başarım sağlamakta olduğu ve MRLB sınırına artan SNR değerleri için yaklaştığı gözlemlenmiştir. Şekil 4.2'de ise Turbo kodlanmış SFBC-OFDM sistemleri için EM-MAP kanal kestirimci kullanma durumda sistemin BER başarımı, EM-ML ve LMMSE kanal kestirimi yapılması durumdaki BER başarımlarıyla karşılaştırmıştır. Ayrıca bu şekilde kanalın tamamiyle bilinmesi (Knl Bil.) durumundaki başarımı ve kanal kestirimi yapılarak elde edilebilecek en iyi başarımda (gönderilen bilginin tamamen pilotlardan oluştuğu durum) "Tüm pilot" şeklinde gösterilmiştir. Şekil 4.2'de EM-MAP algoritmasının bu iki eğriye yaklaştığı görülmektedir.

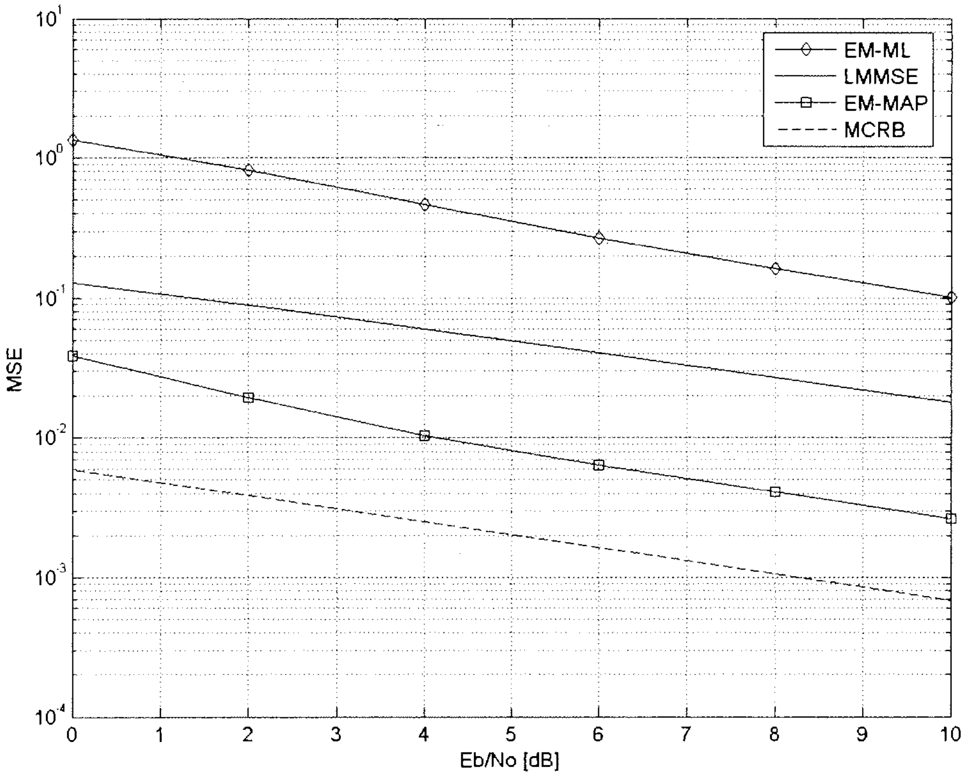
KL açılımının optimum kısaltma özelliği, istatiksel olarak bağımlı bilgiyi ifade etmek için gerekli olan bilgi miktarının enküçüklenmesini sağlamaktadır. Bu özellik sayesinde kanal kestiriminde yapılan işlem karmaşıklığı azaltılabilir. Örneğin KL açılımında en büyük değerlere sahip öz değerler ve bu öz değerlere karşılık gelen öz vektörler sayesinde gerekli olan karmaşık çarpma işlemi azaltılabilmektedir. KL açılımının optimum kısaltma özelliği algoritmada kullanılan toplam KL açılım katsayısına göre Şekil 4.3'te çizdirilmiştir. Bu şekilde, algoritmada kullanılacak 8 KL açılım katsayısının, algoritmanın çalışması için yeterli olduğu görülmektedir. Bir başka deyişle, kullanılacak 8 katsayı ve bu katsayılara karşılık gelen öz vektörler kanalı mükemmel bir biçimde karakterize edebilmektedir. Bu şekilde kanal kestirimindeki karmaşık çarpma işlemleride enaza indirilebilmektedir.

Yüksek başarımlı kanal kodlayıcıların kanal kestirim hatalarına duyarlılığı daha fazladır. Kodlanmış semboller arasındaki ilişkinin fazla olması kanal kodlayıcının kanal kestirimdeki hatalara duyarlı hale getirmekte ve kod çözümü sırasında ciddi hata yayınımları oluşturmaktadır. Bu yüzden turbo ve katlamalı kodlanmış sistemlerin başarımı değişik pilot aralıklarına göre 3 yineleme sonucunda kanal kestiriminde yapılan hataya (MSE) incelenmiş, azalan pilot-sembol yerleştirme oranlarıyla turbo ve konvolüsyonel kodlayıcıların başarımının azaldığı ve bu başarım düşüşünün turbo kodlayıcılarda yukarıda belirtilen sebebten dolayı daha çok olduğu görülmüştür (bkz Şekil 4.4). Düşük SNR değerlerinde Katlamalı kodlayıcı kullanıldığı durumda MSE başarımın biraz daha iyi olması turbo kodun deliklenmiş yapısından kaynaklanmaktadır. Ancak yapılan benzetimlerde bu başarımın artan SNR değerleriyle kaybolduğu sonucuna varılmıştır. Diğer taraftan MSE başarım kriterinin benimsenmesi sistemin genel başarımı hakkında yanıltıcı olabilmektedir. Örneğin, aynı senorya için, artan SNR değerlerinde, MSE'de başarımı farkının pek farkedilemesine karşın, BER grafiğinde bu farkın daha net bir şekilde gözlenebildiği anlaşılmıştır(bkz Şekil 4.5). Şekil 4.5'den görüleceği üzere PIR=(1:32) için konvolüsyonel kodlayıcının başarımı turbo kodlanmış sistemin başarımından fazladır. Ancak ilklendirme işleminde yeterli pilot-semboller kullanıldığında turbo kodlamalı sistemlerin başarımının konvolüsyonel kodlanmış sistemlerden daha üstün olduğu gözlemlenmiştir.

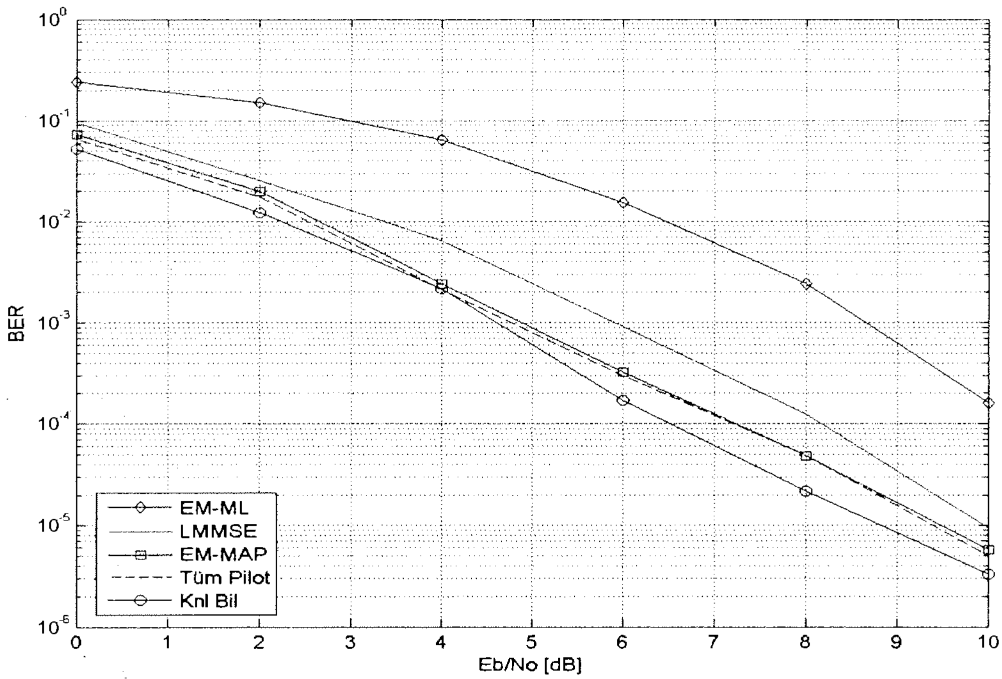
Yineleme sayesinde sistemin başarımındaki artışı göstermek için Şekil 4.6 ve Şekil 4.7'de turbo ve konvolüsyonel kodlanmış sistemlerin başarım eğrileri sırasıyla PIR=1:8 ve PIR=1:16 için yineleme sayısına göre, tüm pilot ve kanal bilindiği durumda elde edilen başarımlarla birlikte verilmiştir. Turbo kodlamasının yapısal üstünlüğünden dolayı yinelemeli yapıda hesaplanan bilgi bitlerinin güvenirliliği katlamalı kodlayıcıya göre daha fazladır. Bu sebeple Şekil 4.6'da $P_e = 10^{-4}$ için turbo kodlayıcıda BER başarımının başlangıç değerine göre düzeltimi 1.3 dB'ken konvolüsyonel kodlayıcıda bu değer sadece 0.3 dB olduğu gözlenmiştir. Sonuçta, turbo kodlamalı yapının başarımının tüm pilot ton kullanımdaki başarıma ulaştığı görülmüştür. Şekil 4.7'de ise yineleme ile başarım, PIR=1:16 için yeniden incelenmiş ve $P_e = 10^{-4}$ için yineleme ile başarım artışının turbo ve katlamalı kodlar için sırasıyla 2.3 dB ve 1 dB olduğu görülmüştür. Turbo kodlanmış sistem başarımının katlamalı kodlayıcaya fazla olmamasına rağmen bu başarımı Şekil 4.6'da verilenlere göre değerlendirilecek olursa, turbo kodlama kullanılması durumunda yineleme kazancındaki fark 1.3dB'den 2.3 dB'ye çıkmışken, katlamalı kodlayıcılarda bu fark 0.3 dB ten 1 dB çıktığı anlaşılmaktadır. Bu sonuçlardan, turbo kodlalama durumunda PIR değerinden etkilenme duyarlılığının, yineleme ile elde edilen başarım kazancında da kendisini gösterdiği açık olarak görülmektedir.

Blok sönümlemeli kanal modelinde kanalın frekans yanıtını oluşturan katsayıların bir OFDM sembolü boyunca değişmediği ancak bir OFDM sembolünden diğerine doppler frekansına bağlı olarak değiştiği varsayılmaktadır. Bu durumda bir OFDM sembolü için kanalın frekans yanıtı, kanalın altbandlarındaki ilişki işlevinden görüleceği üzere Doppler değerinden bağımsızdır. Bu açıdan yapılan bilgisayar benzetimlerinden farklı olarak Şekil 4.8'de Turbo kodlanmış-STBC-OFDM ve Turbo kodlanmış-SFBC-OFDM sistemlerinin başarımları, EM-MAP kanal kestirimci kullanılması durumunda, farklı Doppler değerleri için incelenmiştir. SFBC kodlamalı sistemin yüksek Doppler değerleri için başarımının STBC kodlamalı sisteminden üstün olduğu gösterilmiştir.

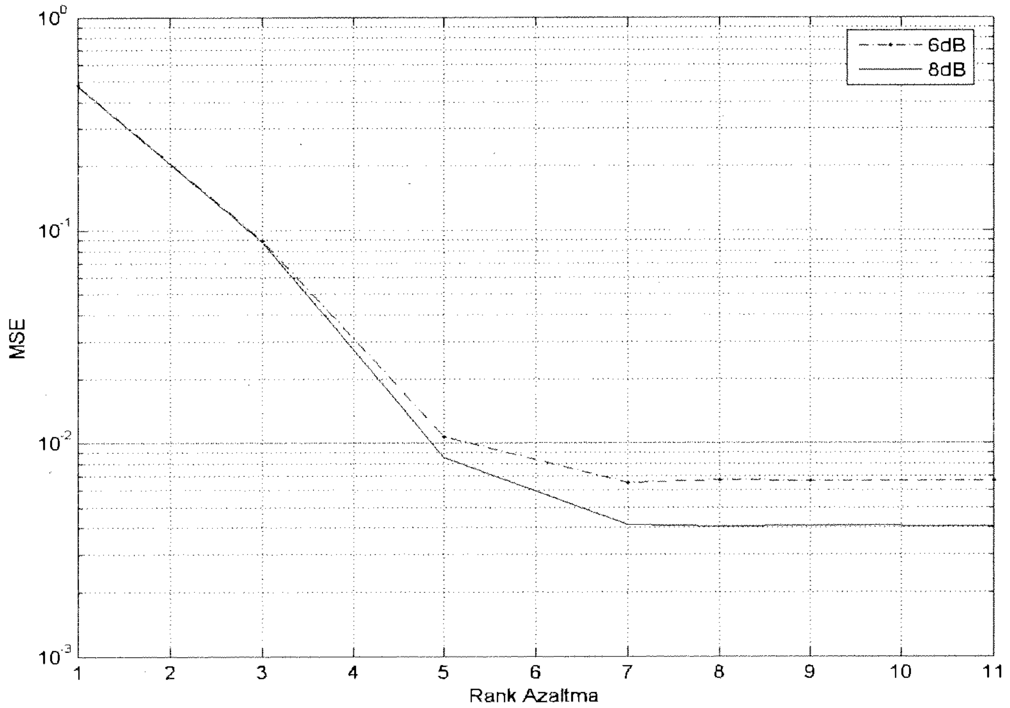
COST-207 kanal modeli için elde edilen bilgisayar benzetim sonuçları ise Şekil 4.9 ve 4.10'da özetlenmektedir. Şekil 4.9 ve 4.10'da sırasıyla BU ve TU kanallar için, EM-MAP, EM-ML ve LMMSE kanal kestirimci kullanan turbo-kodlu STBC-OFDM sistemlerinin BER başarımları kanalın tamamen bilinmesi (Knl Bil.) durumundaki başarımlarıyla karşılaştırılmaktadır. Algoritmanın başlangıç değerinin bulunması için kullanılan pilot aralığı ise PIR=1:8 olarak şeçilmiştir. Şekil 4.9'da EM-MAP algoritmasının EM-ML ve LMMSE algoritmasından daha üstün başarım sağlamakta olduğu ve kanalın bilinmesi durumundaki başarıma artan SNR değerleri için yaklaştığı gözlemlenmektedir. Şekil 4.10'da ise EM-MAP algortimasının başarımının TU kanallarda kanalın bilinmesi durumundaki başarıma daha çok yaklaştığı gözlemlenmişitir. Ancak bu durumda EM-MAP algoritmasının LMMSE kanal kestirimine göre başarım üstünlüğü kaybolmaktadır.



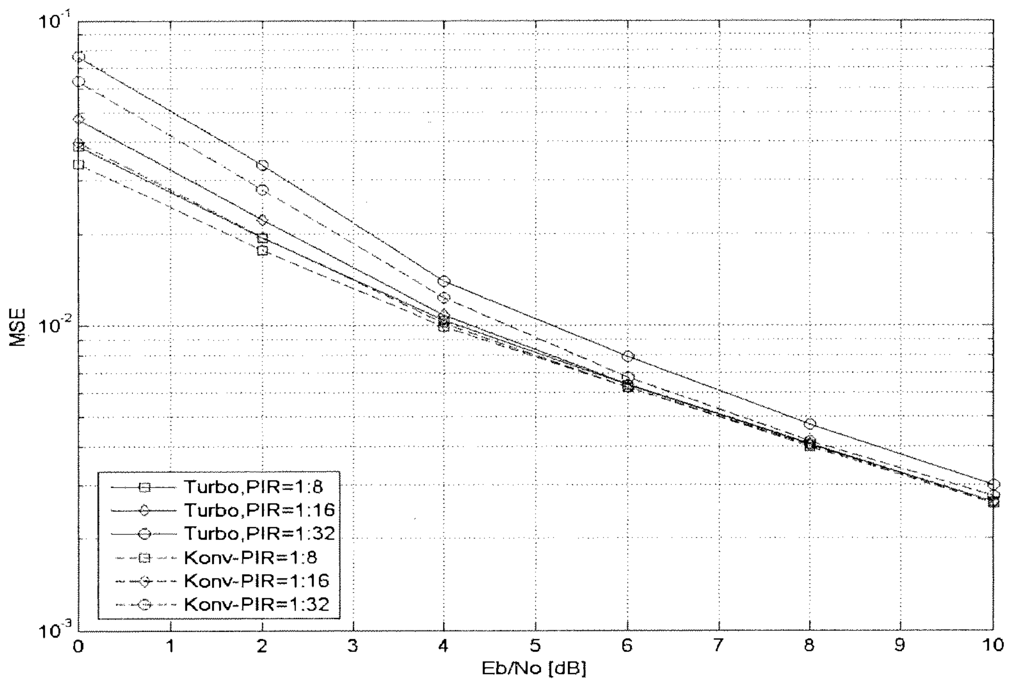
Şekil 4.1 : Turbo kodlanmış SFBC-OFDM sistemleri için çeşitli kanal kestirim algoritmalarının MSE başarımı



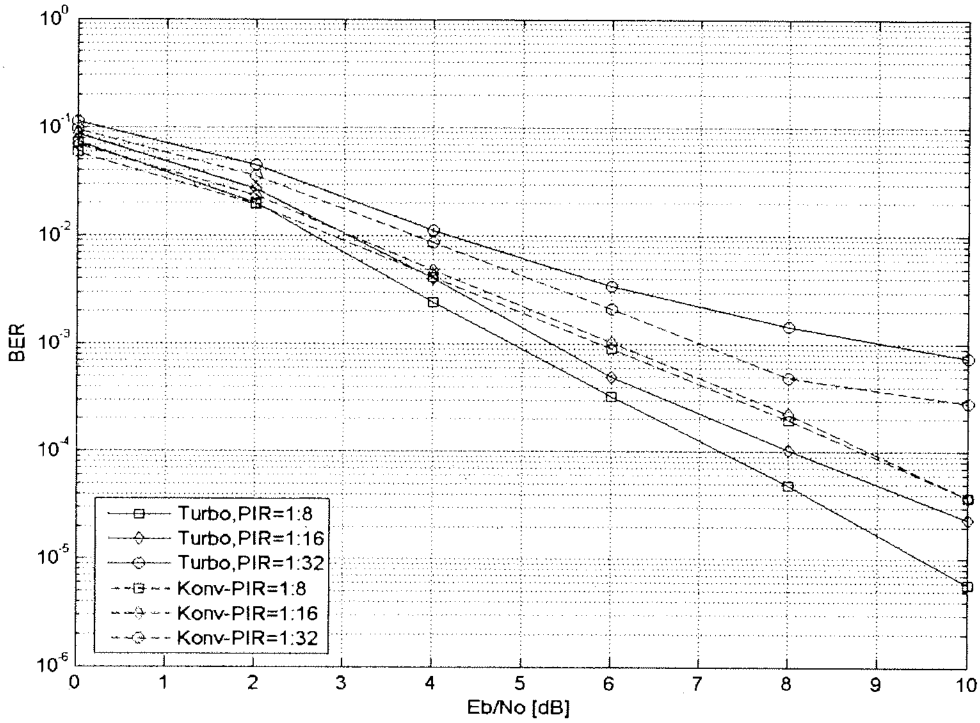
Şekil 4.2 : Turbo kodlanmış SFBC-OFDM sistemlerinde çeşitli kanal kestirim algoritmalarının uygulanması sonucu sistemin BER başarımı



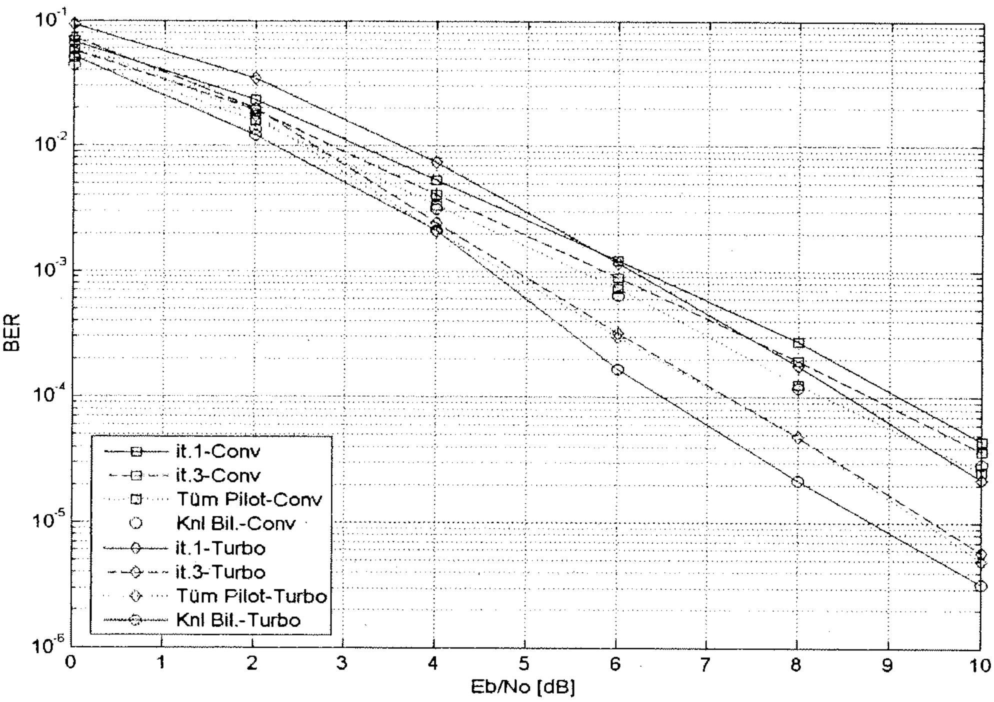
Şekil 4.3 : Turbo kodlanmış SFBC-OFDM sistemlerinde EM-MAP kanal kestimi kullanılması durumunda kullanılan KL katsayılarına göre MSE başarımının incelenmesi



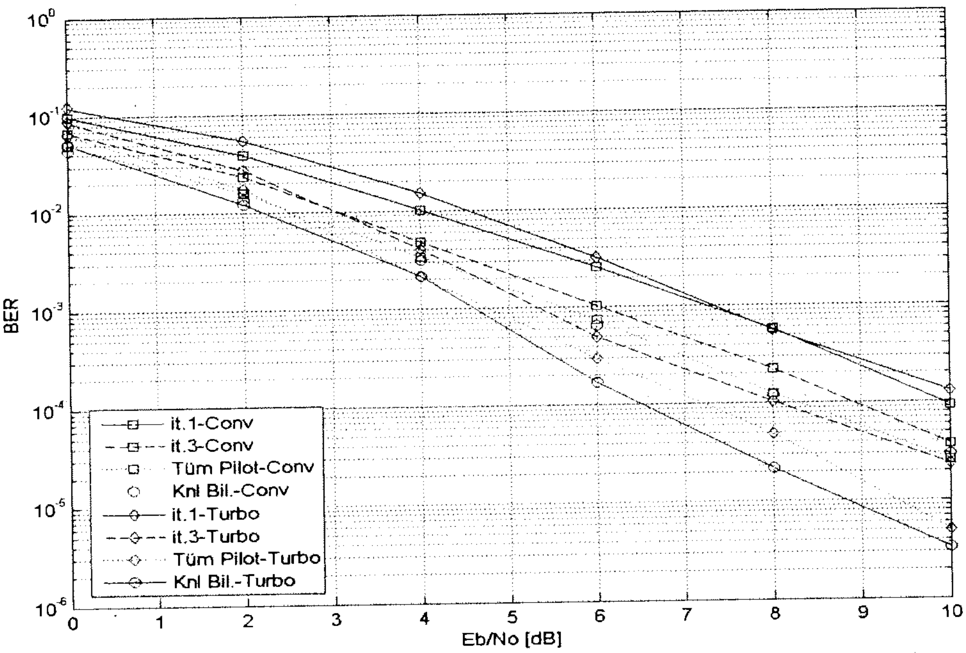
Şekil 4.4: Pilot Koyma Aralığının, Turbo/Katlamalı kodlanmış SFBC-OFDM sistemler için yapılan EM-MAP kanal kestirimine olan etkisinin MSE olarak incelenmesi



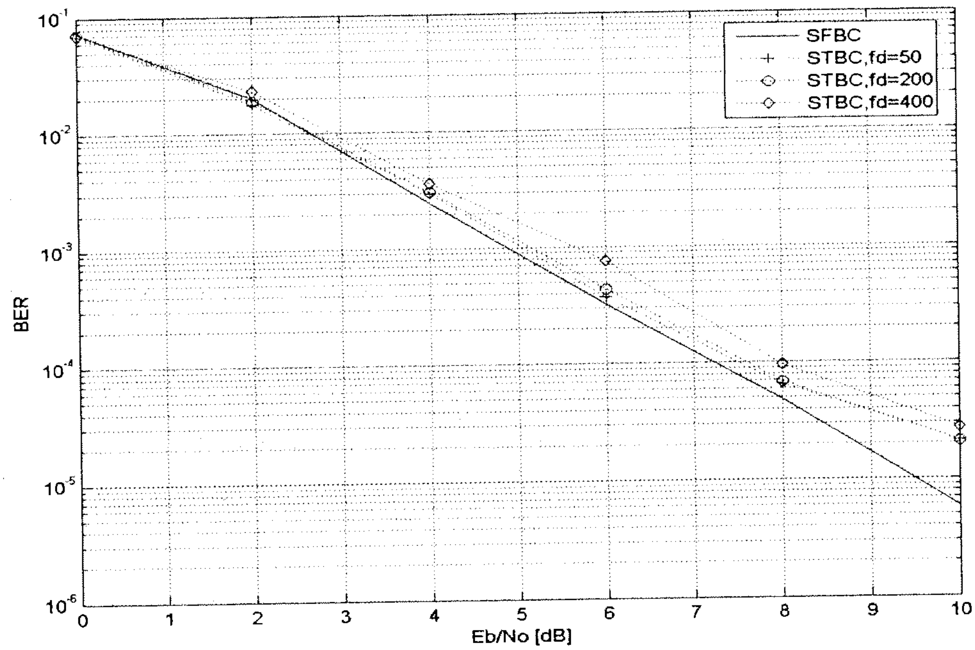
Şekil 4.5: Pilot Koyma Aralığına göre, Turbo/Katlamalı kodlanmış SFBC-OFDM sistemler için yapılan EM-MAP kanal kestirimine olan etkisinin sistemin genel başarımı olan etkisinin BER olarak incelenmesi



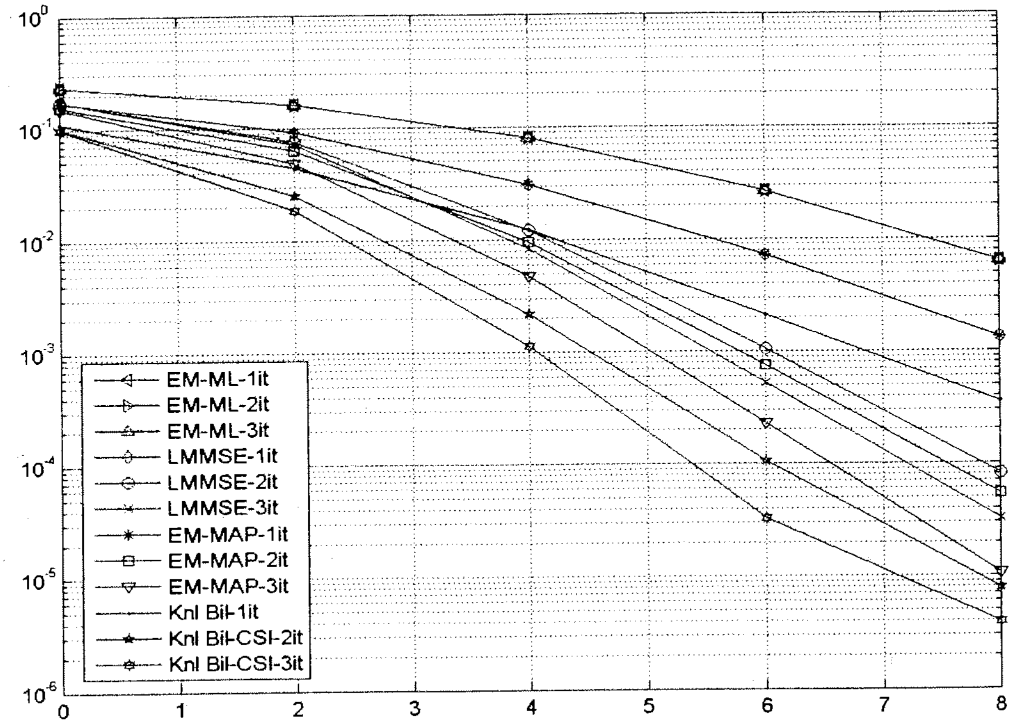
Şekil 4.6: Pilot ton koyma aralığının 8 olması durumunda Turbo/Konvolusyonel kodlanmış SFBC-OFDM sistemlerinin yineleme sayısına göre başarımının tüm pilot (PIR=1) ve Kanalın bilindiği durumdaki başarımlarına göre incelenmesi.



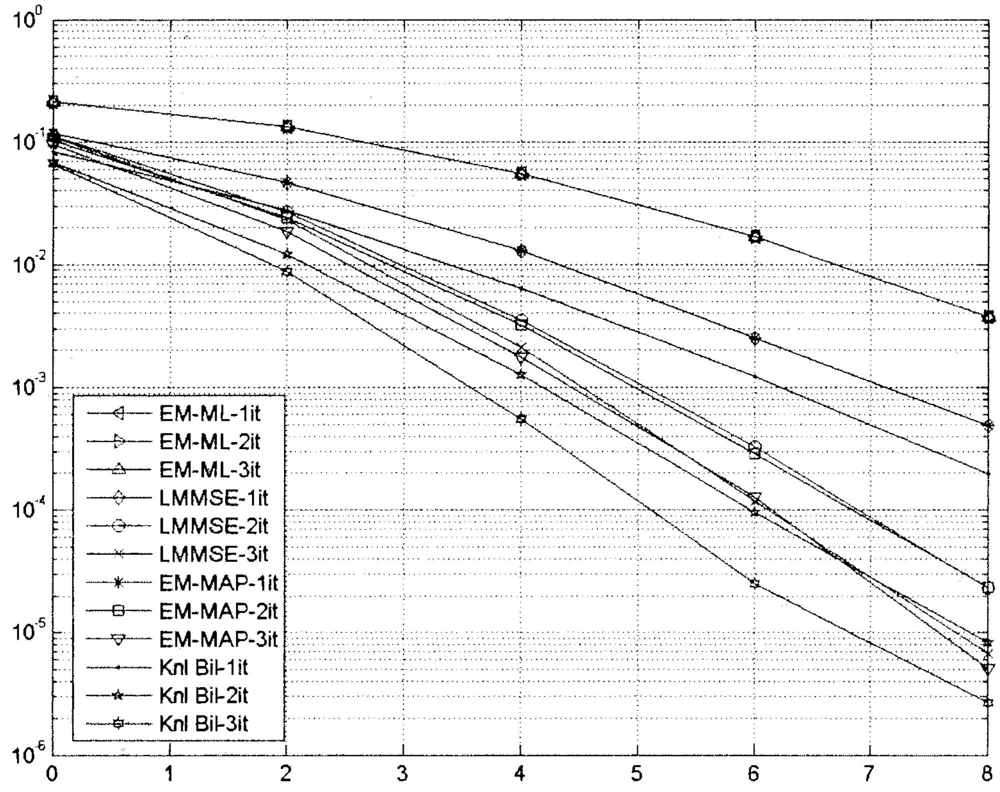
Şekil 4.7: Pilot ton koyma aralığının 16 olması durumunda (PIR=16) Turbo/Konvolusyonel kodlanmış SFBC-OFDM sistemlerinin yineleme sayısına göre başarımının tüm pilot (PIR=1) ve Kanalın bilindiği durumdaki başarımlarına göre incelenmesi.



Şekil 4.8 : Turbo kodlanmış SFBC/STBC-OFDM sistemlerinin çeşitli Doppler frekanslarına göre başarımının BER olarak incelenmesi



Şekil 4.9 COST-207-BU kanallarda Turbo kodlanmış STBC-OFDM sistemlerinde çeşitli kanal kestirim algoritmalarının uygulanması sonucu sistemin BER başarımı

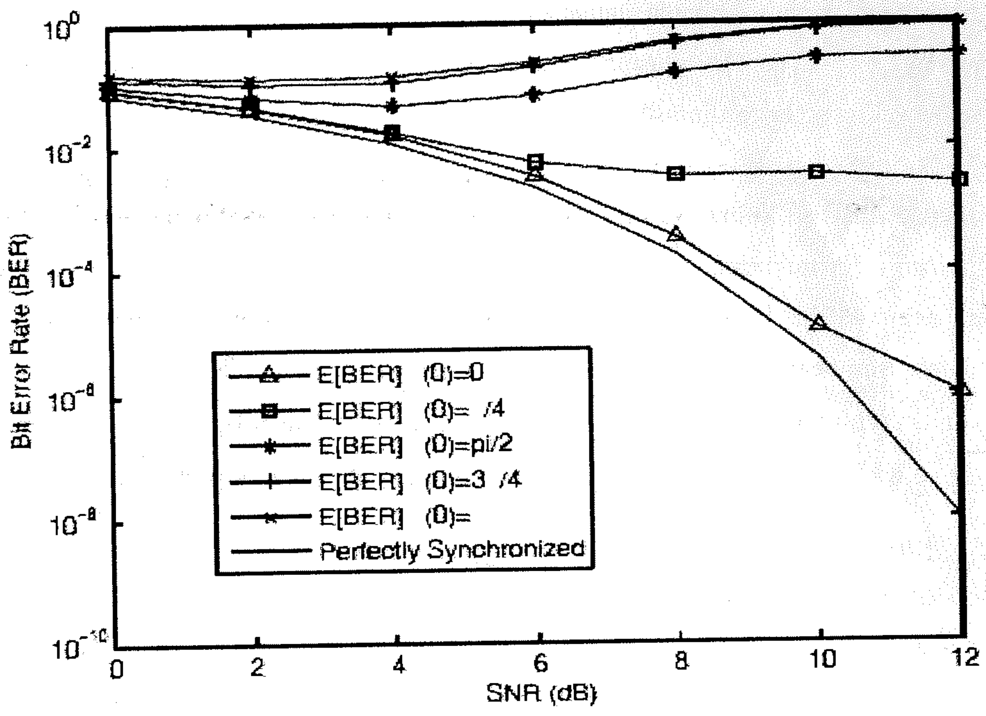


Şekil 4.10 COST-207-TU kanallarda Turbo kodlanmış SFBC-OFDM sistemlerinde çeşitli kanal kestirim algoritmalarının uygulanması sonucu sistemin BER başarımı

4.2 GÜRÜLTÜ VE BİLGİNİN SMC TEKNİĞİ İLE GÖZÜ KAPALI KESTİRİMİ

Bilindiği üzere taşıyıcı sinyalerin faz kestirimi sayısal iletişim sistemleri için kritik bir rol oynamaktadır. Sistemde zamanla değişen veya sabit faz gürültüsü olması durumunda bilgi sezimi işlemi için literatürde pek çok çalışma yapılmıştır. Ancak faz gürültüsü ve bilgi sezimi gerçekleştirme işlemini ortak bir şekilde yapmak matematiksel olarak oldukça karmaşık olduğundan pratik uygulamalarda uygun değildir. Diğer bir açıdan faz kestirim işlemleri bilgi destekli ve desteksiz olmak üzere gerçekleştirmektedir. Bilindiği üzere bilgi destekli kestirim yöntemlerinin, band genişliğini kullanma kapasitesini, kullanılan pilot veya eğitim sembollerini kısıtlaması nedeniyle, bilgi desteksiz yani gözü kapalı kestirimciler uygulamada daha çekici hale getirmiştir. Gözü kapalı kestirimcinin gerçekleştirilmesinde iletilen semboller rasgele değişken olarak varsayılırsa olasılık işlevinin ortalaması bilgi dizisi üzerinden gerçekleştirilebilir. Ancak bu işlem basit bir kaç durum hariç, matematiksel olarak mümkün gözükmemektedir ve yapılan yaklaşımlar sadece yüksek veya düşük SNR değerleri için gerçekleştirilebilmektedir.

Diğer taraftan, faz gürültüsü ve bilginin pratik olarak gerçekleştirilebilmesi için son dönemde EM algoritması uygulanmıştır. Bilindiği üzere EM algoritması, ML çözümüne yinelemeli olarak yakınsamaktadır. Algoritmanın en büyük dezavantajı ise başlangıç değerine olan duyarlılığının oldukça fazla olmasıdır. Ancak sonsal olasılık yoğunluk işlevinin gelen veri ile düzenli olarak yenilenmesi gereken durumlarda algoritma verimli değildir. SMC yöntemi ise bu durumlarda mühendislikte uygulanan güçlü bir yöntemdir. Sistem modelini kapsamadan optimal çözüme yakınsayan yöntem daha önceki gelen verilere bağlı olmadığından hata yayınımı oluşturmamaktadır. Dahası gözü kapalı kestirim olan SMC algoritması Gauss ve Gauss olmayan durumlardada çalışmaktadır. Bu kapsamda faz gürültüsü ve data seziminin ortaklaşa çözümü için SMC algoritması kullanılmış ortak kestirimin gerçeklenebilir karmaşıklıkla sağlanabileceği gösterilmiştir. SMC algoritmasının BPSK modüleli işaret bilgisayar benzetimleri yapılmış ve farklı başlangıç faz hatalarında başarımları Şekil 4.11'te verilmiştir. Algoritmanın alıcıda faz bilgisinin bilindiği durumdaki başarıma yaklaştığı görülmüştür.



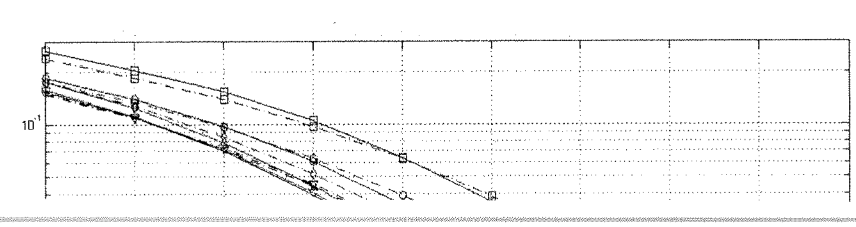
Şekil 4.11 : SMC algoritmasının farklı başlangıç faz hataları için BER başarımının değişik SNR değerleri için gösterimi

4.3 AŞAĞI LİNK MC-CDMA SİSTEMLERİ İÇİN ALICI YAPISI BİLGİSAYAR BENZETİMLERİ

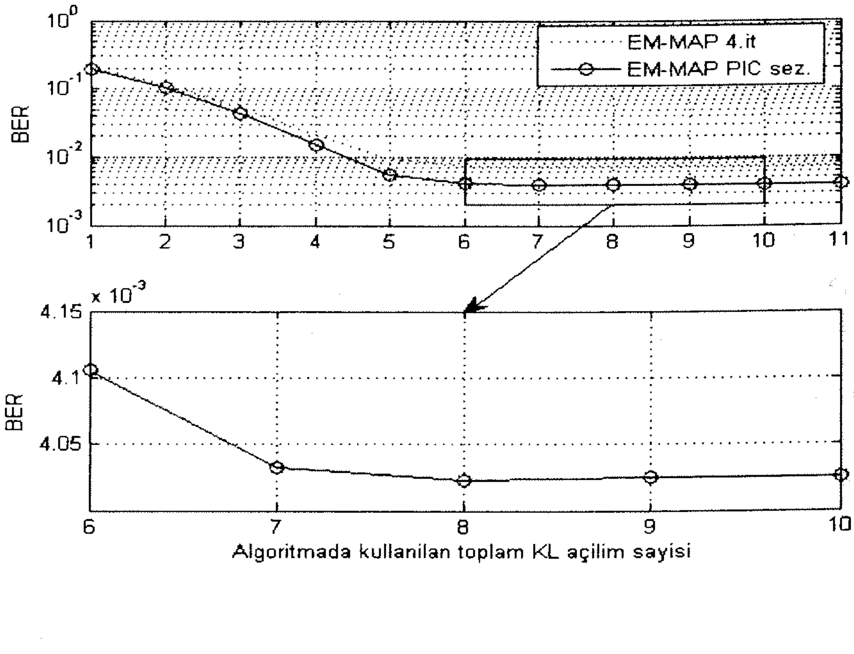
Frekans-seçici kanallar için bilgisayar benzetimi yapılan MC-CDMA sistemleri için herbir kullanıcının eşit miktarda güç aldığı varsayılmıştır. Kodları arasında diklik özelliği bulunan *Gold* dizisi, boyutu kanalda kullanılan altband sayısına eşit olacak şekilde her bir kullanıcı için tanımlanmıştır. Şekil 4.12 ve Şekil 4.13 için sistemin tamamen kullanıldığı yani toplam kullanıcı sayısının kullanılan yayma kodunun uzunluğuna eşit olduğu varsayılarak sistemin benzetimi yapılmıştır ($K = N_c$). Ayrıca algoritmanın ilklendirilmesi için, sekiz OFDM örnek aralığında bir adet pilot ton kullanılmıştır (PIR)=1:8.

Şekil 4.11'de, EM-MAP tabanlı alıcı yapısının BER başarımı, daha önce önerilen LS ve LMMSE kanal kestirimlerinin kullanıldığı MMSE ve tümleşik MMSE-PIC alıcı yapılarının BER başarımlarıyla karşılaştırılmıştır. Yapılan bilgisayar benzetimlerinde, EM-MAP kanal kestirimine dayalı alıcı yapısının LS ve LMMSE kanal kestirimci tabanlı diğer alıcı yapılarından üstünlüğü gözlemlenmiştir. Ayrıca EM-MAP alıcının kanalın alıcıda tamamen bilinmesi (Knl.Bil) ve gönderilen OFDM sembolünün tamamen pilot tonlardan oluşması durumunda (Tüm Pilot) yapılan kanal kestirim başarımına (tüm pilot) yakınsadığı görülmüştür.

KL açılımında kullanılan katsayı kesme özelliği, istatistiksel bağımlı olan kanal parametrelerini ifade etmek için gerekli olan bilgi miktarını en küçükleyebilir. KL açılımında verilen düşük ranklı kanal kestirim yaklaşımı, Şekil 4.12'te 12 dB için incelenmiştir. Kullanılan KL katsayısına göre EM-MAP alıcı yapısının BER başarım benzetimi yapılmış ve kullanılan kanal yapısı için, 8 adet KL katsayısının yeterli olduğu şeklin detaylı verilen bölümünde görülmüştür. Yapılan yaklaşımla, 128×128'lık matris ters alma işlemi yerine 8×8 'lik matris ters alma işleminin yeterli olabileceği gösterilmiştir.



Şekil 4.12: Farklı kanal kestirim algoritmalarının kullanılması sonucu çeşitli alıcı yapılarının BER başarımları



Şekil 4.13: Kullanılan KL katsayısına Algoritmanın BER başarımı

4.4 YUKARI LİNK MC-CDMA SİSTEMLERİ İÇİN ALICI YAPISI BİLGİSAYAR BENZETİMLERİ

Bu bölümde, frekans-seçici kanallar üzerinden çalışan yukarı link MC-CDMA için önerilen alıcı yapısının başarımı, her bir kullanıcının gücünün alıcıya farklı olarak geldiği varsayılarak incelenmiştir. Alıcıya gelen kullanıcılara ait güç miktarları Tablo 1'de verilmiştir. Her bir kullanıcı için, kodları arasında diklik özelliği bulunan Walsh dizisi, boyutu kanalda kullanılan alt band sayısına eşit olacak şekilde tanımlanmıştır (P=16). Alıcı yapılasında EM algoritması için Beta'ların değeri kullanılan yayıcı kodların dikliğinden dolayı 1 olarak alınmıştır. Sistemde aktif kullanıcı sayısı K=16 seçilmiş ve herbir kullanıcının gönderdiği bilgi çerçevesi T adet pilot, F adet bilgi sembolünden oluşmaktadır. Verici antenler ve alıcı anten arasındaki kablosuz haberleşme kanalları, L uzunluğunda karmaşık katsayılardan oluşan ve kanaldaki sekme kazançları Tablo 2'de verilen, TU kanal yapısına sahip olarak modellenmiştir. Ayrıca kanal özilişki matrisi olan Σ_h 'ın alıcıda bilindiği varsayılmıştır.

Tablo 4.1: MC-CDMA Sistemindeki Kullanıcı Güçleri

Kullanıcı	Doğrusal	Logaritmik (dB)
1	1	0
2	0.9560	-0.1954
3	0.9139	-0.3909
4	0.8737	-0.5863
5	0.8353	-0.7817
6	0.7985	-0.9772
7	0.7634	-1.1726
8	0.7298	-1.3680
9	0.6977	-1.5635
10	0.6670	-1.7589
11	0.6376	-1.9543
12	0.6096	-2.1498
13	0.5827	-2.3452
14	0.5571	-2.5406
15	0.5326	-2.7361
16	0.5092	-2.9315

Tablo 4.2. Kanalın Sekme Güçleri

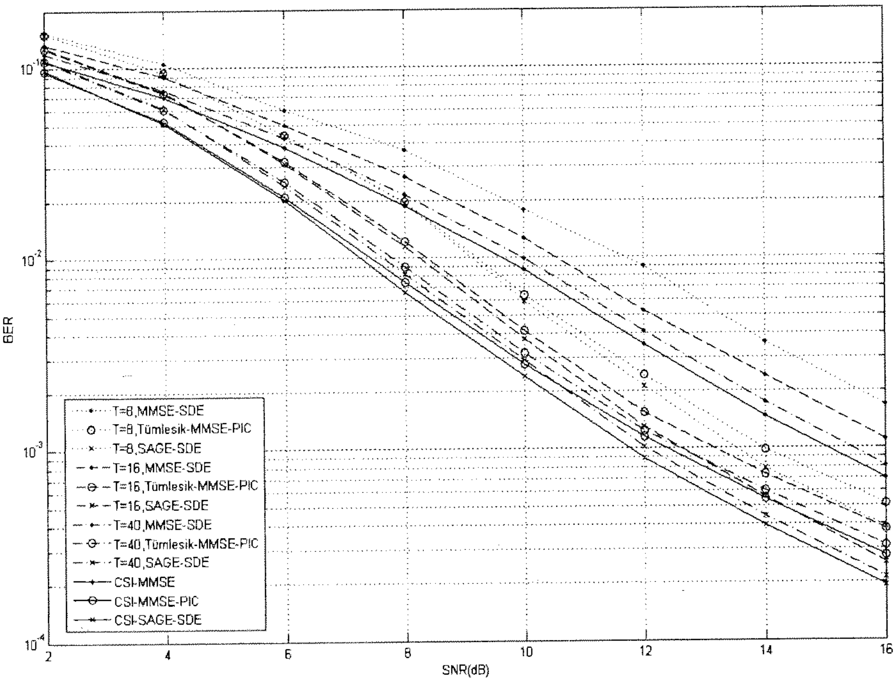
Gecikme (μsn)	Doğrusal	Logaritmik
0	0.6564	-1.8286
0.81	0.2086	-6.8072
1.62	0.0790	-11.0210
2.44	0.0560	-12.5171

Literatürde daha önce önerilen alıcı yapılarına göre; gelen bilgi çervesindeki T adet pilot sembolü kullanılarak MMSE kanal kestirimini gerçekleştiren alıcı, bulduğu bu kanal katsayıları yardımıyla gelen çerçevedeki F adet bilgi sembolünün MMSE kestirimini gerçekleştirmektedir. h ve b'nin ayrı iki adet MMSE kestirimci yardımıyla bulunduğu için bu alıcı yapısı MMSE-SDE (separate detection and estimation) olarak tanımlanmıştır. Ayrıca MMSE-SDE'nin çıkışının PIC ve SAGE alıcılarına uygulanması durumuna, sırasıyla tümleşik MMSE-PIC, SAGE-SDE denilmiştir. Yapılan benzetimlerde birleşik bilgi sezimi ve kanal kestirim işlemini gerçekleştiren EM-JDE (Joint Detection and Estimation) ve SAGE-JDE algoritmalarının başarımları MMSE-SDE, Tümleşik-MMSE-PIC ve SAGE-SDE alıcı yapılarıyla karşılaştırılmıştır. Kanala ait bilgilerinin alıcıda tamamen bilinmesi durumunu gösteren CSI (Channel state information) durumları da CSI-MMSE, CSI-Tümleşik MMSE-PIC, CSI-SAGE-SDE olarak ayrıca incelenmiştir.

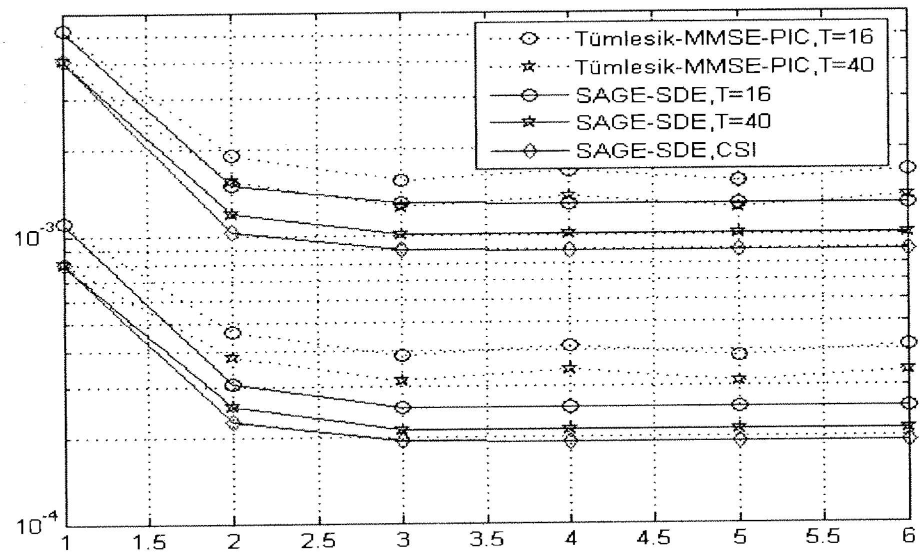
Şekil 4.14'de yukarıda bahsedilen MMSE-SDE, Tümleşik-MMSE-PIC, SAGE-SDE başarımları farklı sayıda kullanılan pilot tonları için başarımı kanalın tamamen alıcıda bilindiği durumlar olan CSI-MMSE, CSI-Tümleşik MMSE-PIC, CSI-SAGE-SDE sistemleri ile karşılaştırılmıştır. Adaletli bir karşılaştırma için Tümleşik-MMSE-PIC, SAGE-SDE, CSI-Tümleşik MMSE-PIC, CSI-SAGE-SDE alıcı yapıları dört iterasyon için çalıştırılmıştır. (Burada EM için yapılan bir iterasyon SAGE için yapılan K iterasyona karşılık gelmektedir) Şekil 4.14'de kullanıcılara ait karışımı engelleyen alıcı yapıları olan tümleşik MMSE-PIC ve SAGE-SDE yapılarının MMSE-SDE yapılarından üstünlüğü gösterilmiştir. Ayrıca yapılan benzetimlerden ancak 40 adet pilot sembol kullanıldığı zaman alıcı başarımlarının kanalın tamamen bilindiği durumdaki alıcı başarımlarına yaklaştığı gözlemlenmiştir. Literatürden beklendiği üzere SAGE-SDE algoritmasının MMSE-PIC lagoritmasından daha önce yakınması ve başarım üstünlüğü Şekil 4.15'te ayrı olarak verilmiştir. Kanalın tamamen bilinmesi durumunda SAGE-SDE başarımı bu sistem için alt sınırı vermektedir.

Şekil 4.16'de 16 adet pilot kullanılması (T=16) durumunda EM-JDE ve SAGE-JDE başarımları SDE algoritmalarıyla karşılaştırılmıştır. Yapılan benzetimlerde JDE tabanlı algoritmalarının SDE tabanlı algoritmalardan başarım üstünlüğü gösterilmiştir. Ayrıca EM-JDE algoritmasının SAGE-JDE algoritmasından başarım sonuçlarının daha iyi olduğu gözlemlenmiştir. Seri sıralı karışım engelleme algoritmalarının başarımının paralel olarak karışım engelleyicilerden üstünlüğü Şekil 4.14'de gösterilmişti. Bu sonuç, DS-CDMA sistemleri içinde geçerli olduğu Kocian tarafından gösterilmiştir[10]. Ancak, Şekil 4.16'de MC-CDMA sistemleri için yapılan benzetimlerde EM-JDE'nin başarımının SAGE-JDE'den üstün olduğu gözlemlenmiştir. Burada SAGE-JDE ilk kanal kestirimi işlemini 1. kullanıcının bilgisini yeniledikten sonra yaparken EM-JDE bütün kullanıcılara ait bilgileri yeniledikten sonra yapmaktadır. Bu sebeble, SAGE-JDE'nin ilk iterasyonda yapılan kanal kestirim sonucu, EM-JDE'de yapılan kestirim işleminden kötü olduğu için, SAGE-JDE alıcı yapısının başarımı EM-JDE alıcı yapısının başarımına göre sınırlı kalmıştır. Açıkçası yenileme işlemi EM-JDE'de daha efektiftir. Yapılan bilgisayar benzetimlerinde EM-JDE alıcısının başarımı MMSE-SDE, Tümleşik-MMSE-PIC, CSI-MMSE, CSI-MMSE-PIC ve SAGE-JDE alıcılarının başarımlarından daha iyi olduğu ve CSI-SAGE-SDE durumundaki başarıma yaklaştığı gözlemlenmiştir.

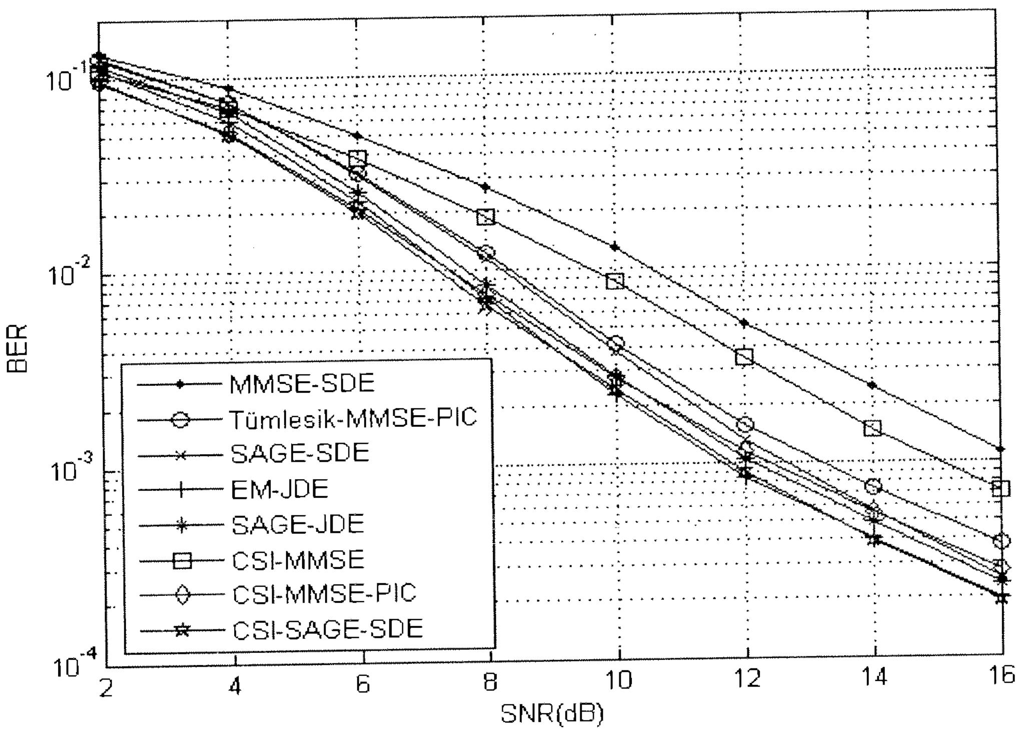
Şekil 4.17'de EM-JDE ve SAGE-JDE algoritmalarının yakınsama sonuçları verilmiştir. SAGE-JDE algoritması yaklaşık 2 iterasyonda yakınsarken, EM-JDE için bu sonuç yaklaşık 4 iterasyondur. Ancak daha öncede belirtildiği üzere EM-JDE algoritması, A matrisinin tamamını yeniledikten sonra kanal kestirimi yaparken, SAGE-JDE algoritması A matrisinin sadece k. sütununu yeniledikten sonra kanal kestirim işlemini yapmaktadır. Bu açıdan yapılan kanal kestirim işleminin güvenirliliği, EM-JDE kadar iyi olmamakta ve başarımı sınırlı kalmaktadır. Ayrıca bu şekilde kullanılan pilot tonların 8 olması durumunda algoritmanın başlangıç değerlerine duyarlı olduğu gözlenmiş ve bu durumda başarım kaybının söz konusu olduğu anlaşılmıştır. Ancak kullanılan pilot sayısına göre, EM tabanlı JDE algoritmalarına ait başarımın, SDE tabanlı algoritmaların başarımından üstünlüğü yapılan benzetimlerde açıkça gösterilmiştir.



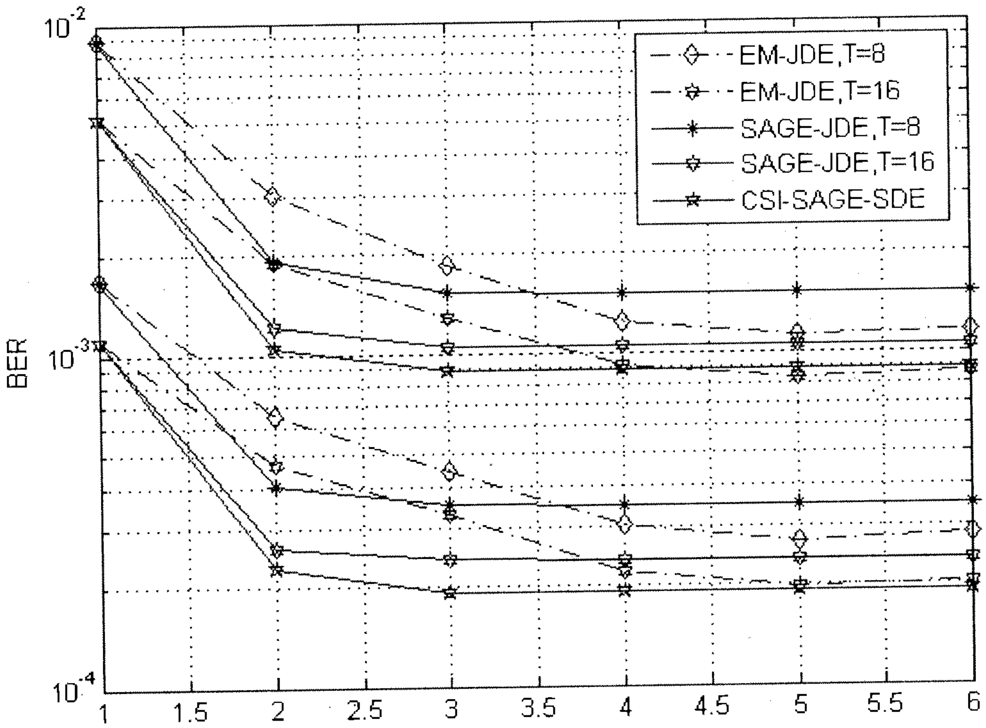
Şekil 4.14 : Yukarı Link MC-CDMA sistemleri için önerilmiş ayrık kanal kestirim ve bilgi sezimine dayalı (SDE) alıcıların BER başarımlarının SNR oranına göre incelenmesi



Şekil 4.15 : Yukarı Link MC-CDMA sistemleri için önerilen SDE algoritmalarının yakınsama başarımları



Şekil 4.16: Yukarı Link MC-CDMA sistemleri için önerilen EM-JDE ve SAGE-JDE algoritmalarının BER başarımları



Şekil 4.17: EM-JDE ve SAGE-JDE algoritmalarının farklı pilot sayıları için yakınsama grafiklerinin BER başarımları

4.5 SONUÇLAR

Proje kapsamında öncelikle Bölüm 3.1'de literatürde önerilen EM algoritlarından farklı olarak dış kodlayıcılı STBC/SFBC-OFDM sistemleri için MAP-EM kanal kestirim tabanlı bir turbo alıcı yapısı önerilmiştir. Daha önce teklif edilen, iletilen veri dizisinin alıcıda bilindiğini varsayan eğitim dizili (traning sequence) yaklaşımlardan farklı olarak, bu çalışmada bilgi-desteksiz (non-data-aided) çalışan EM tabanlı yeni bir kanal kestirim algoritması önerilmiştir. EM algoritması, birleşik bilgi sezimi ve kanal kestirimi için uygulamada çekici gözüksede, yakınsamasının oldukça yavaş olduğu bilinmektedir. Ayrıca algoritma ilklendirme işleminede duyarlıdır. Bu açıdan algoritmanın ilklendirilmesi için az sayıda pilot sembollerin kullanımı yeterlidir. Bilgi-desteksiz kestirim işlemi, eniyileme işleminden önce bilginin istatiksel ortalamasının alınması nedeniyle, çekici bir yöntemdir. Genel olarak uygulamalarda parametre kestirimlerini analitik olarak elde etmek pek mümkün değildir. Ancak, iletim çeşitlemeli STBC ve SFBC sistemlerinin dikgen kodlama özelliği, kanal parametrelerinin kestirimlerinin tam ve basit bir şekilde ifadesini mümkün kılmaktadır.

Bölüm 3.1'de getirilen diğer önemli katkı ise, literatürdeki ML çözümlerinden farklı olarak geliştirilen MAP kanal kestirimi algoritmasının iteratif olarak frekans domeninde gerçeklenmesidir. Ayrıca, algoritmada karmaşık Gauss rastlantısal değişkenler biçiminde modellenen kanal katsayıları, Karhunen-Loeve (KL) birim-dik seri açılımı yardımıyla ilintisiz rastlantısal değişkenlere dönüştürülerek verici çeşitlemeli OFDM sistemleri için önerilen alıcı yapısında kullanılmaktadır. KL açılımıyla birlikte kullanılan Alumouti dikgen kanal kestirim işleminin matris tersi alınmaksızın gerçekleştirilebilmesini sağlamıştır. Ayrıca bu açılımdan hesaplanan öz değerlerden sadece en önemlilerinin kanal kestirimde kullanılmasıyla, kestirim ifadesindeki karmaşıklığın dahada azaltılabileceği gösterilmiştir.

Önerilen kanal kestirimcinin bilgisayar benzetimleri Bölüm 4.1'de yapılarak başarımları, kanal kestirim ortalama karesel hata (MSE) ve bit hata oranı (BER) olarak çeşitli sistem parametreleri için incelenmiştir. Yapılan bilgisayar benzetim çalışmalarında önerilen kanal kestirim algoritmasının daha önce önerilen geleneksel kanal kestirim algortimalarına göre üstünlüğü gösterilmiş ve önerilen alıcı yapısı için turbo kodlanmış sistemlerin kanal kestirim hatalarına duyarlılığının daha fazla olduğu sonucuna varılmıştır. Ayrıca yüksek değerli

Doppler değerleri için turbo kodlanmış SFBC-OFDM sistemlerinin turbo kodlanmış STBC-OFDM sistemlerinden üstün olduğu gösterilmiştir.

Ayrıca proje kapsamında faz gürültüsü ve data seziminin ortaklaşa çözümü için SMC algoritması önerilmiş ve ortak kestirimin işleminin gerçeklenebilir karmaşıklıkla sağlanabileceği gösterilmiştir. SMC algoritmasının BPSK modüleli işaretler için bilgisayar benzetimleri Bölüm 4.2'de yapılmış ve farklı başlangıç faz hatalarındaki başarımının alıcıda faz bilgisinin bilindiği durumdaki başarıma yaklaştığı gösterilmiştir.

Ayrıca proje kapsamında, tek kullanıcılı alıcılar için proje kapsamında önerilen alıcı yapıları çok kullanıcılı alıcılar için de gözönüne alınmış ve bu kapsamda Bölüm 3.3 ve Bölüm 3.4'de sırasıyla yukarı ve aşağı link MC-CDMA sistemleri için alıcı yapıları önerilmiştir. Aşağı link MC-CDMA sistemleri için literatürde önerilmiş, PIC alıcı yapısının başlangıç değerlerini bulmak için EM-MAP tabanlı alıcı yapısı önerilmiştir. Önerilen alıcı yapısı daha önce önerilen LS, LMMSE tabanlı kanal kestirimci yapılarıyla karşılaştırılarak EM-MAP algoritmasının üstünlüğü gösterilmiştir. Algoritmanın kanalın alıcıda tamamen bilindiği durumdaki başarımına yaklaştığı gözlemlenmiştir. Ayrıca, algoritmanın karmaşıklığının azaltılması için kanal değişimlerinin rastgele bir süreç olarak modellenip Karhunen-Loeve (KL) dik seri açılımı uygulanmıştır. Yapılan KL açılımı sayesinde alınması gerekli olan matris ters alma işleminde boyut küçültme işleminin gerçekleştirilebildiği gösterilmiştir. Önerilen kanal kestirimcinin bilgisayar benzetimleri Bölüm 4.3'de yapılarak başarımları bit hata oranı (BER) olarak çeşitli sistem parametreleri için incelenmiştir. Yapılan bilgisayar benzetim çalışmalarında önerilen kanal kestirim algortimasının daha önce önerilen geleneksel kanal kestirim algoritmalarına göre üstünlüğü gösterilmiştir.

Bölüm 3.4'de ise frekans-seçici kanallarda çalışan yukarı link MC-CDMA sistemleri için birleşik bilgi sezimi ve kanal kestirim işlemi problemine yaklaşımda bulunulmuştur. Bu çerçevede Kocian'nın düz sönümlemeli Rayleigh kanallar üzerinden çalışan yukarı link DS-CDMA sistemler için önerdiği EM tabanlı alıcı incelenmiş ve önerilen alıcı yapısı frekans-seçici kanallarda çalışan MC-CDMA sistemleri için genelleştirilerek, kullanıcılara ait bilgi dizilerinin sezimi için parçalı karışım engelleyici ve kanal kestirimci yapısını içeren alıcı yapısı elde edilmiştir. Bölüm 4.4'de yapılan bilgisayar benzetimlerinde, kullanılan az sayıdaki pilotun EM-JDE ve SAGE-JDE algoritmasını ilklendirmek için yeterli olduğu ve bu algoritmaların daha önce önerilen MMSE-SDE, Tümleşik-MMSE-PIC, SAGE-SDE alıcı yapılarından üstünlüğü gösterilmiştir. Ayrıca EM-JDE'nin SAGE-JDE'den üstünlüğü yapılan bilgisayar benzetimlerinde yakınsama grafikleriyle birlikte verilmiştir. Bu grafiklerde, iterasyonla birlikte yapılan kestirim işlemlerinin düzeltildiği ve kanallara ait bilgilerin alıcıda tamamen bilinmesi durumundaki CSI-SAGE-SDE alıcı yapısının başarımına yaklaştığı gözlemlenmiştir.

KAYNAKLAR

1. DEMPSTER A.P., LAIRD N.M. and RUBIN D. B., Maximum Likelihood from Incomplete data via the EM algorithm, *J. Roy. Statist. Soc.*, Vol.39, pp. 1-17, 1977.

2. FEDER M. and WEINSTEIN E., Parameter Estimation of superimposed signals using the EM algorithm, *IEEE Tran. On Acoustic, Speech and Signal Processing,* Vol. 36, pp. 477-489, Nisan 1988.

3. KALEH G.K. AND VALET R., Joint parameter estimation and symbol detection for linear or nonlinear unknown channels, *IEEE Trans. Commun.* vol. 42, No. 7, pp. 2406-2413, Temmuz 1994.

4 Q. Li, GEORGHİADES C.N. and WANG X., An Iterative Receiver for Turbo-Coded Pilot-Symbol Assisted Modulation in Fading Channels, *IEEE Communication Letters*, Vol. 5, No. 4, Nisan 2001.

5. LI Y., GEORGHIADES C.N., and HUANG G., Iterative maximum likelihood sequence estimation for space-time coded systems, *IEEE Transactions on Communications*, Vol. 49, No. 6, pp. 948-951, Haziran 2001.

6 ZHANG Q. and GEORGHIADES C.N., An Interference Rejection Application of the EM Algorithm to Direct-sequence Signals, Kybernetika, Vol. 39, No. 1, pp. 83-91, Mart 1999.

7 Lİ Q., GEORGHIADES C.N., and WANG X., Adaptive Multiuser Detection in Multipath Fading Channels - A Sequential EM Approach, *IEEE Transactions on Communications*, 52(1), pp.71-81, Ocak 2004.

8. LU B., WANG X. and LI Y. (G), Iterative receivers for Space-Time Block-Coded OFDM systems in dispersive fading channels, *IEEE Transactions on Wireless Communications*, Vol. 1, No. 2, pp. 213-225, Nisan 2002.

9 A. T. HUQ, PANAYIRCI E. and C.N. GEOGHIADES, Non-data-aided ML carrier frequency and phase synchronization in OFDM systems, *European Transactions on Telecommunications (ETT)*, vol. 12, no. 2, pp. 83-94 , Mart-Nisan 2001.

10 PANAYIRCI E. and C.N. GEORGHIADES, Carrier phase synchronization of OFDM systems over frequency selective channels via EM algorithm, *Proc. IEEE Vehicular Technology Conference (VTC99)*, Mayıs 16-20, Houston, Texas, USA, 1999.

11 ÇIRPAN H. A. and TSATSANIS M. K., Maximum Likelihood Estimation of FIR Channels Excited by Convolutionally Encoded Inputs, *IEEE Transactions on Communications*, Vol. 49, No. 7, pp. 1125-1128, Haziran 2001.

12 PANAYIRCI E. and ÇIRPAN H.A., Maximum A Posteriori Multipath Fading Channel Estimation for OFDM Systems, *European Trans. on Telecommunications (ETT)*: Special Issue on Multi Carrier Spread Spectrum & Related, Vol. 13, No. 5, pp. 487-494 Eylül/Ekim 2002.

13 ÇIRPAN H.A., PANAYIRCI E. and ÇEKLİ E., Maximum Likelihood Blind Channel Estimation for Space-Time Coding Systems, *EURASIP Journal on Applied Signal Processing: Special Issue on Space-Time Coding and its Applications-Part II*, Vol. 2002, No. 5, pp. 497-506, Mayıs, 2002.

14 ŞEN A., ÇIRPAN H.A., and PANAYIRCI E., Joint Channel Tracking and Symbol Detection for OFDM Systems with Kalman Filtering, *AEÜ International Journal of Electronics and Communications,* Vol. 57, No. 5, sayfa 317-327, 2003.

15 ÇIRPAN H.A., PANAYIRCI E. and DOĞAN H., Iterative Channel Estimation Approach for Space-Frequency Coded OFDM Systems with Transmitter Diversity, chapter in *Multi-Carrier Spread-Spectrum* edited by K. Fazel ve S. Kaiser , pp. 195-202 (2004 Kluwer Academic Publishers, ISBN 1-4020-1837-1).

16 PANAYIRCI E. and ÇIRPAN H.A., Non-Data Aided EM-Based Channel Estimation for OFDM Systems with Time-Varying Fading Channels, chapter in *Multi-Carrier Spread-Spectrum Related Topics* edited by K. Fazel ve S. Kaiser, pp. 293-301ı. (2002 Kluwer Academic Publishers, ISBN 0-7923-7653-6)

17 GUO D., WANG X. and CHEN R., Multilevel Mixture Kalman Filter, *EURASIP Journal of Applied Signal Processing*, special issue on Particle Filtering in Signal Processing, 15 (2004), pp.2255-2266, Kasım 2004.

18 WANG X., Monte Carlo Signal Processing in Digital Communications: Principles and Applications, In *Signal Processing for Mobile Communications Handbook*, Mohamed Ibnkahla, Editor, CRC press, 2004.

19 YANG Z. and WANG X., Blind Detection of OFDM Signals in Multipath Fading Channels via Sequential Monte Carlo, *IEEE Transactions on Signal Processing*, special issue on Monte Carlo Methods for Statistical Signal Processing, 50(2), pp.271-280, Şubat 2002.

20. ALAMOUTI M.S., A simple transmitter diversity scheme for wireless communications, *IEEE J. Selected Areas Commun.*, Vol. 16, pp. 1451-1458, Ekim 1998.

21. TAROKH V., SESHADRI N., and CALDERBANK A. R., Space-time codes for high data rate wireless communications: Performance analysis and code construction, *IEEE Trans. Inform. Theory*, Vol. 44, pp. 744-765, Mart 1998.

22. TAROKH V., JAFARKHANI HAMID, and CALDERBANK A. R., Space-time block Codes from orthogonal designs, *IEEE Trans.Inform. Theory* , Vol. 45, No.5 pp. 1456-1467, Temmuz 1999.

23. VAN NEE R. and PRASAD R., *OFDM Wireless Multimedia Communications*, Artech House Publishers, Boston USA, 0890065306, 2000.

24. LEE KF, WILLIAMS DB., A Space-Frequency Transmitter Diversity Technique for OFDM Systems, *In Proceedings of IEEE Globecom*, 27 Kasım – 1 Aralık 2000, San Francisco, CA, Kasım 2000, pp. 1473–1477.

25. LI Y. (G.), CIMINI L. J., SESHADRI N. and ARIYAVISTAKUL S., Channel estimation for OFDM systems with transmitter diversity in mobile wireless channels, *IEEE Journal Selected Areas on Communications*, Vol. 17, No.3, pp. 461-471, Temmuz 1999.

26. LI Y. (G.), Simplified channel estimation for OFDM systems with multiple transmit antennas, *IEEE Transactions onWireless Communications*, Vol. 1, No. 1, pp. 67-75, Ocak 2002.

27. COZZO C. and HUGHES B.L., Joint channel estimation and data detection in space-time communications, *IEEE Transactions on Communications*, Vol. 51, No. 8, pp 1266-1270, Ağustos 2003.

28. YONGZHE X. and GEORGHIADES C.N., Two EM-type channel estimation algorithms for OFDM with transmitter diversity, *IEEE Transactions on communications*, Vol. 51, No. 1, pp. 106-115, Ocak 2003.

29. MA X., KOYABASHI H. and SCHWARTS S. C., An EM-based channel estimation for Space-Time and Space-Frequency Block Coded OFDM, Nisan 6-10 2003 Hong Kong, *In proceedings of ICASSP '03* , Vol.4 , 2003, pp. 389 -392.

30. MCELIECE R.J. and STARK W.E., Channels with block interference, *IEEE Trans. Inform. Theory*, Vol. 30, No. 1, pp. 44-53, Ocak 1984.

31. GIANNAKIS G. B. and TEPEDELENLIOGLU C., Basis expansion models and diversity techniques for blind identification and equalization of timevarying channels, *Proceedings of the IEEE*, Vol. 86, pp. 1969-1986, Ekim 1998.

32. YIP K. and NG T., Karhunen-Loeve Expansion of the WSSUS Channel Output and its Application to Efficient Simulation, *IEEE Journal on Selected Areas in Communications*, Vol.15, no 4, pp.640-646, Mayıs 1997.

33. BEEK J.-J. VAN DE, EDFORS O., BRJESSON PER O., WAHLQVIST M., ÖSTBERG C., A Conceptual Study of OFDM-based Multiple Access Schemes:Part 2-Channel Estimation

in the Uplink Technical Report Tdoc 116/96, Mayıs, Helsinki, Finland, *ETSI STC SMG2 Meeting,* No 18., 1996.

34. COST (European Cooperation In The Field Of Scientific And Technical Research), 1989, *Digital Land Mobile Radio Communications- COST 207*, Commission of the European Communities, Luxembourg, EUR 12160.

35. BEEK J-J VAN DE, EDFORS O., SANDELL M., WILSON S.K., and BRJESSON PER O., OFDM channel estimation by singular value decomposition, *IEEE Trans. Commun.*, Vol. 46, No.7, pp. 931-936, Eylül 1996.

36. TUCHLER M., KOETTER R., AND SINGER A. C., Turbo Equalization: Principles and New results," *IEEE Trans. Commun.*, Vol. 50, pp. 754-767, Mayıs 2002.

37. ZHU JIE, LEE WOOKWON, A low-complexity channel estimator for OFDM systems in multipath fading channels, *In proceedings of IEEE-PIMRC 2004*, 5-8 Eylül 2004, Barcelona, Spain, Vol.3, 2004, pp. 1978 – 1982.

38. BAOGUO Y., ZHIGANG C., and LETAIEF K., Analysis of low-complexity windowed DFT-based MMSE channel estimator for OFDM systems, *IEEE Trans. Commun.*, Vol. 49, no. 11, pp. 1977-1987, Kasım 2001.

39. EDFORS O., SANDELL M., BEEK J. J. VAN DE, WILSON S. K. and BORJESSON P. O., Analysis of DFT-based channel estimation for OFDM, *Wireless Personal Commun.*, Vol. 12, No. 1, pp. 5570, Ocak 2000.

40. WOODARD J.P., HANZO L., Comparative study of turbo decoding techniques: an overview, *Vehicular Technology, IEEE Transactions on,* Vol. 49, No. 6, pp. 2208 – 2233, Kasım 2000.

41. ÇIRPAN H.A , PANAYIRCI E., DOĞAN H., Nondata-aided Channel Estimation for OFDM Systems with Space-Frequency Transmit Diversity, *IEEE Transactions on Vehicular Technology*, Vol. 55, Issue 2, pp. 449 – 457, Mart 2006.

42. ÇIRPAN H.A, PANAYIRCI E, DOĞAN H, Iterative channel estimation approach for space-time/frequency coded OFDM systems with transmitter diversity, *Eur. Trans. Telecommun.,* 15 (3): 235-248, Haziran 2004.

43. VALENTI, M.C.; WOERNER, B.D., Iterative channel estimation and decoding of pilot symbol assisted turbo codes over flat-fading channels, *IEEE J. Select. Areas Commun.,* Vol. 19, No. 9, pp. 1697 – 1705, Eylül 2001.

44. KAY S., *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice-Hall, Englewood Cliffs NJ. 0-13-345711-7, 1993.

45. PANAYIRCI E. and ÇIRPAN H. A., 2002, Channel estimation for Space-Time Block Coded OFDM systems in the presence of multipath fading, *IEEE Globecom*, Kasım. 17-21 2002, Taiwan, Vol.2, *2002*, pp.1157- 1161.

46 S. HARA and R. PRASAD, Overview of Multicarrier CDMA, *IEEE Communication Magazine*, pp. 126-133, Aralık 1997.

47 K. FAZEL and L. PAPKE, On the performance of convolutionally- coded CDMA/OFDM for mobile communication system, *in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC 93)*, Eylül 1993, pp. 468-472.

48 V. KUHN, Combined MMSE-PIC in coded OFDM-CDMA systems, *IEEE GLOBECOM '01*, pp. 231-235, Kasım 2001.

49 S. IRAJI, T. SIPILA AND J. LILLEBERG, Channel Estimation and Signal Detection for MC-CDMA in Multipath Fading Channels, *in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC)*, Eylül 1993

50 NELSON LB, POOR HV, Iterative multiuser receivers for CDMA channels: An EM-based approach, *IEEE Trans. Commun.*44 (12): 1700-1710, Aralık 1996

51 A. KOCIAN and B. H. FLEURY, EM-based joint data detection and channel estimation of DS-CDMA signals, *IEEE Trans. Commun.*, vol. 51, no. 10, pp. 1709-1720, Ekim 2003.

52 J. A. FESSLER and A. O. HERO, Space-alternating generalized expectation-maximization algorithm, *IEEE Trans. Signal Processing,* vol. 42, pp. 26642677, Ekim 1994.

53 H. DOĞAN, H. A. ÇIRPAN and E. PANAYIRCI, Iterative Channel Estimation and Decoding of Turbo Coded SFBC-OFDM Systems, *IEEE Trans. Wireless Commun.*, vol.6, no.7, Temmuz 2007.

Nondata-Aided Channel Estimation for OFDM Systems With Space-Frequency Transmit Diversity

Hakan A. Çırpan, *Member, IEEE*, Erdal Panayırcı, *Fellow, IEEE*, and Hakan Doğan, *Student Member, IEEE*

***Abstract*—This paper proposes a computationally efficient nondata-aided maximum *a posteriori* (MAP) channel-estimation algorithm focusing on the space-frequency (SF) transmit diversity orthogonal frequency division multiplexing (OFDM) transmission through frequency-selective channels. The proposed algorithm properly averages out the data sequence and requires a convenient representation of the discrete multipath fading channel based on the Karhunen–Loeve (KL) orthogonal expansion and estimates the complex channel parameters of each subcarrier iteratively, using the expectation maximization (EM) method. To further reduce the computational complexity of the proposed MAP algorithm, the optimal truncation property of the KL expansion is exploited. The performance of the MAP channel estimator is studied based on the evaluation of the modified Cramer–Rao bound (CRB). Simulation results confirm the proposed theoretical analysis and illustrate that the proposed algorithm is capable of tracking fast fading and improving overall performance.**

***Index Terms*—Expectation maximization (EM) algorithm, maximum *a posteriori* (MAP) channel estimation, orthogonal frequency division multiplexing (OFDM) systems, space-frequency coding.**

I. Introduction

TRADITIONAL wireless technologies are not very well suited to meet the demanding requirements of providing very high data rates with ubiquity and mobility. Given the scarcity and exorbitant cost of the radio spectrum, such data rates dictate the need for extremely high spectral efficient coding and modulation schemes [1]. The combined application of transmit-antenna diversity and orthogonal frequency division multiplexing (OFDM) modulation appears to be capable of enabling the types of capacities and data rates needed for broadband wireless services [1], [2].

Transmit-antenna diversity has been exploited recently to develop high-performance space-time/frequency codes and simple maximum likelihood (ML) decoders for transmission over flat fading channels [3]–[5]. Unfortunately, their practical application can present a real challenge to channel-estimation algorithms, especially when the signal suffers from frequency-selective multipath channels. One of the solutions for alleviating frequency selectivity is through the use of OFDM together with transmit diversity which combats the long channel impulse response by transmitting parallel symbols over many orthogonal subcarriers, yielding unique reduced complexity physical layer capabilities [1].

Channel estimation for transmit-diversity OFDM systems has attracted much attention with the pioneering studies of Li [6], [7]. Among many other techniques, an iterative procedure based on the expectation maximization (EM) algorithm was also applied to the channel estimation problem in the context of space-time block coding (STBC) [8], [9] as well as transmit-diversity OFDM systems [10]–[13]. In [10], both the ML and the maximum *a posteriori* (MAP) iterative receivers for STBC-OFDM systems based on the EM algorithm are proposed to directly detect transmitted symbols under the assumption that fading processes remain constant across several OFDM words contained in one STBC code word. Note that even this approach pretends to bypass the channel-estimation process; it iterates between the ML data detection and the channel estimation consecutively until the convergence is reached. Although this approach is certainly optimal, its convergence rate is slow; the initial selection of the channel parameters is very critical and its implementation is quite complex.

An EM approach proposed for the general estimation from superimposed signals [15] is applied to the channel estimation for OFDM systems with transmitter- diversity systems and is compared with the space-alternating generalized EM (SAGE) version in [12]. Moreover, in [13], a modified version of [12] is proposed for STBC-OFDM and space-frequency (SF) block-coding (SFBC)-OFDM systems.

Unlike the EM approaches treated in [10]–[13], we adopt a two-step detection procedure: 1) Use the EM algorithm to estimate the channel, and 2) use the estimated channel to perform coherent detection. The major contribution of this paper is to obtain a new efficient nondata-aided MAP EM channel-estimation algorithm for OFDM systems with transmitter diversity using SFBC. A different approach is adapted here to explicitly model the channel parameters by a Karhunen–Loeve (KL) series representation, since a KL expansion allows one to tackle the estimation of correlated parameters as a parameter estimation problem of the uncorrelated coefficients. Note that

Manuscript received August 8, 2003; revised June 17, 2004 and August 2, 2005. This work was conducted within the NEWCOM Network of Excellence in Wireless COMmunications funded through the EC Sixth Framework Programme and was supported in part by the Research Fund of Istanbul University under Projects 220/29042004, UDP-599/28042005, and UDP-599/28072005, and The Scientific and Technological Research Council of Turkey (TUBITAK) under Grant 104E166. The review of this paper was coordinated by Dr. E. Larsson.

H. A. Çırpan and H. Doğan are with the Department of Electrical Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey (e-mail: hcirpan@istanbul.edu.tr; hdogan@istanbul.edu.tr).

E. Panayırcı is with the Department of Electrical and Electronics Engineering, Bilkent University, Bilkent 06800, Ankara, Turkey (e-mail: eepanay@ee.bilkent.edu.tr).

Digital Object Identifier 10.1109/TVT.2005.863427

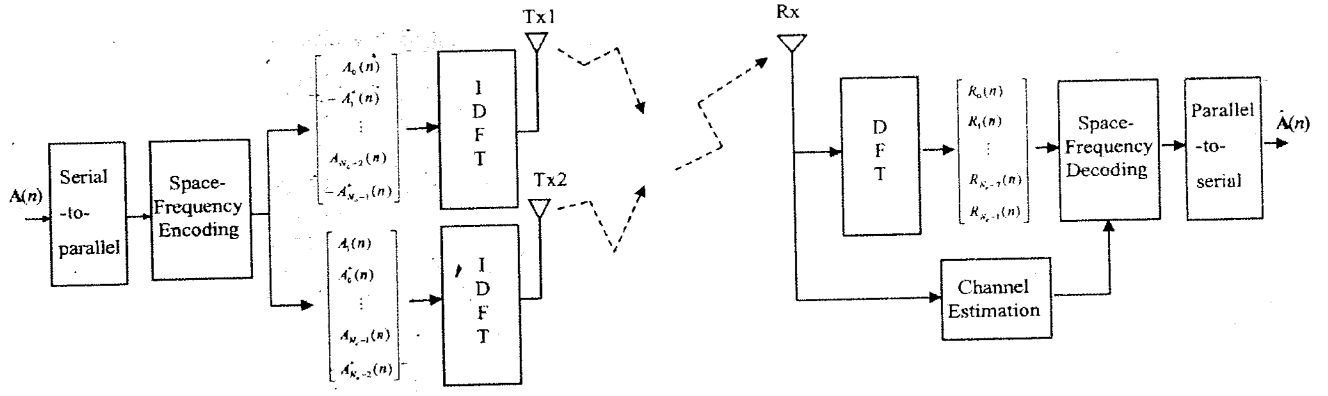


Fig. 1. SF block-coded OFDM scheme.

the KL expansion is well known for its optimal truncation property [19]. That is, the KL expansion requires the minimum number of terms among all possible series expansions in representing a random channel for a given mse. Thus, the optimal truncation property of the KL expansion results in a smaller computational load on the channel-estimation algorithm. Moreover, except for a few pilot symbols for initialization, the technique does not need any training sequence to acquire the channel and more information carrying signals can be transmitted.

Due to the orthogonality of the SFBC system based on the Alamouti orthogonal design, as well as the KL expansion of the multipath channel that yields simple exact iterative expressions for the unknown channel parameters in frequency domain which do not require any matrix inversion [18], [19]. Moreover, the optimal truncation property of the KL expansion can further reduce the computational load on the channel-estimation algorithm.

II. SFBC-OFDM SYSTEMS

Resorting to coding across tones, the set of generally correlated OFDM subchannels is first divided into groups of subchannels. This subchannel grouping with appropriate system parameters preserves the diversity gain while simplifying not only the code construction, but the decoding algorithm as well [14]. A block diagram of a two-branch SF OFDM transmitter-diversity system is shown in Fig. 1. To cast the received signal model, we first define $N_c \times 1$ the data vector $\mathbf{A}(n)$ as $\mathbf{A}(n) = [A(nN_c), A(nN_c+1), \ldots, A(nN_c + N_c - 1)]^T$. Following the notation of [14], let $A_k(n)$ denote the kth forward polyphase component of the serial data symbols, i.e, $A_k(n) = A(nN_c + k)$ for $k = 0, \ldots, N_c - 1$. The polyphase component $A_k(n)$ can also be viewed as the data symbol to be transmitted on the kth tone during the block instant n. The data symbol vector $\mathbf{A}(n)$ can therefore be expressed as $\mathbf{A}(n) = [A_0(n), A_1(n), \ldots, A_{N_c-1}]^T$. Resorting the subchannel grouping, $\mathbf{A}(n)$ is coded into two vectors $\mathbf{A}_e(n)$ and $\mathbf{A}_o(n)$ by the SF encoder as

$$\begin{aligned}\mathbf{A}_e(n) &= [A_0(n), A_2(n), \ldots, A_{N_c-4}(n), A_{N_c-2}(n)]^T\\ \mathbf{A}_o(n) &= [A_1(n), A_3(n), \ldots, A_{N_c-3}(n), A_{N_c-1}(n)]^T \end{aligned} \quad (1)$$

where $\mathbf{A}_e(n)$ and $\mathbf{A}_o(n)$ actually corresponds to the even and odd polyphase component vectors of $\mathbf{A}(n)$. Then, the SF block code transmission matrix may be represented by

$$\begin{array}{cc} & \text{frequency} \rightarrow \\ \text{space} \downarrow & \begin{bmatrix} \mathbf{A}_e(n) & -\mathbf{A}_o^*(n) \\ \mathbf{A}_o(n) & \mathbf{A}_e^*(n) \end{bmatrix} \end{array} \quad (2)$$

where * stands for the complex conjugation.

If the received signal sequence is also parsed in even and odd blocks of N_c tones, $\mathbf{R}_e(n) = [R_0(n), R_2(n), \ldots, R_{N_c-2}(n)]^T$ and $\mathbf{R}_o(n) = [R_1(n), R_3(n), \ldots, R_{N_c-1}(n)]^T$, the received signal can be expressed in vector form as

$$\begin{aligned}\mathbf{R}_e(n) &= \mathcal{A}_e(n)\mathbf{H}_{1,e}(n) + \mathcal{A}_o(n)\mathbf{H}_{2,e}(n) + \mathbf{W}_e(n)\\ \mathbf{R}_o(n) &= -\mathcal{A}_o^\dagger(n)\mathbf{H}_{1,o}(n) + \mathcal{A}_e^\dagger(n)\mathbf{H}_{2,o}(n) + \mathbf{W}_o(n)\end{aligned} \quad (3)$$

where $\mathcal{A}_e(n)$ and $\mathcal{A}_o(n)$ are $N_c/2 \times N_c/2$ diagonal matrices whose elements are $\mathbf{A}_e(n)$ and $\mathbf{A}_o(n)$, respectively, and † denotes the conjugate transpose. $\mathbf{H}_{\mu,e}(n) = [H_{\mu,0}(n), H_{\mu,2}(n), \ldots, H_{\mu,N_c-2}(n)]^T$ and $\mathbf{H}_{\mu,o}(n) = [H_{\mu,1}(n), H_{\mu,3}(n), \ldots, H_{\mu,N_c-1}(n)]^T$ are $N_c/2$ length vectors denoting the even and odd component vectors of the channel attenuations between the μth transmitter and the receiver. Finally, $\mathbf{W}_e(n)$ and $\mathbf{W}_o(n)$ are $N_c/2 \times 1$ zero mean and independent identically distributed (i.i.d.) Gaussian vectors that model additive noise in the N_c tones, with a variance of $\sigma^2/2$ per dimension.

Equation (3) shows that the information symbols $\mathcal{A}_e(n)$ and $\mathcal{A}_o(n)$ are transmitted twice in two consecutive adjacent subchannel groups through two different channels. In order to estimate the channels and decode $\mathcal{A}$ with the embedded diversity gain through the repeated transmission, for each n, we can write the following equation from (3) as

$$\begin{bmatrix} \mathbf{R}_e(n) \\ \mathbf{R}_o(n) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_e(n) & \mathcal{A}_o(n) \\ -\mathcal{A}_o^\dagger(n) & \mathcal{A}_e^\dagger(n) \end{bmatrix} \begin{bmatrix} \mathbf{H}_{1,e}(n) \\ \mathbf{H}_{2,e}(n) \end{bmatrix} + \begin{bmatrix} \mathbf{W}_e(n) \\ \mathbf{W}_o(n) \end{bmatrix} \quad (4)$$

where the complex channel gains between the adjacent subcarriers are assumed to be approximately constant, i.e., $\mathbf{H}_{1,e}(n) \approx \mathbf{H}_{1,o}(n)$ and $\mathbf{H}_{2,e}(n) \approx \mathbf{H}_{2,o}(n)$. The effect of this assumption allows us to omit the dependence of $\mathbf{H}_{1,e}(n)$ and $\mathbf{H}_{2,e}(n)$ on the even channel components. Using (4) and dropping subscript "s," we have

$$\begin{bmatrix} \mathbf{R}_e(n) \\ \mathbf{R}_o(n) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_e(n) & \mathcal{A}_o(n) \\ -\mathcal{A}_o^\dagger(n) & \mathcal{A}_e^\dagger(n) \end{bmatrix} \begin{bmatrix} \mathbf{H}_1(n) \\ \mathbf{H}_2(n) \end{bmatrix} + \begin{bmatrix} \mathbf{W}_e(n) \\ \mathbf{W}_o(n) \end{bmatrix} \tag{5}$$

or in a more succinct form

$$\mathbf{R}(n) = \mathcal{A}(n)\mathbf{H}(n) + \mathbf{W}(n). \tag{6}$$

Based on (6), our main objective in this paper is to develop a channel-estimation algorithm in accordance with the MAP criterion. The channel variations are considered as random processes and the KL orthogonal series expansion is applied. Prompted by the general applicability of the KL expansion, in this paper, we consider the components of $\mathbf{H}_\mu(n)$ to be expressed by a linear combination of orthonormal base vectors as $\mathbf{H}_\mu(n) = \mathbf{\Psi}\mathbf{G}_\mu(n)$, where $\mathbf{\Psi} = [\psi_0, \psi_1, \ldots, \psi_{N_c-1}]$, ψ_i's are the orthonormal basis vectors corresponding to the eigenvectors of the channel autocorrelation matrix $\mathbf{C}_{\mathbf{H}_\mu} = E[\mathbf{H}_\mu \mathbf{H}_\mu^\dagger]$. $\mathbf{G}_\mu(n)$ is an $N_c \times 1$ zero-mean i.i.d. Gaussian vector whose components $\mathbf{G}_\mu(n)[k] = \mathbf{G}_\mu(n,k)$, $k = 0, 1, \ldots, N_c - 1$ correspond to the weights of the KL expansion. Note that the covariance matrix of $\mathbf{G}_\mu(n)$ is $\mathbf{\Lambda} = \mathrm{diag}(\lambda_0, \lambda_1, \ldots, \lambda_{N_c-1})$, where λ_k's are the eigenvalues of $\mathbf{C}_{\mathbf{H}_\mu}$. Therefore, $\mathbf{C}_{\mathbf{H}_\mu}$ can be expressed as

$$\mathbf{C}_{\mathbf{H}_\mu} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^\dagger. \tag{7}$$

Thus, the channel estimation problem in this application is equivalent to estimating the i.i.d. Gaussian vector $\mathbf{G}_\mu$ of the KL expansion coefficients.

III. Nondata-Aided EM-Based MAP Channel Estimation

In the nondata-aided MAP estimation approach, we choose $\widehat{\mathbf{G}}$ to maximize the posterior probability density function (PDF), $\widehat{\mathbf{G}} = \arg\max_{\mathbf{G}} p(\mathbf{G}|\mathbf{R})$ where $\mathbf{G} = [\mathbf{G}_1^T, \mathbf{G}_2^T]^T$. To find the MAP estimator, we must equivalently maximize $p(\mathbf{R}|\mathbf{G})p(\mathbf{G})$. The prior PDF of the KL expansion coefficient r.v.'s of the fading channel can be expressed as $p(\mathbf{G}) \sim \exp(-\mathbf{G}^\dagger \widetilde{\mathbf{\Lambda}}^{-1}\mathbf{G})$, where $\widetilde{\mathbf{\Lambda}} = \mathrm{diag}(\mathbf{\Lambda}\ \mathbf{\Lambda})$.

Hence, the MAP estimator equivalently takes the form

$$\widehat{\mathbf{G}}_{\mathrm{MAP}} = \arg\max_{\mathbf{G}} [\ln p(\mathbf{R}|\mathbf{G}) + \ln p(\mathbf{G})] \tag{8}$$

where $p(\mathbf{R}|\mathbf{G}) = E_{\mathcal{A}}[p(\mathbf{R}|\mathbf{A}, \mathbf{G})$.

Given the transmitted signals $\mathcal{A}$, coded according to the SF transmit-diversity scheme and the discrete channel orthonormal series expansion representation coefficients $\mathbf{G}$ and taking into account the independence of the noise components, the conditional PDF of the received signal $\mathbf{R}$ can be expressed as

$$p(\mathbf{R}|\mathcal{A}, \mathbf{G}) \sim \exp\left[-(\mathbf{R} - \mathcal{A}\widetilde{\mathbf{\Psi}}\mathbf{G})^\dagger \widetilde{\mathbf{\Sigma}}^{-1}(\mathbf{R} - \mathcal{A}\widetilde{\mathbf{\Psi}}\mathbf{G})\right] \tag{9}$$

where $\widetilde{\mathbf{\Sigma}}$ is an $N_c \times N_c$ diagonal matrix with $\widetilde{\mathbf{\Sigma}}[k,k] = \sigma^2$, for $k = 0, 1, \ldots, N_c - 1$ and $\widetilde{\mathbf{\Psi}} = \mathrm{diag}(\mathbf{\Psi}\ \mathbf{\Psi})$.

Obtaining the MAP estimate of $\mathbf{G}$ from (9) is a complicated optimization problem and does not yield a closed-form solution. The solution of such problems usually requires numerical methods, such as methods of scoring, Newton–Raphson, or some other gradient search algorithm. However, for the problem at hand, these numerical methods tend to be computationally complex. Fortunately, the solution can be easily obtained by means of the iterative EM algorithm. Since the EM algorithm has been studied and applied to a number of problems in communications over the years, the details of the algorithm will not be presented in this paper. See [20]–[22] for a general exposition to the EM algorithm and [18] its applications to the estimation problem related to this study. Basically, this algorithm inductively reestimates $\mathbf{G}$ so that a monotonic increase in the *a posteriori* conditional pdf in (9) is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q\left(\mathbf{G}|\mathbf{G}^{(i)}\right) = \sum_{\mathcal{A}} p\left(\mathbf{R}, \mathcal{A}, \mathbf{G}^{(i)}\right) \log p(\mathbf{R}, \mathcal{A}, \mathbf{G}) \tag{10}$$

where $\mathbf{G}^{(i)}$ is the estimation of $\mathbf{G}$ at the ith iteration.

Note that $p(\mathbf{R}, \mathcal{A}, \mathbf{G}) \sim p(\mathbf{R}|\mathcal{A}, \mathbf{G})p(\mathbf{G})$, since the data symbols $\mathcal{A} = \{A_k(n)\}$ are assumed to be independent of each other and are identically distributed and because of the fact that they are independent of $\mathbf{G}$. Therefore, (10) can be easily evaluated compared to a direct computation of (9).

Given the received signal $\mathbf{R}$, the EM algorithm starts with an initial value $\mathbf{G}^0$ of the unknown channel parameter $\mathbf{G}$. The $(i+1)$th estimate of $\mathbf{G}$ is obtained by the maximization step described by $\mathbf{G}^{(i+1)} = \arg\max_{\mathbf{G}} Q(\mathbf{G}|\mathbf{G}^{(i)})$. As described in Appendix I, the expression of the reestimated value of $\mathbf{G}_\mu^{(i+1)}$ ($\mu = 1, 2$) can be obtained as follows:

$$\begin{aligned} \mathbf{G}_1^{(i+1)} &= (\mathbf{I} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{\Psi}^\dagger \left[\mathbf{\Gamma}_1^{(i)\dagger}\mathbf{R}_e(n) - \mathbf{\Gamma}_2^{(i)}\mathbf{R}_o(n)\right] \\ \mathbf{G}_2^{(i+1)} &= (\mathbf{I} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{\Psi}^\dagger \left[\mathbf{\Gamma}_2^{(i)\dagger}\mathbf{R}_e(n) + \mathbf{\Gamma}_1^{(i)}\mathbf{R}_o(n)\right] \end{aligned} \tag{11}$$

where it can be easily seen that

$$(\mathbf{I} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1} = \mathrm{diag}\left(\left[\left(\frac{1+\sigma^2}{\lambda_0}\right)^{-1}, \ldots, \left(\frac{1+\sigma^2}{\lambda_{N_c-1}}\right)^{-1}\right]\right) \tag{12}$$

and $\mathbf{\Gamma}_\mu^{(i)}$ in (11) is an $N_c/2 \times N_c/2$ dimensional diagonal matrix representing the *a posteriori* probabilities of the data symbols at the ith iteration step whose kth component is defined as

$$\begin{aligned} \Gamma_\mu^{(i)}(k) = \sum_{a_1} \sum_{a_2 \in S_k} a_\mu P\Big(A_{2k}(n) = a_1 \\ A_{2k+1}(n) = a_2 |\mathbf{R}, \mathbf{G}^{(i)}\Big), \qquad \mu = 1, 2 \end{aligned} \tag{13}$$

where S_k denotes the alphabet set taken by the kth OFDM symbol.

A truncated expansion $\mathbf{G}_{\mu,r}$ can be formed by selecting r orthonormal basis vectors among all the basis vectors that satisfy $\mathbf{C}_{\mathbf{H}_\mu}\Psi = \Psi\Lambda$. The optimal one that yields the smallest average mean-squared truncation error $1/(N_c/2)\, E[\epsilon_r^\dagger \epsilon_r]$ is the one expanded with the orthonormal basis vectors associated with the first largest r eigenvalues given by

$$\frac{1}{\frac{N_c}{2}-r} E\left[\epsilon_r^\dagger \epsilon_r\right] = \frac{1}{\frac{N_c}{2}-r} \sum_{i=r}^{\frac{N_c}{2}-1} \lambda_i \tag{14}$$

where $\epsilon_r = \mathbf{G}_\mu - \mathbf{G}_{\mu,r}$. For the problem at hand, the truncation property of the KL expansion results in a low-rank approximation. Thus, a rank-r approximation of Λ_r is defined as $\Lambda_r = \mathrm{diag}\{\lambda_0, \lambda_1, \ldots, \lambda_{r-1}, 0, \ldots, 0\}$. Since the trailing $N_c/2 - r$ variances $\{\lambda_l\}_{l=r}^{N_c/2-1}$ are small compared to the leading r variances $\{\lambda_l\}_{l=0}^{r-1}$, then the trailing $N_c/2 - r$ variances are set to zero to produce the approximation. However, the pattern of eigenvalues for Λ typically splits the eigenvectors into dominant and subdominant sets. Then, the choice of r is more or less obvious. The optimal truncated KL (rank-r) estimator of (11) can easily be obtained by replacing Λ_r with Λ in (11).

A. Initialization

In order to choose good initial values for the unknown channel parameters, the N_{PS} data symbols $\{A_k(n)\}$ for $k \in S_{PS}$ in each OFDM frame are inserted as pilot symbols known by the receiver. Corresponding to the pilot symbols, we focus on an under-sampled signal model and employ the least squares (LS) estimate to obtain under-sampled channel parameters. Then, the complete initial channel gains can easily be determined using an interpolation technique, i.e., a lowpass interpolation algorithm [16]. Finally, the initial values of $\mathbf{G}_\mu^{(0)}$ are used in the iterative EM algorithm to avoid divergence. The details of the initialization process is presented in [17] and [18].

B. Computation of $\Gamma_\mu^{(i)}(k)$ for QPSK

As the details are given in Appendix II, $\Gamma_\mu^{(i)} = [\Gamma_\mu^{(i)}(0), \ldots, \Gamma_\mu^{(i)}((N_c/2)-1)]^T$ can be computed for QPSK signaling as follows:

$$\Gamma_\mu^{(i)} = \frac{1}{2}\tanh\left[\frac{1}{\sigma^2}\mathrm{Re}\left(\mathbf{Z}_\mu^{(i)}\right)\right] + \frac{j}{2}\tanh\left[\frac{1}{\sigma^2}\mathrm{Im}\left(\mathbf{Z}_\mu^{(i)}\right)\right] \tag{15}$$

where

$$\mathbf{Z}_1^{(i)} = \mathcal{R}_e \Psi^* \mathbf{G}_1^{*(i)} + \mathcal{R}_o^* \Psi \mathbf{G}_2^{(i)}$$
$$\mathbf{Z}_2^{(i)} = \mathcal{R}_e \Psi^* \mathbf{G}_2^{*(i)} - \mathcal{R}_o^* \Psi \mathbf{G}_1^{(i)}$$

and $\mathcal{R}_e$ and $\mathcal{R}_o$ are $N_c/2 \times N_c/2$ diagonal matrices whose elements are $\mathbf{R}_e$ and $\mathbf{R}_o$, respectively.

IV. Modified Cramer–Rao Bound

In this section, we turn our attention to the analytical performance results and study the performance of the MAP channel estimator based on the evaluation of the modified CRB.

The mean-squared estimation error for the unbiased estimation of a nonrandom parameter has a lower bound, the CRB, which defines the ultimate accuracy of the unbiased estimation procedure. Suppose $\hat{\mathbf{G}}$ is an unbiased estimator of a vector of unknown parameters $\mathbf{G}$ (i.e., $E\{\hat{\mathbf{G}}\} = \mathbf{G}$), then the mse matrix is lower bounded by the inverse of the Fisher information matrix (FIM) $E\{(\mathbf{G}-\hat{\mathbf{G}})(\mathbf{G}-\hat{\mathbf{G}})^\dagger\} \geq \mathbf{J}^{-1}(\mathbf{G})$.

Since the estimation of unknown random parameters $\mathbf{G}'$ via the MAP approach is considered in this paper, the modified FIM needs to be taken into account in the derivation of the stochastic CRB [24]. Fortunately, the modified FIM can be obtained by a straightforward modification of the FIM as

$$\mathbf{J}_M(\mathbf{G}) \triangleq \mathbf{J}(\mathbf{G}) + \mathbf{J}_P(\mathbf{G}) \tag{16}$$

where $\mathbf{J}_P(\mathbf{G})$ represents the *a priori* information.

Under the assumption that $\mathbf{G}$ and $\mathbf{W}(n)$ are independent of each other and $\mathbf{W}(n)$ is a zero-mean, the A_k's are adopting finite complex values, from [24] and (9), the conditional PDF is given by

$$p(\mathbf{R}|\mathbf{G}) = E_{\mathcal{A}}\{p(\mathbf{R}|\mathcal{A},\mathbf{G})\}$$
$$\sim \frac{1}{\sigma^2}E_{\mathcal{A}}\left\{(\mathbf{R}-\mathcal{A}\tilde{\Psi}\mathbf{G})^\dagger(\mathbf{R}-\mathcal{A}\tilde{\Psi}\mathbf{G})\right\}. \tag{17}$$

Since $\ln p(\mathbf{R}|\mathbf{G})$ is required for the computation of $\mathbf{J}(\mathbf{G})$, it is unfortunately computationally intensive. However, an approximate of $\ln p(\mathbf{R}|\mathbf{G})$ can still be obtained from $\ln p(\mathbf{R}|\mathcal{A},\mathbf{G})$. Since the logarithmic function is a concave, by Jensen's inequality, we have

$$\ln p(\mathbf{R}|\mathbf{G}) \leq E_{\mathcal{A}} \ln\{p(\mathbf{R}|\mathcal{A},\mathbf{G})\}. \tag{18}$$

Therefore, we get a valid $\mathbf{J}(\mathbf{G})$ from $E_{\mathbf{A}}\{\ln p(\mathbf{R}|\mathcal{A},\mathbf{G})\}$ which may not be tight, but is much easier to compute. From (17), the derivatives follow as

$$\frac{\partial \ln p(\mathbf{R}|\mathbf{G})}{\partial \mathbf{G}^T} = \frac{1}{\sigma^2}(\mathbf{R}-\mathcal{A}\tilde{\Psi}\mathbf{G})^\dagger \mathcal{A}\tilde{\Psi} \tag{19}$$

$$\frac{\partial^2 \ln p(\tilde{\mathbf{R}}|\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T} = -\frac{1}{\sigma^2}\tilde{\Psi}^\dagger \mathcal{A}^\dagger \mathcal{A}\tilde{\Psi}. \tag{20}$$

Since the Alamouti's scheme imposes an orthogonal structure on the transmitted symbols $\mathcal{A}^\dagger\mathcal{A} = \mathbf{I}$ and using $\tilde{\Psi}^\dagger\tilde{\Psi} = \mathbf{I}$ and taking the expected values yields the simple form

$$\mathbf{J}(\mathbf{G}) = -E\left[\frac{\partial^2 \ln p(\tilde{\mathbf{R}}|\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T}\right] = \frac{1}{\sigma^2}\mathbf{I}. \tag{21}$$

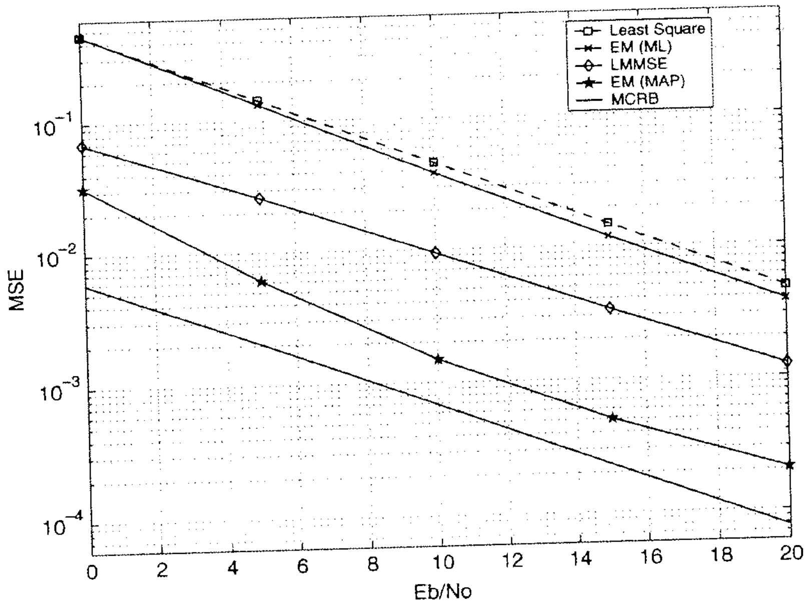


Fig. 2. Channel-estimation mse as a function of the average E_b/N_0.

The second term in (16) is easily obtained as follows. Consider the prior PDF $p(\mathbf{G}) \sim \exp[(-\mathbf{G}^{\dagger}\widetilde{\Lambda}^{-1}\mathbf{G})]$. The respective derivatives are found as

$$\frac{\partial \ln p(\mathbf{G})}{\partial \mathbf{G}^T} = -\mathbf{G}^{\dagger}\Lambda^{-1}, \qquad \frac{\partial^2 \ln p(\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T} = -\Lambda^{-1}. \tag{22}$$

Upon taking the negative expectations, the second term in (16) becomes $\mathbf{J}_P(\mathbf{G}) = \Lambda^{-1}$. Substituting $\mathbf{J}(\mathbf{G})$ and $\mathbf{J}_P(\mathbf{G})$ in (16) produces the modified FIM as follows:

$$\mathbf{J}_M(\mathbf{G}) = \mathbf{J}(\mathbf{G}) + \mathbf{J}_P(\mathbf{G}) = \frac{1}{\sigma^2}\mathbf{I} + \Lambda^{-1}. \tag{23}$$

Inverting the matrix $\mathbf{J}_M(\mathbf{G})$ yields $\mathrm{CRB}(\hat{\mathbf{G}}) = \mathbf{J}_M^{-1}(\mathbf{G})$. $\mathrm{CRB}(\hat{\mathbf{G}})$ is a diagonal matrix with the elements on the main diagonal equaling the reciprocal of that of the $\mathbf{J}(\mathbf{G})$ matrix. Because of the zero-valued off-diagonal entries in the FIM, the errors between the corresponding estimates are not independent.

V. Simulations

In this section, we present some simulation results in order to verify the performance of the channel estimation via the EM algorithm for SFBC-OFDM systems. The diversity scheme with two transmit and one receive antenna is considered. The channels between the transmitter and receiver are generated according to the doubly-selective fading channel model. In this model, $H_\mu(k)$'s are with an exponentially decaying power delay profile $\theta(\tau_\mu) = C\exp(-\tau_\mu/\tau_{\rm rms})$ and delays τ_μ that are uniformly and independently distributed over the length of the cyclic prefix. C is a normalizing constant. Note that the normalized discrete channel correlations for different subcarriers and blocks of this channel model were presented in [17] as follows:

$$r_1(k,k') = \frac{1-\exp\left[-L\left[\frac{1}{\tau_{\rm rms}} + \frac{2\pi j(k-k')}{N_c}\right]\right]}{\tau_{\rm rms}\left(1-\exp\left(\frac{-L}{\tau_{\rm rms}}\right)\right)\left(\frac{1}{\tau_{\rm rms}} + \frac{j2\pi(k-k')}{N_c}\right)}.$$

The scenario for the SFBC-OFDM simulation study consists of a wireless QPSK-OFDM system. The system has a 2.28-MHz bandwith (for the pulse roll-off factor $\alpha = 0.2$) and is divided into $N_c = 512$ tones with a total period Ts of 136 μs, of which 1.052 μs constitutes the cyclic prefix ($L = 4$). The uncoded data rate is 7.6 Mb/s. We assume that the rms width is $\tau_{\rm rms} = 1$ sample (0.263 μs) for the power-delay profile.

The proposed EM-based iterative channel estimator of (11) is implemented and compared with the previously reported SFBC-OFDM channel estimator [13] in terms of average mse for a wide range of signal-to-noise ratio (E_b/N_0) levels. The average mse is defined as the norm of the difference between the vectors $\mathbf{G} = [\mathbf{G}_1^T, \mathbf{G}_2^T]$ and $\widehat{\mathbf{G}}_{\rm map}$, representing the true and the estimated values of the channel parameters, respectively. Namely, $\mathrm{mse} = 1/2N_c\|\mathbf{G} - \widehat{\mathbf{G}}_{\rm map}\|^2$. In order to obtain good initial values for the unknown channel parameters, $N_{PS} = 64$ equally spaced pilot tones are inserted into the data symbols. Corresponding to the pilot symbols, we employed the LS estimate to obtain under-sampled channel parameters. Then, the complete initial channel gains are determined using a lowpass interpolation technique [16]. Finally, the initial values of $\mathbf{G}_\mu^{(0)}$ are used in the iterative EM algorithm to avoid divergence.

Fig. 2 compares the performance of the proposed EM-MAP channel-estimation approach with an EM-ML [13] which is the modified version of [12] and both used LS for initialization. The proposed EM-based approach is also compared to other widely

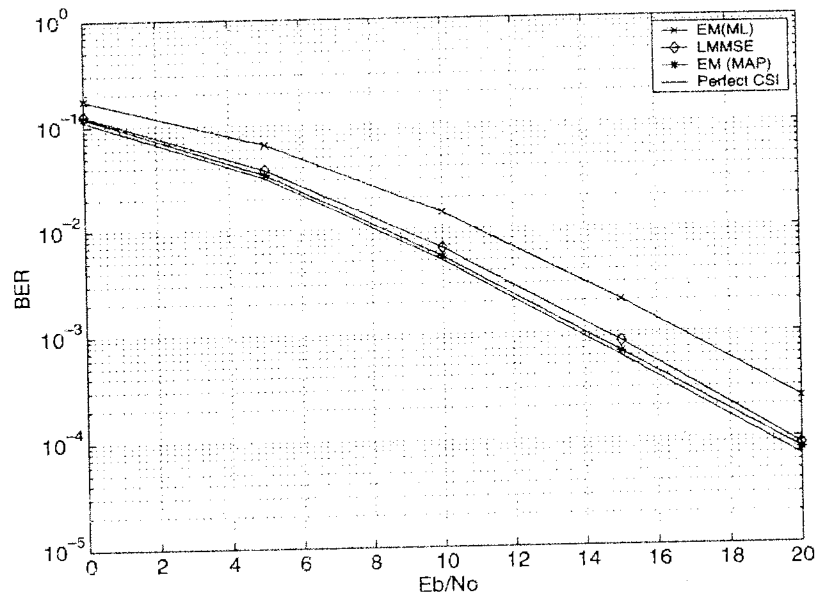


Fig. 3. BER performance of the EM algorithms as a function of the average E_b/N_0.

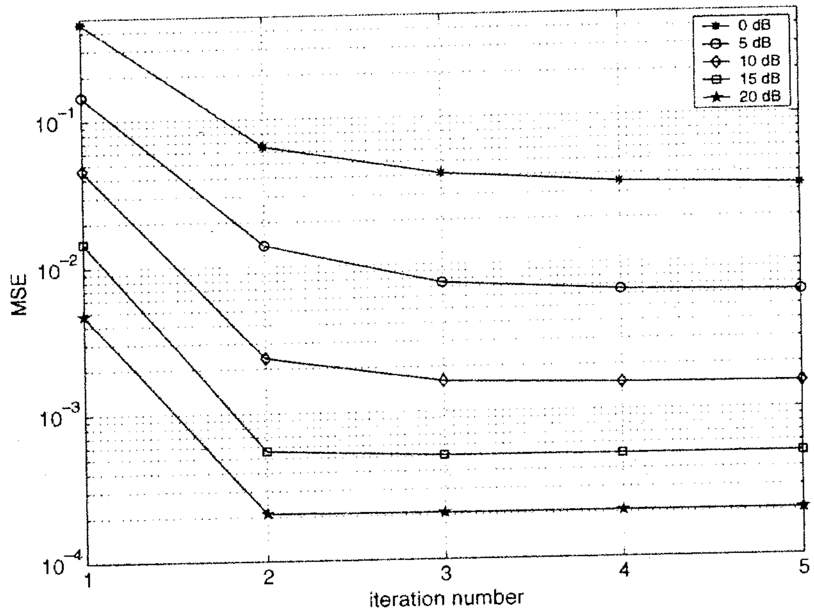


Fig. 4. Convergence of the mse with respect to the number of iterations.

used linear mmse (Lmmse) and LSE pilot symbol assisted modulation (PSAM) channel-estimation techniques [23]. It can be seen that the proposed EM-MAP significantly outperforms the EM ML as well as the PSAM techniques.

Assuming the channel parameters are estimated accurately, the SF block constructs the decision estimate vector in [14]. Therefore, we used channel estimates for symbol decoding and compared the bit error rate (BER) performance of the proposed iterative EM-MAP estimator with the EM-ML and the Lmmse ones. Fig. 3 shows the average results of 1000 Monte Carlo runs. We observe from the BER performance simulation results that the EM-MAP BER performance still outperforms the EM-ML and the Lmmse approaches, especially for high SNRs.

In Fig. 4, the average mse performance of the EM-MAP algorithm is presented as a function of the number of iterations. It is concluded from these curves that the mse performance of the EM-based algorithm converges within 2–4 iterations, depending on the average SNR.

Apart from the simulated BER performance, the truncated estimator performance is also studied as a function of the number of KL coefficients. Fig. 5 presents the mse result of the truncated EM-MAP estimator. If only a few expansion coefficients are employed to reduce the complexity of the proposed estimator, then the mse between channel parameters becomes large. However, if the number of parameters in the expansion is increased to include the dominant eigenvalues, we

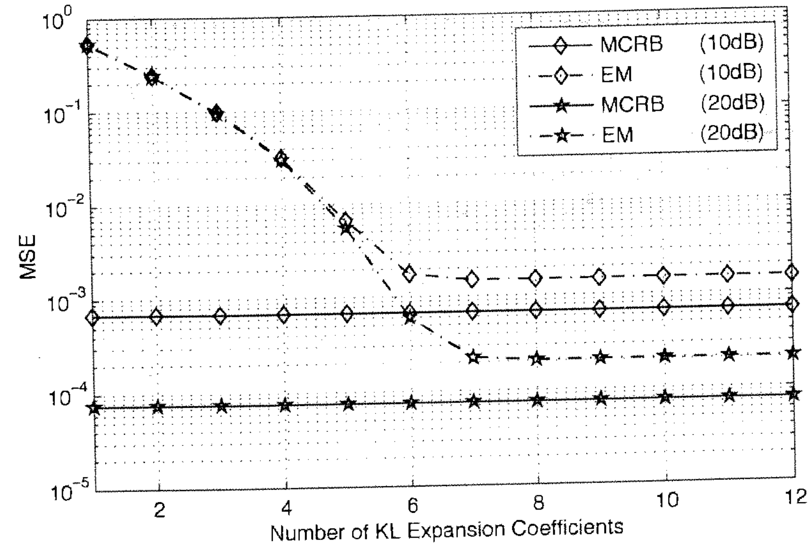


Fig. 5. Truncated EM algorithm mse performance.

are able to obtain a good approximation with a relatively small number of KL coefficients. For instance, by replacing 256 × 256 diagonal Λ in (11) with a 8 × 8 diagonal, Λ_r decreases the computational complexity enormously.

VI. Conclusion

In this paper, we proposed an efficient nondata-aided EM-based channel-estimation algorithm for SFBC-OFDM systems, which is crucial for the decoding of SF codes. This algorithm performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing the M-PSK modulation scheme with additive Gaussian noise. The likelihood ratio is properly averaged out over the data sequence so that the resulting algorithm does not need a training sequence to acquire the channel; thus, the throughput of the system improves substantially compared to the existing channel-estimation algorithms based on the data-aided schemes in literature. The performance of our channel-estimation algorithm is confirmed by corroborating simulations and is compared with existing EM-ML alternatives. It has been shown that the EM-MAP estimator performs well over the EM-ML. Moreover, the truncation property of the KL expansion significantly reduces the complexity of the EM-based algorithms.

Appendix I
Derivation of (11)

In (10), the term $\log p(\mathbf{R}, \mathcal{A}, \mathbf{G})$ can be expressed as

$$\log p(\mathbf{R}, \mathcal{A}, \mathbf{G}) \sim \log p(\mathcal{A}, \mathbf{G}) + \log p(\mathbf{R}|\mathcal{A}, \mathbf{G}) + \log p(\mathbf{G}). \tag{24}$$

The first term in (24) is constant, since the data sequences $\mathcal{A}$ have an equal *a priori* probability and $\mathcal{A}$ and $\mathbf{G}$ are independent of each other. Also, since the noise samples are independent, from (3) and (9), the second and third terms in (24) can be written as

$$\begin{aligned}\log p(\mathbf{R}|\mathcal{A}, \mathbf{G}) \sim & - \left[\mathbf{R}_e(n) - \mathbf{A}_e(n)\mathbf{H}_1 - \mathbf{A}_o(n)\mathbf{H}_2\right]^\dagger \\ & \times \mathbf{\Sigma}^{-1}\left[\mathbf{R}_e(n) - \mathbf{A}_e(n)\mathbf{H}_1 - \mathbf{A}_o(n)\mathbf{H}_2\right] \\ & - \left[\mathbf{R}_o(n) + \mathbf{A}_o^\dagger(n)\mathbf{H}_1 - \mathbf{A}_e^\dagger(n)\mathbf{H}_2\right]^\dagger \\ & \times \mathbf{\Sigma}^{-1}\left[\mathbf{R}_o(n) + \mathbf{A}_o^\dagger(n)\mathbf{H}_1 - \mathbf{A}_e^\dagger(n)\mathbf{H}_2\right] \\ \log p(\mathbf{G}) \sim & - \mathbf{G}_1^\dagger\mathbf{\Lambda}^{-1}\mathbf{G}_1 - \mathbf{G}_2^\dagger\mathbf{\Lambda}^{-1}\mathbf{G}_2.\end{aligned} \tag{25}$$

Taking the derivatives in (10) with respect to $\mathbf{G}_1$ and $\mathbf{G}_2$, along with the fact that $\|\mathbf{A}_e(n)\|^2 = |\mathbf{A}_o(n)|^2 = (1/2)\mathbf{I}$, and equating the resulting equations to zero, we have

$$\begin{aligned}\frac{\partial Q}{\partial \mathbf{G}_1} = & \sum_{\mathcal{A}} p\left(\mathbf{R}, \mathcal{A}, \mathbf{G}^{(i)}\right) \\ & \times \left[\mathbf{\Sigma}^{-1}\mathbf{\Psi}^\dagger\left(\mathbf{A}_e^\dagger(n)\mathbf{R}_e(n)\right.\right. \\ & \left.\left. - \mathbf{A}_o(n)\mathbf{R}_o(n) - \mathbf{H}_1\right) - \mathbf{\Lambda}^{-1}\mathbf{G}_1\right] = 0 \\ \frac{\partial Q}{\partial \mathbf{G}_1} = & \sum_{\mathcal{A}} p\left(\mathbf{R}, \mathcal{A}, \mathbf{G}^{(i)}\right) \\ & \times \left[\mathbf{\Sigma}^{-1}\mathbf{\Psi}^\dagger\left(\mathbf{A}_o^\dagger(n)\mathbf{R}_e(n)\right.\right. \\ & \left.\left. + \mathbf{A}_e(n)\mathbf{R}_o(n) - \mathbf{H}_2\right) - \mathbf{\Lambda}^{-1}\mathbf{G}_2\right] = 0.\end{aligned} \tag{26}$$

Since $p(\mathbf{R}, \mathcal{A}, \mathbf{G}^{(i)})$ may be replaced by $p(\mathcal{A}|\mathbf{R}, \mathbf{G}^{(i)})$ without violating the equalities in (26), defining the conditional probabilities as

$$\begin{aligned}\Gamma_\mu^{(i)}(k) = & \sum_{a_1}\sum_{a_2 \in S_k} a_\mu P \\ & \times \left(A_{2k}(n) = a_1, A_{2k+1}(n) = a_2 | \mathbf{R}, \mathbf{G}^{(i)}\right)\end{aligned} \tag{27}$$

$$\Gamma_\mu^{(i)}(k) = \frac{\sum_{a_1,a_2\in S_k} a_\mu p\left(\mathbf{R}|A_{2k}(n)=a_1, A_{2k+1}(n)=a_2, \mathbf{G}^{(i)}\right) P\left(A_{2k}(n)=a_1, A_{2k+1}=a_2\right)}{\sum_{a_1,a_2\in S_k} p\left(\mathbf{R}|A_{2k}(n)=a_1, A_{2k+1}(n)=a_2, \mathbf{G}^{(i)}\right) P\left(A_{2k}(n)=a_1, A_{2k+1}=a_2\right)} \tag{30}$$

and the $N_c/2 \times N_c/2$ diagonal matrix

$$\mathbf{\Gamma}_\mu^{(i)} = \mathrm{diag}\left(\Gamma_\mu^{(i)}(0), \ldots, \Gamma_\mu^{(i)}\left(\frac{N_c}{2}-1\right)\right) \tag{28}$$

the equations in (26) can be expressed as

$$\begin{aligned} \mathbf{\Sigma}^{-1}\mathbf{\Psi}^\dagger\left(\mathbf{\Gamma}_1^{(i)\dagger}\mathbf{R}_e(n) - \mathbf{\Gamma}_2^{(i)}\mathbf{R}_o(n) - \mathbf{H}_1\right) &= \mathbf{\Lambda}^{-1}\mathbf{G}_1 \\ \mathbf{\Sigma}^{-1}\mathbf{\Psi}^\dagger\left(\mathbf{\Gamma}_2^{(i)\dagger}\mathbf{R}_e(n) + \mathbf{\Gamma}_1^{(i)}\mathbf{R}_o(n) - \mathbf{H}_2\right) &= \mathbf{\Lambda}^{-1}\mathbf{G}_2 \end{aligned} \tag{29}$$

from which, the final expression for $\mathbf{G}_\mu^{(i+1)}$, $\mu = 1, 2$, given by (11) easily follows.

Appendix II
Exact Computation of $\Gamma_\mu^{(i)}(k)$ for QPSK Signaling

Let $a = (\pm 1 \pm j)/2$ represent the unit power and the independent and identically distributed data sequence modulating the QPSK carrier, $\Gamma_\mu^{(i)}(k)$ in (13) can be expressed (30), shown at the top of the page. From (9), it follows that

$$\Gamma_\mu^{(i)}(k) = \frac{\sum_{a_1,a_2\in S_k} a_\mu \exp\left(\frac{1}{\sigma^2}\mathrm{Re}\left[a_\mu^* Z_\mu^{(i)}(k)\right]\right)}{\sum_{a_1,a_2\in S_k} \exp\left(\frac{1}{\sigma^2}\mathrm{Re}\left[a^* Z_\mu^{(i)}(k)\right]\right)} \tag{31}$$

where

$$\begin{aligned} Z_1^{(i)}(k) &= R_{e,k}\sum_m G_1^{(i)*}(m)\psi_m^*(k) + R_{o,k}^*\sum_m G_2^{(i)}(m)\psi_m(k) \\ Z_2^{(i)}(k) &= R_{e,k}\sum_m G_2^{(i)*}(m)\psi_m^*(k) - R_{o,k}^*\sum_m G_1^{(i)}(m)\psi_m(k) \end{aligned}$$

Then, taking summations in the numerator and the denominator of (31) over the values of the QPSK symbols a, we have the final result as follows:

$$\Gamma_\mu^{(i)}(k) = \frac{1}{2}\tanh\left[\frac{1}{\sigma^2}\mathrm{Re}\left(Z_\mu^{(i)}(k)\right)\right] + \frac{j}{2}\tanh\left[\frac{1}{\sigma^2}\mathrm{Im}\left(Z_\mu^{(i)}(k)\right)\right]. \tag{32}$$

Acknowledgment

The authors would like to thank the anonymous reviewers for their insightful comments and suggestions that improved the quality of this paper.

References

[1] R. Van Nee and R. Prasad, *OFDM Wireless Multimedia Communications*. Boston, MA: Artech House, 2000.
[2] Z. Liu, Y. Xin, and G. B. Giannakis, "Space-time-frequency coded OFDM for frequency selective fading channels," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2465–2476, Oct. 2002.
[3] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.
[4] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communications: Performance analysis and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.
[5] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456–1467, Jul. 1999.
[6] Y. Li, L. J. Cimini, N. Seshadri, and S. Ariyavistakul, "Channel estimation for OFDM systems with transmitter diversity in mobile wireless channels," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 3, pp. 461–471, Mar. 1999.
[7] Y. Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," *IEEE Trans. Wireless Commun.*, vol. 1, no. 1, pp. 67–75, Jan. 2002.
[8] Y. Li, C. N. Georghiades, and G. Huang, "Iterative maximum likelihood sequence estimation for space-time coded systems," *IEEE Trans. Commun.*, vol. 49, no. 6, pp. 948–951, Jun. 2001.
[9] C. Cozzo and B. L. Hughes, "Joint channel estimation and data detection in space-time communications," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1266–1270, Aug. 2003.
[10] B. Lu, X. Wang, and Y. Li, "Iterative receivers for space-time block-coded ODFM systems in dispersive fading channels," *IEEE Trans. Wireless Commun.*, vol. 1, no. 2, pp. 213–225, Apr. 2002.
[11] B. Lu, X. Wang, and K. R. Narayanan, "LDPC-based space-time coded OFDM over correlated fading channels," *IEEE Trans. Commun.*, vol. 50, no. 1, pp. 74–88, Jan. 2002.
[12] X. Yongzhe and C. N. Georghiades, "Two EM-type channel estimation algorithms for OFDM with transmitter diversity," *IEEE Trans. Commun.*, vol. 51, no. 1, pp. 106–115, Jan. 2003.
[13] X. Ma, H. Kobayashi, and S. C. Schwarts, "An EM-based channel estimation for space-time and space-frequency block coded OFDM," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP)*, Hong Kong, Apr. 6–10, 2003, vol. 4, pp. IV-389–IV-392.
[14] K. F. Lee and D. B. Williams, "A space-frequency transmitter diversity technique for OFDM Systems," in *Proc. IEEE Global Telecommunications (GLOBECOM)*, San Francisco, CA, Nov. 2000, pp. 1473–1477.
[15] M. Feder and E. Weinstein, "Parameter estimation of superimposed signals using the EM algorithm," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 36, no. 4, pp. 477–489, Apr. 1988.
[16] S. Coleri, M. Ergen, A. Puri, and A. Bahai, "Channel estimation techniques based on pilot arrangement in OFDM systems," *IEEE Trans. Broadcast.*, vol. 48, no. 3, pp. 223–229, Sep. 2002.
[17] J.-J. van de Beek, O. Edfors, M. Sandell, S. K. Wilson, and P. O. Börjesson, "OFDM channel estimation by singular value decomposition," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 931–936, Jul. 1998.
[18] E. Panayirci and H. A. Cirpan, "Channel estimation for space-time block coded OFDM systems in the presence of multipath fading," in *Proc. IEEE Global Telecommunications (GLOBECOM)*, Taipei, Taiwan, R.O.C., Nov. 17–21, 2002, pp. 1157–1161.
[19] K. Yip and T. Ng, "Karhunen–Loeve expansion of the WSSUS channel

output and its application to efficient simulation," *IEEE J. Sel. Areas Commun.*, vol. 15, no. 4, pp. 640–646, May 1997.

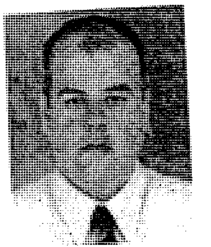
[20] A. P. Dempster, N. M. Laird, and D. B. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," *J. R. Stat. Soc.*, vol. 39, no. 1, pp. 1–17, 1977.

[21] G. K. Kaleh and R. Valet, "Joint parameter estimation and symbol detection for linear and nonlinear unknown channels," *IEEE Trans. Commun.*, vol. 42, no. 7, pp. 2406–2413, Jul. 1994.

[22] E. Panayirci and C. N. Georghiades, "Carrier phase synchronization of OFDM systems over frequency selective channels via EM algorithm," in *Proc. IEEE Vehicular Technology Conf. (VTC)*, Houston, TX, May 16–20, 1999, pp. 675–679.

[23] M. Morelli and U. Mengali, "A comparison of pilot-aided channel estimation methods for OFDM systems," *IEEE Trans. Signal Process.*, vol. 49, no. 12, pp. 3065–3073, Dec. 2001.

[24] H. L. Van Trees, *Detection, Estimation and Modulation Theory, Part I*. New York: Wiley, 1993.



Hakan A. Çırpan (M'97) received the B.S. degree from Uludag University, Bursa, Turkey, in 1989, the M.S. degree from the University of Istanbul, Istanbul, Turkey, in 1992, and the Ph.D. degree from Stevens Institute of Technology, Hoboken, NJ, in 1997, all in electrical engineering.

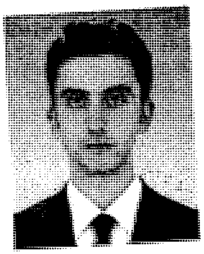
From 1995 to 1997, he was a Research Assistant at Stevens Institute of Technology, working on signal processing algorithms for wireless communication systems. In 1997, he joined the faculty of the Department of Electrical and Electronics Engineering, University of Istanbul. His general research interests cover wireless communications, statistical signal and array processing, system identification, and estimation theory. His current research activities are focused on signal processing and communication concepts with specific attention to channel estimation and equalization algorithms for space-time coding and multicarrier (orthogonal frequency-division multiplexing) systems.

Dr. Cirpan is a member of Sigma Xi. He received the Peskin Award from Stevens Institute of Technology as well as the Prof. Nazim Terzioglu Award from the Research Fund of the University of Istanbul.

Erdal Panayırcı (F'03) received the Diploma Engineering degree in electrical engineering from the Istanbul Technical University, Istanbul, Turkey, in 1964 and the Ph.D. degree in electrical engineering and system science from Michigan State University, East Lansing, in 1970.

From 1970 to 2000, he was with the Faculty of Electrical and Electronics Engineering, Istanbul Technical University, where he was a Professor and the Head of the Telecommunications Chair. He has also been a part-time Consultant to several leading companies in telecommunications in Turkey. From 1979 to 1981, he was with the Department of Computer Science, Michigan State University, as a Fulbright-Hays Fellow and a NATO Senior Scientist. Between 1983 and 1986, he served as a NATO Advisory Committee Member for the Special Panel on Sensory Systems for Robotic Control. From August 1990 to December 1991, he was a Visiting Professor at the Center for Communications and Signal Processing, New Jersey Institute of Technology, Newark, and took part in the research project on interference cancellation by array processing. Between 1998 and 2000, he was a Visiting Professor at the Department of Electrical Engineering, Texas A&M University, College Station, and took part in research on developing efficient synchronization algorithms for orthogonal frequency-division multiplexing (OFDM) systems. He is currently a Visiting Professor at the Department of Electrical and Electronics Engineering, Bilkent University, Ankara, Turkey. He is engaged in research and teaching in digital communications and wireless systems, equalization and channel estimation in multicarrier (OFDM) communication systems, and efficient modulation and coding techniques (TCM and turbo coding).

Prof. Panayırcı is a member of Sigma Xi. He was the Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS in the fields of synchronization and equalizations from 1995 to 1999. He is currently the Head of the Turkish Scientific Commission on Signals, Systems, and Communications of the International Union of Radio Science.



Hakan Doğan (S'02) was born in Istanbul, Turkey, on 1979. He received the B.S. and M.S. degrees in electronics engineering from Istanbul University, Istanbul, Turkey, in 2001 and 2003, respectively, and is currently working toward the Ph.D. degree from the same university.

Since 2001, he has been a Research Assistant at the Department of Electrical and Electronics Engineering, Istanbul University. His general research interests cover communication theory, estimation theory, statistical signal processing, and information theory. His current research activities are focused on wireless communication concepts with specific attention to equalization and channel estimation for spread-spectrum and multicarrier (orthogonal frequency-division multiplexing) systems.

Iterative Channel Estimation and Decoding of Turbo Coded SFBC-OFDM Systems

Hakan Doğan, *Student Member, IEEE,* Hakan A. Çırpan, *Member, IEEE,* and Erdal Panayırcı, *Fellow, IEEE*

***Abstract*— We consider the design of turbo receiver structures for space-frequency block coded orthogonal frequency division multiplexing (SFBC-OFDM) systems in the presence of unknown frequency and time selective fading channels. The Turbo receiver structures for SFBC-OFDM systems under consideration consists of an iterative MAP Expectation/Maximization (EM) channel estimation algorithm, soft MMSE-SFBC decoder and a soft MAP outer-channel-code decoder. MAP-EM employs iterative channel estimation and it improves receiver performance by re-estimating the channel after each decoder iteration. Moreover, the MAP-EM approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. The optimal truncation property of the KL expansion can reduce computational load on the iterative estimation approach. The performance of the proposed approaches are studied in terms of mean square error and bit-error rate. Through computer simulations, the effect of a pilot spacing on the channel estimator performance and sensitivity of turbo receiver structures on channel estimation error are studied. Simulation results illustrate that receivers with turbo coding are very sensitive to channel estimation errors compared to receivers with convolutional codes. Moreover, superiority of the turbo coded SFBC-OFDM systems over the turbo coded STBC-OFDM systems is observed especially for high Doppler frequencies.**

***Index Terms*—EM algorithm, MAP channel estimation, OFDM systems, space-frequency coding, turbo receiver.**

I. Introduction

THE goal of the developments for the future generations of broadband wireless mobile systems is to provide a wide range of high quality enhanced and integrated services with high data rates. Several key enabling techniques capable of achieving the highest possible spectrum efficiency are therefore currently being investigated. An important area that has to be focused on to make this goal accomplished is related to spectrally efficient and flexible modulation and coding techniques. Specifically, the combined application of orthogonal frequency division multiplexing (OFDM) and transmit antenna diversity appears to be capable of enabling the types of capacities and data rates needed for broadband wireless services.

Manuscript received January 5, 2006; revised May 11, 2006; accepted August 16, 2006. The associate editor coordinating the review of this paper and approving it for publication was D. Huang. This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 6th Framework Programme and the Research Fund of Istanbul University under Projects 513/05052006, T-856/02062006, UDP-838/27072006, and UDP-732/05052006. This work was also supported in part by the Turkish Scientific and Technical Research Institute (TUBITAK) under Grant 104E166. Part of the results of this paper were presented at the the 4th International Symposium on Turbo Codes & Related Topics, Munich, Germany, April 3-7, 2006.

H. Doğan and H. A. Çırpan are with the Department of Electrical and Electronics Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey (e-mail: {hdogan,hcirpan}@istanbul.edu.tr).

E. Panayırcı is with the Department of Electronics Enginering, Kadir Has University, Cibali 34083, Istanbul, Turkey (e-mail: eepanay@khas.edu.tr).

Digital Object Identifier 10.1109/TWC.2007.06006.

Transmit antenna diversity has been exploited recently to develop high-performance space-time/frequency codes and simple maximum likelihood (ML) decoders for transmission over flat-fading channels [1]–[3]. Unfortunately, their practical application can present a real challenge to channel estimation algorithms, especially when the signal suffers from frequency selective multipath channels. One of the solutions alleviating the frequency selectivity is the use of OFDM together with transmit diversity which combats long channel impulse response by transmitting parallel symbols over many orthogonal subcarriers yielding a unique reduced-complexity physical layer capabilities [4].

The continued increase in demand for all types of services further necessitates the need for higher capacity and data rates. In this context, emerging technology that improves the wireless systems spectrum efficiency is error control coding. Recent trends in coding favor parallel and/or serially concatenated coding and probabilistic soft-decision iterative (turbo-style) decoding. Such codes are able to exhibit near-Shannon-limit performance with reasonable complexities in many cases and are of significant interest for communications applications that require moderate error rates. An outer channel code is therefore applied in addition to transmit diversity to further improve the receiver performance. We therefore consider the combination of turbo codes with the transmit diversity OFDM systems. Especially we address the design of iterative channel estimation approach for transmit diversity OFDM systems employing an outer channel code.

Channel estimation for transmit diversity OFDM systems has attracted much attention with pioneering works by Li [5], [6]. However, most of the early work on channel estimation for transmit diversity OFDM systems focused on uncoded systems. Since most practical systems use error control coding, more recent work have addressed the coded transmit diversity OFDM systems. Among many other techniques, an iterative procedures based on Expectation-Maximization (EM) algorithm was also applied to channel estimation problem in the context of space-time block-coding (STBC) [7], [8] as well as transmit diversity OFDM systems with or without outer channel coding (e.g. convolutional code or Turbo code) [10]–[13]. In [10], maximum a *posteriori* (MAP) EM based iterative receivers for STBC-OFDM systems with Turbo code are proposed to directly detect transmitted symbols under the assumption that fading processes remain constant across several OFDM symbols contained in one STBC code-word.

An EM approach proposed for the general estimation of the superimposed signals [9] is applied to the channel estimation for transmit diversity OFDM systems with outer channel code (convolutional code) and compared with the SAGE version presented in [12]. Moreover in [13], a modified version of [12] is proposed for the STBC-OFDM and space-frequency block-coding (SFBC)-OFDM systems. Unlike the EM approaches treated in [10]–[13], we propose in the paper a new Turbo receiver based on MAP-EM channel estimation algorithm for SFBC-OFDM systems employing outer channel coding. The Turbo receiver scheme under consideration employs iterative channel estimation and it improves receiver performance by re-estimating the channel after each decoder iteration. The paper has several major novelties and contributions. The main contribution of the paper mainly comes from the fact that the channel estimation technique presented in our work is an EM based *non-data-aided* approach as opposed to the existing works in the literature which are mostly assumed either the data is known at the receiver through a training sequence or a joint data detection and the channel estimation. Note that very small number of pilots used in our approach is necessary only for initialization of the EM algorithm leading to channel estimation. Although, the joint data and channel estimation technique with EM algorithm seems to be attractive in practice, it is known that the convergency of the algorithm is much slower, it is more sensitive to the initial selection of the parameters and the algorithm is more computationally complex than the techniques that deal with only channel estimation. As it is known in the estimation literature, non data-aided estimation techniques are more challenging mainly due to a data averaging process which must be performed prior to optimization step. Most of the time this may not lead to a simple analytical expression for the estimates. Thanks to the orthogonal space/frequency coding techniques which made possible to derive exact and simple analytical expressions for the unknown channel parameters in our work.

Another significant contribution of the paper comes from the fact that the channel parameter estimation technique proposed in our paper is for the SFBC-OFDM transmitter diversity systems with outer channel coding. The estimation algorithm performs an iterative estimation of the fading channel parameters in frequency domain according to the maximum a posteriori criterion (MAP) as opposed to the ML approaches adopted in many publications appeared in the literature. Furthermore, our approach is based on a novel representation of the fading channel by means of the Karhunen-Loeve (KL) expansion and the application of this expansion to the turbo receiver structures for SFBC-OFDM systems. Note that, KL orthogonal expansion together with space-frequency coded system based on the Alamouti orthogonal design enable us to estimate the channel in a very simple way without taking inverse of large dimensional matrices, yielding a computationally efficient iterative analytical expressions [15]–[17]. Moreover, optimal truncation property of the KL expansion is exploited in our paper resulting in a further reduction in computational load on the channel estimation algorithm. In order to explore the performance of the proposed turbo receivers, we first investigate the effect of a pilot spacing on the turbo receiver performance by considering average MSE

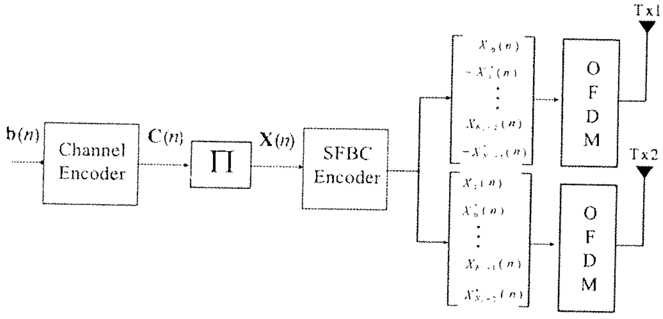


Fig. 1. Transmitter structure for turbo coded SFBC-OFDM systems.

as well as bit-error-rate (BER). We also analyze the sensitivity of turbo receiver structures on channel estimation errors.

The rest of the paper is organized as follows. In Section II, a system model for turbo coded SFBC-OFDM is introduced. In Section III, multipath channel and its orthogonal series representation based on the Karhunen-Loeve expansion is presented. Vector representation of the received signal is formulated in Section IV, while an EM based MAP channel estimation algorithm is developed in Section V. In addition to iterative MAP channel estimation approach, iterative channel equalization and decoding structures are proposed in Section VI. Some computer simulation are provided in Section VII. Finally, conclusions are drawn in Section VIII.

II. Turbo Coded SFBC-OFDM System Model

We consider a SFBC-OFDM system with outer channel coding. Turbo code is applied, in addition to a SFBC system, to further improve the error performance of the SFBC-OFDM system. A block diagram of a transmitter structure for a turbo coded two-branch SFBC transmitter diversity OFDM system is shown in Fig. 1.

A. Turbo Encoder

Turbo codes are a class of powerful error correction codes that enable reliable communications with power efficiencies close to the theoretical Shannon channel capacity limit. In particular, a turbo code is formed from the parallel or serial concatenation of codes separated by an interleaver. In general, Turbo codes are low-rate codes which require considerable bandwidth expansion for high rate data transmission. In order to improve spectral efficiency, it is necessary to combine turbo codes with a bandwidth efficient transmit diversity systems. Thus combinations of implicit (turbo coding) and external (i.e. multiple transmit antenna) diversity can be used to improve the performance of the communication system in fading environments.

As illustrated in Fig. 1, the block of binary data bits of length $N_c/2$, $\mathbf{b}(n) = [b_0(n), b_1(n), ..., b_{\frac{N_c}{2}-1}(n)]^T$ at time n are encoded by an 1/2 rate outer-channel-encoder, resulting in a BPSK-coded symbol stream $\mathbf{C}(n) = [C_0(n), C_1(n), ..., C_{N_c-1}(n)]^T$ of length N_c. The coded symbols are then interleaved by a random permutation resulting in a stream of independent symbols(of length N_c), denoted by $\{\mathbf{X}(n)\}$. A code-bit interleaver reduces probability of burst error bursts and removes correlation in the coded symbol stream. Finally, the modulated BPSK symbols are encoded by a SFBC encoder and transmitted from two transmit antennas on corresponding OFDM subcarriers.

B. SFBC-OFDM Encoder

In this paper, we consider a transmitter diversity OFDM scheme in conjunction with inner channel coding. In order to compensate for the reduced data rate of turbo codes, some space-time codes having data rates greater than one could be employed. However it is well known from literature that the Alamouti antenna modulation configuration is the only scheme which retain orthogonality and full rate when for the complex-valued data as well as the low complexity. As will be seen shortly, orthogonality property is essential and required condition for the channel estimation algorithm in our paper. Moreover, orthogonality structure of Alamouti allows decoupling of the channel and reduces the equalizer complexity. Furthermore, the Alamouti's schemes has been adopted in several wireless standards such as WCDMA and CDMA2000. It imposes an orthogonal spatio-temporal structure on the transmitted symbols that guarantees full (i.e., order 2) spatial diversity. In addition to the spatial level, to realize multipath diversity gains over frequency selective channels, the Alamouti block coding scheme is implemented at a block level in frequency domain. Thus the use of OFDM in transmitter diversity systems also offers the possibility of coding in a form of space-frequency OFDM [18], [19]. Under the assumption that the channel responses are known or can be estimated accurately at the receiver, it was shown that the SFBC-OFDM system has the same performance as a previously reported STBC-OFDM scheme in slow fading environments but shows better performance in the more difficult fast fading environments [18]. Also, since SFBC-OFDM transmitter diversity scheme performs decoding within one OFDM block, it requires only half of the decoder memory needed for the STBC-OFDM system of the same block size. Similarly, the decoder latency for SFBC-OFDM is also half of the STBC-OFDM implementation. In SFBC-OFDM systems, the OFDM subchannels are divided into certain number of groups. This subchannel grouping with appropriate system parameters does preserve diversity gain while simplifying not only the code construction but decoding algorithm significantly as well [18].

Adopting the notation of [18], let N_c turbo coded, interleaved and BPSK modulated symbols, taking values $\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}$, be represented by a vector $\mathbf{X}(n)$= $[X(nN_c), X(nN_c+1), \cdots, X(nN_c+N_c-1)]^T$, where $X_k(n) = X(nN_c+k)$ denotes the kth forward polyphase component of the serial data symbols, for $k = 0, \cdots, N_c-1$. Polyphase component $X_k(n)$ can also be viewed as the coded symbol to be transmitted on the kth tone during the block instant n. The coded symbol vector $\mathbf{X}(n)$ can therefore be expressed as $\mathbf{X}(n)$= $[X_0(n), X_1(n), \cdots, X_{N_c-1}]^T$. Resorting subchannel grouping, $\mathbf{X}(n)$ is coded into two vectors $\mathbf{X}_e(n)$ and $\mathbf{X}_o(n)$ by the space-frequency encoder as

$$\mathbf{X}_e(n) = [X_0(n), X_2(n), \cdots, X_{N_c-4}(n), X_{N_c-2}(n)]^T,$$
$$\mathbf{X}_o(n) = [X_1(n), X_3(n), \cdots, X_{N_c-3}(n), X_{N_c-1}(n)]^T, \quad (1)$$

where $\mathbf{X}_e(n)$ and $\mathbf{X}_o(n)$ actually corresponds to even and odd polyphase component vectors of $\mathbf{X}(n)$,respectively. Then the space-frequency block-coded transmission matrix may be represented by

$$\begin{array}{c} \text{frequency} \rightarrow \\ \text{space} \downarrow \begin{bmatrix} \mathbf{X}_e(n) & -\mathbf{X}_o^*(n) \\ \mathbf{X}_o(n) & \mathbf{X}_e^*(n) \end{bmatrix}, \end{array} \quad (2)$$

where * stands for complex conjugation.

III. Channel: KL-Basis Expansion Model

Dispersive fading channels are modeled widely by the block fading channel model[20]. According to this model, the channel is assumed to remain constant over a block of a given size and successive blocks may be correlated or independent. This is an approximate model that would be applied to some of the practical communication systems such as OFDM, frequency-hopped spread-spectrum (FHSS) and time-division multiple access (TDMA).

In this paper, it is assumed that the channel is frequency selective during each OFDM symbol [21]and exhibits time selectivity over the OFDM symbols according to Doppler frequency. We consider the Alamouti transmitter diversity coding scheme, employed in an OFDM system utilizing N_c subcarrier per antenna transmissions. Note that N_c is chosen as an even integer. The fading channel between the μth transmit antenna and the receive antenna is described by the baseband equivalent discrete frequency response $\mathbf{H}_\mu(n)$ at the nth time slot.

In wireless mobile communications, channel variations arise mainly due to multipath effect. Consequently, these variations evolve in a progressive fashion and hence fit in some evolution models. It appears that a basis expansion approach would be a natural way of modelling the channel variation [22]. Fourier and Taylor series expansions as well as the polynomial expansion have played a prominent role in deterministic modelling. In contrast, a convenient choice for bases expansion of random processes is Karhunen-Loeve (KL) series. Moreover, the KL expansion methodology has been also used for efficient simulation of the multipath fading environments [16]. Prompted by the general applicability of the KL expansion, we consider in this paper the parameters of $\mathbf{H}_\mu(n)$ to be expressed by a linear combination of orthonormal bases.

An orthonormal expansion of the vector $\mathbf{H}_\mu(n)$ involves expressing the $\mathbf{H}_\mu(n)$ as a linear combination of the orthonormal basis vectors as follows:

$$\mathbf{H}_\mu(n) = \mathbf{\Psi}\mathbf{G}_\mu(n), \quad (3)$$

where $\mathbf{\Psi} = [\psi_0, \psi_1, \cdots, \psi_{N_c-1}]$, ψ_i's are the orthonormal basis vectors, and $\mathbf{G}_\mu(n) = [G_{\mu,0}(n), \cdots, G_{\mu,N_c-1}(n)]^T$ is the vector representing the weights of the expansion. By using different basis functions $\mathbf{\Psi}$, we can generate sets of coefficients with different properties. The autocorrelation matrix $\mathbf{C}_{\mathbf{H}_\mu} = E\left[\mathbf{H}_\mu \mathbf{H}_\mu^\dagger\right]$ can be decomposed as

$$\mathbf{C}_{\mathbf{H}_\mu} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^\dagger, \quad (4)$$

where $\mathbf{\Lambda} = E\{\mathbf{G}_\mu \mathbf{G}_\mu^\dagger\}$ and $\dagger$ denotes the complex transpose. The KL expansion yields $\mathbf{\Lambda}$ in (4) to be a diagonal matrix (i.e., the coefficients are uncorrelated). Then (4) represents an *eigendecomposition* of $\mathbf{C}_{\mathbf{H}_\mu}$. As a result, diagonalization of $\mathbf{C}_{\mathbf{H}_\mu}$ leads to a desirable property that the KL coefficients are uncorrelated. Furthermore, in the Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem becomes equivalent to estimating the i.i.d. Gaussian vector $\mathbf{G}_\mu$, whose coefficients are the KL expansion coefficients.

As mentioned earlier, the channels between transmitter and receiver in this paper are assumed to be doubly-selective where, $\mathbf{H}_\mu(n)$'s have exponentially decaying power delay profiles, described by $\theta(\tau_\mu) = C\exp(-\tau_\mu/\tau_{rms})$. The delays τ_μ are uniformly and independently distributed over the length of the cyclic prefix. τ_{rms} determines the decay of the power-delay profile and C is the normalizing constant. Note that the normalized discrete channel-correlations for different subcarriers and blocks of this channel model were presented in [23] as follows,

$$r_2(k,k') = \frac{1-\exp\left[-L\left(\frac{1}{\tau_{rms}}+\frac{2\pi j(k-k')}{N_c}\right)\right]}{\tau_{rms}(1-\exp(-\frac{L}{\tau_{rms}}))\left(\frac{1}{\tau_{rms}}+\frac{j2\pi(k-k')}{N_c}\right)}, \quad (5)$$

$$r_1(n,n') = J_0(2\pi(n-n')f_dT_s), \quad (6)$$

where, (k,k') denotes different subcarriers, L is the cyclix prefix, N_c is the total number of subcarriers. Also in (6) (n,n') denotes the discrete times for the different OFDM symbols, $J_0(.)$ is the zeroth-order Bessel function of the first kind and f_d is the Doppler frequency.

IV. Received Signal Model

At receiver, after matched filtering and symbol rate sampling, the discrete Fourier transform is applied to the received discrete time signal to obtain $\mathbf{R}(n)$. If $\mathbf{R}(n)$ is parsed into even and odd blocks of $N_c/2$ tones each as , $\mathbf{R}_e(n) = [R_0(n), R_2(n), \cdots, R_{N_c-2}(n)]^T$ and $\mathbf{R}_o(n) = [R_1(n), R_3(n), \cdots, R_{N_c-1}(n)]^T$, the received signal can be expressed in vector form as follows.

$$\begin{aligned}\mathbf{R}_e(n) &= \mathcal{X}_e(n)\mathbf{H}_{1,e}(n) + \mathcal{X}_o(n)\mathbf{H}_{2,e}(n) + \mathbf{W}_e(n) \quad (7)\\ \mathbf{R}_o(n) &= -\mathcal{X}_o^\dagger(n)\mathbf{H}_{1,o}(n) + \mathcal{X}_e^\dagger(n)\mathbf{H}_{2,o}(n) + \mathbf{W}_o(n),\end{aligned}$$

where $\mathcal{X}_e(n)$ and $\mathcal{X}_o(n)$ are $N_c/2 \times N_c/2$ diagonal matrices whose elements are $\mathbf{X}_e(n)$ and $\mathbf{X}_o(n)$, respectively. $\mathbf{H}_{\mu,e}(n) = [H_{\mu,0}(n), H_{\mu,2}(n), \cdots, H_{\mu,N_c-2}(n)]^T$ and $\mathbf{H}_{\mu,o}(n) = [H_{\mu,1}(n), H_{\mu,3}(n) \cdots, H_{\mu,N_c-1}(n)]^T$ are $N_c/2$ length vectors denoting the even and odd component vectors of the channel coefficients between the μth transmitter and the receiver. Finally, $\mathbf{W}_e(n)$ and $\mathbf{W}_o(n)$ are an $N_c/2 \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive Gaussian noise in the N_c tones, with variance σ^2 per dimension.

Equation (7) shows that the information symbols $\mathcal{X}_e(n)$ and $\mathcal{X}_o(n)$ are transmitted twice in two consecutive adjacent subchannel groups through two different channels. In order to estimate the channels and decode $\mathbf{X}(n)$ with the embedded diversity gain through repeated transmission, for each n, we can write the following from (7), assuming the complex channel gains between adjacent subcarriers are approximately constant, i.e., $\mathbf{H}_{1,e}(n) \approx \mathbf{H}_{1,o}(n)$ and $\mathbf{H}_{2,e}(n) \approx \mathbf{H}_{2,o}(n)$.

$$\begin{bmatrix}\mathbf{R}_e(n)\\ \mathbf{R}_o(n)\end{bmatrix} = \begin{bmatrix}\mathcal{X}_e(n) & \mathcal{X}_o(n)\\ -\mathcal{X}_o^\dagger(n) & \mathcal{X}_e^\dagger(n)\end{bmatrix}\begin{bmatrix}\mathbf{H}_{1,e}(n)\\ \mathbf{H}_{2,e}(n)\end{bmatrix} + \begin{bmatrix}\mathbf{W}_e(n)\\ \mathbf{W}_o(n)\end{bmatrix}. \quad (8)$$

The effect of this assumption allows us to omit dependencies between $\mathbf{H}_{1,e}(n)$ and $\mathbf{H}_{2,e}(n)$ on even channel components. Using (8) and dropping subscript "$_e$" in $\mathbf{H}_{1,e}(n)$ and $\mathbf{H}_{2,e}(n)$, we have

$$\begin{bmatrix}\mathbf{R}_e(n)\\ \mathbf{R}_o(n)\end{bmatrix} = \begin{bmatrix}\mathcal{X}_e(n) & \mathcal{X}_o(n)\\ -\mathcal{X}_o^\dagger(n) & \mathcal{X}_e^\dagger(n)\end{bmatrix}\begin{bmatrix}\mathbf{H}_1(n)\\ \mathbf{H}_2(n)\end{bmatrix} + \begin{bmatrix}\mathbf{W}_e(n)\\ \mathbf{W}_o(n)\end{bmatrix}. \quad (9)$$

Finally, (9) can be expressed in a more succinct form as

$$\mathbf{R}(n) = \mathcal{X}(n)\mathbf{H}(n) + \mathbf{W}(n). \quad (10)$$

V. Iterative Channel Estimation

In recent years, inspired by the development of turbo coding, various types of iterative channel estimation, detection and decoding schemes have been proposed in the literature. These approaches have shown that iterative receivers can offer significant performance improvements over the noniterative counterparts. We therefore consider an EM based MAP iterative channel estimation technique in frequency domain for turbo coded SFBC-OFDM systems. Frequency domain estimator presented in this paper was inspired by the conclusions in [24]-[25], where it has been shown time domain channel estimators based on a Discrete Fourier Transform (DFT) approach for non sample-spaced channels cause aliased spectral leakage and result in an error floor.

Details of the algorithm will not be presented here since the EM algorithm has been studied and applied to a number of problems in communications over the years. The reader is suggested to refer [27], [28] for a general exposition to EM algorithm and [17] for its applications to the estimation problem related to the work herein. Basically, this algorithm inductively reestimate $\mathbf{G}$ so that a monotonic increase in the *a posteriori* conditional pdf $p(\mathbf{R}|\mathbf{G})$ is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q(\mathbf{G}|\mathbf{G}^{(q)}) = \sum_{\mathcal{X}} p(\mathbf{R}, \mathcal{X}, \mathbf{G}^{(q)}) \log p(\mathbf{R}, \mathcal{X}, \mathbf{G}), \quad (11)$$

where $\mathbf{G}^{(q)}$ is the estimation of $\mathbf{G}$ at the qth iteration.

Note that, the term $\log p(\mathbf{R}, \mathcal{X}, \mathbf{G})$ in (11) can be expressed as [35],

$$\log p(\mathbf{R},\mathcal{X},\mathbf{G}) = \log p(\mathcal{X}\,|\,\mathbf{G}) + \log p(\mathbf{R}\,|\,\mathcal{X},\mathbf{G}) + \log p(\mathbf{G}). \quad (12)$$

The first term on the right hand side of (12) is constant, since, the data sequence $\mathcal{X} = \{X_k(n)\}$ and $\mathbf{G}$ are independent of each other and $\mathcal{X}$ have equal *a priori* probability. Moreover, the *a priori* PDF of the KL expansion coefficients $\mathbf{G}$ can be expressed as $p(\mathbf{G}) \sim \exp(-\mathbf{G}^\dagger\widetilde{\mathbf{\Lambda}}^{-1}\mathbf{G})$ where $\mathbf{G} = [\mathbf{G}_1^T, \mathbf{G}_2^T]^T$, $\widetilde{\mathbf{\Lambda}} = \mathrm{diag}(\mathbf{\Lambda}\ \mathbf{\Lambda})$. Also, since the noise samples are independent, it follows from (7) that the second and third terms on the right hand side of (12) can be written as

$$\begin{aligned}\log p(\mathbf{R}|\mathcal{X},\mathbf{G}) \sim & -[\mathbf{R}_e(n) - \mathcal{X}_e(n)\mathbf{H}_1 - \mathcal{X}_o(n)\mathbf{H}_2]^\dagger \\ & \times\mathbf{\Sigma}^{-1}[\mathbf{R}_e(n) - \mathcal{X}_e(n)\mathbf{H}_1 - \mathcal{X}_o(n)\mathbf{H}_2] \\ & - [\mathbf{R}_o(n) + \mathcal{X}_o^\dagger(n)\mathbf{H}_1 - \mathcal{X}_e^\dagger(n)\mathbf{H}_2]^\dagger \\ & \times\mathbf{\Sigma}^{-1}[\mathbf{R}_o(n) + \mathcal{X}_o^\dagger(n)\mathbf{H}_1 - \mathcal{X}_e^\dagger(n)\mathbf{H}_2],\end{aligned}$$

$$\log p(\mathbf{G}) \sim -\mathbf{G}_1^\dagger\mathbf{\Lambda}^{-1}\mathbf{G}_1 - \mathbf{G}_2^\dagger\mathbf{\Lambda}^{-1}\mathbf{G}_2, \tag{13}$$

where $\mathbf{\Sigma}$ is an $N_c/2 \times N_c/2$ diagonal matrix with $\mathbf{\Sigma}[k,k] = \sigma^2$, for $k = 0, 1, \cdots, N_c/2 - 1$.

Taking derivatives in (11) with respect to $\mathbf{G}_1$ and $\mathbf{G}_2$, along with the fact that $\|\mathcal{X}_e(n)\|^2 = \|\mathcal{X}_o(n)\|^2 = \frac{1}{2}\mathbf{I}$, and equating the resulting equations to zero, the expression of the reestimate $\hat{\mathbf{G}}_\mu^{(q+1)}$ ($\mu = 1, 2$) for SFBC-OFDM can be obtained as follows:

$$\begin{aligned}\hat{\mathbf{G}}_1^{(q+1)} &= (\mathbf{I}+\mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{\Psi}^\dagger\left[\hat{\mathcal{X}}_e^{\dagger(q)}\mathbf{R}_e(n) - \hat{\mathcal{X}}_o^{(q)}\mathbf{R}_o(n)\right] \\ \hat{\mathbf{G}}_2^{(q+1)} &= (\mathbf{I}+\mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{\Psi}^\dagger\left[\hat{\mathcal{X}}_o^{\dagger(q)}\mathbf{R}_e(n) + \hat{\mathcal{X}}_e^{(q)}\mathbf{R}_o(n)\right]\end{aligned} \tag{14}$$

It can be easily seen that $(\mathbf{I} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1} = \text{diag}([(1 + \sigma^2/\lambda_0)^{-1}, \cdots, (1 + \sigma^2/\lambda_{\frac{N_c}{2}-1})^{-1}])$ and $\hat{\mathcal{X}}_e^{(q)}$, $\hat{\mathcal{X}}_o^{(q)}$ in (14) are an $\frac{N_c}{2} \times \frac{N_c}{2}$ dimensional diagonal matrices whose diagonal elements are estimated values of the coded symbols $\hat{\mathbf{X}}_e^{(q)}$, $\hat{\mathbf{X}}_o^{(q)}$ obtained at the qth iteration step.

Initialization: For initialization of the EM algorithm leading to channel estimation, a small number of pilot symbols are inserted in each OFDM frame, known by the receiver. Corresponding to pilot symbols, we focus on an under-sampled signal model and employ the linear minimum mean-square error (LMMSE) estimate to obtain the under-sampled channel coefficients. Then the complete initial channel coefficients are easily determined using an interpolation technique, i.e., Lagrange interpolation algorithm. Finally, the initial values for the $\mathbf{G}_\mu^{(0)}$ are used in the iterative EM algorithm to avoid divergence. The details of the initialization process is presented in [15], [23].

Truncation property: The truncated basis vector $\mathbf{G}_{\mu,r}$ can be formed by selecting r orthonormal basis vectors among all basis vectors that satisfy $\mathbf{C}_{\mathbf{H}_\mu}\mathbf{\Psi} = \mathbf{\Psi}\mathbf{\Lambda}$. The optimal solution that yields the smallest average mean-squared truncation error $\frac{1}{N_c/2}E[\epsilon_r^\dagger\epsilon_r]$ is the one expanded with the orthonormal basis vectors associated with the first largest r eigenvalues as given by

$$\frac{1}{N_c/2 - r}E[\epsilon_r^\dagger\epsilon_r] = \frac{1}{N_c/2 - r}\sum_{i=r}^{N_c/2-1}\lambda_i, \tag{15}$$

where $\epsilon_r = \mathbf{G}_\mu - \mathbf{G}_{\mu,r}$. For the problem at hand, truncation property of the KL expansion results in a low-rank approximation as well. Thus, a rank-r approximation of $\mathbf{\Lambda}$ can be defined as $\mathbf{\Lambda}_r = diag\{\lambda_0, \lambda_1, \cdots, \lambda_{r-1}\}$ by ignoring the trailing $N_c/2 - r$ variances $\{\lambda_l\}_{l=r}^{N_c/2-1}$, since they are very small compared to the leading r variances $\{\lambda_l\}_{l=0}^{r-1}$. Actually, the pattern of eigenvalues for $\mathbf{\Lambda}$ typically splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious.

Complexity: Based on the approach presented in [23], the traditional LMMSE estimation for $\mathbf{H}_\mu$ can be easily expressed as

$$\hat{\mathbf{H}}_\mu = \underbrace{\mathbf{C}_{\mathbf{H}_\mu}(\mathbf{\Sigma} + \mathbf{C}_{\mathbf{H}_\mu})^{-1}}_{Precomputed}\mathbf{P}_\mu, \qquad \mu = 1, 2$$

where $\mathbf{P}_1 = \hat{\mathcal{X}}_e^{\dagger(q)}\mathbf{R}_e(n) - \hat{\mathcal{X}}_o^{(q)}\mathbf{R}_o(n)$ and $\mathbf{P}_2 = \hat{\mathcal{X}}_o^{\dagger(q)}\mathbf{R}_e(n) + \hat{\mathcal{X}}_e^{(q)}\mathbf{R}_o(n)$. Since $\mathbf{C}_{\mathbf{H}_\mu}(\mathbf{\Sigma}+\mathbf{C}_{\mathbf{H}_\mu})^{-1}$ does not change with data symbols, its inverse can be pre-computed and stored during each OFDM block. Since $\mathbf{C}_{\mathbf{H}_\mu}$ and $\mathbf{\Sigma}$ are assumed to be known at the receiver, the estimation algorithm in (16) requires $N_c^2/4$ complex multiplications [1] after precomputation. However, this direct approach has high computational complexity due to required large-scale matrix inversion [2] of the precomputation matrix. Moreover, the error caused by the small fluctuations in $\mathbf{C}_{\mathbf{H}_\mu}$ and $\mathbf{\Sigma}$ have an amplified effect on the channel estimation due to the matrix inversion. Furthermore, this effect becomes more severe as the dimension of the matrix, to be inverted, increases [26]. Therefore, the KL based approach is need to avoid matrix inversion. Using the equations (3) and (14), the iterative estimate of $\mathbf{H}_\mu$ with KL expansion can be obtained as

$$\hat{\mathbf{H}}_\mu^{(q+1)} = \mathbf{\Psi}((\mathbf{I} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1})\mathbf{\Psi}^\dagger\mathbf{P}_\mu. \tag{16}$$

To reduce the complexity of the estimator further, we proceed with the low-rank approximations by considering only r column vectors of $\mathbf{\Psi}$ corresponding to the r largest eigenvalues of $\mathbf{\Lambda}$.

$$\hat{\mathbf{H}}_\mu^{(q+1)} = \mathbf{\Psi}_r\underbrace{((\mathbf{I} + \mathbf{\Sigma}_r\mathbf{\Lambda}_r^{-1})^{-1})\mathbf{\Psi}_r^\dagger}_{Precomputed}\mathbf{P}_\mu, \tag{17}$$

where $((\mathbf{I} + \mathbf{\Sigma}_r\mathbf{\Lambda}_r^{-1})^{-1}) = diag(\frac{\lambda_0}{\lambda_0+\sigma^2}, \cdots, \frac{\lambda_{r-1}}{\lambda_{r-1}+\sigma^2})$. $\mathbf{\Sigma}_r$ in (17) is a $r \times r$ diagonal matrix whose elements are equal to σ^2 and $\mathbf{\Psi}_r$ is an $N_c/2 \times r$ matrix which can be formed by omitting the last $N_c/2 - r$ columns of $\mathbf{\Psi}$. The low-rank estimator is shown to require $N_c r$ complex multiplications [3]. In comparison with the estimator (Traditional) the number of multiplications has been reduced from $N_c/4$ to r per tone.

VI. Iterative Channel Equalization and Decoding

We now consider the SFBC-OFDM decoding algorithm and the MAP outer channel code decoding to complete the description of the Turbo receiver.

A. SFBC-OFDM Decoding Algorithm

Since the channel vectors or equivalently the KL-expansion coefficients are estimated through EM based iterative approach, it is possible to decode $\mathbf{R}$ with diversity gains by

[1] Multiplication of $N_c/2 \times N_c/2$ precomputation matrix with $N_c/2 \times 1$ P_μ vector.

[2] The computational complexity of an $N_c/2 \times N_c/2$ matrix inversion, using Gaussian elimination is $O((N_c/2)^3)$.

[3] First, multiplication of precomputation matrix with P_μ, has $\frac{N_c r}{2}$ complex multiplications and then multiplication with $\mathbf{\Psi}_r$ has $\frac{N_c r}{2}$ complex multiplication which totally requires $N_c r$ complex multiplication.

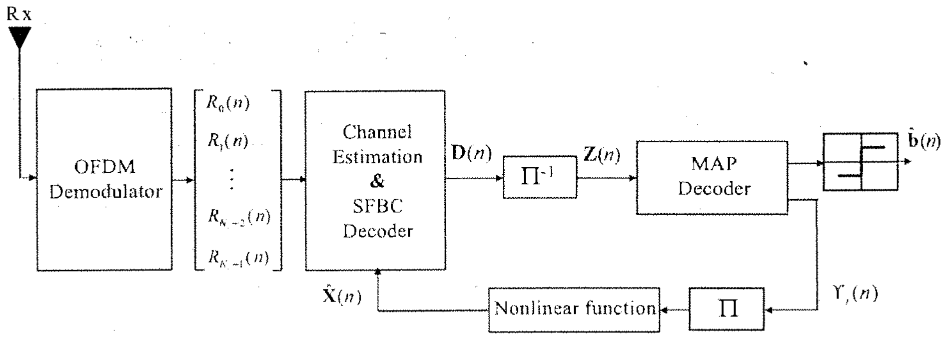


Fig. 2. Turbo receiver structure for SFBC-OFDM systems.

a simple matrix multiplication. Before dealing with how we resolve decoding, let us first re-express the received signal model (7) as follows.

$$\tilde{\mathbf{R}}(n) = \mathcal{H}(n)\tilde{\mathbf{X}}(n) + \tilde{\mathbf{W}}(n), \tag{18}$$

where $\tilde{\mathbf{R}}(n) = [\mathbf{R}_e^T(n), \mathbf{R}_o^\dagger(n)]^T$, $\tilde{\mathbf{X}}(n) = [\mathbf{X}_e^T(n), \mathbf{X}_o^T(n)]^T$, $\tilde{\mathbf{W}}_k(n) = [\mathbf{W}_e^T(n), \mathbf{W}_o^\dagger(n)]^T$ and

$$\mathcal{H}(n) = \begin{bmatrix} \mathcal{H}_{1,e}(n) & \mathcal{H}_{2,e}(n) \\ \mathcal{H}_{2,o}^\dagger(n) & -\mathcal{H}_{1,o}^\dagger(n) \end{bmatrix}. \tag{19}$$

Here, $\mathcal{H}_{\mu,e}(n)$ and $\mathcal{H}_{\mu,o}(n)$ $\mu = 1,2$ are $N_c/2 \times N_c/2$ diagonal matrices whose elements are $\mathbf{H}_{\mu,e}(n) = [H_{\mu,0}(n), H_{\mu,2}(n), \cdots, H_{\mu,N_c-2}(n)]^T$ and $\mathbf{H}_{\mu,o}(n) = [H_{\mu,1}(n), H_{\mu,3}(n) \cdots, H_{\mu,N_c-1}(n)]^T$ respectively.

Depending on complexity versus performance tradeoffs, any linear equalizer can be applied to retrieve $\tilde{\mathbf{X}}(n)$ from (18). In this paper, we consider a linear equalizer where the parameters are updated using the MMSE criterion. Given the observation $\tilde{\mathbf{R}}(n)$, the linear MMSE estimate $\mathbf{D}(n)$ of $\tilde{\mathbf{X}}(n)$ is given as follows [29].

$$\begin{aligned} \mathbf{D}(n) &= \bar{\tilde{\mathbf{X}}} + \mathbf{C}_{\tilde{\mathbf{X}}}\mathcal{H}(\mathbf{n})^\dagger(\mathcal{H}(\mathbf{n})^\dagger\mathbf{C}_{\tilde{\mathbf{X}}}\mathcal{H}(\mathbf{n}) + \mathbf{C}_{\tilde{\mathbf{W}}})^{-1} \\ &\times (\tilde{\mathbf{R}}(n) - \mathcal{H}(n)\bar{\tilde{\mathbf{X}}}(\mathbf{n})), \end{aligned} \tag{20}$$

where $\bar{\tilde{\mathbf{X}}}$ and $\mathbf{C}_{\tilde{\mathbf{X}}}$ are the mean and covariance of $\tilde{\mathbf{X}}(n)$, respectively. $\mathbf{C}_{\tilde{\mathbf{W}}}$ is the covariance of $\tilde{\mathbf{W}}(n)$.

With a scaled unitary matrix $\mathcal{H}(n)$ and approximately constant complex channel gains with $\mathcal{H}_{1,e,k}^2 + \mathcal{H}_{2,e,k}^2 \approx 1$ assumptions, $\mathcal{H}^\dagger(n)\mathcal{H}(n)$ can be simplified as

$$\mathcal{H}(n)^\dagger\mathcal{H}(n) = \begin{bmatrix} \mathcal{H}_{1,e}^2 + \mathcal{H}_{2,e}^2 & \mathbf{0} \\ \mathbf{0} & \mathcal{H}_{1,e}^2 + \mathcal{H}_{2,e}^2 \end{bmatrix} = \mathbf{I}_{N_c \times N_c}, \tag{21}$$

where $\mathbf{I}_{N_c \times N_c}$ is the $N_c \times N_c$ identity matrix. Moreover, following the assumptions used in [29], $\bar{\tilde{\mathbf{X}}} = \mathbf{0}$ and $\mathbf{C}_{\tilde{\mathbf{X}}} = \frac{1}{2}\mathbf{I}$, then (20) becomes

$$\mathbf{D}(n) = (\mathbf{I} + 2\sigma_n^2\mathbf{I})^{-1}\mathcal{H}^\dagger\tilde{\mathbf{R}}(n). \tag{22}$$

If we set $\mathbf{C}_{\tilde{\mathbf{W}}} = \mathbf{0}$ in (20), a further simplified form of the linear equalizer is obtained as follows.

$$\mathbf{D}(n) = \mathcal{H}^\dagger(n)\tilde{\mathbf{R}}(n) = \mathcal{H}^\dagger(n)\mathcal{H}(n)\tilde{\mathbf{X}}(n) + \eta(n), \tag{23}$$

where $\eta(n) = \mathcal{H}^\dagger(n)\tilde{\mathbf{W}}(n)$.

The Turbo receiver structure proposed in this paper for SFBC-OFDM systems consists of three submodules: (i) an iterative MAP-EM channel estimator, (ii) SFBC decoder and (iii) a soft MAP outer-channel-code decoder. As shown in Fig. 2 [34], first the EM based channel estimator computes the channel coefficients by means of the pilot symbols as described in the initialization step to use in the SFBC demodulator (23). Then, the equalized symbol sequence $\{\mathbf{D}(n)\}$ is passed through a channel deinterleaver, resulting in a deinterleaved equalized symbols sequence $\{\mathbf{Z}(n)\}$. Finally, $\{\mathbf{Z}(\mathrm{n})\}$ is applied to the MAP decoder submodule along with the deinterleaved estimated channel gains where the log-likelihood ratio's (LLRs) of the posteriori probabilities are computed based on the coded symbols and the uncoded bits[31], [32]. In the next iteration step, the LLRs of coded symbols $\{\Upsilon_j\}$ are deinterleaved and passed through a nonlinearity (see Appendix II for details.) yielding a soft estimate of $\mathbf{X}(n)$ as $\hat{\mathbf{X}}(n)$ as shown in Fig. 2. $\hat{\mathbf{X}}(n)$ is used in the form of $\hat{\mathcal{X}}_e^{(q)}$ and $\hat{\mathcal{X}}_o^{(q)}$ in (17) for the $(q+1)$th iteration. Thus, the MAP-EM channel estimator iteratively generates the channel estimates by taking the received signals from receiver antennas and the interleaved soft values of the LLRs which are computed by the outer channel code decoder at the previous iteration. Then, SFBC-OFDM decoder takes the channel estimates together with the received signals and computes the equalized symbol sequence for the next turbo iteration. Iterative operation is realized among these three submodules.

B. MAP Outer Channel Code Decoder

Maximum a Posterior (MAP) outer channel decoder takes the deinterleaved equalized symbol sequence and the corresponding fading amplitude values [32] as input and compute the extrinsic LLRs of the outer channel code bits as well as the hard decisions of the information bits $\{\hat{\mathbf{b}}_i\}$, at the last turbo iteration [31].

1) Details of The MAP Algorithm : The MAP algorithm provides the conditional probability of each coded symbol, C_k, taking values $-a$ and $+a$, given that the deinterleaved equalized symbol sequence is $\mathbf{Z}_0^{N_c-1} = [Z_0, \cdots, Z_k, \cdots, Z_{N_c-1}]^T$. Then, the LLR of these probabilities is calculated as

$$L(C_k) = \log\frac{P(C_k = -a|\mathbf{Z}_0^{N_c-1})}{P(C_k = a|\mathbf{Z}_0^{N_c-1})}. \tag{24}$$

Let S_k and C_k be the encoder state at time k and the encoded symbol, associated with the transition from step $k-1$ to step k. And, let trellis states at step $k-1$ and step k be indexed by the integer m' and m , respectively. Then, (24) can be expressed as

$$L(C_k) = \log \frac{\sum_{m'}\sum_{m} P(C_k = -a, S_{k-1} = m', S_k = m, \mathbf{Z}_0^{N_c-1})}{\sum_{m'}\sum_{m} P(C_k = a, S_{k-1} = m', S_k = m, \mathbf{Z}_0^{N_c-1})}. \quad (25)$$

Using the Bayes' rule and the fact that the channel is assumed memoryless, the joint probability $P(C_k, S_{k-1} = m', S_k = m, \mathbf{Z}_0^{N_c-1})$ may be written as a product of three independent probabilities

$$\begin{aligned} &= P(C_k, S_{k-1} = m', S_k = m, \mathbf{Z}_0^{k-1}, Z_k, \mathbf{Z}_{k+1}^{N_c-1}) \quad (26)\\ &= \underbrace{P(\mathbf{Z}_{k+1}^{N_c-1} | S_k = m, Z_k, S_{k-1} = m', \mathbf{Z}_0^{k-1}, C_k)}_{P(\mathbf{Z}_{k+1}^{N_c-1}|S_k=m,C_k)} \\ &\quad \times \underbrace{P(S_k = m, Z_k | S_{k-1} = m', \mathbf{Z}_0^{k-1}, C_k)}_{P(S_k=m,Z_k|S_{k-1}=m',C_k)} \\ &\quad \times P(S_{k-1} = m', \mathbf{Z}_0^{k-1}, C_k) . \end{aligned}$$

Applying now the Markovian property, (26) may be simplified as follows.

$$\begin{aligned} P(C_k, S_{k-1} = m', S_k = m, \mathbf{Z}_0^{N_c-1}) &= P(\mathbf{Z}_{k+1}^{N_c-1} | S_k = m, C_k) \quad (27)\\ \times P(S_k = m, Z_k | S_{k-1} = m', C_k) &P(S_{k-1} = m', \mathbf{Z}_0^{k-1}, C_k). \end{aligned}$$

The joint likelihood function in (27) can be recursively computed by means of the forward and backward variables $\alpha_k(m)$, $\beta_k(m)$ and the transition probabilities $\gamma_k(m', m)$, which are defined as

$$\begin{aligned} \alpha_k(m) &\triangleq P(S_k = m, \mathbf{Z}_0^k, C_{k+1}) \quad (28)\\ \beta_k(m) &\triangleq P(\mathbf{Z}_{k+1}^{N_c-1} | S_k = m, C_k) \\ \gamma_k(m', m) &\triangleq P(S_k = m, Z_k | S_{k-1} = m', C_k) . \end{aligned}$$

Substituting $\alpha_k(m)$, $\beta_k(m)$ and $\gamma_k(m', m)$ in (24), the conditional LLRs of C_k, given the received sequence $\mathbf{Z}_0^{N_c-1}$, can be rewritten

$$L(C_k) = \log \frac{\sum_{m}\sum_{m'}\limits_{C_k=-a} \alpha_{k-1}(m')\, \gamma(m', m)\, \beta_k(m)}{\sum_{m}\sum_{m'}\limits_{C_k=+a} \alpha_{k-1}(m')\, \gamma(m', m)\, \beta_k(m)} . \quad (29)$$

Finally, the recursive computations of $\alpha_k(m)$, $\beta_k(m)$ are given by

$$\begin{aligned} \alpha_k(m) &= \sum_{m'} \alpha_{k-1}(m') \gamma_k(m', m) \qquad k = 0, 1, \ldots, N_c - 1 \\ \beta_k(m) &= \sum_{m'} \beta_{k+1}(m') \gamma_{k+1}(m, m') \quad k = N_c - 1, \ldots, 0 \quad (30) \end{aligned}$$

and $\gamma_k(m', m)$ can be computed from

$$\begin{aligned} \gamma_k(m', m) &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(Z_k - (\hat{H}_k^\dagger \hat{H}_k) C_k(m', m))^2}{2\sigma^2}\right] \\ &\times \quad P(S_k = m | S_{k-1} = m'), \quad (31) \end{aligned}$$

where C_k, Z_k and $\hat{H}_k$ are the encoded symbol, deinterleaved equalized codeword and the estimated channel coefficient, respectively. σ^2 is the Gaussian noise variance.

2) Log-MAP: The equations for the MAP algorithm presented above may be written in the logarithmic domain. Firstly, we express $\alpha_k(m)$, $\beta_k(m)$ and $\gamma_k(m', m)$ in the logarithmic domain as

$$\begin{aligned} \mathrm{A}_k(m) &= \log \alpha_k(m) \quad (32)\\ \mathrm{B}_k(m) &= \log \beta_k(m) \\ \Gamma_k(m', m) &= \log \gamma_k(m', m). \end{aligned}$$

Then the recursive equations can be derived for $\mathrm{A}_k(m)$ and $\mathrm{B}_k(m)$ easily as follows.

$$\begin{aligned} \mathrm{A}_k(m) &= \log \sum_{m'} e^{\mathrm{A}_{k-1}(m') + \Gamma_k(m', m)} \quad (33)\\ \mathrm{B}_k(m) &= \log \sum_{m'} e^{\mathrm{B}_{k+1}(m') + \Gamma_{k+1}(m, m')} . \end{aligned}$$

Substituting (33) in (29) yields,

$$\begin{aligned} L(C_k) &= \underbrace{\log \sum_{m'}\sum_{m} e^{\Gamma_k(m', m) + \mathrm{A}_{k-1}(m') + \mathrm{B}_k(m)}}_{for\ C_k=-a} \\ &\quad \underbrace{- \log \sum_{m'}\sum_{m} e^{\Gamma_k(m', m) + \mathrm{A}_{k-1}(m') + \mathrm{B}_k(m)}}_{for\ C_k=a} . \quad (34) \end{aligned}$$

If we define the $\mathbb{E}$ operator as $\mathbb{E}(x_m) = \log_m \sum_m e^{x_n}$ then, $L(C_k)$ can be expressed as,

$$\begin{aligned} L(C_k) = &\mathop{\mathbb{E}}_{\substack{m',m \\ C_k=-a}} \left[\Gamma_k(m', m) + \mathrm{A}_{k-1}(m') + \mathrm{B}_k(m)\right] \\ - &\mathop{\mathbb{E}}_{\substack{m',m \\ C_k=a}} \left[\Gamma_k(m', m) + \mathrm{A}_{k-1}(m') + \mathrm{B}_k(m)\right]. \quad (35) \end{aligned}$$

Calculation of the LLRs according to equation (35) for encoded bits with generator matrices g(5,7) and g(1,5/7) are demonstrated in Appendix III

VII. Computer Simulations

In this section, computer simulations are carried out to evaluate the performance of the proposed turbo receiver structure for SFBC-OFDM systems. To understand the behavior of different channel encoders, we simulated both turbo and convolutionally coded SFBC-OFDM systems. In case of Turbo Encoder, two identical recursive systematic convolutional component codes (RSC) with generator ($1,5_8/7_8$) concatenated in parallel via a pseudorandom interleaver formed the encoder [31], [32]. For the convolutionally coded system, a ($5_8,7_8$) code with rate 1/2 code was used.

The scenario for SFBC-OFDM simulation study is as follows: A BPSK modulation format is employed and the number of transmitting and receiving antennas are chosen as $T = 2$ and $R = 1$ respectively. The system has a 2.4 MHz bandwidth (for the pulse roll-off factor $\alpha = 0.2$) and is divided into $N_c = 512$ tones with a total period T_s of 136 μs, of which 8 μs constitute the cyclic prefix ($L = 32$). The data rate is 1.9 Mbit/s. We assume that the rms value of the multipath width is $\tau_{rms} = 1$ sample (0.25 μs) for the power-delay profile. In all simulations, three iterations are employed.

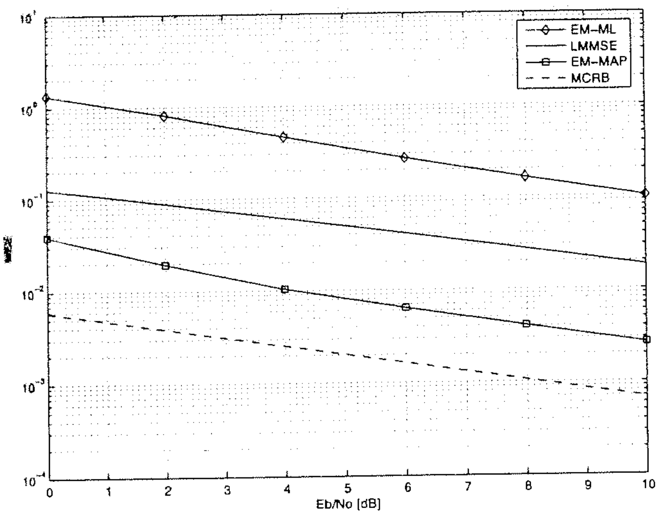


Fig. 3. MSE performance of SFBC-Turbo-OFDM turbo receivers, PIR=1:8.

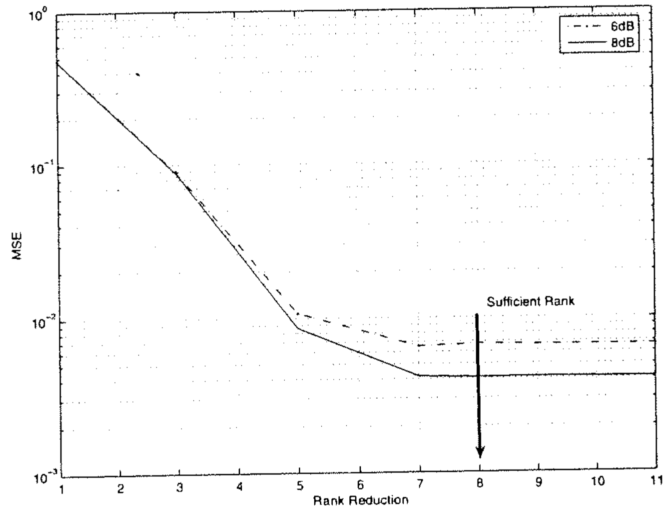


Fig. 5. MSE performance of SFBC-Turbo-OFDM Systems according to number of used KL coefficients, PIR=1:8.

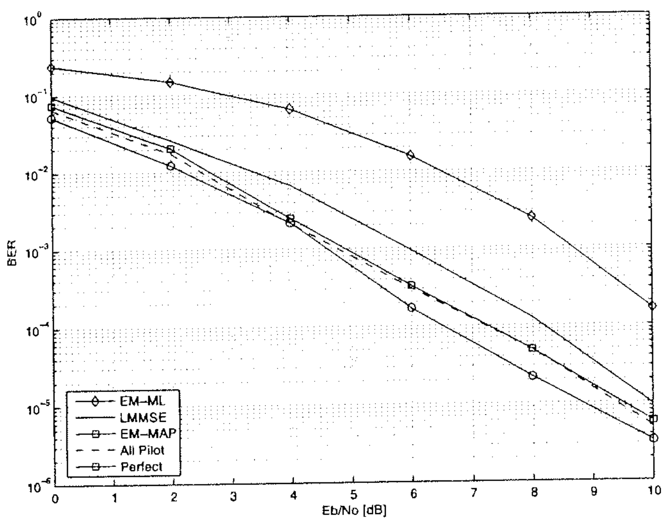


Fig. 4. BER performance of SFBC-Turbo-OFDM turbo receivers, PIR=1:8.

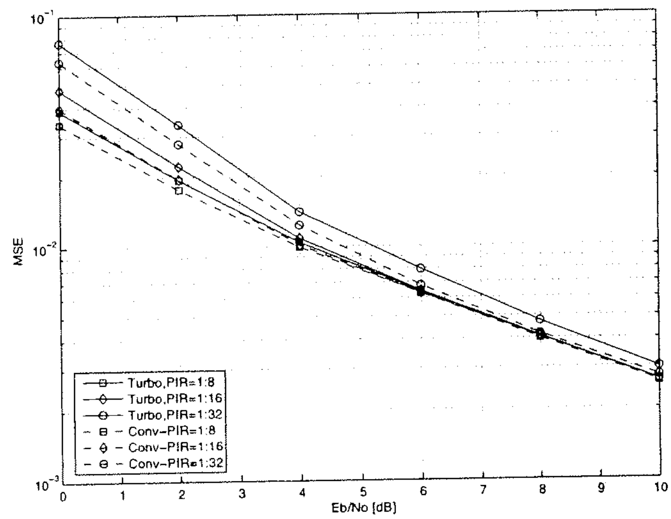


Fig. 6. MSE performance of the EM algorithms as a function of PIRs.

Fig. 3 compares the MSE performance of the EM-MAP channel estimation approach with EM-ML [13] and a widely used LMMSE pilot symbol assisted modulation (LMMSE-PSAM)schemes [14], as well as with the Modified Cramer Rao Bound (MCRB) for turbo coded SFBC-OFDM systems. Pilot Insertion Rate was chosen as (PIR) =1:8. That is one pilot is inserted for every 8 data symbols. It is observed that the proposed EM-MAP significantly outperforms the EM-ML as well as PSAM techniques and approaches the MCRB for higher Eb/No values. Moreover, in Fig. 4, the BER performance of proposed system is compared with the all-pilot and perfect channel cases for the turbo coded and the convolutionally coded systems.

The optimal truncation property of KL expansion minimizes the amount of information required to represent the statistically dependent data. Thus, this property can further reduce computational load on the channel estimation algorithm. If the number of parameters in the expansion include dominant eigenvalues (Rank=8), it is possible to obtain an excellent approximation with a relatively small number of KL coefficients. The optimal truncation property of the KL expansion is exploited in Fig. 5 and the simulation results are presented for the MSE performance.

It is clear that good channel codes are more sensitive to the poorly estimated channel. With high correlation between the coded bits, a well designed channel code is more sensitive to channel estimation errors which might cause severe error propagation in the decoding process. Therefore, it is expected that BER performance of turbo coded structures degrades more than convolutionally coded structures. Thus, lower pilot insertion rates provide poor initial estimates, resulting in a more BER performance degradation in turbo coded systems as compared to convolutionally encoded systems. This effect is demonstrated in Fig. 6 by the MSE performance curves for different values of PIR=1:8,1:16,1:32. Although a slight MSE performance difference is observed in Fig. 6,for chosen different pilot densities, its effect on the BER performance is more obvious especially at higher SNR values as seen

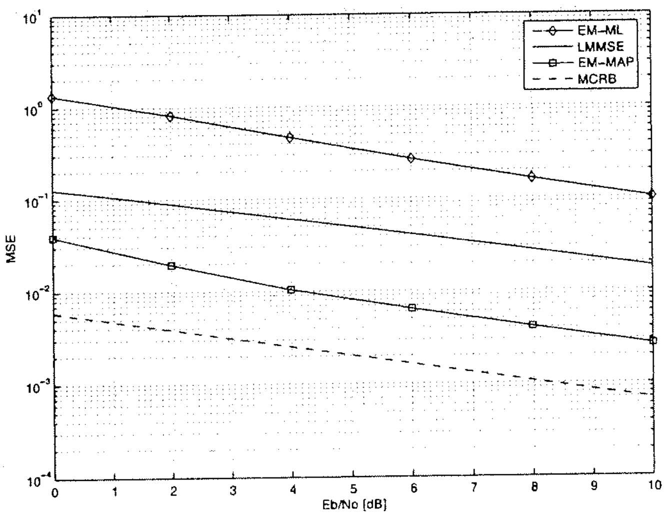


Fig. 3. MSE performance of SFBC-Turbo-OFDM turbo receivers, PIR=1:8.

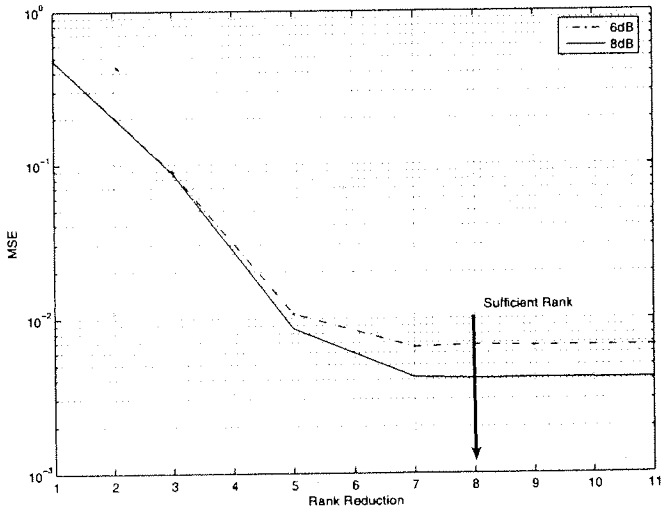


Fig. 5. MSE performance of SFBC-Turbo-OFDM Systems according to number of used KL coefficients, PIR=1:8.

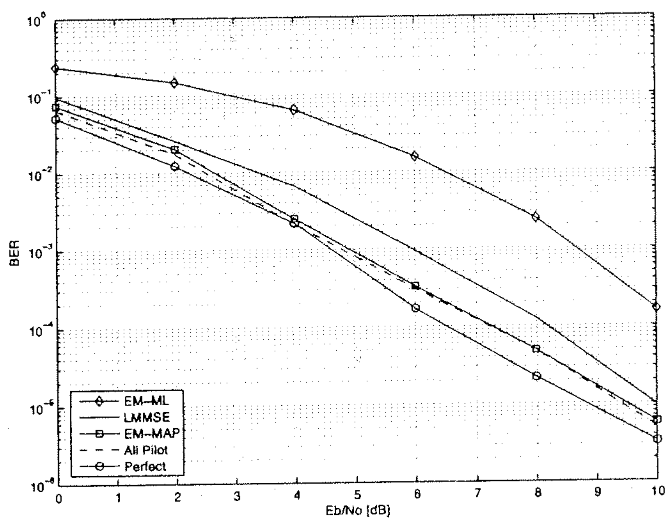


Fig. 4. BER performance of SFBC-Turbo-OFDM turbo receivers, PIR=1:8.

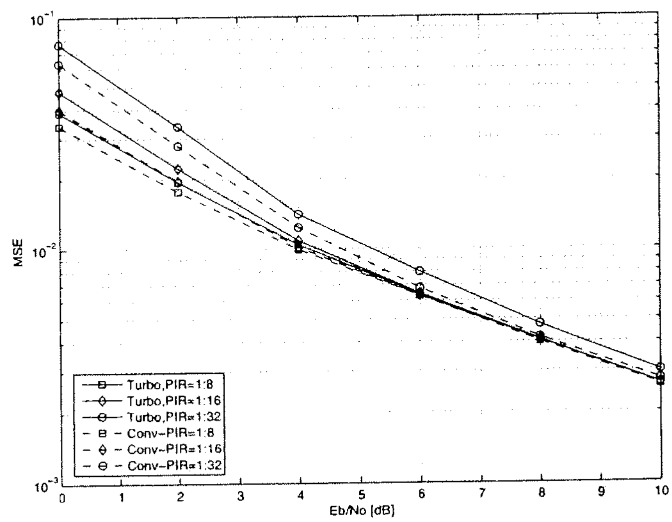


Fig. 6. MSE performance of the EM algorithms as a function of PIRs.

Fig. 3 compares the MSE performance of the EM-MAP channel estimation approach with EM-ML [13] and a widely used LMMSE pilot symbol assisted modulation (LMMSE-PSAM)schemes [14], as well as with the Modified Cramer Rao Bound (MCRB) for turbo coded SFBC-OFDM systems. Pilot Insertion Rate was chosen as (PIR) =1:8. That is one pilot is inserted for every 8 data symbols. It is observed that the proposed EM-MAP significantly outperforms the EM-ML as well as PSAM techniques and approaches the MCRB for higher Eb/No values. Moreover, in Fig. 4, the BER performance of proposed system is compared with the all-pilot and perfect channel cases for the turbo coded and the convolutionally coded systems.

The optimal truncation property of KL expansion minimizes the amount of information required to represent the statistically dependent data. Thus, this property can further reduce computational load on the channel estimation algorithm. If the number of parameters in the expansion include dominant eigenvalues (Rank=8), it is possible to obtain an excellent approximation with a relatively small number of KL coefficients. The optimal truncation property of the KL expansion is exploited in Fig. 5 and the simulation results are presented for the MSE performance.

It is clear that good channel codes are more sensitive to the poorly estimated channel. With high correlation between the coded bits, a well designed channel code is more sensitive to channel estimation errors which might cause severe error propagation in the decoding process. Therefore, it is expected that BER performance of turbo coded structures degrades more than convolutionally coded structures. Thus, lower pilot insertion rates provide poor initial estimates, resulting in a more BER performance degradation in turbo coded systems as compared to convolutionally encoded systems. This effect is demonstrated in Fig. 6 by the MSE performance curves for different values of PIR=1:8,1:16,1:32. Although a slight MSE performance difference is observed in Fig. 6,for chosen different pilot densities, its effect on the BER performance is more obvious especially at higher SNR values as seen

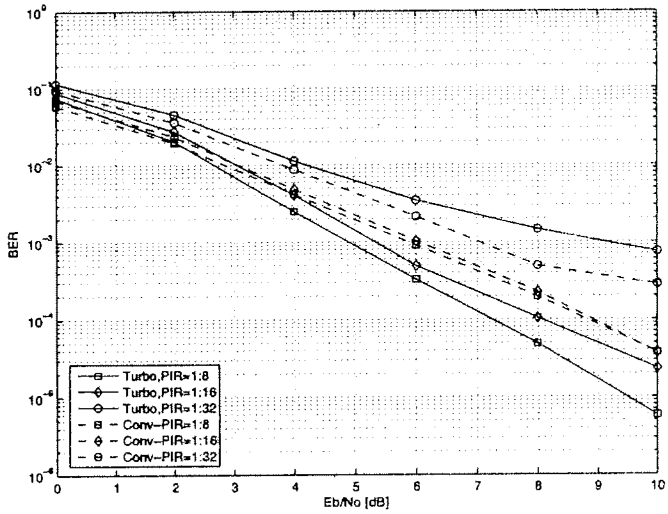


Fig. 7. BER performance of the EM algorithms as a function of PIRs.

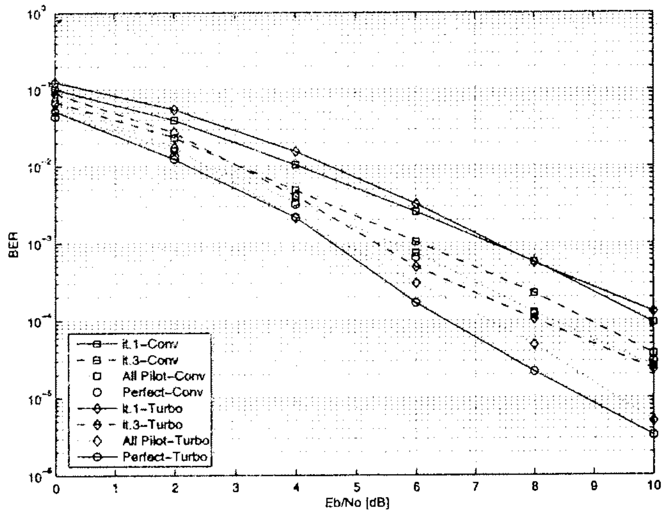


Fig. 9. BER performance of the turbo receiver structures as a function of average Eb/No, PIR=1:16.

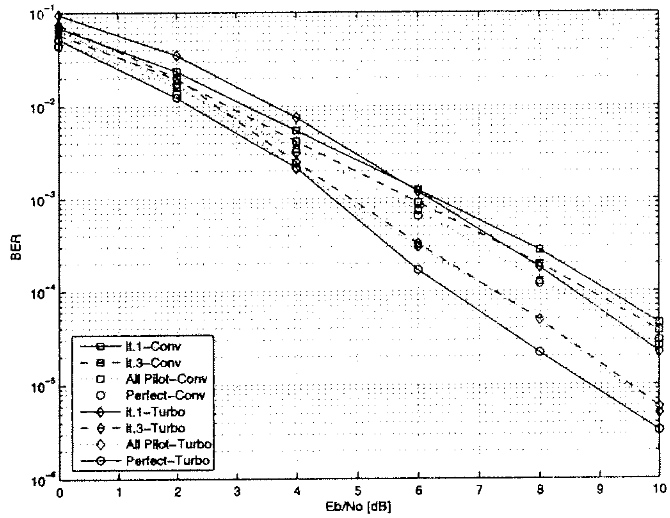


Fig. 8. BER performance of the turbo receiver structures as a function of average Eb/No, PIR=1:8.

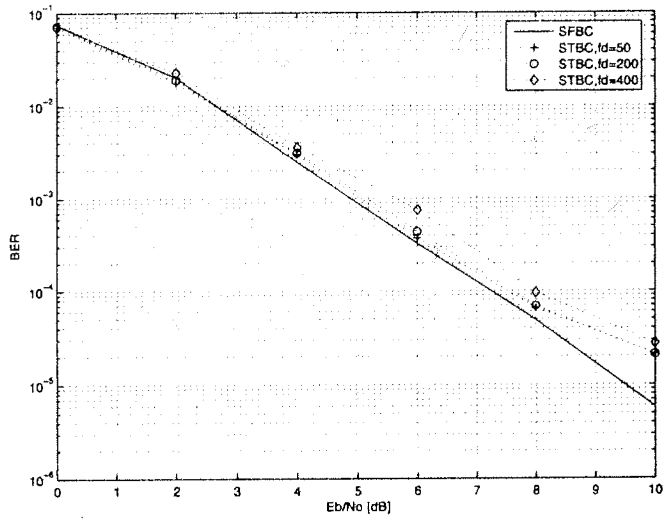


Fig. 10. BER performance of Transmit Diversity Coded Turbo-OFDM Systems, PIR=1:8.

in Fig. 7. Moreover, a slightly better MSE performance is observed in convolutional codes compared to the turbo code due to punctured structure of turbo codes.

The BER performances of the proposed systems are presented together with initial values for $PIR = 1:8$ and $PIR = 1:16$ in Fig. 8 and Fig. 9 respectively. All pilot and perfect channel cases for the turbo and convolutionally coded systems are included as well. Due to its structure, the turbo decoding evidently provides more reliable priori information for iterative processing compared to the convolutional decoding. Therefore, as seen in Fig. 8, the BER performance improvement observed due to iterative turbo coding structure is 1.3 dB for $Pe = 10^{-4}$, meanwhile it is 0.3 dB for convolutionally coded systems.

Although overall performance degrades by decreasing pilot insertion rate, iterative decoding provides better performance improvements on each iteration for the lower pilot insertion rate (e.g. $PIR = 1:16$). From simulation studies, it is observed in Fig. 9 that iterative performance gain is increased for PIR=1:16 as 2.3 dB and 1 dB, for turbo and convolutionally coded systems, respectively. As previously mentioned, turbo coding is more sensitive to channel estimation performance than the convolutional coding. The iterative performance gain increment for turbo coded structure is therefore lower than the convolutionally coded systems.

In space-frequency coding, detection is accomplished completely within a single OFDM block. Therefore the Doppler shift will not affect SFBC-OFDM system for the channel model we consider in this work. In Fig. 10, the computer simulation results are presented for the BER performances of both space-time and space-frequency block coded turbo aided receivers in the presence of different Doppler frequencies. We observe in Fig. 10 that at higher Doppler frequencies, the space-frequency block coded structure is superior and an error floor effect is observed in the BER performance of the space-time structure. Thus, we conclude that the space-frequency coded OFDM turbo receiver structure is a good candidate for doubly selective block fading channels.

VIII. Conclusions

We consider the design of turbo receiver structures for SFBC-OFDM systems in unknown frequency selective fading channels. By using extrinsic information of the transmitted symbols, both the channel estimation and the decoding process can be improved. The turbo structure performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing BPSK modulation scheme. Moreover, the MAP-EM approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. The optimal truncation property of the KL expansion can reduce computational load on the iterative estimation approach. The performance merits of our channel estimation algorithm is confirmed by corroborating computer simulations. It is observed that the proposed EM-MAP significantly outperforms the EM-ML as well as PSAM techniques. Furthermore, sensitivity to channel estimation errors of turbo receivers are investigated for turbo coded and convolutionally coded SFBC-OFDM systems. One of the important conclusions of the paper is that turbo codes are more sensitive to channel parameter estimation errors than the convolutional coded systems. It has been shown that receiver with turbo codes performs well over the convolutional receiver structure when the quality of the channel estimation performance is high. Moreover, superiority of the turbo coded SFBC-OFDM systems over the turbo coded STBC-OFDM systems is observed especially for high Doppler frequencies.

Acknowledgement

The authors would like to thank the editor, anonymous reviewers and Prof. Ümit Aygölü of The Istanbul Technical University for their insightful comments and suggestions that improved the quality of this paper.

Appendix I
Modified Cramer-Rao Bound

The modified Fisher information matrix (FIM) can be obtained by a straightforward modification of FIM as[15],

$$\mathbf{J}_M(\mathbf{G}) \triangleq \underbrace{-E[\frac{\partial^2 \ln p(\mathbf{R}|\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T}]}_{\mathbf{J}(\mathbf{G})} \underbrace{-E[\frac{\partial^2 \ln p(\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T}]}_{\mathbf{J}_P(\mathbf{G})}, \quad (36)$$

where $\mathbf{J}_P(\mathbf{G})$ represents the *a priori* information.

Under the assumption that $\mathbf{G}$ and $\mathbf{W}(n)$ are independent of each other and $\mathbf{W}(n)$ is a zero-mean, the transmitted symbols become uncorrelated due to the channel interleaver. The conditional PDF $\mathbf{R}$ given $\mathbf{G}$ is given as

$$\begin{aligned} p(\mathbf{R}|\mathbf{G}) &= E_{\mathcal{X}}\{p(\mathbf{R}|\mathcal{X},\mathbf{G})\} \\ &\sim \frac{1}{\sigma^2} E_{\mathcal{X}}\left\{(\mathbf{R}-\mathcal{X}\widetilde{\mathbf{\Psi}}\mathbf{G})^{\dagger}(\mathbf{R}-\mathcal{X}\widetilde{\mathbf{\Psi}}\mathbf{G})\right\}, (37)\end{aligned}$$

where $\widetilde{\mathbf{\Psi}} = \mathrm{diag}(\mathbf{\Psi}\ \mathbf{\Psi})$. From (37), the derivatives can be taken as follows.

$$\frac{\partial \ln p(\mathbf{R}|\mathbf{G})}{\partial \mathbf{G}^T} = \frac{1}{\sigma^2}(\mathbf{R}-\mathcal{X}\widetilde{\mathbf{\Psi}}\mathbf{G})^{\dagger}\mathcal{X}\widetilde{\mathbf{\Psi}} \quad (38)$$

$$\frac{\partial^2 \ln p(\widetilde{\mathbf{R}}|\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T} = -\frac{1}{\sigma^2}\widetilde{\mathbf{\Psi}}^{\dagger}\mathcal{X}^{\dagger}\mathcal{X}\widetilde{\mathbf{\Psi}}\ . \quad (39)$$

Since, space frequency transition matrix provide $\mathcal{X}^{\dagger}\mathcal{X} = \mathbf{I}$ and using $\widetilde{\mathbf{\Psi}}^{\dagger}\widetilde{\mathbf{\Psi}} = \mathbf{I}$ and taking the expected values yields the following simple form:

$$\mathbf{J}(\mathbf{G}) = \frac{1}{\sigma^2}\mathbf{I}\,. \quad (40)$$

Second term in (36) is easily obtained as follows. Consider the prior PDF $p(\mathbf{G}) \sim \exp(-\mathbf{G}^{\dagger}\widetilde{\mathbf{\Lambda}}^{-1}\mathbf{G})$. The respective derivatives are found as

$$\frac{\partial \ln p(\mathbf{G})}{\partial \mathbf{G}^T} = -\mathbf{G}^{\dagger}\widetilde{\mathbf{\Lambda}}^{-1}, \qquad \frac{\partial^2 \ln p(\mathbf{G})}{\partial \mathbf{G}^* \partial \mathbf{G}^T} = -\widetilde{\mathbf{\Lambda}}^{-1}\,. \quad (41)$$

Upon taking the negative expectations, the second term in (41) becomes $\mathbf{J}_P(\mathbf{G}) = \widetilde{\mathbf{\Lambda}}^{-1}$. Substituting $\mathbf{J}(\mathbf{G})$ and $\mathbf{J}_P(\mathbf{G})$ in (36) produces for the modified FIM as follows

$$\mathbf{J}_M(\mathbf{G}) = \frac{1}{\sigma^2}\mathbf{I} + \widetilde{\mathbf{\Lambda}}^{-1}. \quad (42)$$

Inverting the matrix $\mathbf{J}_M(\mathbf{G})$ yields $MCRB(\hat{\mathbf{G}}) = \mathbf{J}_M^{-1}(\mathbf{G})$. $MCRB(\hat{\mathbf{G}})$ is a diagonal matrix with the elements on the main diagonal equaling the reciprocal of those $\mathbf{J}(\mathbf{G})$ matrix.

Appendix II
Nonlinear Function

To use soft decision of the LLRs in the turbo receiver structure [34], let us find firstly $P(X_k = a)$ and $P(X_k = -a)$ in terms of LLRs ($a = 1/\sqrt{2}$).

$$L(X_k) = \log_e\left[\frac{P(X_k = -a)}{P(X_k = +a)}\right] = \log_e\left[\frac{P(X_k = -a)}{1 - P(X_k = -a)}\right]. \quad (43)$$

From (43), $P(X_k = \pm a)$ can be determined as

$$P(X_k = -a) = \frac{e^{L(X_k)}}{1 + e^{L(X_k)}} \quad (44)$$

$$P(X_k = a) = 1 - \frac{e^{L(X_k)}}{1 + e^{L(X_k)}} = \frac{1}{1 + e^{L(X_k)}}. \quad (45)$$

For BPSK signalling, the mean value of X_k is

$$\begin{aligned} \bar{X}_k &= \sum_{x_b \in \{+a,-a\}} x_b . P(X_k = x_b) \\ &= a(P(X_k = +a) - P(X_k = -a)). \quad (46)\end{aligned}$$

Finally, substituting $P(X_k = +a)$ and $P(X_k = -a)$ into (46), the mean value of X_k could be written as

$$\begin{aligned} \bar{X}_k &= a\left(\frac{e^{L(X_k)}}{1 + e^{L(X_k)}} - \frac{1}{1 + e^{L(X_k)}}\right) \\ &= \frac{e^{L(X_k)} - 1}{e^{L(X_k)} + 1} = a \tanh(L(X_k)/2) \quad (47)\end{aligned}$$

Appendix III
Calculation of LLRs for Encoded Symbols

A. LLRs of Encoded Symbols ($C_{e,k}, C_{o,k}$) for Generator Matrix (1,5/7)

$$\begin{aligned} L(C_{e,k}) &= \underset{\substack{m,m' \\ C_{e,k}=-a}}{\mathbb{E}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\ &- \underset{\substack{m,m' \\ C_{e,k}=+a}}{E} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)]\end{aligned}$$

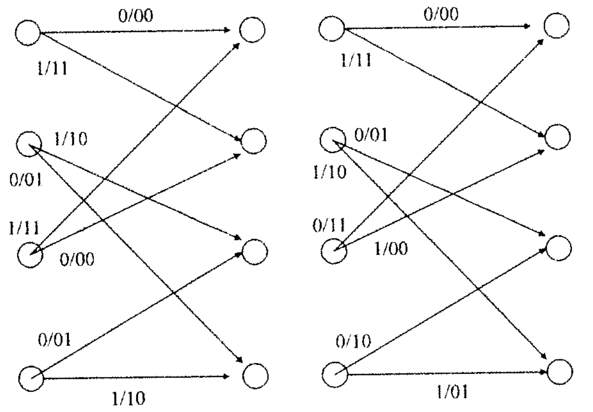


Fig. 11. (A) State transition diagram for the generator matrix (1,5/7); (B) shows state transition diagram for the generator matrix (5,7).

$$
\begin{aligned}
&= \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,4)+\mathrm{B}_k(4)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,3)+\mathrm{B}_k(3)]) \\
&\quad - \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,3)+\mathrm{B}_k(3)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,4)+\mathrm{B}_k(4)])
\end{aligned}
\tag{48}
$$

$$
\begin{aligned}
L(C_{o,k}) &= \underset{\substack{m,m' \\ C_{o,k}=-a}}{\mathbb{E}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&\quad - \underset{\substack{m,m' \\ C_{o,k}=+a}}{\mathbb{E}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&= \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,3)+\mathrm{B}_k(3)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,4)+\mathrm{B}_k(4)]) \\
&\quad - \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,4)+\mathrm{B}_k(4)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,3)+\mathrm{B}_k(3)])
\end{aligned}
\tag{49}
$$

B. LLRs of Encoded Symbols $(C_{e,k}, C_{o,k})$ for Generator Matrix (5,7)

$$
\begin{aligned}
L(C_{e,k}) &= \underset{\substack{m,m' \\ C_{e,k}=-a}}{\mathbb{E}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&\quad - \underset{\substack{m,m' \\ C_{e,k}=+a}}{\mathbb{E}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&= \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,3)+\mathrm{B}_k(3)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,4)+\mathrm{B}_k(4)]) \\
&\quad - \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,4)+\mathrm{B}_k(4)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,3)+\mathrm{B}_k(3)])
\end{aligned}
\tag{50}
$$

$$
\begin{aligned}
L(C_{o,k}) &= \underset{\substack{m,m' \\ C_{o,k}=-a}}{\mathbb{E}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&\quad - \underset{\substack{m,m' \\ C_{o,k}=+a}}{\mathbb{E}} [\mathrm{A}_{k-1}(m') + \Gamma_k(m',m) + \mathrm{B}_k(m)] \\
&= \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,4)+\mathrm{B}_k(4)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,3)+\mathrm{B}_k(3)]) \\
&\quad - \mathbb{E}([\mathrm{A}_{k-1}(1)+\Gamma_k(1,2)+\mathrm{B}_k(2)], [\mathrm{A}_{k-1}(2)+\Gamma_k(2,3)+\mathrm{B}_k(3)] \\
&\quad , [\mathrm{A}_{k-1}(3)+\Gamma_k(3,1)+\mathrm{B}_k(1)], [\mathrm{A}_{k-1}(4)+\Gamma_k(4,4)+\mathrm{B}_k(4)])
\end{aligned}
\tag{51}
$$

REFERENCES

[1] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451-1458, Oct. 1998.

[2] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communications: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744-765, Mar. 1998.

[3] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456-1467, July 1999.

[4] R. Van Nee and R. Prasad, *OFDM Wireless Multimedia Communications*. Boston, MA: Artech House Publishers, 2000.

[5] Y. (G.) Li, L. J. Cimini, N. Seshadri, and S. Ariyavistakul, "Channel estimation for OFDM systems with transmitter diversity in mobile wireless channels," *IEEE J. Select. Areas Commun.*, vol. 17, no. 3, pp. 461-471, July 1999.

[6] Y. (G.) Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," *IEEE Trans. Wireless Commun.*, vol. 1, no. 1, pp. 67-75, Nov. 1999.

[7] Y. Li, C. N. Georghiades, and G. Huang, "Iterative maximum likelihood sequence estimation for space-time coded systems," *IEEE Trans. Commun.*, vol. 49, no. 6, pp. 948-951, June 2001.

[8] C. Cozzo and B. L. Hughes, "Joint channel estimation and data detection in space-time communications," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1266-1270, Aug. 2003.

[9] M. Feder and E. Weinstein, "Parameter estimation of superimposed signals using the EM algorithm," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 36, pp. 477-489, Apr. 1988.

[10] B. Lu, X. Wang, and Y. (G) Li, "Iterative receivers for space-time block-coded OFDM systems in dispersive fading channels," *IEEE Trans. Wireless Commun.*, vol. 1, no. 2, pp. 213-225, Apr. 2002.

[11] B. Lu, X. Wang, and K. Narayanan, "LDPC-based space-time coded OFDM over correlated fading channels," *IEEE Trans. Commun.*, vol. 50, no. 1, pp 74-88, Jan. 2002.

[12] X. Yongzhe and C. N. Georghiades, "Two EM-type channel estimation algorithms for OFDM with transmitter diversity," *IEEE Trans. Commun.*, vol. 51, no. 1, pp. 106-115, Jan. 2003.

[13] X. Ma, H. Koyabashi, and S. C. Schwartz, "An EM-based channel estimation for space-time and space-frequency block coded OFDM," in *Proc. IEEE ICASSP*, Apr. 2003, vol. 4, pp. 389-392.

[14] M. Morelli and U. Mengali, "A comparison of pilot-aided channel estimation methods for OFDM systems," *IEEE Trans. Signal Processing*, vol. 49, pp. 3065-3073, Dec. 2001.

[15] H. A Cırpan, E. Panayırcı, and H. Doğan, "Nondata-aided channel estimation for OFDM systems with space-frequency transmit diversity," *IEEE Trans. Veh. Technol.*, vol. 55, no. 2, pp. 449-457, Mar. 2006.

[16] K. Yip and T. Ng, "Karhunen-Loeve expansion of the WSSUS channel output and its application to efficient simulation," *IEEE J. Select. Areas Commun.*, vol. 15, no. 4, pp. 640-646, May 1997.

[17] E. Panayırcı and H. A. Cırpan, "Channel estimation for space-time block coded OFDM systems in the presence of multipath fading," in *Proc. IEEE Globecom*, Nov. 2002, vol. 2, pp. 1157-1161.

[18] K. F. Lee and D. B. Williams, "A space-frequency transmitter diversity technique for OFDM systems," in *Proc. IEEE Globecom*, Nov.-Dec. 2000, vol. 3, pp. 1473-1477.

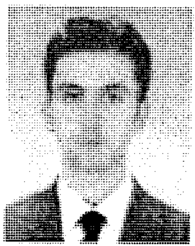
[19] Z. Liu, Y. Xin, and G. B. Giannakis, "Space-time-frequency coded OFDM over frequency selective fading channels," *IEEE Trans. Signal Processing*, vol. 50, no. 10, pp. 2465-2476, Oct. 2002.

[20] R. J. McEliece and W. E. Stark, "Channels with block interference," *IEEE Trans. Inform. Theory*, vol. 30, no. 1, pp. 44-53, Jan. 1984.

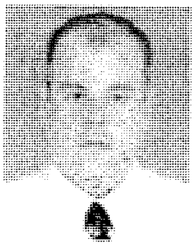
[21] J.-J. van de Beek, O. Edfors, P. O. Börjesson, M. Wahlqvist, and C. Östberg, "A conceptual study of OFDM-based multiple access schemes: Part 2 - Channel estimation in the uplink," Technical Report Tdoc 116/96, ETSI STC SMG2 meeting no. 18, May 1996.

[22] G. B. Giannakis and C. Tepedelenlioglu, "Basis expansion models and diversity techniques for blind identification and equalization of time-varying channels," *Proc. IEEE*, vol. 86, pp. 1969-1986, Oct. 1998.

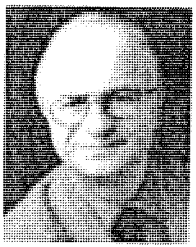
[23] O. Edfors, M. Sandell, J.-J. van de Beek, S. K. Wilson, and P.O. Börjesson, "OFDM channel estimation by singular value decomposition," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 931-936, July 1998.
[24] Y. Baoguo, C. Zhigang, and K. Letaief, "Analysis of low-complexity windowed DFT-based MMSE channel estimator for OFDM systems," *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 1977-1987, Nov. 2001.
[25] O. Edfors, M. Sandell, J. J. van de Beek, S. K. Wilson, and P. O. Börjesson, "Analysis of DFT-based channel estimation for OFDM," *Wireless Pers. Commun.*, vol. 12, no. 1, pp. 55-70, Jan. 2000.
[26] J. Zhu and W. Lee, "A low-complexity channel estimator for OFDM systems in multipath fading channels," in *Proc. IEEE PIMRC*, Sep. 2004, vol. 3, pp. 1978-1982.
[27] A. P. Dempster, N. M. Laird, and D. B. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," *J. Roy. Statist. Soc.*, vol. 39, no. 1, pp. 1-17, 1977.
[28] G. K. Kaleh and R. Valet, "Joint parameter estimation and symbol detection for linear or nonlinear unknown channels," *IEEE Trans. Commun.* vol. 42, no. 7, pp. 2406-2413, July 1994.
[29] M. Tuchler, R. Koetter, and A. C. Singer, "Turbo equalization: Principles and new results," *IEEE Trans. Commun.*, vol. 50, pp. 754-767, May 2002.
[30] P. Robertson, E. Villebrun, and P. Hoeher, "A comparison of optimal and sub-optimal MAP decoding algorithms operating in the log domain," in *Proc. IEEE International Conf. Commun.*, June 1995, vol. 2, pp. 1009-1013.
[31] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error-correcting coding: Turbo codes," in *Proc. IEEE Int. Conf. Commun.*, May 1993, pp. 1064-1070.
[32] J. P. Woodard and L. Hanzo, "Comparative study of turbo decoding techniques: An overview," *IEEE Trans. Veh. Technol.*, vol. 49, no. 6, pp. 2208-2233, Nov. 2000
[33] H. A Cırpan, E. Panayırcı, and H. Doğan, "Iterative channel estimation approach for space-time/frequency coded OFDM systems with transmitter diversity," *Eur. Trans. Telecommun.*, vol. 15, no. 3, pp. 235-248, May-June 2004.
[34] M. C. Valenti and B. D. Woerner, "Iterative channel estimation and decoding of pilot symbol assisted turbo codes over flat-fading channels," *IEEE J. Select. Areas Commun.*, vol. 19, no. 9, pp. 1697-1705, Sep. 2001.
[35] S. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory.* Englewood Cliffs, NJ: Prentice Hall, 1993.



Hakan Doğan (S'02) was born in Istanbul, Turkey, in 1979. He received the B.S. and M.S. degrees in electronics engineering from Istanbul University, Istanbul, Turkey, in 2001 and 2003, respectively, and is currently working toward the Ph.D. degree from the same university. Since 2001, he has been a Research Assistant at the Department of Electrical and Electronics Engineering, Istanbul University. His general research interests cover communication theory, estimation theory, statistical signal processing, and information theory. His current research areas are focused on wireless communication concepts with specific attention to equalization and channel estimation for spread-spectrum and multicarrier (orthogonal frequency-division multiplexing) systems.



Hakan A. Çırpan (M'97) received the B.S. degree from Uludag University, Bursa, Turkey, in 1989, the M.S. degree from the University of Istanbul, Istanbul, Turkey, in 1992, and the Ph.D. degree from Stevens Institute of Technology, Hoboken, NJ, in 1997, all in electrical engineering. From 1995 to 1997, he was a Research Assistant at Stevens Institute of Technology, working on signal processing algorithms for wireless communication systems. In 1997, he joined the faculty of the Department of Electrical and Electronics Engineering, University of Istanbul. His general research interests cover wireless communications, statistical signal and array processing, system identification, and estimation theory. His current research activities are focused on signal processing and communication concepts with specific attention to channel estimation and equalization algorithms for space-time coding and multicarrier (orthogonal frequency-division multiplexing) systems. Dr. Çirpan is a member of Sigma Xi. He received the Peskin Award from Stevens Institute of Technology as well as the Prof. Nazım Terzioglu Award from the Research Fund of the University of Istanbul.



Erdal Panayırcı (F'03) received the Diploma Engineering degree in electrical engineering from the Istanbul Technical University, Istanbul, Turkey, in 1964 and the Ph.D. degree in electrical engineering and system science from Michigan State University, East Lansing, in 1970. From 1970 to 2000, he was with the Faculty of Electrical and Electronics Engineering, Istanbul Technical University, where he was a Professor and the Head of the Telecommunications Chair. He has also been a part-time Consultant to several leading companies in telecommunications in Turkey. From 1979 to 1981, he was with the Department of Computer Science, Michigan State University, as a Fulbright-Hays Fellow and a NATO Senior Scientist. Between 1983 and 1986, he served as a NATO Advisory Committee Member for the Special Panel on Sensory Systems for Robotic Control. From August 1990 to December 1991, he was a Visiting Professor at the Center for Communications and Signal Processing, New Jersey Institute of Technology, Newark, and took part in the research project on interference cancellation by array processing. Between 1998 and 2000, he was a Visiting Professor at the Department of Electrical Engineering, Texas A&M University, College Station, and took part in research on developing efficient synchronization algorithms for orthogonal frequency-division multiplexing (OFDM) systems. In 2005 he was a Visiting Professor at the Department of Electrical and Electronics Engineering, Bilkent University, Ankara, Turkey. He is currently a Professor and Department Head at the Electronics Engineering Department, Kadir Has University, Istanbul, Turkey. He is engaged in research and teaching in digital communications and wireless systems, equalization and channel estimation in multicarrier (OFDM) communication systems, and efficient modulation and coding techniques (TCM and turbo coding). Prof. Panayırcı is a member of Sigma Xi. He was the Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS in the fields of synchronization and equalizations from 1995 to 1999. He is currently the Head of the Turkish Scientific Commission on Signals, Systems, and Communications of the International Union of Radio Science.

EUROPEAN TRANSACTIONS ON TELECOMMUNICATIONS
Eur. Trans. Telecomms. 2006; **17**:685–693
Published online in Wiley InterScience
(www.interscience.wiley.com). DOI: 10.1002/ett.1143

Special Issue on MC-SS

Blind-Phase Noise Estimation in OFDM Systems by Sequential Monte Carlo Method

Erdal Panayırcı[1], Hakan A. Çırpan[2,*], Marc Moeneclaey[3] and Nele Noels[3]

[1]*Department of Electronics Engineering, Kadir Has University, 34230 Cibali, Istanbul, Turkey*
[2]*Department of Electrical Engineering, Istanbul University, Avcilar 34320, Istanbul, Turkey*
[3]*TELIN/DIGCOM Department, Ghent University, B9000 Gent, Belgium*

SUMMARY

One of the main drawbacks of orthogonal frequency division multiplexing (OFDM) systems is the phase noise (PN) caused by the oscillator instabilities. Unfortunately, due to the PN, the most valuable feature namely orthogonality between the carriers, is destroyed resulting in a significant degradation in the performance of OFDM systems. In this paper, based on a sequential Monte Carlo method (particle filtering), a computationally efficient algorithm is presented for estimating the residual phase noise, blindly, generated at the output of the phase tracking loop employed in OFDM systems. The basic idea is to treat the transmitted symbols as 'missing data' and draw samples sequentially of them based on the observed signal samples up to time *t*. This way, the Bayesian estimates of the phase noise is obtained through these samples, sequentially drawn, together with their importance weights. The proposed receiver structure is seen to be ideally suited for highspeed parallel implementation using VLSI technology. The performance of the proposed approaches are studied in terms of average mean square error. Through experimental results, the effects of an initialisation on the tracking performance are also explored. Copyright © 2006 AEIT.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has lately been extensively considered for use in wireless/mobile communications systems, mainly in WLAN standards such as the IEEE802.11a and its European equivalent ETSI HIPERLAN/2 due to its robustness to multipath, its high-data rates and its efficient use of bandwidth [1, 2]. The attractiveness of OFDM systems stems from the fact that these systems transform the frequency-selective channel into a set of parallel flat-fading channels. The information is thus split into different streams sent over different sub-carriers thereby reducing intersymbol interference (ISI) and allowing for high-data rates without adding complexity to the equalizers [3, 4].

One of the main drawbacks of OFDM systems is the phase noise (PN) caused by the oscillator instabilities [5]. Unfortunately, due to the PN, the most valuable feature namely orthogonality between the carriers, is destroyed resulting in a significant degradation in the performance of OFDM systems [5]. Random PN causes two effects on OFDM systems, rotating each symbol by a random phase that is referred to as the common phase error (CPE) and producing intercarrier interference (ICI) term that adds to the channel noise due to the loss of orthogonality between subcarriers [6]. Several methods have been

* Correspondence to: Hakan A. Çırpan, Department of Electrical Engineering, Istanbul University, Avcilar 34320, Istanbul, Turkey.
E-mail: hcirpan@istanbul.edu.tr

Contract/grant sponsor: The Scientific & Technological Research Council of Turkey (TÜBİTAK); contract/grant number: 104E166.
Contract/grant sponsor: University of Istanbul; contract/grant number: UDP-732/05052006.

Accepted 14 June 2006

proposed in the literature for the estimation and compensation of the PN in OFDM systems [7, 8]. Most of the approaches however only addresses the estimation of the CPE by assuming ICI terms are approximated by a Gaussian distribution and these techniques are implemented after the DFT process at the receiver [8]. The main drawback of these approaches is the data-dependent ICI which introduces an additional random noise on top of the additive Gaussian channel noise causes a significant degradation in the CPE estimator performance. In contrast to these approaches, we try to solve PN estimation problem in the time domain before the DFT process at the OFDM receiver. As it will be seen shortly this approach will not be faced with ICI effect during the estimation procedure resulting in more accurate random phase estimation.

A considerable amount of research has been carried out for online estimation of the timevarying as well as the fixed phase offset at digital receivers in the presence of data [6]. Estimating the phase offset by maximum likelihood (ML) technique does not seem to be analytically tractable. Even if the likelihood function can be evaluated offline; however, it is invariably a nonlinear function of the parameter to be estimated, which makes the maximisation step computationally infeasible. Phase synchronisation is typically implemented by a decision directed (or data-aided) or nondecision directed (or nondata aided). Decision-directed schemes depend on availability of reliably detected symbol for obtaining the phase estimate, and therefore, they usually require transmission of pilot or training data. However, in applications where bandwidth is the most precious resource, training data can significantly reduce the overall system capacity. Thus blind or nondata-aided techniques become an attractive alternative [9, 10]. Unlike data-aided techniques, nondata-aided methods do not require knowledge of the transmitted data, and instead, they exploit statistics of digital-transmitted signal. ML estimation techniques can also be used in nondecision-directed methods if the symbols transmitted are treated as random variables with known statistics so that the likelihood function can be averaged over the data sequence received. Unfortunately, except for few simple cases, this averaging process is mathematically impracticable and it can be obtained only by some approximations which are valid only either at high- or low-SNR values [11].

On the other hand, in order to provide an implementable solution, recently there have been a substantial amount of work on iterative formulation of the parameter estimation problem based on the expectation-maximization (EM) technique [12]. It is known that the EM algorithm derives iteratively and converges to the true ML estimation of these unknown parameters. The main drawbacks of this approach are that the algorithm is sensitive to the initial starting values chosen for the parameters, it does not necessarily converge to the global extremum and the convergence can be slow. Furthermore, in situation where the posterior distribution must be constantly updated with arrival of the new data with missing parts, EM algorithm can be highly inefficient, because the whole iteration process must be redone with additional data. The sequential Monte Carlo (SMC) methodology, also called *particle filtering*, [14] that has emerged in the field of statistics and engineering has shown great promise to solve such problems. This technique can approximate the optimal solution directly without compromising the system model. Additionally, the decision made at time t does not depend on any decisions made previously, and thus, no error is propagated in their implementation. More importantly, the SMC yields a fully blind algorithm and allows for both Gaussian and non-Gaussian ambient noise as well as high-speed parallel implementations.

The main objective of this paper is to solve the PN estimation problem by means of the SMC technique. The basic idea is to treat the transmitted data as 'missing data' and to sequentially draw samples of them based on the current observation and computing appropriate importance sampling weights. Based on sequentially drawn samples, the Kalman filter is used to estimate the unknown phase from an extended Kalman state-space model of the underlying system. Furthermore, the tracking of the timevarying PN and the data detection are naturally integrated. The algorithm is self-adaptive and no training/pilot symbols or decision feedback are needed [13].

In the following, the system and the main model for the PN are described in Section 2, the solution of the blind-phase noise estimation problem by means of the SMC method is presented in Section 3. Resampling method is detailed in Section 4. Finally, the simulation results and the main conclusions of the paper are given in Sections 5 and 6 respectively.

2. SYSTEM DESCRIPTION

We consider an OFDM system with N subcarriers operating over a frequency-selective Rayleigh fading channel. In this paper, we assume that the multipath intensity profile has exponential distribution and the delay spread T_d is less than or equal to the guard interval L. With the aid of the discrete time channel model [3], the output of the frequency-selective channel can be written as

$$y_t = \sum_{k=0}^{L} h_k s_{t-k} \tag{1}$$

where the $h_k, k = 0, 1, \ldots, L$ denotes the kth tap gain and we assume to have ideal knowledge of these channel tap gains. Moreover, $s_t = \sum_{n=0}^{N-1} d_n e^{-j\frac{2\pi t n}{N}}$ where $\{d_n\}$ denotes the independent data symbols transmitted on the nth subcarrier of an OFDM symbol. Hence, s_t is a linear combination of independent, identically distributed random variables. If the number of subcarriers is sufficiently large, s_t can be modelled as a complex Gaussian process whose real and imaginary parts are independent. It has zero mean and variance $\sigma_s^2 = E\{|s_t|^2\} = E_s$, where E_s is the symbol energy per subcarrier.

Also, assuming perfect frequency and timing synchronisation, the received signal, r_t, corrupted by the additive Gaussian noise n_t and distorted by the timevarying phase noise θ_t can be expressed as

$$r_t = y_t e^{j\theta_t} + n_t, \quad t = 1, \ldots, T_0 \tag{2}$$

where n_t is the complex envelope of the additive white Gaussian noise with variance $\sigma_n^2 = E\{|n_t(k)|^2\}$. θ_t is the sample of the PN process at the output of the free-running local oscillator representing the phase noise. It is shown in Reference [16] that in the case of free-running oscillators, PN can be modelled as a Wiener process defined as

$$\theta_t = \theta_{t-1} + u_t$$

$$\theta_0 \sim \text{uniform}(-\pi, \pi) \tag{3}$$

where u_t is zero-mean Gaussian random variable with variance $\sigma_u^2 = 2\pi BT_s$ where T_s is the sampling period of the OFDM receiver A/D converter and BT refers to the PN rate, where $T = T_s(N + L)$. It is assumed that u_t and n_t are independent of each other. Defining the vectors $\boldsymbol{R}_t = [r_0, r_1, \ldots, r_t]^{\mathrm{T}}$, $\boldsymbol{S}_t = [s_0, s_1, \ldots, s_t]^{\mathrm{T}}$, $\boldsymbol{s}_t = [s_t, s_{t-1}, \ldots, s_{t-L}]^{\mathrm{T}}$, and $\boldsymbol{h}_t = [h_0, h_1, \ldots, h_L]^{\mathrm{T}}$ combining Equations (2) and (3), and taking into account the structure of s_t, we obtain the following dynamic state-space representation of the communication system,

$$\theta_t = \theta_{t-1} + u_t$$

$$\boldsymbol{s}_t = \mathbf{F}\boldsymbol{s}_{t-1} + \boldsymbol{v}_t$$

$$r_t = \boldsymbol{h}_t^T \boldsymbol{s}_t e^{j\theta_t} + n_t \tag{4}$$

where

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 \\ 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & . \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix} \tag{5}$$

is a $(L+1) \times (L+1)$ shifting matrix, and $\boldsymbol{v}_t = [s_t, 0, \ldots, 0]$ is a $(L+1) \times 1$ perturbation vector that contains the new symbol s_t.

3. SMC FOR BLIND-PHASE NOISE ESTIMATION

Since we are interested in estimating the phase noise θ_t blindly at time t based on the observation $\boldsymbol{R}_t$, the Bayes solution requires the posterior distribution

$$p(\theta_t|\boldsymbol{R}_t) = \int p(\theta_t|\boldsymbol{R}_t, \boldsymbol{S}_t) p(\boldsymbol{S}_t|\boldsymbol{R}_t) \mathrm{d}\boldsymbol{S}_t \tag{6}$$

Note that with a given $\boldsymbol{S}_t$, the nonlinear (Kalman filter) model (4) can be converted into a linear model by linearising the observation Equation (2) as follows [15]:

$$\theta_t = \theta_{t-1} + u_t$$

$$r_t = \boldsymbol{h}_t^{\mathrm{T}} \boldsymbol{s}_t (V_t \theta_t + Q_t) + n_t \tag{7}$$

where

$$V_t = j e^{j\hat{\theta}_{t|t-1}}$$

$$Q_t = (1 - j\hat{\theta}_{t|t-1}) e^{j\hat{\theta}_{t|t-1}} \tag{8}$$

$\hat{\theta}_{t|t-1}$ denotes the estimator of θ_t based on the observations $\boldsymbol{R}_{t-1} = (r_0, r_1, \ldots, r_{t-1})$. Then the state-space model (4) becomes a linear Gaussian system. Hence, $p(\theta_t|\boldsymbol{S}_t, \boldsymbol{R}_t) \sim N(\mu_{\theta_t}(\boldsymbol{S}_t), \sigma^2_{\theta_t}(\boldsymbol{S}_t))$, where the mean $\mu_{\theta_t}(\boldsymbol{S}_t)$ and the variance $\sigma^2_{\theta_t}(\boldsymbol{S}_t)$ can be obtained as follows. Denoting $\mu_{\theta_t}(\boldsymbol{S}_t) \triangleq {}_{t|t}\theta_t$, and $\sigma^2_{\theta_t}(\boldsymbol{S}_t) \triangleq M_{t|t}$.

$\theta_{t|t}$ and $M_{t|t}$ can be calculated recursively by using the Extended Kalman Technique [[15], pages 449–452] with the given $\boldsymbol{S}_t$ as:

$$\hat{\theta}_{t|t} = \hat{\theta}_{t|t-1} + K_t(r_t - \boldsymbol{h}^{\mathrm{T}} \boldsymbol{s}_t e^{j\hat{\theta}_{t|t-1}}) \tag{9}$$

$$M_{t|t} = (1 - K_t \boldsymbol{h}_t^{\mathrm{T}} \boldsymbol{s}_t V_t) M_{t|t-1}$$

where

$$K_t = \frac{M_{t|t-1}(\boldsymbol{h}_t^{\mathrm{T}} \boldsymbol{s}_t V_t)^*}{|\boldsymbol{h}_t^{\mathrm{T}} \boldsymbol{s}_t|^2 M_{t|t-1} + \sigma_n^2}$$

DOI: 10.1002/ett

$$\hat{\theta}_{t|t-1} = \hat{\theta}_{t-1|t-1}$$

$$M_{t|t-1} = M_{t-1|t-1} + \sigma_u^2 \qquad (10)$$

We can now make timely estimates of θ_t based on the currently available observation $\boldsymbol{R}_t$, up to time t, blindly, as follows. With the Bayes theorem, we realise that the optimal solution to this problem is

$$\hat{\theta}_t = E\{\theta_t|\boldsymbol{R}_t\}$$
$$= \int_{S_t} \underbrace{\left[\int_{\theta_t} \theta_t p(\theta_t|\boldsymbol{S}_t, \boldsymbol{R}_t)\mathrm{d}\theta_t\right]}_{\mu\theta_t(S_t)} p(\boldsymbol{S}_t|\boldsymbol{R}_t).\,\mathrm{d}\boldsymbol{S}_t \qquad (11)$$

In most cases, an exact evaluation of the expectation (11) is analytically intractable. Sequential Monte Carlo technique can provide us an alternative way for the required computation. Specifically, following the notation adopted in Reference [4], if we can draw m independent random samples $\{\boldsymbol{S}_t^{(j)}\}_{j=1}^{m}$ from the distribution $p(\boldsymbol{S}_t|\boldsymbol{R}_t)$, then we can approximate the quantity of interest $E\{\theta_t|\boldsymbol{R}_t\}$ in Equation (11) by

$$E\{\theta_t|\boldsymbol{R}_t\} \cong \frac{1}{m}\sum_{j=1}^{m} \mu_{\theta_t}(\boldsymbol{S}_t^{(j)}) \qquad (12)$$

But, usually drawing samples from $p(\boldsymbol{S}_t|\boldsymbol{R}_t)$ directly is difficult. Instead, sample generation from some *trial distribution* may be easier. In this case, the idea of *importance sampling* can be used [4]. By associating the weight $w_t^{(j)} = \frac{p(\boldsymbol{S}_t^{(j)}|\boldsymbol{R}_t)}{q(\boldsymbol{S}_t^{(j)}|\boldsymbol{R}_t)}$ to the samples, the quantity of interest, $E\{\theta_t|\boldsymbol{S}_t\}$ can be approximated as follows:

$$E\{\theta_t|\boldsymbol{R}_t\} \cong \frac{1}{W_t}\sum_{j=1}^{m} \mu_{\theta_t}(\boldsymbol{S}_t^{(j)})w_t^{(j)} \qquad (13)$$

with $W_t = \sum w_t^{(j)}$. The pair $(\boldsymbol{S}_t^{(j)}, w_t^{(j)}), j = 1, 2, \ldots, m$ is called a properly weighted sample with respect to distribution $p(\boldsymbol{S}_t|\boldsymbol{R}_t)$.

Specifically, it was shown in Reference [4] that a suitable choice for the trial distribution is of the form $q(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)}) = p(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)})$. For this trial sampling distribution, it is shown in Reference [4] that the importance weight is updated according to

$$w_t^{(j)} = w_{t-1}^{(j)} p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}), \quad t = 1, 2, \ldots \qquad (14)$$

The optimal trial distribution in $p(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)})$ can be computed as follows:

$$p(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)}) = p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t)P(s_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) \quad (15)$$

Furthermore, it can be shown from the state and observation equations in (4) that $p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t) \sim \mathcal{N}(\mu_{r_t}^{(j)}, \sigma_{r_t}^{2(j)})$ with mean and variance given by

$$\begin{aligned}\mu_{r_t}^{(j)} &= E\{r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t\}\\ &= \boldsymbol{h}_t^{\mathrm{T}} s_t(V_t\hat{\theta}_{t|t-1}^{(j)} + Q_t)\end{aligned} \qquad (16)$$

$$\begin{aligned}\sigma_{r_t}^{2(j)} &= \mathrm{Var}\{r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t\}\\ &= |\boldsymbol{h}_t^T s_t|^2 M_{t|t-1}^{(j)} + \sigma_n^2\end{aligned} \qquad (17)$$

where the quantities $\hat{\theta}_{t|t-1}^{(j)}$ and $M_{t|t-1}^{(j)}$ in Equations (16) and (17) respectively can be computed recursively for the Extended Kalman equations given in Equations (9) and (10). Also since s_t is independent of $\boldsymbol{S}_{t-1}$ and $\boldsymbol{R}_{t-1}$, the second term in Equation (15), it can be written as $p(s_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) = p(s_t)$ where it was pointed out earlier that $p(s_t) \sim \mathcal{N}(0, \sigma_s^2)$.

Note that dependency of the $\sigma_{r_t}^{2(j)}$ in (16) to s_t precludes combining the product of Gaussian densities in Equation (15) into a single Gaussian, hence obtaining a tractable sampling distribution. This problem can be circumvented by approximating the $\sigma_{r_t}^{2(j)}$ as follows. From Equation (4), we can use the approximation $s_t \approx \mathbf{F}s_{t-1}$ in Equation (16) to obtain

$$\sigma_{r_t}^{2(j)} \cong |\boldsymbol{h}_t^T \mathbf{F} s_{t-1}^{(j)}|^2 M_{t|t-1}^{(j)} + \sigma_n^2 \qquad (18)$$

Similarly using Equations (11) in (16), the mean $\mu_{r_t}^{(j)}$ can be expressed as

$$\mu_{r_t}^{(j)} = (\boldsymbol{h}_t^{\mathrm{T}}\mathbf{F}s_{t-1}^{(j)} + h_0 s_t)G_t^{(j)} \quad \text{where} \quad G_t^{(j)} \triangleq V_t\hat{\theta}_{t|t-1}^{(j)} + Q_t \qquad (19)$$

Then, the true trial sampling distribution $p(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)})$ in Equation (15) can be expressed as follows:

$$p(s_t|\boldsymbol{R}_t, \boldsymbol{S}_{t-1}^{(j)}) \sim \mathcal{N}(\mu_{s_t}^{(j)}, \sigma_{s_t}^{2(j)}) \qquad (20)$$

where

$$\mu_{s_t}^{(j)} = \frac{(r_t - \boldsymbol{h}_t^{\mathrm{T}}\mathbf{F}s_{t-1}^{(j)}G_t^{(j)})}{h_0 G_t^{(j)}}\left(\frac{\sigma_{r_t}^{2(j)}}{|h_0 G_t^{(j)}|^2\sigma_s^2} + 1\right)^{-1}$$

$$\sigma_{s_t}^{2(j)} = \frac{\sigma_{r_t}^{2(j)}\sigma_s^2}{\sigma_{r_t}^{2(j)} + |h_0 G_t^{(j)}|^2\sigma_s^2}$$

and $\sigma_{r_t}^{2(j)}$ is defined as Equation (16).

In order to obtain the recursion for the weighting factor $w_t^{(j)}$, the predictive distribution $p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)})$ in Equation (15) should be evaluated. It is given by

$$p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) = \int_{s_t} p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t) P(s_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) \mathrm{d}s_t$$
$$= \int_{s_t} p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t) p(s_t) \mathrm{d}s_t \tag{21}$$

where Equation (21) holds because s_t is independent of $\boldsymbol{S}_{t-1}$ and $\boldsymbol{R}_{t-1}$. Since the both terms in the integrand of Equation (21), are Gaussian densities, the product of the Gaussian densities are integrated with respect to s_t is also Gaussian. Therefore the predictive distribution is found to be

$$p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) \sim \mathcal{N}(M_{r_t}^{(j)}, \Sigma_{r_t}^{2(j)}) \tag{22}$$

where

$$M_{r_t}^{(j)} = \boldsymbol{h}^{\mathrm{T}} \mathbf{F} \boldsymbol{s}_{t-1}^{(j)} G_t^{(j)}$$

$$\Sigma_{r_t}^{2(j)} = |h_0 G_t^{(j)}|^2 \sigma_s^2 + |\boldsymbol{h}^{\mathrm{T}} \mathbf{F} \boldsymbol{s}_{t-1}^{(j)}|^2 M_{t|t-1}^{(j)} + \sigma_n^2 \tag{23}$$

We now summarise the SMC blind data phase noise estimation algorithm in Table 1:

The proposed SMC approach perform three basic operations: generation of new particles (sampling from the space of unobserved states), computation of particle weights (probability masses associated with the particles) and resampling (a process of removing particles with small weights and replacing them with particles with large weights). Particle generation and weight computation steps are computationally the most intensive ones. The particle filtering speed can be increased through both algorithmic modifications and architecture development [4]. On the algorithmic level, the main challenges for speed increase include reducing the number of operations and exploiting operational concurrency between the particle

Table1. SMC algorithm for blind-phase noise estimation.
Given $\{h_0, h_1, \ldots, h_L\}$

- *Initialisation*:

— *Initialise the extended Kalman filter*: Choose the initial mean and the variance of the estimated θ_t as

$$\mu_{\theta_0}^{(j)} = \hat{\theta}_{0|0}^{(j)} = 0, \quad \sigma_{\theta_0}^{2(j)} = M_{0|0}^{(j)} = \pi^2/12, \quad j = 1, 2, \ldots, m. \tag{24}$$

— *Initialise the importance weights*: All importance weights are initialised as $w_0^{(j)} = 1, j = 1, 2, \ldots, m$.

For $j = 1, m$

For $t = 1, T_0$

- *Compute* $\hat{\theta}_{t|t-1}, M_{t|t-1}^{(j)}$ *from Equation (8).*
- *Compute* $\mu_{r_t}^{(j)}, \sigma_{r_t}^{2(j)}$ *from Equations (16).*
- *Compute sampling distribution mean/variance* $\mu_{s_t}^{(j)}, \sigma_{s_t}^{2(j)}$ *from the Equation (20).*
- *Sample* $s_t^{(j)} \sim N(\mu_{s_t}^{(j)}, \sigma_{s_t}^{(j)})$ *and Append* $s_t^{(j)}$ to $\boldsymbol{S}_{t-1}^{(j)}$ *to obtain* $\boldsymbol{S}_t^{(j)} = (s_t^{(j)}, \boldsymbol{S}_{t-1}^{(j)})$.
- *Compute the importance weights*:

$$w_t^{(j)} = w_{t-1}^{(j)} p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}),$$

where $p(r_t|\boldsymbol{R}_{t-1}, \boldsymbol{S}_{t-1}^{(j)})$ *is computed from Equation (22).*

- *Update the a posteriori mean and variance of the phase noise according to Kalman equations (7–8)*
If the samples drawn up to time t is S_t, *set*

$$\mu_{\theta_t}(\boldsymbol{S}_t^{(j)}) \triangleq \mu_{\theta_t}^{(j)} = \hat{\theta}_{t|t}^{(j)}$$

$$\sigma_{\theta_t}^{2(j)}(\boldsymbol{S}_t^{(j)}) \triangleq \sigma_{\theta_t}^{2(j)} = M_{t|t}^{(j)} \; j = 1, 2, \ldots, m.$$

- *Do the re-sampling as described in Equation [4].*

next j

- *Estimate phase noise* $\hat{\theta}_t = \frac{1}{m} \sum_{j=1}^{m} \mu_{\theta_t}(\boldsymbol{S}_t^{(j)})$

next t

DOI:10.1002/ett

generation and weight computation steps. Moreover, a parallel implementation with multiple processing elements can be employed to increase speed further [4].

4. RESAMPLING METHOD

A major problem in the practical implementation of the SMC method described so far is that after a few iteration most of the importance weights have negligible values that is $\tilde{w}_t^{(j)} \approx 0$. A relatively small weight implies that the sample is drawn far from the main body of the posterior distribution and has a small contribution in the final estimation. Such sample is said to be ineffective. The SMC algorithm becomes ineffective if there are too many ineffective samples. The common solution to this problem is *resampling*. Resampling is an algorithmic step that stochastically eliminates those samples with small weights. Basically, the resampling method takes the samples, to be generated sequentially $\Xi_t = \{S_t^{(j)}, \mu_{\theta_t}^{(j)}, \sigma_{\theta_t}^{2(j)}\}_{j=1}^{m}$ with corresponding weights $\{w_t^{(j)}\}_{j=1}^{m}$ as an input and generates a new set of samples $\tilde{\Xi}_t = \{\tilde{S}_t^{(j)}, \tilde{\mu}_{\theta_t}^{(j)}, \tilde{\sigma}_{\theta_t}^{2(j)}\}_{j=1}^{m}$ with equal weights, that is $\{w_t^{(j)} = 1/m\}_{j=1}^{m}$, assuming they are normalised to $\sum_{j=1}^{m} w_t^{(j)} = 1$. A simple procedure to achieve this goal is, for each $j = 1, 2, \ldots, m$, to choose $(\tilde{S}_t^{(j)}, \tilde{\mu}_{\theta_t}^{(j)}, \tilde{\sigma}_{\theta_t}^{2(j)}) = (S_t^{(i)}, \mu_{\theta_t}^{(i)}, \sigma_{\theta_t}^{2(i)})$ with probability $w_t^{(i)}$.

In this paper, a resampling technique suggested by Reference [13] is employed. This technique forms a new set of weighted samples $\tilde{\Xi}_t = \{\tilde{S}_t^{(j)}, \tilde{\mu}_{\theta_t}^{(j)}, \tilde{\sigma}_{\theta_t}^{2(j)}\}_{j=1}^{m}$ according to the following algorithm. (assume that $\sum_{j=1}^{m} w_t^j = m$)

(1) For $j = 1, 2, \ldots, m$, retain $\ell_j = w_t^j$ copies of the samples $(S_t^{(j)}, \mu_{\theta_t}^{(i)}, \sigma_{\theta_t}^{2(i)})$. Denote $L_r = m - \sum_{j=1}^{m} \ell_j$.
(2) Obtain L_r i.i.d. draws from the original sample set $\{(S_t^{(j)}, \mu_{\theta_t}^{(i)}, \sigma_{\theta_t}^{2(i)})\}_{j=1}^{m}$, with probabilities proportional to $(w_t^j - \ell_j), j = 1, 2, \ldots, m$.
(3) Assign equal weights, that is set $w_t^j = 1$, for each new sample.

It is shown in Reference [13] that the samples drawn by the above procedure are properly weighted with respect to $p(S_t|Y_t)$, provided that m is sufficiently large. Note that resampling at every time step is not needed in general. In one way the resampling can be done every k_0 recursions where k_0 is a prefixed resampling interval. On the other hand, the resampling can be carried out whenever the effective sample size, approximated as

$$\hat{N}_{\text{eff}} = \frac{1}{\sum_{j=1}^{m} (w_t^j)^2} \leq m \tag{25}$$

goes below a certain threshold, typically a fraction of m. Intuitively, $\hat{N}_{\text{eff}}$ reflects the equivalent size of i.i.d samples from the true posterior densities of interest for the set of m weighted ones. It is suggested in Reference [4] that resampling should be performed when $\hat{N}_{\text{eff}} < m/10$. Alternatively, one can conduct the first approach to conduct resampling at every fixed-length time interval say every five steps.

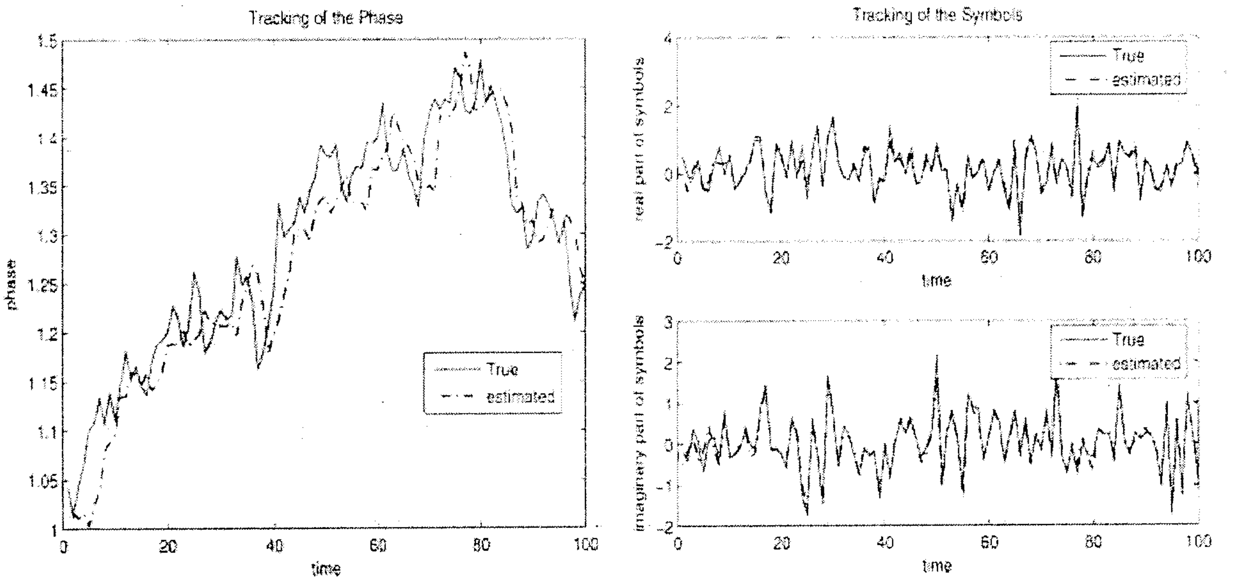


Figure 1. Tracking performance.

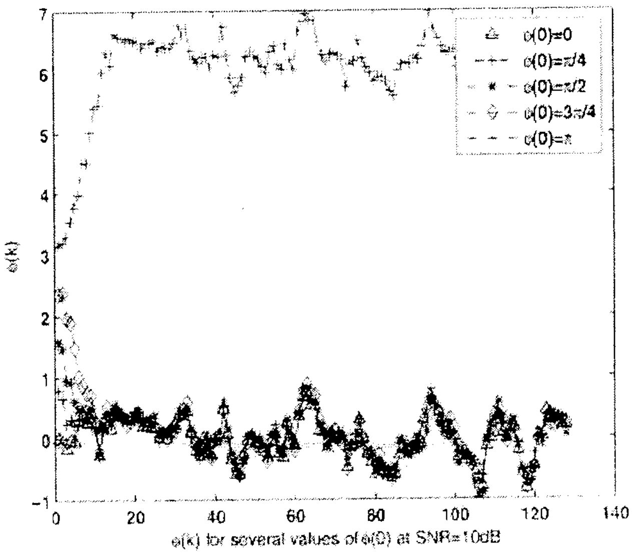


Figure 2. Tracking performance for different initialisations at SNR = 10 dB.

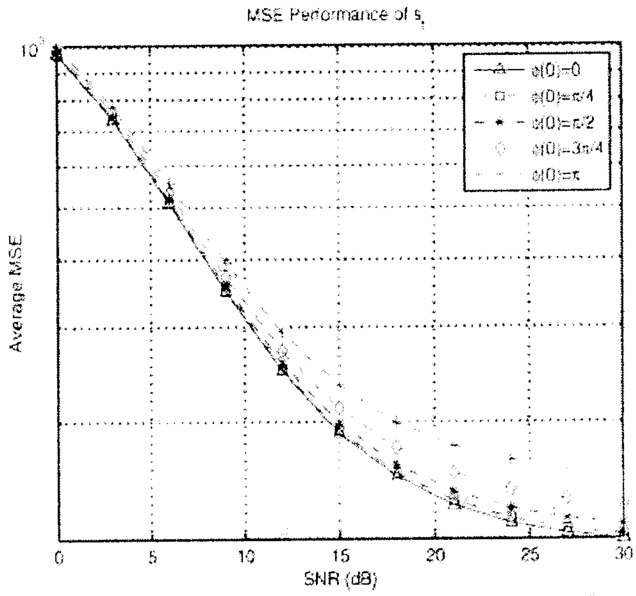


Figure 4. Average MSE performance of s_t for different initialization.

5. SIMULATION RESULTS

In this section, we provide some computer simulation examples to demonstrate the performance of the proposed SMC approach for blind-phase noise estimation and data detection in OFDM systems. The phase process is modelled by AR process driven by a white Gaussian noise with $\sigma_u^2 = 0.1$. s_t is modelled as a complex Gaussian process

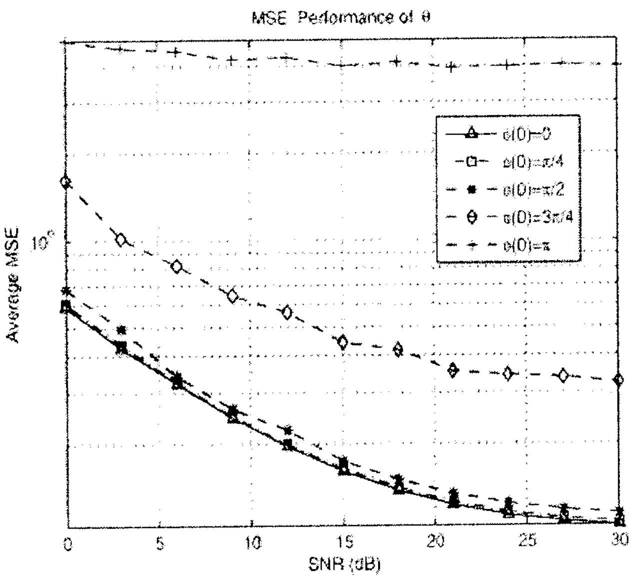


Figure 3. Average MSE performance of phase noise for different initialisations.

which has zero mean and variance $\sigma_s^2 = 1$. The impulse response of the channel has five uniformly distributed taps with spacing equal to the sampling period and with exponentially decaying profile.

In order to demonstrate the performance of the adaptive SMC approach, we first present the tracking performance for both phase and symbols at SNR = 20 dB in Figure 1. It is shown through simulations that the performance of the proposed SMC algorithm can track the phase as well as transmitted symbols close to the true values.

We then consider the performance (in terms of the phase error $\phi(k) = (\theta_t - \hat{\theta}_t)$ for 1000 Monte Carlo trials for different initial phase errors $\phi(k) = 0, \pi/4, \pi/2, 3\pi/4, \pi$. The phase error for several values of $\phi(0)$ for a wide range of SNR values. The results are shown in Figure 2.

The performance of the proposed algorithm is further exploited by the evaluation of average MSE over observed subcarriers for different SNRs and different initial phase errors. The average MSE performance of this adaptive approach for both phase and symbols are plotted in Figures 3 and 4.

Our simulations indicate that as the initial phase error $\phi(0)$ approaches π, the probability that the phase error converges to the dual equilibrium point becomes very high.

Moreover, the relevant simulation results show that the proposed scheme enables to perform blind reliable phase tracking with relatively good initialisation.

6. CONCLUSIONS

We have developed a new adaptive Bayesian approach for blind-phase noise estimation and data detection for OFDM systems based on sequential Monte Carlo methodology. The optimal solutions to joint symbol detection and phase noise estimation problem is computationally prohibitive to implement by conventional methods. Thus the proposed sequential approach offers an novel and powerful approach to tackling this problem at a reasonable computational cost. The performance merits of our blind-phase noise estimation algorithm is confirmed by corroborating simulations. Sensitivity to initialisation of the proposed algorithm are investigated for OFDM systems. It is observed from simulations that as the initial phase error $\phi(0)$ approaches π, the probability that the phase error converges to the dual equilibrium point becomes very high.

ACKNOWLEDGEMENTS

This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 6th Framework Programme and by the The Scientific & Technological Research Council of Turkey (TÜBİTAK), Project No: 104E166. This work was also supported in part by the Research Fund of the University of Istanbul. Project number: UDP-732/05052006.

REFERENCES

1. Van Nee R, Prasad R. *OFDM Wireless Multimedia Communications*. Artech House Publishers: London 2000.
2. Sari H, Karam G, Jeanclaude I. Transmission techniques for digital terrestrial TV broadcasting. *IEEE Communication Magazine* 1995; **33**:100–109.
3. van de Beek JJ, Edfors O, Sandell M, Wilson SK, Börjesson PO. On channel estimation in OFDM systems. In *Proceedings of 45th IEEE Vehicular Technology Conference (VTC'96)*, Atlanta, GA, April 1996.
4. Yang Z, Wang X. A sequential Monte Carlo blind receiver for OFDM systems in frequencyselective fading channels. *IEEE Transactions on Signal Processing* 2003; **50**(2):271–280.
5. Pollet MVBT, Moeneclaey M. BER sensitivity of OFDM systems to carrier frequency offset and wiener phase noise. In *IEICE Transactions on Communications* 1995; **43**:192–193.
6. Meyr H, Moeneclaey M, Fechtel SA. *Digital Communications Receivers*. John Wiley: New York, 1998.
7. Wu S, Bar-Ness Y. A phase noise suppression algorithm for OFDM-based WLANs. *IEEE Communications Letters* 1998; **44**(3):535–537.
8. Petrovic D, Rave W, Fetweis G. Phase noise suppression in OFDM including intercarrier interference. In *Proceedings of International OFDM Workshop* Hamburg, Germany, 2003; 219–224.
9. Huq AT, Panayirci E, Georghiades CN. ML NDA carrier phase recovery for OFDM systems with M-PSK signaling. In *Proceedings of IEEE International Conference on Communications (ICC'99)*, June 6–10, 1999,Vancouver, Canada.
10. Panayırcı E, Georghiades CN. Non-data-aided ML carrier frequency and phase synchronization in OFDM systems. *European Transactions on Telecommunications (ETT)* 2000;**12**(2):83–94.
11. Mengali U, D'Andrea AL. *Synchronization Techniques for Digital Receivers*. Plenum Press: New York, 1997.
12. Dempster AP, Laird NM, Rubin DB. Maximum likelihood from incomplete data via the EM algorithm. *Annual Royal Statistical Society*, 1–38, December 1977.
13. Liu SJ, Chen R. Sequential Monte Carlo method for dynamical systems. *Journal of American Statistical Association* 1998; **93**:1031–1044.
14. Pitt M, Shephard N. Filtering via simulation: auxiliry particle filter. *Journal of American Statistical Society B* 2001; **94**(446):590–599.
15. Kay SM. Fundamentals of statistical signal processing: estimation theory. *Prentice Hall* NJ, USA 1993.
16. Demir A, Mehrotra A, Roychowdhury J. Phase noise oscillators: a unifying theory and numerical methods for characterization. *IEEE Transaction Circuits System I* 2000; **47**(5):655-674.

AUTHORS' BIOGRAPHIES

Erdal Panayırcı received the Diploma Engineering degree in Electrical Engineering from Istanbul Technical University, Istanbul, Turkey in 1964 and the Ph.D. degree in Electrical Engineering and system science from Michigan State University, East Lansing, Michigan, U.S.A., in 1970. From 1970 to 2000 he was with the Faculty of Electrical and Electronics Engineering at the Istanbul Technical University, where he was a Professor and Head of the Telecommunications Chair. Currently, he is a Professor at the Department of Electronics Engineering, Kadir Has University, Istanbul, Turkey. He is engaged in research and teaching in digital communications and wireless systems, equalisation and channel estimation in multicarrier(OFDM) communication systems, and efficient modulation and coding techniques (TCM and turbo coding). He has also been, a part-time consultant for the several leading companies in telecommunications in Turkey. He spent two years (1979–1981) with the Department of Computer Science, Michigan State University, as a Fulbright–Hays Fellow and a NATO Senior Scientist. During 1983–1986 he served as a NATO Advisory Committee Member for the Special Panel on Sensory Systems for Robotic Control. From August 1990 to December 1991, he was with the Center for Communications and Signal Processing, New Jersey Institute of Technology, as a Visiting Professor, and took part in the research project on Interference Cancellation by Array Processing. During 1998–2000, he was a Visiting Professor at the Department of Electrical Engineering, Texas A&M University and took part in research on developing efficient synchronisation algorithms for OFDM systems. During 1995–1999, he was an Editor for *IEEE Transactions on Communications* in the fields of synchronisation and equalisation.

6. CONCLUSIONS

We have developed a new adaptive Bayesian approach for blind-phase noise estimation and data detection for OFDM systems based on sequential Monte Carlo methodology. The optimal solutions to joint symbol detection and phase noise estimation problem is computationally prohibitive to implement by conventional methods. Thus the proposed sequential approach offers an novel and powerful approach to tackling this problem at a reasonable computational cost. The performance merits of our blind-phase noise estimation algorithm is confirmed by corroborating simulations. Sensitivity to initialisation of the proposed algorithm are investigated for OFDM systems. It is observed from simulations that as the initial phase error $\phi(0)$ approaches π, the probability that the phase error converges to the dual equilibrium point becomes very high.

ACKNOWLEDGEMENTS

This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 6th Framework Programme and by the The Scientific & Technological Research Council of Turkey (TÜBİTAK), Project No: 104E166. This work was also supported in part by the Research Fund of the University of Istanbul. Project number: UDP-732/05052006.

REFERENCES

1. Van Nee R, Prasad R. *OFDM Wireless Multimedia Communications*. Artech House Publishers: London 2000.
2. Sari H, Karam G, Jeanclaude I. Transmission techniques for digital terrestrial TV broadcasting. *IEEE Communication Magazine* 1995; **33**:100–109.
3. van de Beek JJ, Edfors O, Sandell M, Wilson SK, Börjesson PO. On channel estimation in OFDM systems. In *Proceedings of 45th IEEE Vehicular Technology Conference (VTC'96)*, Atlanta, GA, April 1996.
4. Yang Z, Wang X. A sequential Monte Carlo blind receiver for OFDM systems in frequencyselective fading channels. *IEEE Transactions on Signal Processing* 2003; **50**(2):271–280.
5. Pollet MVBT, Moeneclaey M. BER sensitivity of OFDM systems to carrier frequency offset and wiener phase noise. In *IEICE Transactions on Communications* 1995; **43**:192–193.
6. Meyr H, Moeneclaey M, Fechtel SA. *Digital Communications Receivers*. John Wiley: New York, 1998.
7. Wu S, Bar-Ness Y. A phase noise suppression algorithm for OFDM-based WLANs. *IEEE Communications Letters* 1998; **44**(3):535–537.
8. Petrovic D, Rave W, Fetweis G. Phase noise suppression in OFDM including intercarrier interference. In *Proceedings of International OFDM Workshop* Hamburg, Germany, 2003; 219–224.
9. Huq AT, Panayirci E, Georghiades CN. ML NDA carrier phase recovery for OFDM systems with M-PSK signaling. In *Proceedings of IEEE International Conference on Communications (ICC'99)*, June 6–10, 1999, Vancouver, Canada.
10. Panayırcı E, Georghiades CN. Non-data-aided ML carrier frequency and phase synchronization in OFDM systems. *European Transactions on Telecommunications (ETT)* 2000; **12**(2):83–94.
11. Mengali U, D'Andrea AL. *Synchronization Techniques for Digital Receivers*. Plenum Press: New York, 1997.
12. Dempster AP, Laird NM, Rubin DB. Maximum likelihood from incomplete data via the EM algorithm. *Annual Royal Statistical Society*, 1–38, December 1977.
13. Liu SJ, Chen R. Sequential Monte Carlo method for dynamical systems. *Journal of American Statistical Association* 1998; **93**:1031–1044.
14. Pitt M, Shephard N. Filtering via simulation: auxiliry particle filter. *Journal of American Statistical Society B* 2001; **94**(446):590–599.
15. Kay SM. Fundamentals of statistical signal processing: estimation theory. *Prentice Hall* NJ, USA 1993.
16. Demir A, Mehrotra A, Roychowdhury J. Phase noise oscillators: a unifying theory and numerical methods for characterization. *IEEE Transaction Circuits System I* 2000; **47**(5):655-674.

AUTHORS' BIOGRAPHIES

Erdal Panayırcı received the Diploma Engineering degree in Electrical Engineering from Istanbul Technical University, Istanbul, Turkey in 1964 and the Ph.D. degree in Electrical Engineering and system science from Michigan State University, East Lansing, Michigan, U.S.A., in 1970. From 1970 to 2000 he was with the Faculty of Electrical and Electronics Engineering at the Istanbul Technical University, where he was a Professor and Head of the Telecommunications Chair. Currently, he is a Professor at the Department of Electronics Engineering, Kadir Has University, Istanbul, Turkey. He is engaged in research and teaching in digital communications and wireless systems, equalisation and channel estimation in multicarrier(OFDM) communication systems, and efficient modulation and coding techniques (TCM and turbo coding). He has also been, a part-time consultant for the several leading companies in telecommunications in Turkey. He spent two years (1979–1981) with the Department of Computer Science, Michigan State University, as a Fulbright–Hays Fellow and a NATO Senior Scientist. During 1983–1986 he served as a NATO Advisory Committee Member for the Special Panel on Sensory Systems for Robotic Control. From August 1990 to December 1991, he was with the Center for Communications and Signal Processing, New Jersey Institute of Technology, as a Visiting Professor, and took part in the research project on Interference Cancellation by Array Processing. During 1998–2000, he was a Visiting Professor at the Department of Electrical Engineering, Texas A&M University and took part in research on developing efficient synchronisation algorithms for OFDM systems. During 1995–1999, he was an Editor for *IEEE Transactions on Communications* in the fields of synchronisation and equalisation.

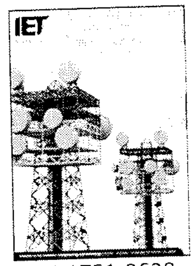
He is currently the Head of the Turkish Scientific Commission on Signals, Systems and Communications of URSI (International Union of Radio Science). He is a fellow of IEEE and member of Sigma Xi.

Nele Noels received the diploma in Electrical Engineering from Ghent University, Gent, Belgium in 2001. She is currently a Ph.D. student at the Department of Telecommunications and Information Processing, Ghent University. Her main research interests are in carrier and symbol synchronisation. She is the author of several papers in international journals and conference proceedings.

Marc Moeneclaey received a Diploma in Electrical Engineering and Ph.D. degree in Electrical Engineering from Ghent University, Gent, Belgium in 1978 and 1983, respectively. He is a Professor at the Department of Telecommunications and Information Processing (TELIN), Gent University. His main research interests are in statistical communication theory, carrier and symbol synchronisation, bandwidth-efficient modulation and coding, spread-spectrum, satellite and mobile communication. He is the author of more than 300 scientific papers in international journals and conference proceedings. Together with Prof. H. Meyr (RWTH Aachen) and Dr. S. Fechtel (Siemens AG), he co-authored the book *Digital Communication Receivers--Synchronisation, Channel Estimation, and Signal Processing* (John Wiley, 1998). He is also a co-recipient of the Mannesmann Innovations Prize 2000. Since 2002, he has been a fellow of IEEE. During the period 1992–1994, he was an Editor of Synchronisation for the *IEEE Transactions on Communications*. He served as a co-guest editor for special issues of the *Wireless Personal Communications Journal* (on Equalisation and Synchronisation in Wireless Communications) and the *IEEE Journal on Selected Areas in Communications* (on Signal Synchronisation in Digital Transmission Systems) in 1998 and 2001, respectively.

Hakan Ali Çırpan received his B.S. degree in 1989 from Uludag University, Bursa, Turkey, M.S. degree in 1992 from the University of Istanbul, Istanbul, Turkey, and Ph.D. degree in 1997 from the Stevens Institute of Technology, Hoboken, NJ, U.S.A., all in Electrical Engineering. From 1995 to 1997, he was a Research Assistant with the Stevens Institute of Technology, working on signal processing algorithms for wireless communication systems. In 1997, he joined the faculty of the Department of Electrical and Electronics Engineering at The University of Istanbul. His general research interests cover wireless communications, statistical signal and array processing, system identification and estimation theory. His current research activities are focused on signal processing and communication concepts with specific attention to channel estimation and equalisation algorithms for space–time coding and multicarrier(OFDM) systems. He received Peskin Award from Stevens Institute of Technology as well as Prof. Nazim Terzioglu award from the Research fund of The University of Istanbul. He is a member of IEEE and member of Sigma Xi.

Published in IET Communications
Received on 25th August 2007
Revised on 16th November 2007
doi: 10.1049/iet-com:20070387



ISSN 1751-8628

An efficient joint channel estimation and decoding algorithm for turbo-coded space–time orthogonal frequency division multiplexing receivers

H. Doğan[1] H.A. Çırpan[1] E. Panayirci[2]
[1]Department of Electrical and Electronics Engineering, Istanbul University, Avcilar, Istanbul 34850, Turkey
[2]Department of Electronics Engineering, Kadir Has University, Cibali, Istanbul 34230, Turkey
E-mail: hdogan@istanbul.edu.tr

Abstract: The challenging problem in the design of digital receivers of today's and future high-speed, high data-rate wireless communication systems is to implement the optimal decoding and channel estimation processes jointly in a computationally feasible way. Without realising such a critical function perfectly at receiver, the whole system will not work properly within the desired performance limits. Unfortunately, direct implementation of such optimal algorithms is not possible mainly due to their mathematically intractable and computationally prohibitive nature. A novel algorithm that reaches the performance of the optimal maximum a posteriori (MAP) algorithm with a feasible computational complexity is proposed. The algorithm makes use of a powerful statistical signal processing tool called the expectation–maximisation (EM) technique. It iteratively executes the MAP joint channel estimation and decoding for space–time block-coded orthogonal frequency division multiplexing systems with turbo channel coding in the presence of unknown wireless dispersive channels. The main novelty of the work comes from the facts that the proposed algorithm estimates the channel in a non-data-aided fashion and therefore except a small number of pilot symbols required for initialisation, no training sequence is necessary. Also the approach employs a convenient representation of the discrete multipath fading channel based on the Karhunen–Loeve (KL) orthogonal expansion and finds MAP estimates of the uncorrelated KL series expansion coefficients. Based on such an expansion, no matrix inversion is required in the proposed MAP estimator. Moreover, optimal rank reduction is achieved by exploiting the optimal truncation property of the KL expansion resulting in a smaller computational load on the iterative estimation approach.

1 Introduction

Future wireless communication systems aim to provide various multimedia services, where high-speed data transmission needs to be reliably transmitted. The attainment to succeed high data rates reliably on wireless channels is severely restricted by effects of channel fading. In such a channel, increasing the quality or reducing the effective error rate is extremely difficult.

Space–time block coding (STBC) was introduced in [1] as an effective transmit diversity technique for mitigating the detrimental effects of channel fading and it was later expanded in [2] for an arbitrary number of antennas. Unfortunately, their practical application can present a real challenge to channel estimation algorithms, especially when the signal suffers from frequency-selective multipath channels. One of the solutions alleviating the frequency selectivity is the use of orthogonal frequency division multiplexing (OFDM) together with transmit diversity which combats long channel impulse response by transmitting parallel symbols over many orthogonal subcarriers yielding a unique reduced complexity physical layer capabilities

STBC is not designed to achieve an additional coding gain. Therefore an outer channel code is applied in addition to transmit diversity to further improve the receiver performance. Recent trends in coding favour parallel and/or serially concatenated coding and probabilistic soft-decision iterative (turbo-style) decoding. Such codes are able to exhibit near-Shannon-limit performance with reasonable complexities in many cases and are of significant interest for communications applications that require moderate error rates. We therefore consider the combination of turbo codes with the transmit diversity OFDM systems. Especially we address the design of iterative channel estimation approach for transmit diversity OFDM systems employing an outer channel code.

There have been several studies to estimate channel for transmit diversity OFDM systems. Most of the early studies focus on channel estimation for transmit diversity OFDM systems subjected on uncoded systems. Since the necessity or desirability of using error-correcting codes to protect data is crucial for wireless communication systems, more recent work have addressed coded transmit diversity OFDM systems. Since direct calculation of estimation is computationally prohibitive, expectation–maximisation (EM) algorithm is a good candidate that can iteratively approximate the maximum-likelihood (ML) estimate with practical complexity. Therefore an iterative procedure based on EM applied to channel estimation problem in the context of STBC [3–5] as well as transmit diversity OFDM systems with or without outer channel coding (e.g. convolutional code or turbo code) [6–9]. In [6], maximum a posteriori (MAP) EM-based iterative receivers for STBC-OFDM systems with turbo code are proposed to detect transmitted symbols, directly, assuming that the fading process remains constant across several OFDM words contained in one STBC code word. In [7], an EM approach, proposed for the general estimation from superimposed signals [10], is applied to the channel estimation for transmit diversity OFDM systems with outer channel code (convolutional code) and compared with SAGE version. In [8], a modified version of [7] is suggested for STBC-OFDM and space–frequency block-coding (SFBC)-OFDM systems. Moreover, Kashima *et al.* [9] proposed two new types of MAP receivers for multiple-input-multiple-output and OFDM systems with a channel coding such as the low-density parity-check code. One proposed receiver employs the EM algorithm so as to improve the performance of the approximated MAP detection.

Assuming that the channel varies between every adjacent OFDM symbols, a non-data-aided EM-MAP channel estimation algorithm was proposed [11] for SFBC-OFDM systems unlike the EM approaches used in [6–8]. This receiver structure was also extended to the turbo/convolutionally coded SFBC-OFDM systems [12] assuming that the complex channel gains between adjacent subcarriers are approximately constant. In [12], it was shown that this approach was more effective for time-varying channels. On the other hand, for highly frequency-selective channels, it is obvious that the bit error rate (BER) performances of the turbo/convolutionally coded SFBC-OFDM systems degrade substantially and cause error floors for high SNR values. Therefore in this paper, we extend the work presented in [12] to the turbo receiver structures of the turbo/convolutionally coded STBC-OFDM systems and propose an EM-MAP channel estimation algorithm that reaches the performance of the optimal MAP algorithm with a feasible computational complexity. The algorithm makes use of the EM technique. It iteratively executes the MAP joint channel estimation and decoding for STBC-OFDM systems with turbo channel coding in the presence of unknown wireless dispersive channels. In [12], binary phase shift keying (BPSK) modulation was considered for the channel model having an exponentially decaying power profile [13]. In the current work, both BPSK and high-level modulation schemes are considered in the presence of COST-207 channel model. Moreover, the effects of the correlation mismatch and increasing frame length of the turbo-coded structure are also investigated and important conclusions are drawn for practical considerations.

The rest of the paper is organised as follows. In Section 2, we introduce the signal model for encoded STBC-OFDM systems and corresponding channel model is established. In Section 3, the proposed channel estimation and iterative equalisation and decoding algorithms are presented. Computer simulation results are given with detailed discussion in Section 4, and finally conclusions are drawn in Section 5.

Notation: Vectors (matrices) are denoted by bold italics lower (upper) case letters; all vectors are column vectors; $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^\dagger$ denote the conjugate, transpose and conjugate transpose, respectively; $\mathbf{I}_L$ denotes the $L \times L$ identity matrix.

2 Received signal model for encoded STBC-OFDM systems

In order to compensate for the reduced data rate of turbo codes, some ST codes having data rates greater than one could be employed. However, it is well known from literature that the Alamouti antenna modulation configuration is the only scheme that retains orthogonality and full rate when the complex-valued data are involved and the low complexity is concerned [1]. As will be seen shortly, orthogonality property is an essential and a required condition for the channel estimation algorithm in our work. Moreover, orthogonality structure of Alamouti allows the decoupling of the channel and reduces the equaliser complexity. Note that the Alamouti's schemes has been adopted in several wireless standards such as WCDMA and CDMA2000. It imposes an orthogonal spatio-temporal structure on the transmitted symbols that guarantees full (i.e. order 2) spatial diversity. In addition to the spatial level, to realise multipath diversity gains over

IET Commun., 2008, Vol. 2, No. 7, pp. 886–894
doi: 10.1049/iet-com:20070387

frequency-selective channels, the Alamouti block-coding scheme is implemented at a block level in frequency domain.

As illustrated in Fig. 1, the binary information bits $\boldsymbol{b}$ of random data are encoded by an outer-channel code, resulting in a coded symbol stream $\{\boldsymbol{C}\}$. The coded symbols are then interleaved by a random permutation resulting in a stream of independent symbols. A code-bit interleaver reduces the probability of error bursts and removes correlation in coded symbol stream. The coded and interleaved symbols form a block of independent symbol stream of length $2N_c$, denoted by $\boldsymbol{X} = [X_0, X_1, \ldots, X_{2N_c-1}]^{\mathrm{T}}$. Resorting $\boldsymbol{X}$ into two consecutive disjoint subchannel blocks $\boldsymbol{X}(n)$ and $\boldsymbol{X}(n+1)$ as

$$X(n) = \left[X_0, X_2, \ldots, X_{2N_c-2}\right]^{\mathrm{T}},\qquad X(n+1) = \left[X_1, X_3, \ldots, X_{2N_c-1}\right]^{\mathrm{T}} \tag{1}$$

the ST encoder maps them to the following $2N_c \times 2$ matrix

$$\text{space} \downarrow \text{antenna}\#1 \begin{bmatrix} \boldsymbol{X}(n) & -\boldsymbol{X}^*(n+1) \\ \boldsymbol{X}(n+1) & \boldsymbol{X}^*(n) \end{bmatrix}$$

whose columns are transmitted in successive time intervals with the upper and lower blocks in a given column simultaneously through the first and second transmitted antennas, respectively as shown in Fig. 1.

The frequency-selective wireless channel is assumed to be a quasi-static so that path gains are constant over a frame of length L_{frame} and vary from one frame to another. If the frequency response of the channel between the lth ($l = 1, 2$) transmitter and the receiver is denoted by $H_l(f, t)$, then the discrete channel coefficients $\{H_l(k, n)\}$ at the kth subcarrier frequency and nth time instant is given as

$$H_l(k, n) = H_l\left(\frac{k}{N_c T_s}, n\right), \quad k = 0, 1, \ldots, N_c - 1 \tag{2}$$

where T_s is the sampling period of the OFDM symbol. They are correlated samples, in frequency, of a complex Gaussian process. Moreover, $\boldsymbol{H}_l(n) = [H_l(0, n), \ldots, H_l(N_c - 1, n)]^{\mathrm{T}}$ denotes the channel vector between the lth transmitter and the receiver.

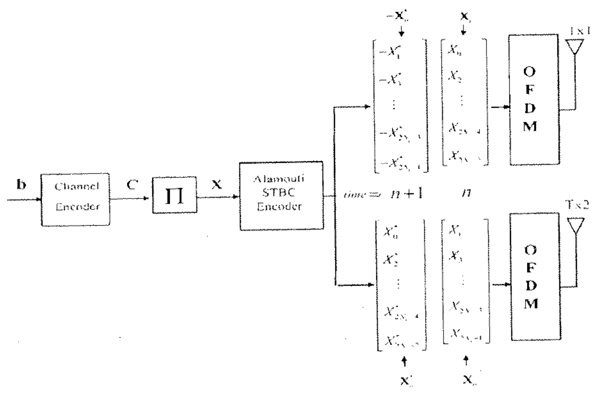


Figure 1 *Transmitter structure for STBC-OFDM with outer-channel coding*

Since the Alamouti's two-branch transmit diversity scheme with one receiver is employed here, each pair of the two consecutive received signal can be expressed in vector form as

$$\begin{aligned} \boldsymbol{R}(n) &= \mathcal{X}(n)\boldsymbol{H}_1(n) + \mathcal{X}(n+1)\boldsymbol{H}_2(n) + \boldsymbol{W}(n) \\ \boldsymbol{R}(n+1) &= -\mathcal{X}^*(n+1))\boldsymbol{H}_1(n+1) + \mathcal{X}^*(n)\boldsymbol{H}_2(n+1) \\ &\quad + \boldsymbol{W}(n+1) \end{aligned} \tag{3}$$

where $\boldsymbol{R}(n) = [R_0, R_2, \ldots, R_{2N_c-2}]^{\mathrm{T}}$, $\boldsymbol{R}(n+1) = [R_1, R_3, \ldots, R_{2N_c-1}]^{\mathrm{T}}$. $\mathcal{X}(n)$ and $\mathcal{X}(n+1)$ are $N_c \times N_c$ diagonal matrices whose elements are $\boldsymbol{X}_e$ and $\boldsymbol{X}_o$, respectively. Finally, $\boldsymbol{W}(n)$ and $\boldsymbol{W}(n+1)$ are $N_c \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive noise in the N_c tones.

Equation (3) shows that the information symbols $\mathcal{X}(n)$ and $\mathcal{X}(n+1)$ are transmitted twice in two consecutive time intervals through two different channels. To simplify the problem, we assume that the complex channel gains remain constant over the duration of one ST-OFDM code word, that is, $\boldsymbol{H}_1(n) \simeq \boldsymbol{H}_1(n+1)$ and $\boldsymbol{H}_2(n) \simeq \boldsymbol{H}_2(n+1)$. In order to estimate the channels and decode $\boldsymbol{X}$ with the embedded diversity gain through the repeated transmission, for each n, we define, $\boldsymbol{R} = [\boldsymbol{R}^{\mathrm{T}}(n) \quad \boldsymbol{R}^{\mathrm{T}}(n+1)]^{\mathrm{T}}$ and write (3) into a matrix form

$$\boldsymbol{R} = \mathcal{X}\boldsymbol{H} + \boldsymbol{W} \tag{4}$$

where $\boldsymbol{H} = [\boldsymbol{H}_1^{\mathrm{T}} \quad \boldsymbol{H}_2^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{W} = [\boldsymbol{W}^{\mathrm{T}}(n) \quad \boldsymbol{W}^{\mathrm{T}}(n+1)]^{\mathrm{T}}$ and

$$\mathcal{X} = \begin{bmatrix} \mathcal{X}(n) & \mathcal{X}(n+1) \\ -\mathcal{X}^*(n) & \mathcal{X}^*(n+1) \end{bmatrix} \tag{5}$$

Obviously, channel estimation is very essential in digital receivers for decoding of the STBC-OFDM systems with outer-channel encoder. In this paper, a novel channel estimation algorithm is presented by representing the discrete multipath channel based on the Karhunen–Loeve (KL) orthogonal representation and making use of the EM technique.

Modelling the frequency-selective fading channels as random processes, we employ a linear expansion based on the KL series representation involving a complete set of orthogonal deterministic vectors with the corresponding uncorrelated random coefficients. An orthonormal expansion of the vector $\boldsymbol{H}_l(n)$ involves expressing the $\boldsymbol{H}_l(n)$ as a linear combination of the orthonormal basis vectors as $\boldsymbol{H}_l(n) = \boldsymbol{\Psi}\boldsymbol{G}_l(n)$, where $\boldsymbol{\Psi} = [\psi_0, \psi_1, \ldots, \psi_{N_c-1}]$, ψ_i's are the orthonormal basis vectors, $\boldsymbol{G}_l(n) = [G_l(0, n), \ldots, G_l(N_c - 1, n)]^{\mathrm{T}}$, and $G_l(k, n)$ represent the weights of the expansion. The autocorrelation matrices of all the channels between transmitter antennas and receiver are same and it can be decomposed as

$$C_H = \boldsymbol{\Psi}\boldsymbol{\Lambda}\boldsymbol{\Psi}^{\dagger} \tag{6}$$

where $\boldsymbol{\Lambda} = E\{G_l G_l^{\dagger}\}$. The KL expansion is one where $\boldsymbol{\Lambda}$ of (6) is a diagonal matrix (i.e. the coefficients are uncorrelated). If $\boldsymbol{\Lambda}$ is diagonal, then (6) must be eigendecomposition of C_H. The fact that only the eigenvectors diagonalise C_{H_l} leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in the Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem in this application equivalent to estimating the i.i.d. Gaussian vector G_l KL expansion coefficients.

The channels between transmitter and receiver in this paper are assumed to be doubly selective, where $H_l(n)$'s modelled based on realistic channel model determined by COST-207 project in which typical urban (TU), bad urban (BU), hilly terrain (HT) and rural area (RA) channel model are considered and their channel transfer functions are given.

3 Turbo receiver

Iterative decoding and detection has been a topic of recent interest since the introduction of turbo codes. Iterative receiver structure for coded STBC-OFDM system is illustrated in Fig. 2 which uses three submodules to carry on iterative process: channel estimation, STBC-OFDM decoding and the MAP outer-channel code decoding. We therefore consider an EM-based MAP iterative channel estimation technique in frequency domain for turbo-coded STBC-OFDM systems. Frequency-domain estimator presented in this paper was inspired by the conclusions in [14] in which it has been shown that the time-domain channel estimators based on a discrete Fourier transform approach for non-sample-spaced channels cause aliased spectral leakage and result in an error floor.

3.1 EM-based MAP channel estimation

The MAP criterion is used in the fading channel as seen at the FFT output of the OFDM receiver. Then, the probability density function of the uncorrelated channel vectors G_l's can be expressed as

$$p(G_l) \sim \exp\left(-G_l^{\dagger}\widetilde{\Lambda}^{-1}G_l\right), \quad l = 1, 2 \tag{7}$$

where $\widetilde{\boldsymbol{\Lambda}} = \mathrm{diag}(\boldsymbol{\Lambda}\ \ \boldsymbol{\Lambda})$. Since we assumed that the channel covariance matrix C_h is known by the receiver, the diagonal matrix $\boldsymbol{\Lambda}$ can be determined easily by the eigendecomposition of C_h in (6). Given the transmitted signals X as coded according to Alamouti's scheme and the discrete channel representations $G = [G_1^{\mathrm{T}}, G_2^{\mathrm{T}}]^{\mathrm{T}}$ and taking into account the independence of the noise components, the conditional probability density function of the received signal R can be expressed as,

$$p(R|\mathcal{X}, G) \sim \exp\left[-(R - \mathcal{X}\widetilde{\boldsymbol{\Psi}}G)^{\dagger}\widetilde{\boldsymbol{\Sigma}}^{-1}(R - \mathcal{X}\widetilde{\boldsymbol{\Psi}}G)\right] \tag{8}$$

where $\widetilde{\boldsymbol{\Psi}} = \mathrm{diag}(\boldsymbol{\Psi}\ \ \boldsymbol{\Psi})$ and $\widetilde{\boldsymbol{\Sigma}} = \mathrm{diag}(\boldsymbol{\Sigma}\ \ \boldsymbol{\Sigma})$ and $\boldsymbol{\Sigma}$ is an $N_c \times N_c$ diagonal matrix with $\boldsymbol{\Sigma}[k, k] = \sigma^2$, for $k = 0, 1, \ldots, N_c - 1$.

The MAP estimate $\widehat{G}$ is given by

$$\widehat{G} = \arg\max p(G|R) \tag{9}$$

Directly solving this equation is mathematically intractable. However, the solution can be obtained easily by means of the iterative EM algorithm. This algorithm inductively reestimate G so that a monotonic increase in the a posteriori conditional pdf in (9) is guaranteed. The monotonic increase is realised via the maximisation of the auxiliary function

$$Q(G|G^{(q)}) = \sum_{X} p(R, \mathcal{X}, G)\log p(R, \mathcal{X}, G^{(q)}) \tag{10}$$

where the sum is taken over all possible transmitted data-coded signals and $G^{(q)}$ is the estimation of G at the qth iteration.

Note that the term $\log p(R, \mathcal{X}, G)$ in (11) can be expressed as

$$\log p(R, \mathcal{X}, G) = \log p(\mathcal{X}|G) + \log p(R|\mathcal{X}, G) + \log p(G) \tag{11}$$

The first term on the right-hand side of (12) is constant, since the data sequence $\mathcal{X} = \{X_k(n)\}$ and G are independent of each other and $\mathcal{X}$ have equal a priori probability. Therefore (11) can be evaluated by means of (7) and (8). Given the received signal R, the EM algorithm starts with an initial value G^0 of the unknown channel parameters G. The $(q+1)$th estimate of G s obtained by the maximisation step described by

$$G^{(q+1)} = \arg\max_{G} Q(G|G^{(q)})$$

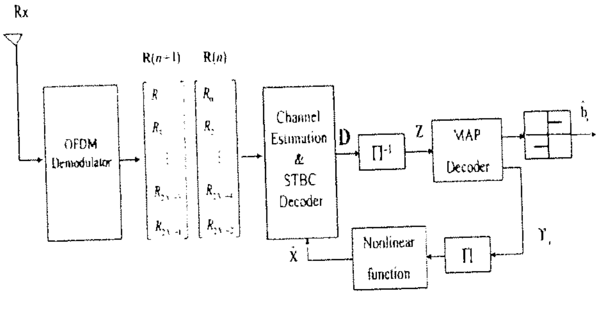


Figure 2 *Turbo-coded STBC-OFDM receiver*

IET Commun., 2008, Vol. 2, No. 7, pp. 886–894
doi: 10.1049/iet-com:20070387

After long algebraic manipulations, the expression of the re-estimate $\hat{G}_l^{(q+1)}(l = 1, 2)$ can be obtained as follows [11]

$$\hat{G}_1^{(q+1)} = (I + \Sigma\Lambda^{-1})^{-1}\Psi^{\dagger}\left[\hat{\mathcal{X}}_e^{\dagger(q)}R(n) - \hat{\mathcal{X}}_o^{(q)}R(n+1)\right]$$
$$\hat{G}_2^{(q+1)} = (I + \Sigma\Lambda^{-1})^{-1}\Psi^{\dagger}\left[\hat{\mathcal{X}}_o^{\dagger(q)}R(n) + \hat{\mathcal{X}}_e^{(q)}R(n+1)\right] \quad (12)$$

where it can be easily seen that

$$((I + \Sigma\Lambda^{-1})^{-1}) = \mathrm{diag}\left\{\frac{\lambda_0}{\lambda_0 + \sigma^2}, \ldots, \frac{\lambda_{N_c-1}}{\lambda_{N_c-1} + \sigma^2}\right\}$$

and $\hat{\mathcal{X}}_e^q$ and $\hat{\mathcal{X}}_o^q$ in (12) is an $N_c \times N_c$-dimensional diagonal matrix representing the a posteriori probabilities of the data symbols at the qth iteration step.

3.1.1 Truncation property: The truncated basis vector $G_{l,r}$ can be formed by selecting r orthonormal basis vectors among all basis vectors that satisfy $C_{H_l}\Psi = \Psi\Lambda$. The optimal solution that yields the smallest average mean-squared truncation error $1/N_c E[\epsilon_r^{\dagger}\epsilon_r]$ is the one expanded with the orthonormal basis vectors associated with the first largest r eigenvalues as given by

$$\frac{1}{N_c - r}E[\epsilon_r^{\dagger}\epsilon_r] = \frac{1}{N_c - r}\sum_{i=r}^{N_c-1}\lambda_i \quad (13)$$

where $\epsilon_r = G_l - G_{l,r}$. For the problem at hand, truncation property of the KL expansion results in a low-rank approximation as well. Thus, a rank-r approximation of Λ can be defined as $\Lambda_r = \mathrm{diag}\{\lambda_0, \lambda_1, \ldots, \lambda_{r-1}\}$ by ignoring the trailing $N_c - r$ variances $\{\lambda_l\}_{l=r}^{N_c-1}$, since they are very small compared with the leading r variances $\{\lambda_l\}_{l=0}^{r-1}$. Actually, the pattern of eigenvalues for Λ typically splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious.

3.1.2 Complexity: Based on the approach presented in [13], the traditional linear minimum mean squared error (LMMSE) estimation for H_l can be easily expressed as

$$\hat{H}_l = \underbrace{C_{H_l}(\Sigma + C_{H_l})^{-1}}_{\text{Precomputed}}P_{\mu}, \quad \mu = 1, 2 \quad (14)$$

where $P_1 = \hat{\mathcal{X}}_e^{\dagger(q)}R(n) - \hat{\mathcal{X}}_o^{(q)}R(n+1)$ and $\mathbf{P}_2 = \hat{\mathcal{X}}_o^{\dagger(q)}R(n) + \hat{\mathcal{X}}_e^{(q)}R(n+1)$. Since $C_{H_l}(\Sigma + C_{H_l})^{-1}$ does not change with data symbols, its inverse can be pre-computed and stored during each OFDM block. Since C_{H_l} and Σ are assumed to be known at the receiver, the estimation algorithm in (14) requires N_c^2 complex multiplications after precomputation (Multiplication of $N_c \times N_c$ precomputation matrix with $N_c \times 1 P_l$ vector.). However, this direct approach has high computational complexity due to the requirement of large-scale matrix inversion of the precomputation matrix (The computational complexity of an $N_c \times N_c$ matrix inversion, using Gaussian elimination, is $O((N_c)^3)$.). Moreover, the error caused by the small fluctuations in C_{H_l} and Σ have an amplified effect on the channel estimation due to the matrix inversion. Furthermore, this effect becomes more severe as the dimension of the matrix, to be inverted, increases [15]. Therefore the KL-based approach is need to avoid matrix inversion. Using (6) and (14), iterative estimate of the H_l with KL expansion can be obtained as

$$\hat{H}_l^{(q+1)} = \Psi((I + \Sigma\Lambda^{-1})^{-1})\Psi^{\dagger}P_1 \quad (15)$$

To reduce the complexity of the estimator further, we proceed with the low-rank approximations by considering only r column vectors of Ψ corresponding to the r largest eigenvalues of Λ, yielding

$$\hat{H}_l^{(q+1)} = \Psi_r\underbrace{((I + \Sigma_r\Lambda_r^{-1})^{-1})\Psi_r^{\dagger}}_{\text{precomputation}}\mathrm{P}_1 \quad (16)$$

where $((I + \Sigma_r\Lambda_r^{-1})^{-1}) = \mathrm{diag}(\lambda_0/(\lambda_0 + \sigma^2), \ldots, \lambda_{r-1}/(\lambda_{r-1} + \sigma^2))$. Σ_r in (16) is a $r \times r$ diagonal matrix whose elements are equal to σ^2 and Ψ_r is an $N_c \times r$ matrix which can be formed by omitting the last $N_c - r$ columns of Ψ. The low-rank estimator is shown to require $2N_c r$ complex multiplications (First, multiplication of precomputation matrix with P_{μ} has $N_c r$ complex multiplications and then multiplication with Ψ_r has $N_c r$ complex multiplication which totally requires $2N_c r$ complex multiplication.). In comparison with the estimator (traditional), the number of multiplications has been reduced from N_c to $2r$ per tone.

3.2 Iterative equalisation and decoding

We now consider the STBC-OFDM decoding algorithm and the MAP outer-channel code decoding to complete the description of the turbo receiver. Since the channel vectors or equivalently expansion coefficients are estimated through EM-based iterative approach, it is possible to decode R with diversity gains by a simple matrix multiplication. Before dealing with how we resolve decoding, let us first re-express received signal model (3) as

$$\tilde{R} = \mathcal{H}\tilde{X} + \tilde{W} \quad (17)$$

where $\tilde{R} = [R^{\mathrm{T}}(n), R^{\dagger}(n+1)]^{\mathrm{T}}, \tilde{X} = [X_e^{\mathrm{T}}, X_o^{\mathrm{T}}]^{\mathrm{T}}, \tilde{W} = [W^{\mathrm{T}}(n), W^{\dagger}(n+1)]^{\mathrm{T}}$ and

$$\mathcal{H} = \begin{bmatrix} \mathcal{H}_1(n) & \mathcal{H}_2(n) \\ \mathcal{H}_2^{\dagger}(n+1) & -\mathcal{H}_1^{\dagger}(n+1) \end{bmatrix} \quad (18)$$

where $\mathcal{H}_l(n)$ and $\mathcal{H}_l(n+1)$, $l = 1, 2$, are $N_c \times N_c$ diagonal matrices

Depending on complexity against performance tradeoffs, any linear equaliser can be applied to retrieve $\tilde{X}$ from (17). In this paper, we consider linear equaliser where the parameters are updated using the MMSE criterion. We define the linear MMSE estimate D of $\tilde{X}$ given the

IET Commun., 2008, Vol. 2, No. 7, pp. 886–894
doi: 10.1049/iet-com:20070387

observation $\tilde{R}$ by

$$D = \bar{\tilde{X}} + C_{\tilde{X}}\mathcal{H}^{\dagger}(\mathcal{H}^{\dagger}C_{\tilde{X}}\mathcal{H} + C_{\tilde{W}})^{-1} \times (\tilde{R} - \mathcal{H}\bar{\tilde{X}}) \quad (19)$$

where $\bar{\tilde{X}}$, $C_{\tilde{X}}$ and $C_{\tilde{W}}$ are the mean of $\tilde{X}$, the covariance matrix of $\tilde{X}$ and the covariance matrix of $\tilde{W}$, respectively.

With a scaled unitary matrix $\mathcal{H}$ and approximately constant complex channel gains with $\mathcal{H}_1^2(n) + \mathcal{H}_2^2(n) \simeq 1$ assumptions, we can simplify $\mathcal{H}^{\dagger}\mathcal{H}$ as

$$\mathcal{H}^{\dagger}\mathcal{H} = I_{2N_c \times 2N_c} \quad (20)$$

where $I_{2N_c \times 2N_c}$ is the $2N_c \times 2N_c$ identity matrix. Moreover, following the assumptions used in [16], $\bar{\tilde{X}} = 0$, $C_{\tilde{X}} = I$, (19) becomes

$$D = (I + \sigma_n^2 I)^{-1}\mathcal{H}^{\dagger}\tilde{R} \quad (21)$$

If we set $C_{\tilde{W}} = 0$ in (19), further simplified form of linear equaliser called zero forcing equaliser is obtained resulting in

$$D = \mathcal{H}^{\dagger}\tilde{R} = \mathcal{H}^{\dagger}\mathcal{H}\tilde{X} + \eta \quad (22)$$

where $\eta = \mathcal{H}^{\dagger}\tilde{W}$.

We propose a turbo receiver structure for STBC-OFDM systems in this paper, which consists of an iterative MAP-EM channel estimation algorithm, STBC decoder and a soft MAP outer-channel-code decoder. As shown in Fig. 2, first EM-based channel estimator computes channel gains according to pilot symbols. Output of the estimator is used in the STBC demodulator (22). Next, the equalised symbol sequence $\{D\}$ is passed through a channel de-interleaver, resulting in de-interleaved equalised symbols sequence $\{Z\}$. Finally, $\{Z\}$ is applied to a MAP decoder by de-interleaved estimated channel gains. In the MAP decoder submodule, log-likelihood ratios (LLRs) of a posteriori probabilities on the coded and uncoded bits are yielded. In the next iteration, LLRs of coded bits $\{Y_j\}$ are re-interleaved and passed through a nonlinearity [12]. Output of the nonlinearity computes soft value estimation of X as $\hat{X}$ in Fig. 2.

$\hat{X}$ is used as $\hat{\mathcal{X}}_e^{(q)}$ and $\hat{\mathcal{X}}_o^{(q)}$ in (12) for next iteration. Thus, the MAP-EM channel estimator iteratively generates the channel estimates by taking the received signals from receiver antennas and interleaved soft value of LLRs which are computed by the outer-channel code decoder in the previous iteration. Then, STBC-OFDM decoder takes channel estimates together with the received signals and computes equalised symbol sequence for next turbo iteration. Iterative operation is fulfilled between these three submodules. In all simulations, three iterations are employed.

4 Simulations

In this section, BER performances of channel estimators are presented through simulations for turbo-coded STBC-OFDM systems. Moreover, to investigate the sensitivity to channel estimation errors, we also considered convolutionally coded STBC-OFDM systems. In case of the turbo encoder, two identical recursive systematic convolutional component codes with generator $(1,5_8/7_8)$ were concatenated in parallel via a pseudorandom interleaver form the encoder. For the convolutionally coded system, a $(5_8, 7_8)$ code with rate 1/2 was used. The channels between transmitter and receiver in this paper are assumed to be doubly selective where $H_i(n)$'s were modelled, based on a realistic channel model determined by COST-207 project in which TU, BU, HT and RA channel models are considered.

The scenario for the STBC-OFDM study, with outer channel codes simulation, consists of BPSK and QPSK modulation formats. The system has 1.07 MHz bandwidth (for the raised cosine pulse roll-off factor $\alpha = 0.2$) and is divided into $N_c = 512$ tones with a total period T_s of 328 μs, of which 40 μs constitute the cyclic prefix. The data rate is about 0.8 and 1.6 Mbps for BPSK and QPSK modulation techniques, respectively. In order to choose good initial values for the unknown channel parameters, pilot tones known by the receiver are inserted in OFDM symbol. One pilot tone is inserted for every K data symbol denoted as a pilot insertion rate (PIR = 1:K). The details of the initialisation process are presented in [13, 17]. In all simulations, three iterations are employed.

Fig. 3 compares the BER performances of the EM-MAP channel estimation approach with the EM-ML [7] and widely used LMMSE-PSAM (LMMSE pilot-symbol-assisted modulation) approaches [18], for the turbo-coded STBC-OFDM systems over the BU channels for BPSK-modulated signals. It can be seen from Fig. 3 that the performance of the proposed EM-MAP algorithm outperforms EM-ML as well as PSAM techniques while

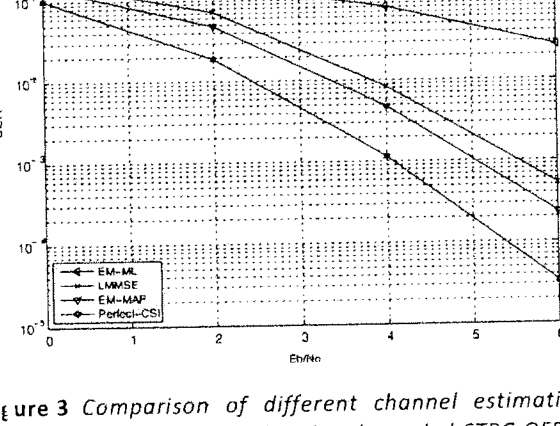


Figure 3 *Comparison of different channel estimation algorithms for BPSK-modulated turbo-coded STBC-OFDM systems over BU channels (f_{Dm} = 100Hz, PIR = 1:8*

IET Commun., 2008, Vol. 2, No. 7, pp. 886–894
doi: 10.1049/iet-com:20070387

approaching the perfect channel state information (CSI) case for higher SNR values.

In Fig. 4, by using the same bandwidth and total number of subcarriers, QPSK modulation technique is used for BU channels and the overall data rate is increased to 1.6 Mbit/s. Since the turbo-coded data frame length is doubled when compared with BPSK case, it is observed that the performance difference between EM-MAP and LMMSE is decreased, as shown in Fig. 4, and the overall performance is degraded when compared with the BPSK case for PIR = 1:8 case. Therefore it is concluded that the EM-MAP channel estimator precedence could be faded for larger turbo-coded data frames. However, the turbo-coded data frame length is usually limited by the total number of subcarriers for OFDM applications. The coding process over several OFDM symbols is not practical due to the complexity and implementation issues and requires more decoder memory. On the other hand, the superiority of the EM-MAP over the LMMSE algorithm is clearly seen for PIR = 1:16 corresponding to PIR = 1:8, while the overall performance falls.

Computer simulation results with the same simulation parameters for TU channels are presented in Fig. 5. In this figure, it can be observed that the performance difference between the EM-MAP and the perfect CSI scenarios for BU channels is about 1 dB, and in TU channels, about 0.5 dB corresponding to $Pe = 10^{-4}$ for PIR = 1:8. However, the performances of the EM-MAP and the LMMSE are almost similar to each other since estimation accuracy is sufficient for the turbo decoder to work well. On the other hand, the superiority of the EM-MAP over the LMMSE algorithm is clearly seen for PIR = 1:16 corresponding to PIR = 1:8, while overall performance falls. In these figures, it can be concluded that the EM-MAP algorithm performs far better than other methods for the efficient bandwidth usage (i.e. lower PIRs).

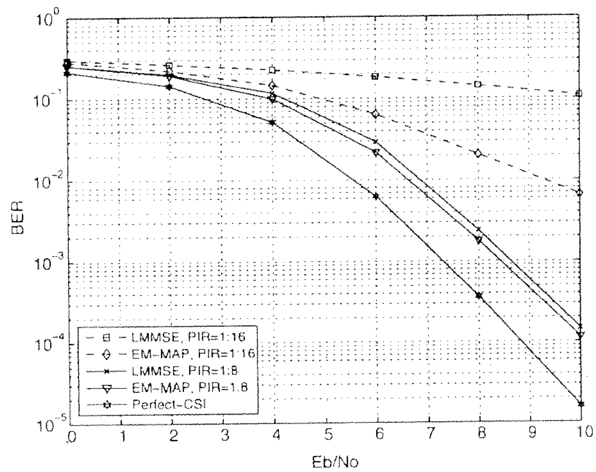


Figure 4 *Comparison of different channel estimation algorithms for QPSK-modulated turbo-coded STBC-OFDM systems over BU channels (f_{Dm} = 100 Hz)*

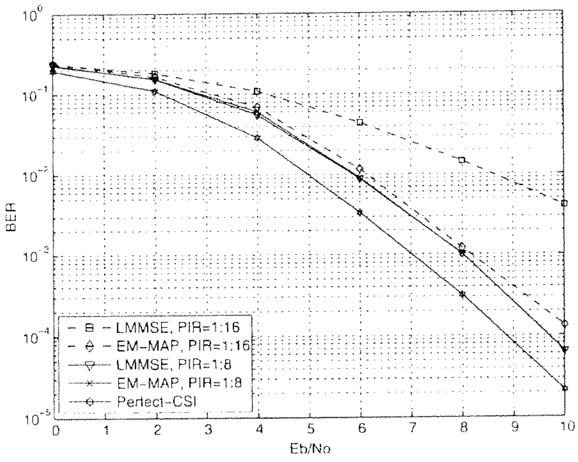


Figure 5 *Comparison of different channel estimation algorithms for QPSK-modulated turbo-coded STBC-OFDM systems over TU channels (f_{Dm} = 100 Hz)*

Moreover, in Fig. 6, the EM-MAP algorithm performance for turbo/convolutionally coded STBC-OFDM systems are investigated as a function of PIRs (1:8, 1:16, 1:32) for the TU channel model. Generally, it is known that a good channel code is more sensitive to channel estimation errors with high dependency among the coded bits that might cause severe error propagation during decoding process. Our simulation results support this argument. We observe that more performance degradation is experienced in the turbo-coded systems when compared with the convolutionally encoded systems in the presence of the TU channels since lower PIRs provide poor initial estimates.

The amount of information required to represent the statistically dependent channel parameters could be minimised by using the optimal truncation property. It is

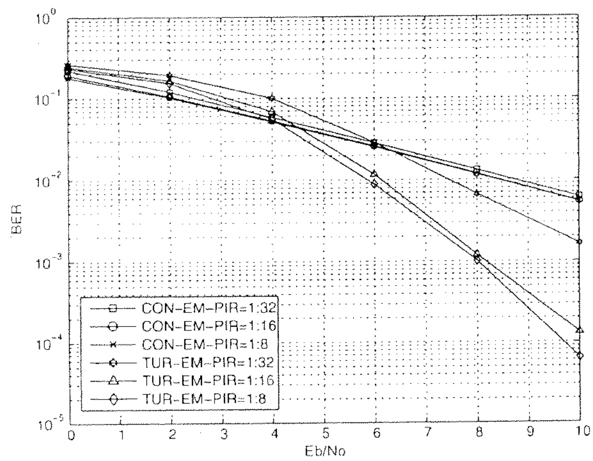


Figure 6 *EM-MAP channel estimator BER performance of the QPSK-modulated turbo/convolutionally coded STBC-OFDM systems as a function of PIRs for TU channels (f_{Dm} = 100 Hz)*

IET Commun., 2008, Vol. 2, No. 7, pp. 886–894
doi: 10.1049/iet-com:20070387

possible to obtain an excellent approximation, if the dominant eigenvalues are selected in the KL expansion. In this way, more reduction in computational complexity could be achieved on the channel estimation algorithm as explained in Section 3. Since the frequency selectivity of the BU channels is more than the TU channels, low-rank representation of the BU channel requires more KL coefficients than TU channels. In Fig. 7, the BER performance of the EM-MAP algorithm is investigated for SNR = 6dB as a function of the number of KL coefficients employed. It is observed that 15 KL coefficients are sufficient to represent for TU channels, whereas it is about 25 coefficients for the BU channels.

In the preceding simulations, the autocorrelation matrix was assumed to be available as a priori information at the receiver for the COST-207 channel model. However, in practice, the true channel correlation is not known and it is important to analyse the performance degradation due to the mismatch between the approximated and the actual autocorrelation matrices. In this scope, the autocorrelation matrix $\boldsymbol{C} = [c_{m,n}]$ of the TU channel was approximated by

$$c_{m,n} = \begin{cases} 1 & \text{if} \quad m = n \\ \dfrac{1 - e^{-j2\pi L(m-n)/N}}{2\pi j L(m-n)/N} & \text{if} \quad m \neq n \end{cases} \tag{23}$$

where N is the number of subcarriers and L the length of the cyclix prefix. Note that (23) can be obtained from a profile having a uniform power-delay [13]. The simulation results concerning the mismatch analysis are presented in Fig. 8 for turbo-coded STBC-OFDM systems. It is concluded that the using the approximate $\boldsymbol{C}$ does not degrade the BER performance significantly for the TU channels in the case of correct channel length L. Moreover, it can be shown that the performance degradation will be visible when L is less or more than the correct channel length.

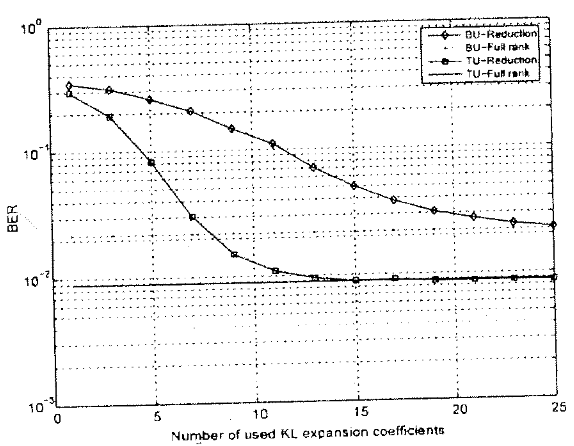


Figure 7 *Optimal truncation property of the KL expansion (PIR = 1:8, f_{Dm} = 100 Hz, QPSK)*

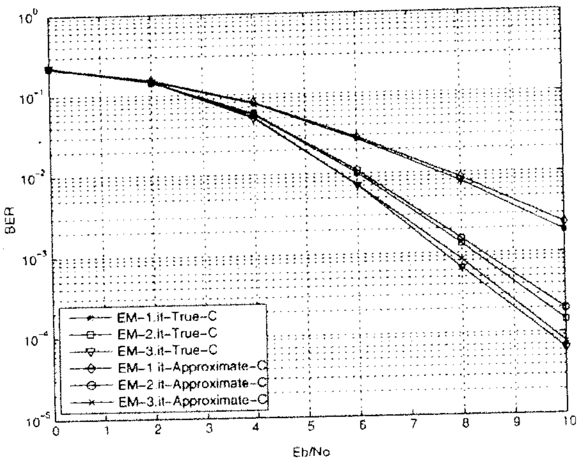


Figure 8 *Mismatch analysis for autocorrelation matrix (PIR = 1:8, f_{Dm} = 10 Hz, QPSK)*

5 Conclusion

In this paper, we proposed the EM-based MAP channel estimation algorithm for turbo/convolutionally coded STBC-OFDM systems, which is crucial for turbo receiver structure. This algorithm iteratively estimates the channel variations according to the MAP criterion, using the EM algorithm based on the orthogonal expansion representation of the channel via KL transform. It is observed that the proposed EM-MAP outperforms the EM-ML as well as PSAM techniques at lower pilot insertion rates. Based on such representation, we show that no matrix inversion is needed in the channel estimation algorithm. Moreover, a simplified approach with rank reduction is also proposed. It has been shown that in comparison with the traditional estimators the number of complex multiplications has been reduced from N_c to $2r$ per tone. Furthermore, sensitivity to channel estimation errors of turbo receivers is investigated. It was concluded that the receiver structures with turbo codes outperform the convolutional coded receiver structures assuming that channel estimation error is sufficient low. Moreover, it is observed that the performance difference between the EM-MAP and LMMSE decreases as the length of turbo-coded frames increases. Therefore it was concluded that the EM-MAP channel estimator precedence could fade out for longer turbo-coded data frames. However, the length of turbo-coded frames is usually limited by the total number of subcarriers for OFDM applications. Finally, we also concluded that the coding process over several OFDM symbols is not practical due to the complexity and implementation issues and requires more decoder memory. Consequently, the EM-MAP receiver structure is very efficient for turbo-coded STBC-OFDM systems.

6 Acknowledgments

The authors would like to thank the editor Prof. Habib F. Rashvand and the anonymous reviewers whose insightful comments and suggestions improved the quality of the paper. This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 7th Framework Programme. This work was also supported in part by the Turkish Scientific and Technical Research Institute(TUBITAK) under Grant 104E166. Part of the results of this paper was presented at the NEW-ACORN Joint Workshop, 20 – 22 September 2006, in Vienna, Austria.

7 References

[1] ALAMOUTI S.M.: 'A simple transmitter diversity scheme for wireless communications', *IEEE J. Sel. Areas Commun.*, 1998, **16**, pp. 1451–1458

[2] TAROKH V., SESHADRI N., CALDERBANK A.R.: 'Space–time codes for high data rate wireless communications: performance criterion and code construction', *IEEE Trans. Inform. Theory*, 1998, **44**, pp. 744–765

[3] LI Y., GEORGHIADES C.N., HUANG G.: 'Iterative maximum likelihood sequence estimation for space-time coded systems', *IEEE Trans. Commun.*, 2001, **49**, (6), pp. 948–951

[4] COZZO C., HUGHES B.L.: 'Joint channel estimation and data detection in space-time communications', *IEEE Trans. Commun.*, 2003, **51**, (8), pp. 1266–1270

[5] KARKHANECHI H.M., LEVY B.C.: 'EM-based channel estimation for space–time block coded MIMO OFDM Systems'. IEEE Workshop on Signal Processing Systems Design and Implementation, SIPS'06, October 2006, pp. 177–181

[6] LU B., WANG X., LI Y. (G): 'Iterative receivers for space-time block-coded OFDM systems in dispersive fading channels', *IEEE Trans. Wirel. Commun.*, 2002, **1**, (2), pp. 213–225

[7] YONGZHE X., GEORGHIADES C.N.: 'Two EM-type channel estimation algorithms for OFDM with transmitter diversity', *IEEE Trans. Commun.*, 2003, **51**, (1), pp. 106–115

[8] MA X., KOYABASHI H., SCHWARTS S.C.: 'An EM-based channel estimation for space-time and space-frequency block coded OFDM'. Proc. ICASSP'03, April 2003, vol. 4, pp. 389–392

[9] KASHIMA T., FUKAWA K., SUZUKI H.: 'Adaptive MAP receiver via the EM algorithm and message passings for MIMO-OFDM mobile communications', *IEEE J. Sel. Areas Commun.*, 2006, **24**, (3), pp. 437–447

[10] FEDER M., WEINSTEIN E.: 'Parameter estimation of superimposed signals using the EM algorithm', *IEEE Trans. Acoust., Speech Signal Process.*, 1988, **36**, pp. 477–489

[11] CIRPAN H.A., PANAYIRCI E., DOGAN H.: 'Nondata-aided channel estimation for OFDM systems with space–frequency transmit diversity', *IEEE Trans. Veh. Technol.*, 2006, **55**, (2), pp. 449–457

[12] DOGAN H., ÇIRPAN H.A., PANAYIRCI E.: 'Iterative channel estimation and decoding of turbo coded SFBC-OFDM systems', *IEEE Trans. Wirel. Commun.*, 2007, **6**, (7), pp. 3090–3101

[13] EDFORS O., SANDELL M., VAN DE BEEK J.-J., WILSON S.K., BORJESSON P.O.: 'OFDM channel estimation by singular value decomposition', *IEEE Trans. Commun.*, 1998, **46**, (7), pp. 931–936

[14] BAOGUO Y., ZHIGANG C., LETAIEF K.: 'Analysis of low-complexity windowed DFT-based MMSE channel estimator for OFDM systems', *IEEE Trans. Commun.*, 2001, **49**, (11), pp. 1977–1987

[15] ZHU J., LEE W.: 'A low-complexity channel estimator for OFDM systems in multipath fading channels'. Proc. IEEE-PIMRC 2004, September 2004, vol. 3, pp. 1978–1982

[16] TUCHLER M., KOETTER R., SINGER A.C.: 'Turbo equalization: principles and new results', *IEEE Trans. Commun.*, 2002, **50**, pp. 754–767

[17] COLERI S., ERGEN M., PURI A., BAHAI A.: 'Channel estimation techniques based on pilot arrangement in OFDM systems', *IEEE Trans. Broadcast.*, 2002, **48**, (3), pp. 223–229

[18] MORELLI M., MENGALI U.: 'A comparison of pilot-aided channel estimation methods for OFDM systems', *IEEE Trans. Signal Process.*, 2001, **49**, pp. 3065–73

doi: 10.1049/iet-com:20070387

Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2008, Article ID 570624, 9 pages
doi:10.1155/2008/570624

Research Article

MAP Channel-Estimation-Based PIC Receiver for Downlink MC-CDMA Systems

Hakan Doğan,[1] Erdal Panayırcı,[2] Hakan A. Çırpan,[1] and Bernard H. Fleury[3]

[1] *Department of Electrical and Electronics Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey*
[2] *Department of Electronics Enginering, Kadir Has University, Cibali 34083, Istanbul, Turkey*
[3] *Section Navigation and Communications, Department of Electronic Systems, Aalborg University Fredrik Bajers Vej 7A3 DK-9000 Aalborg, Denmark and The Telecommunications Research Center, Donau City Strasse 1, 1220 Vienna (ftw.), Austria*

Correspondence should be addressed to Hakan Doğan, hdogan@istanbul.edu.tr

Received 15 May 2007; Revised 10 September 2007; Accepted 2 October 2007

Recommended by Arne Svensson

We propose a joint MAP channel estimation and data detection technique based on the expectation maximization (EM) method with paralel interference cancelation (PIC) for downlink multicarrier (MC) code division multiple access (CDMA) systems in the presence of frequency selective channels. The quality of multiple access interference (MAI), which can be improved by using channel estimation and data estimation of all active users, affects considerably the performance of PIC detector. Therefore, data and channel estimation performance obtained in the initial stage has a significant relationship with the performance of PIC. So obviously it is necessary to make excellent joint data and channel estimation for initialization of PIC detector. The EM algorithm derived estimates the complex channel parameters of each subcarrier iteratively and generates the soft information representing the data a posterior probabilities. The soft information is then employed in a PIC module to detect the symbols efficiently. Moreover, the MAP-EM approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. The performance of the proposed approach is studied in terms of bit-error rate (BER) and mean square error (MSE). Throughout the simulations, extensive comparisons with previous works in literature are performed, showing that the new scheme can offer superior performance.

1. INTRODUCTION

Traditional wireless technologies are confronted with new challenges in meeting the ubiquity and mobility requirements of cellular systems. Extensive attemps have therefore been made in recent years to provide promising avenue that makes efficient utilization of the limited bandwidth and cope with the adverse access environments. These include the development of several modulation and multiple access techniques. Among these, multicarrier (MC) and code division multiple access (CDMA) have gained considerable interest due to their considerable performance [1, 2].

MC modulation technique, known also as OFDM (orthogonal frequency division multiplexing), has emerged as an attractive and powerful alternative to conventional modulation schemes in the recent past due to its various advantages. The advantages of MC which lie behind such a success are robustness in the case of multipath fading, a very reduced system complexity due to equalization in the frequency domain and the capability of narrow-band interference rejection. OFDM has already been chosen as the transmission method for the European radio (DAB) and TV (DVB-T) standard and is used in xDSL systems as well. Supporting multiple users can be achieved in a variety of ways. One popular multiple access scheme is CDMA. CDMA makes use of spread spectrum modulation and distinct spreading codes to separate different users using the same channel. It is well known that CDMA system has an ability to reduce user's signal power during transmission using a spreading so that the user can communicate with a low-level transmitted signal closed to noise power level. As a combination of MC and CDMA techniques, it combines the advantages of both MC and CDMA [1–3].

To evaluate the performance of these systems, ideal knowledge of transmission parameters is often assumed known. Iterative receivers for coded MC-CDMA promise a significant performance gain compared to conventional noniterative receivers by using combined minimum mean square error-parallel interference cancellation (combined MMSE-PIC) detector [5] assuming perfectly known channel impulse response. However, the performance of MC-CDMA-based transmission systems under realistic conditions critically depends on good estimate of the parameters, such as the channel parameters. In [4], different detection schemes were considered for least square estimation case as well as perfect channel case.

The quality of multiple access interference (MAI), which can be improved by using channel estimation and data estimation of all active users, affects considerably the performance of PIC detector. Therefore, data and channel estimation performance is obtained in the initial stage has a significant relationship with the performance of PIC. So obviously it is necessary to make excellent joint data and channel estimation for initialization of PIC detector. Inspired by the conclusions in [4, 5], including channel estimation into the iterative receiver yields further improvements. We therefore consider iterative channel estimation techniques based on the expectation-maximization (EM) algorithm in this paper.

The EM algorithm is a broadly applicable approach to the iterative computation of parameters from intractable and high complexity likelihood functions. An EM approach proposed for the general estimation from superimposed signals [6] is applied to the channel estimation for OFDM systems and compared with SAGE version in [7]. For CDMA systems, Nelson and Poor [8] extend the EM and SAGE algorithms for detection, rather than for estimation of continuous parameters. Moreover, EM-based channel estimation algorithms were investigated in [9, 10] for synchronous uplink DS-CDMA and asynchronous uplink DS-CDMA systems, respectively. Unlike the EM approaches, we adopt a two-step detection procedure: (i) use the EM algorithm to estimate the channel (frequency domain estimation) and (ii) use the estimated channel to perform coherent detection [11, 12]. The paper has several major novelties and contributions. The major contribution of the paper is to obtain EM-based channel estimation algorithm approach as opposed to the existing works in the literature which mostly assumed that the data is known at the receiver through a training sequence. Note that very small number of pilots used in our approach is necessary only for initialization of the EM algorithm leading to channel estimation. Although, the joint data and channel estimation technique with EM algorithm seems to be attractive in practice, it is known that the convergency of the algorithm is much slower, it is more sensitive to the initial selection of the parameters and the algorithm is more computationally complex than the techniques that deal with only channel estimation. As it is known in the estimation literature, non-data-aided estimation techniques are more challenging mainly due to a data-averaging process which must be performed prior to optimization step. The proposed EM-MAP receiver compared with the combined MMSE-PIC receiver in the case of LS, LMMSE, and perfect channel estimation [4].

Another significant contribution of the paper comes from the fact that the proposed approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. It was shown that KL expansion enable us to estimate the channel in a very simple way without taking inverse of large-dimensional matrices for OFDM system [11, 12]. However, this property will not help to avoid matrix inversion for the signal model in this paper as shown in Section 4. On the other hand, we show that optimal truncation property of the KL expansion help to decrease inverse matrix dimension so that reduction in computational load on the channel estimation algorithm can be done.

The rest of this paper is organized as follows. In Sections 2 and 3 we introduce the model of a downlink MC-CDMA system and the corresponding channel model established, respectively. Using the discrete-time model, the maximum a posteriori (MAP) channel estimation algorithm is derived in Section 4. Moreover, in this section, truncation property of the KL expansion and complexity calculation of the proposed algorithm are also given. In the next section, PIC-detection scheme is then developed for the proposed channel estimation algorithm. Finally, computer simulation results are presented with detailed discussions in Section 6, and conclusions are drawn in Section 7.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; $(\cdot)^T$, $(\cdot)^\dagger$ and $(\cdot)^{-1}$ denote the transpose, conjugate transpose, and matrix inversion, respectively; $\mathbf{I}_L$ denotes the $L \times L$ identity matrix; diag$\{\cdot\}$ denotes a diagonal matrix.

2. DOWNLINK MC-CDMA

Transmission of MC-CDMA signals from the base station to mobile stations forms the downlink transmission. The Base station must detect all the signals while each mobile is related with its own signal. In the downlink applications, all the signals arriving from the base station to specific user propagate through the same channel. Therefore, channel estimation methods that is developed for OFDM systems can be appliciable for downlink application of MC-CDMA systems [11].

Let b^k's denote the QPSK modulated symbols that would be send for kth user within mobile cell $k = 1, \ldots, K$ where K is the number of mobile users which are simultaneously active. The base station spread the data b^k's over chips of length N_c by means of specific orthogonal spreading sequences, $\mathbf{c}^k = (c_1^k, c_2^k, \ldots, c_{N_c}^k)^T$ where each chip, c_i^k, takes values in the set $\{-1/\sqrt{N_c}, 1/\sqrt{N_c}\}$. Then, the spreaded sequences of all users $\mathbf{c}^k b^k$ are summed together to form the input sequences of the OFDM block. After summation process, OFDM modulator block takes inverse discrete Fourier transform (IDFT) and inserts cyclic prefix (CP) of length equal to at least the channel memory (L). Pilot tones uniformly inserted in OFDM modulated data for the initial channel estimation [19]. In this work, to simplify the notation, it is assumed that the spreading factor equals to the number of subcarriers and all users have the same spreading factor.

At the receiver, CP is removed and DFT is then applied to the received discrete time signal to obtain the received vector expressed as

$$\mathbf{R} = \mathcal{H}\mathbf{C}\mathbf{b} + \mathbf{W}, \tag{1}$$

where $\mathbf{C} = [\mathbf{c}^1, \ldots, \mathbf{c}^K]$ is the $N_c \times K$ spreading code matrix, $\mathbf{b} = [b^1, \ldots, b^K]^T$ is the $K \times 1$ vector of the transmitted symbols by the K users. $\mathcal{H}$ is the $N_c \times N_c$ diagonal channel matrix whose elements representing the fading of the subcarriers are modeled in the next section, $\mathbf{W}$ is the $N_c \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive noise in the N_c tones, with variance $\sigma^2/2$ per dimension. Note that due to orthogonality property of the spreading sequences, $\mathbf{C}^T\mathbf{C} = \mathbf{I}_K$.

In this study, our major focus lies on the development of a MAP-EM channel estimation algorithm based on the observation model (1). However, in the sequel we will first present the channel model based on KL expansions.

3. CHANNEL: BASIS EXPANSION MODEL

The fading channel between the transmit and the receive antenna is assumed to be frequency and time selective and the fading process is assumed to be constant during each OFDM symbol. Let $\mathbf{H} = [H_1, H_2, \ldots, H_{N_c}]^T$ denote the correlated channel coefficients corresponding to the frequency response of the channel between the transmit and the receive antenna. The KL expansion methodology has been applied for efficient simulation of multipath fading channels [14]. Prompted by the general applicability of KL expansion, we consider in this paper the parameters of $\mathbf{H}$ to be expressed by a linear combination of orthonormal bases,

$$\mathbf{H} = \mathbf{\Psi}\mathbf{G}, \tag{2}$$

where $\mathbf{\Psi} = [c\psi_1, \psi_2, \ldots, \psi_{N_c}]$, ψ_i's are the orthonormal basis vectors, $\mathbf{G} = [G_1, \ldots, G_{N_c}]^T$, and G_i is the vector representing the weights of the expansion. By using different basis functions $\mathbf{\Psi}$, we can generate sets of coefficients with different properties. The autocorrelation matrix $\mathbf{C_H} = E[\mathbf{H}\mathbf{H}^\dagger]$ can be decomposed as

$$\mathbf{C_H} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^\dagger, \tag{3}$$

where $\mathbf{\Lambda} = E\{\mathbf{G}\mathbf{G}^\dagger\}$ is a diagonal. Then (3) represents the *eigendecomposition* of $\mathbf{C_H}$. The fact that only the eigenvectors diagonalize $\mathbf{C_H}$ leads to the desirable property that the KL coefficients $(G_1, \ldots, G_{N_c})$ are uncorrelated. Furthermore, in the Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem in this study is equivalent to estimating the i.i.d. Gaussian vector $\mathbf{G}$, namely, the KL expansion coefficients.

4. EM BASED MAP CHANNEL ESTIMATION

In MC-CDMA system, channel equalization is moved from the time domain to the frequency domain, that is, the channel frequency response is estimated. Note that, it is possible to estimate the channel parameters from the time-domain channel model (channel impulse response), in our work, time-domain approach introduces additional complexity mainly because the frequency domain channel parameters are required and directly employed in the detection process. Moreover, the frequency domain estimator presented in this paper was inspired by the conclusions in [15, 16], where it has been shown that time domain channel estimators based on a Discrete Fourier Transform (DFT) approach for non sample-spaced channels cause aliased spectral leakage and result in an error floor. Furthermore, our proposed frequency domain iterative channel estimation technique employs the KL expansion which reduces the overall computational complexity significantly.

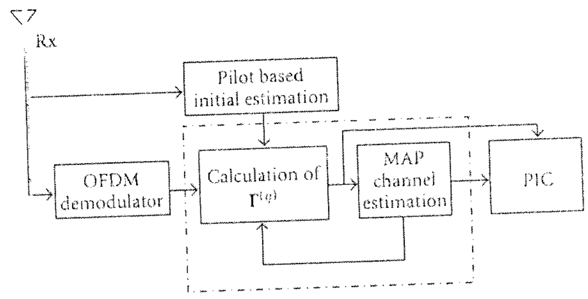


Figure 1: Receiver structure for MC-CDMA systems.

To find MAP estimate of $\hat{\mathbf{G}}$, (1) can be rewritten by using the channel KL expansion as follows:

$$\mathbf{R} = \mathrm{diag}(\mathbf{C}\mathbf{b})\mathbf{\Psi}\mathbf{G} + \mathbf{W}. \tag{4}$$

The MAP estimate $\hat{\mathbf{G}}$ is then given by

$$\hat{\mathbf{G}} = \arg\max_{\mathbf{G}} p(\mathbf{G} \mid \mathbf{R}) \tag{5}$$

Direct maximization of (5) is mathematically intractable. However, the solution can be obtained easily by means of the iterative EM algorithm. A natural choice for the complete data for this problem is $\chi = \{\mathbf{R}, \mathbf{b}\}$. The vector to be estimated is $\mathbf{G}$, and the incomplete data is $\mathbf{R}$. The EM algorithm stated above is equivalent to determining the parameter set $\mathbf{G}$ that maximize the Kullback-Leibler information measure defined by

$$Q(\mathbf{G}|\mathbf{G}^{(q)}) = \sum_{b} p(\mathbf{R}, \mathbf{b}, \mathbf{G}^{(q)}) \log p(\mathbf{R}, \mathbf{b}, \mathbf{G}), \tag{6}$$

where $\mathbf{G}^{(q)}$ is the estimation of $\mathbf{G}$ at the qth iteration. This algorithm inductively reestimate $\mathbf{G}$ so that a monotonic increase in the *a posteriori* conditional pdf (probability density function) in (5) is guaranteed.

Note that, the term $\log p(\mathbf{R}, \mathbf{b}, \mathbf{G})$ in (6) can be expressed as

$$\log p(\mathbf{R}, \mathbf{b}, \mathbf{G}) = \log p(\mathbf{b} \mid \mathbf{G}) + \log p(\mathbf{R} \mid \mathbf{b}, \mathbf{G}) + \log p(\mathbf{G}). \tag{7}$$

The first term on the right-hand side of (7) is constant, since the data sequence $\mathbf{b}$ and $\mathbf{G}$ are independent of each other and b have equal *a priori* probability. The probability density function of G is known as a priori by the receiver and can be expressed as

$$p(\mathbf{G}) \sim \exp\left(-\mathbf{G}^{\dagger}\Lambda^{-1}\mathbf{G}\right). \tag{8}$$

Also, given the transmitted symbols b and the discrete channel representation G and taking into account the independence of the noise components, the conditional probability density function of the received signal R can be expressed as

$$p(\mathbf{R} \mid \mathbf{b}, \mathbf{G}) \sim \exp\left[-\left(\mathbf{R} - \mathrm{diag}(\mathbf{C}\mathbf{b})\Psi\mathbf{G}\right)^{\dagger}\Sigma^{-1}\left(\mathbf{R} - \mathrm{diag}(\mathbf{C}\mathbf{b})\Psi\mathbf{G}\right)\right], \tag{9}$$

where Σ is an $N_c \times N_c$ diagonal matrix with $\Sigma[k,k] = \sigma^2$, for $k = 1, 2, \ldots, N_c$.

Taking derivatives in (6) with respect to G and equating the resulting equations to zero, we have

$$\sum_{b} p(\mathbf{R}, \mathbf{b}, \mathbf{G}^{(q)})\left(\Psi^{\dagger}\mathrm{diag}(\mathbf{b}^{\dagger}\mathbf{C}^{T})\right) \times \Sigma^{-1}\left(\mathbf{R} - \mathrm{diag}(\mathbf{C}\mathbf{b})\Psi\mathbf{G}\right) - \Lambda^{-1}\mathbf{G}) = 0. \tag{10}$$

Note that $p(\mathbf{R}, \mathbf{b}, \mathbf{G}^{(q)})$ may be replaced by $p(\mathbf{b} \mid \mathbf{R}, \mathbf{G}^{(q)})$ without violating the equalities in (10). Solving (10) for G, after taking average over b, the final expression of reestimate of $\hat{\mathbf{G}}^{(q+1)}$ can be obtained as follows:

$$\hat{\mathbf{G}}^{(q+1)} = \left(\mathbf{T}^{\dagger^{(q)}}\mathbf{T}^{(q)} + \Sigma\Lambda^{-1}\right)^{-1}\mathbf{T}^{\dagger^{(q)}}\mathbf{R}, \tag{11}$$

where

$$\mathbf{T}^{(q)} = \mathrm{diag}(\mathbf{C}\Gamma^{(q)})\Psi. \tag{12}$$

$\Gamma^{(q)} = \left[\Gamma^{(q)}(1), \Gamma^{(q)}(2), \ldots, \Gamma^{(q)}(K)\right]$ represents the *a posteriori probabilities* of the data symbols at the qth iteration step defined as

$$\Gamma^{(q)}(k) = \sum_{b \in S_k} bP\left(b^k = \mathbf{b} \mid \mathbf{R}, \mathbf{G}^{(q)}\right). \tag{13}$$

$\Gamma^{(q)}$ can be computed for QPSK signaling as follows [11]:

$$\Gamma^{(q)} = \frac{1}{\sqrt{2}}\tanh\left[\frac{\sqrt{2}}{\sigma^2}\mathrm{Re}(\hat{\mathbf{Z}}^{(q)})\right] + \frac{j}{\sqrt{2}}\tanh\left[\frac{\sqrt{2}}{\sigma^2}\mathrm{Im}(\hat{\mathbf{Z}}^{(q)})\right], \tag{14}$$

where

$$\hat{\mathbf{Z}}^{(q)} = \mathbf{C}^{T}\left(\widehat{\mathcal{H}}^{\dagger^{(q)}}\widehat{\mathcal{H}}^{(q)} + \sigma^2\mathbf{I}_{N_c}\right)^{-1}\widehat{\mathcal{H}}^{\dagger^{(q)}}\mathbf{R}. \tag{15}$$

Finally, the data b transmitted by each user can be estimated at the qth iteration step as

$$\hat{\mathbf{b}}^{(q)} = \frac{1}{\sqrt{2}}\mathrm{csign}(\Gamma^{(q)}), \tag{16}$$

where "csign" is defined as $\mathrm{csign}(a + jb) = \mathrm{sign}(a) + j\,\mathrm{sign}(b)$.

Truncation property

A truncated expansion vector $\mathbf{G}_r$ be formed from G by selecting r orthonormal basis vectors among all basis vectors that satisfy $\mathbf{C_H}\Psi = \Psi\Lambda$. The optimal solution that yields the smallest average mean-squared truncation error $(1/N_c)E[\epsilon_r^{\dagger}\epsilon_r]$ is the one expanded with the orthonormal basis vectors associated with the first largest r eigenvalues as given by

$$\frac{1}{N_c - r}E[\epsilon_r^{\dagger}\epsilon_r] = \frac{1}{N_c - r}\sum_{i=r}^{N_c}\lambda_i, \tag{17}$$

where $\epsilon_r = \mathbf{G} - \mathbf{G}_r$. For the problem at hand, truncation property of the KL expansion results in a low-rank approximation as well. Thus, a rank-r approximation of Λ can be defined as $\Lambda_r = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_r\}$ by ignoring the trailing $N_c - r$ variances $\{\lambda_l\}_{l=r}^{N_c}$, since they are very small compared to the leading r variances $\{\lambda_l\}_{l=1}^{r}$. Actually, the pattern of eigenvalues for Λ typically splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious. For instance, if the number of parameters in the expansion include dominant eigenvalues, it is possible to obtain a good approximation with a relatively small number of KL coefficients.

Complexity

Based on the approach presented in [17], the traditional LMMSE estimation for H can be easily expressed as

$$\hat{\mathbf{H}} = \mathbf{C_H}\underbrace{\left[\mathbf{C_H} + \Sigma\left(\mathrm{diag}(\mathbf{C}\mathbf{b})\mathrm{diag}(\mathbf{C}\mathbf{b})\right)^{-1}\right]^{-1}}_{\text{"}O(N_c^3)\text{" computational complexity}} \times \left[\mathrm{diag}(\mathbf{C}\mathbf{b})\right]^{-1}\mathbf{R}. \tag{18}$$

Since $\left[\mathbf{C_H} + \Sigma(\mathrm{diag}(\mathbf{C}\mathbf{b})\mathrm{diag}(\mathbf{C}\mathbf{b})^{-1}\right]^{-1}$ changes with data symbols, its inverse cannot be precomputed and has high computational complexity due to required large-scale matrix inversion.[1] Moreover, the error caused by the small fluctuations in $\mathbf{C_H}$ and Σ have an amplified effect on the channel estimation due to the matrix inversion. Furthermore, this effect becomes more severe as the dimension of the matrix, to be inverted, increases [18]. Therefore, the KL-based approach is needed to avoid large-scale matrix inversion. Using (2) and (11), the iterative estimate of H with KL expansion can be obtained as

$$\hat{\mathbf{H}}^{(q+1)} = \Psi\left(\mathbf{T}^{\dagger^{(q)}}\mathbf{T}^{(q)} + \Sigma\Lambda^{-1}\right)^{-1}\mathbf{T}^{\dagger^{(q)}}\mathbf{R}. \tag{19}$$

However, in this form, complexity of channel estimate is greater than the traditional LMMSE estimate. Therefore, to reduce the complexity of the estimator further we rewrite (19) as

$$\hat{\mathbf{H}}^{(q+1)} = \Psi\Lambda\left(\Lambda\mathbf{T}^{\dagger^{(q)}}\mathbf{T}^{(q)}\Lambda + \Sigma\Lambda\right)^{-1}\Lambda\mathbf{T}^{\dagger^{(q)}}\mathbf{R} \tag{20}$$

[1] The computational complexity of an $N_c \times N_c$ matrix inversion, using Gaussian elimination is $O(N_c^3)$.

TABLE 1

Algorithm	Computational complexity
LMMSE	$2N_c^2 + 5N_c + N_cK + O(N_c^3)$
KL	$N_c^3 + 4N_c^2 + N_cK + 2N_c + O(N_c^3)$
KL-truncated	$N_cr^2 + 3N_cr + r^2 + N_cK + 2r + O(r^3)$

and proceed with the low-rank approximations by considering only r column vectors of Ψ and $\mathbf{T}$ corresponding to the r largest eigenvalues of Λ, yielding

$$\hat{\mathbf{H}}^{(q+1)} = \Psi_r\Lambda_r \underbrace{\left(\Lambda_r\mathbf{T}_r^{\dagger(q)}\mathbf{T}_r^{(q)}\Lambda_r + \Sigma_r\Lambda_r\right)^{-1}}_{\text{"}O(r^3)\text{" computational complexity}} \Lambda_r\mathbf{T}_r^{\dagger(q)}\mathbf{R}, \qquad (21)$$

where Σ_r is an $r \times r$ diagonal matrix whose elements are equal to σ^2. Ψ_r and $\mathbf{T}_r$ are in (21) an $N_c \times r$ matrices which can be formed by omitting the last $N_c - r$ columns of Ψ and $\mathbf{T}$, respectively. Equation (21) can then be rearranged as follows:

$$\hat{\mathbf{H}}^{(q+1)} = \Psi_r\left(\mathbf{T}_r^{\dagger(q)}\mathbf{T}_r^{(q)} + \Sigma_r\Lambda_r^{-1}\right)^{-1}\mathbf{T}_r^{\dagger(q)}\mathbf{R}. \qquad (22)$$

Thus, the low-rank expansion yields an excellent approximation with a relatively small number of KL coefficients. Computational complexity has been evaluated quantitatively and summarized in Table 1.

5. PARALEL INTERFACE CANCELLATION (PIC)

The estimated complex QPSK vector $\hat{\mathbf{b}}$ given by (16) is passed to a PIC module after last iteration. In this module, the calculation of all interfering signals for user k can be written as

$$\mathbf{R}_{\text{int}}^k = \hat{\mathcal{H}}\mathbf{C}\hat{\mathbf{b}} \quad \text{for } \hat{b}^k = 0. \qquad (23)$$

Interfering signals for user k subtracted from the received signal $\mathbf{R}$, then passed to the single user detector. Finally, the PIC detector for kth user can be written as

$$b_{\text{pic}}^k = (\mathbf{c}^k)^T[\hat{\mathcal{H}}(\mathbf{R} - \mathbf{R}_{\text{int}}^k)] \quad \text{for } k = 1,\ldots,K. \qquad (24)$$

For the last iteration, detected symbols for QPSK modulation are

$$\hat{b}_{\text{pic}}^k = \frac{1}{\sqrt{2}}\text{csign}\,(b_{\text{pic}}^k) \quad \text{for } k = 1,\ldots,K. \qquad (25)$$

Initialization

Given the received signal R, the EM algorithm starts with an initial value $\mathbf{G}^{(0)}$ of the unknown channel parameters G. Corresponding to pilot symbols, we focus on a under-sampled signal model and employ the linear minimum mean-square error (LMMSE) estimate to obtain the under-sampled channel parameters. Then the complete initial channel gains can easily be determined using an interpolation technique, that is, Lagrange interpolation algorithm. Finally, the initial values of $\mathbf{G}_\mu^{(0)}$ are used in the iterative EM algorithm to avoid divergence. The details of the initialization process are presented in [11, 17].

6. MODIFIED CRAMER-RAO BOUND

The modified Fisher information matrix (FIM) can be obtained by a straightforward modification of FIM as [11],

$$\mathbf{J}_M(\mathbf{G}) \triangleq \underbrace{-E\left[\frac{\partial^2 \ln p(\mathbf{R} \mid \mathbf{G})}{\partial\mathbf{G}^*\partial\mathbf{G}^T}\right]}_{\mathbf{J}(\mathbf{G})} \underbrace{- E\left[\frac{\partial^2 \ln p(\mathbf{G})}{\partial\mathbf{G}^*\partial\mathbf{G}^T}\right]}_{\mathbf{J}_P(\mathbf{G})}, \qquad (26)$$

where $\mathbf{J}_p(\mathbf{G})$ represents the *a priori* information.

Under the assumption that G and W are independent of each other and W is a zero-mean Gaussian vector, the transmitted signals become uncorrelated due to the orthogonal spreading codes. The conditional PDF of R given G can be obtained by averaging $p(\mathbf{R} \mid \mathbf{b}, \mathbf{G})$ over b as follows

$$p(\mathbf{R} \mid \mathbf{G}) = E_{\mathbf{b}}\{p(\mathbf{R} \mid \mathbf{b}, \mathbf{G})\}. \qquad (27)$$

From (27), the derivatives can be taken as follows:

$$\frac{\partial \ln p(\mathbf{R} \mid \mathbf{G})}{\partial\mathbf{G}^T} = \frac{1}{\sigma^2}(\mathbf{R} - \text{diag}(\mathbf{C}\mathbf{b})\Psi\mathbf{G})^\dagger\text{diag}(\mathbf{C}\mathbf{b})\Psi,$$
$$\frac{\partial^2 \ln p(\tilde{\mathbf{R}} \mid \mathbf{G})}{\partial\mathbf{G}^*\partial\mathbf{G}^T} = -\frac{1}{\sigma^2}\tilde{\Psi}^\dagger\text{diag}(\mathbf{b}^T\mathbf{C}^T)\text{diag}(\mathbf{C}\mathbf{b})\tilde{\Psi}. \qquad (28)$$

Second term in (26) is easily obtained as follows:

$$\frac{\partial \ln p(\mathbf{G})}{\partial\mathbf{G}^T} = -\mathbf{G}^\dagger\Lambda^{-1}, \qquad \frac{\partial^2 \ln p(\mathbf{G})}{\partial\mathbf{G}^*\partial\mathbf{G}^T} = -\Lambda^{-1}. \qquad (29)$$

Taking the negative expectations, the first and the second term in (26) becomes $\mathbf{J}(\mathbf{G}) = (1/\sigma^2)\mathbf{I}_{N_c}$ and $\mathbf{J}_P(\mathbf{G}) = \Lambda^{-1}$, respectively. Finally, (26) produces for the modified FIM as follows:

$$\mathbf{J}_M(\mathbf{G}) = \frac{1}{\sigma^2}\mathbf{I}_{N_c} + \Lambda^{-1}. \qquad (30)$$

Inverting the matrix $\mathbf{J}_M(\mathbf{G})$ yields $\text{MCRB}(\hat{\mathbf{G}}) = \mathbf{J}_M^{-1}(\mathbf{G})$. $\text{MCRB}(\hat{\mathbf{G}})$ is a diagonal matrix with the elements on the main diagonal equaling the reciprocal of those $\mathbf{J}(\mathbf{G})$ matrices.

7. SIMULATIONS

In this section, performance of the MC-CDMA system based on the proposed receivers is investigated by computer simulations operating over frequency selective channels. In simulation, we assume that all users receive the same power. The orthogonal Gold sequence code is selected as spreading code and the processing gain equals to the number of subcarriers. The assumption of a full-load system is made throughout the simulations except Figure 4, that is the number of active users K, is equal to the length of the spreading code $N_c = 128$.

The correlative channel coefficients, H, have exponentially decaying power delay profiles, described by $\theta(\tau_\mu) = C\exp(-\tau_\mu/\tau_{\text{rms}})$. The delays τ_μ are uniformly and independently distributed over the length of the cyclic prefix. τ_{rms} determines the decay of the power-delay profile and C is

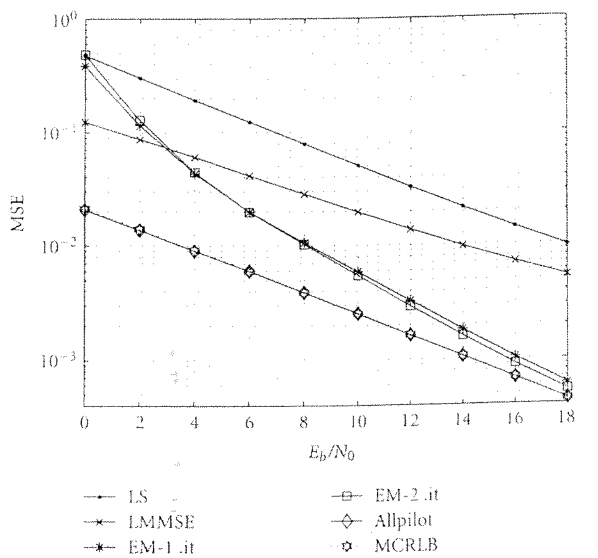


FIGURE 2: Comparison of different channel estimation algorithms (MSE).

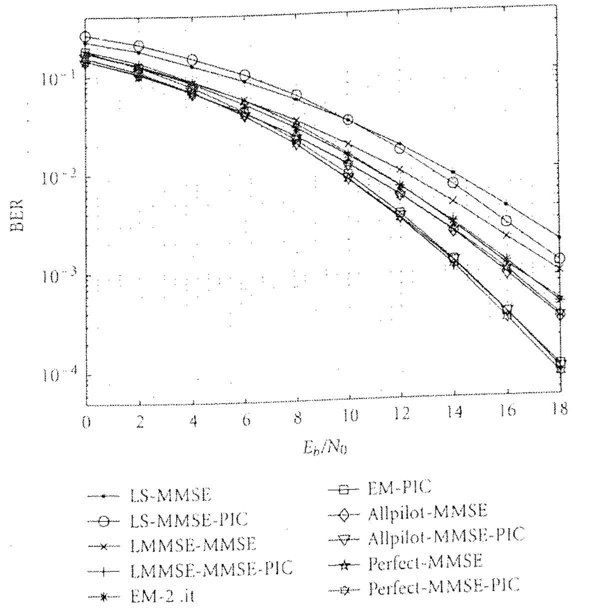


FIGURE 3: BER performances of receiver structures for full load system.

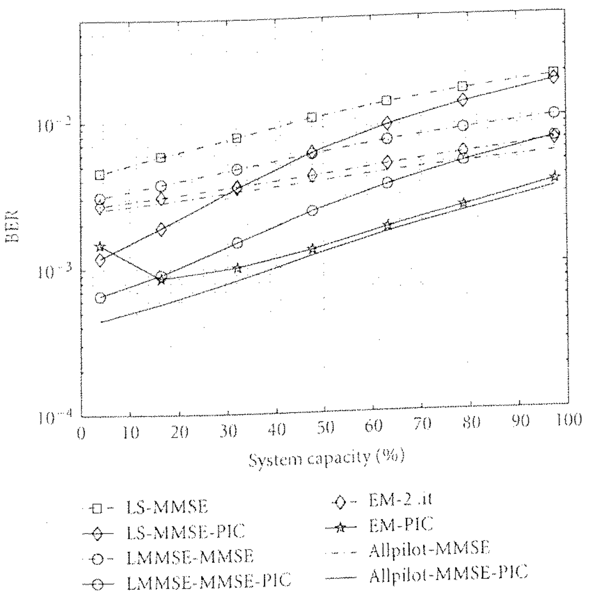


FIGURE 4: BER performances of receiver structures in terms of system capacity usage.

the normalizing constant. Note that the normalized discrete channel-correlations for different subcarriers and blocks of this channel model were presented in [17] as follows:

$$C_H(k,k') = \frac{1-\exp\left[-L\left(1/\tau_{rms}+2\pi j(k-k')/N_c\right)\right]}{\tau_{rms}\left(1-\exp\left(-L/\tau_{rms}\right)\right)\left(1/\tau_{rms}+j2\pi(k-k')/N_c\right)} \tag{31}$$

where (k, k') denotes different subcarriers, L is the cyclix prefix, N_c is the total number of subcarriers. The system has an 800 KHz bandwith and is divided into $N_c = 128$ tones with a total period T_s of 165 microseconds, of which 5 microseconds constitute the cyclic prefix ($L = 4$). We assume that the rms value of the multipath width is $\tau_{rms} = 1$ sample (1.25 microseconds) for the power-delay profile. With the τ_{rms} value chosen and to avoid ISI, the guard interval duration is chosen to be equal to 4 sample (5 microseconds)[17].

7.1. *Performance evaluation*

The performance merits of the proposed structure over other candidates are confirmed by corroborating simulations. Figure 2 compares the MSE performance of the EM-MAP channel estimation approach with a widely used LS and LMMSE pilot symbol assisted modulation (PSAM) schemes [14], as well as all-pilot estimation for MC-CDMA systems. Pilot insertion rate (PIR) was chosen as PIR =1 : 8 That is one pilot is inserted for every 8 data symbols. It is observed that the proposed EM-MAP significantly outperforms the LS as well as LMMSE techniques and approaches the all-pilot estimation case and the MCRLB at higher E_b/N_0 values. Moreover, the BER performance of the proposed system is also studied for different detection schemes in Figure 3. It is shown that the BER performance of the proposed receiver structure is much better that the combined MMSE-PIC receiver in the case of LS, LMMSE while approaches the performance of the all-pilot and perfect channel estimation cases.

We also determined BER performance of the algorithm as a function of the system capacity usage for $E_b/N_0 = 12$ dB. As shown in Figure 4, the BER performance will degrade as the total capacity usage approaches full load for both two detection schemes. On the other hand, our simulation results

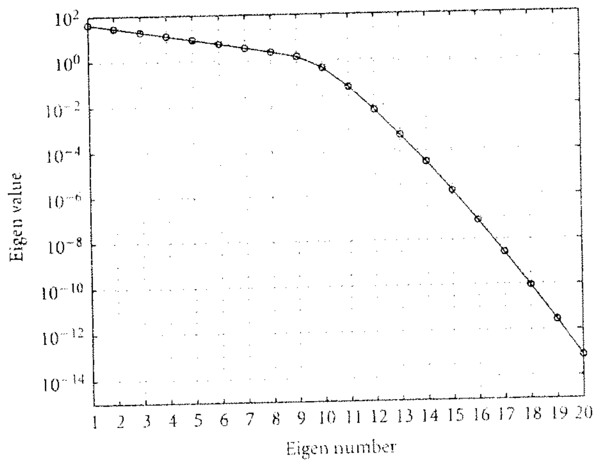


FIGURE 5: Eigenvalue spectrum.

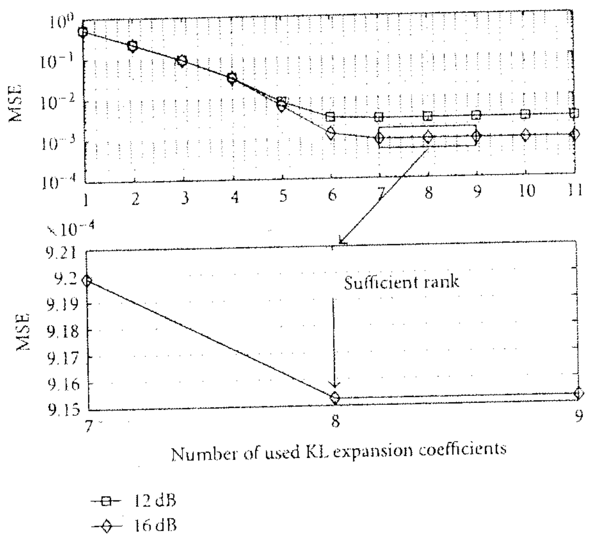


FIGURE 6: Optimal truncation property of the KL expansion.

show that the performance difference between MMSE and MMSE-PIC detection becomes more distinguishable as the total active users decreases.

7.2. The optimal truncation property

The KL expansion minimizes the amount of information required to represent the statistically dependent data. Thus, this property can further reduce the computational load of the channel estimation algorithm. An example of the Eigenspectrum is shown in Figure 5 for the correlation matrix of the channel given in (31). Since the Eigenspectrum of the correlation matrix among different frequencies has an exponential profile, a reduced set of channel parameters can be employed. Therefore, the optimal truncation property of the KL expansion is exploited in Figure 6 where MSE performances versus the number of coefficient used KL expansion are given for 12 dB and 16 dB. If the number of parameters in the expansion includes dominant eigenvalues (Rank = 8), it is possible to obtain an excellent approximation with a relatively small number of KL coefficients.

7.3. Mismatch simulations

Once the true frequency-domain correlation, characterizing the channel statistics and the SNR values, is known, the channel estimator can be designed as indicated in Section 4. In the previous simulations, the autocorrelation matrix and the SNR were assumed to be available as *a priori* information at the receiver. However, in practice the true channel correlation and the SNR are not known. It is then important to analyze the performance degradation due to a mismatch of the estimator to the channel statistics to check its robustness to the variation of these parameters.

Correlation mismatch

We designed the estimator for a uniform channel correlation which gives the worst MSE performance among all channel models and evaluated it for an exponentially decaying power delay profile. Note that as τ_{rms} goes to infinity, the power delay profile of the channel given by (31) approaches to the uniform power delay profile with autocorrelation

$$\tilde{C}_H(k,k') = \begin{cases} 1 & \text{if } k = k' \\ \dfrac{1 - e^{-j2\pi L(k-k')/N_c}}{2\pi j L(k-k')/N_c} & \text{if } k \neq k'. \end{cases} \quad (32)$$

Figure 7 demonstrates the estimator's sensitivity to the channel statistics as a function of the average MSE performance for the following mismatch cases.

Case 1. True statistic $\Rightarrow \tau_{rms} = 1, L = 4, N_c = 128$.
Mismatch $\Rightarrow \tau_{rms} = \infty, L = 4, N_c = 128$.

Case 2. True statistic $\Rightarrow \tau_{rms} = 1, L = 8, N_c = 128$.
Mismatch $\Rightarrow \tau_{rms} = \infty, L = 8, N_c = 128$.

From the mismatch curves presented in Figure 7, it is seen that for Case 1, practically there is no mismatch degradation observed when the estimator is designed for mismatched channel statistics specified above. Thus, we conclude that the estimator is quite robust against the channel correlation mismatch for Case 1. For Case 2 frequency selectivity of the channel is increased by increasing the channel length L. In this case, we observed that the mismatch performance of the estimator was degraded moderately. In fact, the performance degradation between true and mismatch cases is approximately 0.9 dB for BER $= 10^{-3}$.

In Figure 8, we investigate again sensitivity of estimator to the channel statistics between the true correlation with $\tau_{rms} = 1$ and the effective channel length $L = 4$ against $\tau_{rms} = \infty$ and for $L = 2, 3, 4, 5, 6$. We conclude from the mismatch curves presented in Figure 8 that the mismatch affects substantially on the MSE performance when L is less than the correct channel length, and affects less when L is greater than the correct channel length.

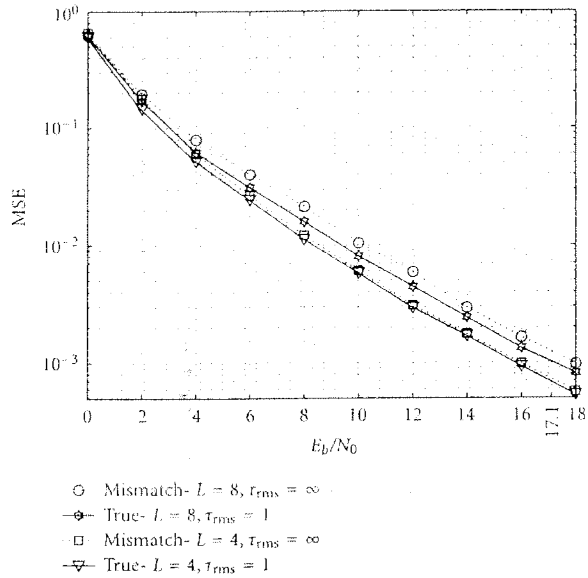


FIGURE 7: Correlation mismatch for τ_{rms}.

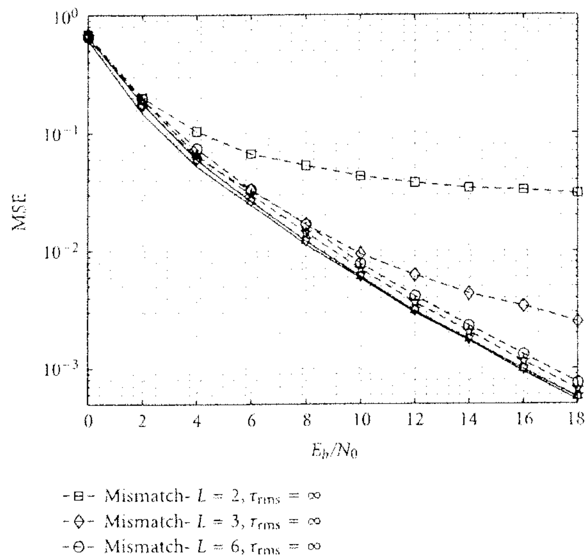


FIGURE 8: Correlation mismatch for L and τ_{rms}.

SNR mismatch

The BER curves for a design SNR of 5 dB, 10 dB, and 15 dB are shown in Figure 9 with the true SNR performance. The performance of the EM-MAP estimator for high SNR (15 dB) design is better than low-SNR (5 dB) design across a range of SNR values (10–18 dB). These results confirm that the channel estimation error is concealed in noise for low SNR whereas it tends to dominate for high SNR. Thus, the system performance degrades especially at low-SNR region.

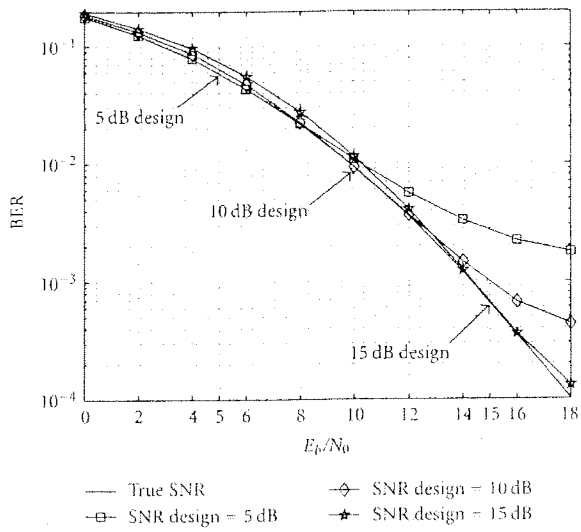


FIGURE 9: SNR mismatch.

8. CONCLUSIONS

In this work we have presented an efficient EM-MAP channel-estimation-based PIC receiver structure for downlink MC-CDMA systems. This algorithm performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing MPSK modulation scheme with additive Gaussian noise. Furthermore, the advantage of this algorithm, besides its simple implementation, is that the channel estimation is instantaneous, since the signal and the pilot are orthogonal code division multiplexed (OCDM) and they are distorted at the same time. Moreover, it was shown that KL expansion without optimal truncation property did not enable us to estimate the channel in a very simple way without taking inverse of large dimensional matrices for MC-CDMA systems. Computer simulation results have indicated that the MSE and BER performance of the proposed algorithm is well over the conventional algorithms and approaches to the MCRLB by iterative improvement. Finally, we have also investigated the effect of modelling mismatch on the estimator performance. It was concluded that the performance degradation due to such mismatch is negligible especially at low SNR values.

ACKNOWLEDGMENTS

This work was supported in part by the Turkish Scientific and Technical Research Institute (TUBITAK) under Grant no. 104E166 and the Research Fund of Istanbul University under Projects UDP-889/22122006, UDP- 921/09052007, T-856/02062006. This research has been also conducted within the NEWCOM++ Network of Excellence in Wireless Communications funded through the EC 7th Framework Programme. Part of the results of this paper was presented at the IEEE Wireless Communications and Networking Conference (WCNC-2007), March 11–15 2007, Hong Kong.

REFERENCES

[1] N. Yee, J.-P. Linnarz, and G. Fettweis, "Multi-carrier CDMA in indoor wireless radio networks," in *Proceedings of the 4th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC '93)*, pp. 109–113, Yokohama, Japan, September 1993.

[2] K. Fazel and L. Papke, "On the performance of convolutionally-coded CDMA/OFDM for mobile communication system," in *Proceedings of the 4th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC '93)*, pp. 468–472, Yokohama, Japan, September 1993.

[3] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Communications Magazine*, vol. 35, no. 12, pp. 126–133, 1997.

[4] S. Iraji, T. Sipila, and J. Lilleberg, "Channel estimation and signal detection for MC-CDMA in multipath fading channels," in *Proceedings of the 4th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC '93)*, Yokohama, Japan, September 1993.

[5] V. Kuhn, "Combined MMSE-PIC in coded OFDM-CDMA systems," in *Proceedings of the IEEE Global Conference on Telecommunications (Globecom '01)*, pp. 231–235, San Antonio, Tex, USA, November 2001.

[6] M. Feder and E. Weinstein, "Parameter Estimation of superimposed signals using the EM algorithm," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 36, no. 4, pp. 477–489, 1988.

[7] Y. Xie and C. N. Georghiades, "Two EM-type channel estimation algorithms for OFDM with transmitter diversity," *IEEE Transactions on Communications*, vol. 51, no. 1, pp. 106–115, 2003.

[8] L. B. Nelson and H. V. Poor, "Iterative multiuser receivers for CDMA channels: an EM-based approach," *IEEE Transactions on Communications*, vol. 44, no. 12, pp. 1700–1710, 1996.

[9] A. Kocian and B. H. Fleury, "EM-based joint data detection and channel estimation of DS-CDMA signals," *IEEE Transactions on Communications*, vol. 51, no. 10, pp. 1709–1720, 2003.

[10] A. A. D'Amico, U. Mengali, and M. Morelli, "Channel estimation for the uplink of a DS-CDMA system," *IEEE Transactions on Wireless Communications*, vol. 2, no. 6, pp. 1132–1137, 2003.

[11] H. A. Çirpan, E. Panayirci, and H. Dogan, "Nondata-aided channel estimation for OFDM systems with space-frequency transmit diversity," *IEEE Transactions on Vehicular Technology*, vol. 55, no. 2, pp. 449–457, 2006.

[12] H. Dogan, H. A. Çirpan, and E. Panayirci, "Iterative channel estimation and decoding of turbo coded SFBC-OFDM systems," *IEEE Transactions on Wireless Communications*, vol. 6, no. 8, pp. 3090–3101, 2007.

[13] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1993.

[14] K. W. Yip and T.-S. Ng, "Karhunen-Loeve expansion of the WSSUS channel output and its application to efficient simulation," *IEEE Journal on Selected Areas in Communications*, vol. 15, no. 4, pp. 640–646, 1997.

[15] B. Yang, Z. Cao, and K. B. Letaief, "Analysis of low-complexity windowed DFT-based MMSE channel estimator for OFDM systems," *IEEE Transactions on Communications*, vol. 49, no. 11, pp. 1977–1987, 2001.

[16] O. Edfors, M. Sandell, J. J. van de Beek, S. K. Wilson, and P. O. Borjesson, "Analysis of DFT-based channel estimation for OFDM," *Wireless Personal Communications*, vol. 12, no. 1, pp. 55–70, 2000.

[17] O. Edfors, M. Sandell, J.-J. van de Beek, S. K. Wilson, and P. O. Borjesson, "OFDM channel estimation by singular value decomposition," *IEEE Transactions on Communications*, vol. 46, no. 7, pp. 931–936, 1998.

[18] J. Zhu and W. Lee, "A low-complexity channel estimator for OFDM systems in multipath fading channels," in *Proceedings of 15th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC '04)*, vol. 3, pp. 1978–1982, Barcelona, Spain, September 2004.

[19] S. Coleri, M. Ergen, A. Puri, and A. Bahai, "Channel estimation techniques based on pilot arrangement in OFDM systems," *IEEE Transactions on Broadcasting*, vol. 48, no. 3, pp. 223–229, 2002.

LECTURES NOTES IN ELECTRICAL ENGINEERING | 1

Simon Plass
Stefan Kaiser
Armin Dammann
Khaled Fazel
Editors

Multi-Carrier Spread Spectrum 2007

Plass · Dammann
Kaiser · Fazel · *Eds.*

Multi-Carrier Spread Spectrum 200

78-1-4020-6128-8

In the most favorable cases given in Tab. 1 only N_{CL} extra bits are needed. For a data rate of 24 Mbits/s, when the PSDU consists of 1498 bytes, the efficiency loss is $v = 0.07\%$. In the worst case N_{DBPS} bits are needed, resulting in 0.79% efficiency loss. In view of the potential bit error rate performance improvement seen in Fig. 3 b), notably for large E_b/N_0, our proposal seems to be justified.

7. Conclusions and Outlook

We described a method to construct a postamble symbol in OFDM block-oriented transmission from the Tail and Pad bits of the payload and the replacement of a few user data bits by zero bits. Channel estimation is then carried out by a WF, which interpolates between the ML estimates of the pre- and postamble. Compared to channel estimation techniques based on smoothing across successive preambles, the proposed scheme does not extend processing beyond a single frame and therefore reduces processing delay and complexity. The experimental results showed significant bit error rate improvements for large E_b/N_0 compared to channel estimation schemes utilizing only the preamble.

In future research we try to improve the performance by inserting non-zero bits to increase the energy of the generated postamble and taking into account KF based estimates instead of instantaneous estimates. The effects of non-perfect synchronization and phase noise have to be investigated, too.

References

[1] Z. Wang and G. Giannakis, *Wireless multicarrier communications*, IEEE Signal Processing Magazine, vol. 17, no. 3, pp. 29-48, 2000.

[2] A. R. S. Bahai, B. R. Saltzberg and M. Ergen, *Multi-Carrier Digital Communications: Theory and Applications of OFDM*, Springer, 2004.

[3] IEEE, *Part11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications - High-speed Physical Layer in the 5 GHz Band*, IEEE Std. 802.11a-1999.

[4] B. O'Hara and A. Petrick, *IEEE 802.11 Handbook - A Designer's Companion*, Standards Information Network IEEE Press, 2001.

[5] K.-D. Kammeyer, *Nachrichtenuebertragung*, Teubner, Ed. 3, 2004.

[6] K.-D. Kammeyer. *Berichtigungen und Ergaenzungen zum Buch Nachrichtenuebertragung*, http://www.ant.uni-bremen.de/pub/books/nue/correction/index.html.

[7] T. Roman, M. Enescu and V. Koivunen, *Time domain method for tracking dispersive channels in OFDM systems*, Proc. IEEE VTC Spring 2003, Jeju, Korea.

JOINT DATA DETECTION AND CHANNEL ESTIMATION FOR UPLINK MC-CDMA SYSTEMS OVER FREQUENCY SELECTIVE CHANNELS

Erdal Panayirci[1]*, Hakan Doğan[2], Hakan A. Çirpan[2] and Bernard H. Fleury[3]

[1] *Kadir Has University, Department of Electronics Enginering, Cibali 34083, Istanbul, Turkey*
epanayirci@khas.edu.tr

[2] *Istanbul University, Department of Electrical and Electronics Engineering, Avcilar 34850, Istanbul, Turkey*
hdogan@istanbul.edu.tr,hcirpan@istanbul.edu.tr

[3] *Aalborg University, Section Navigation and Communications, Department of Electronic Systems, Fredrik Bajers Vej 7A3 DK-9000 Aalborg, Denmark and Forschungszentrum Telekommunikation Wien (ftw.) Donau-City-Strasse 1 A-1220 Vienna, Austria*
bfl@kom.auc.dk

Abstract This paper is concerned with joint channel estimation and data detection for uplink multicarrier code-division multiple-access (MC-CDMA) systems in the presence of frequency fading channel. The detection and estimation, implemented at the receiver, are based on a version of the expectation maximization (EM) algorithm which is very suitable for the the multicarrier signal formats. Application of the EM-based algorithm to the problem of iterative data detection and channel estimation leads to a receiver structure that also incorporates a partial interference cancelation. Computer simulations show that the proposed algorithm has excellent BER end estimation performance.

Keywords: Joint data detection and channel estimation, MC-CDMA Systems, EM algorithm, Superimposed signals

*This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 6th Framework Programme and the Research Fund of Istanbul University under Projects T-856/02062006. This work was also supported in part by the Turkish Scientific and Technical Research Institute(TUBITAK) under Grant 104E166.

1. Introduction

MC-CDMA transmission through the frequency-selective fading channels causes the signal-to-noise ratio (SNR) degradation and the occurrence of the multiple-access interference (MAI). To eliminate or reduce the resulting performance degradations, equalization and detection of the received signal can be performed at the receiver based on the complete channel information. In conventional MC-CDMA systems, MAI mitigation is accomplished at the receiver using single-user or multi-user detection schemes [1]. However, multiuser detection scheme is known to increase the bandwidth efficiency of the system drastically, although its detection complexity grows exponentially with the number of users and the number of multipaths, which makes it infeasible to implement. Several suboptimal detection techniques have proposed in literature such as linear multiuser detection[2] and iterative cancelation of MAI, either in a successive or parallel way in the received signal prior to data detection [3]. Therefore data detection and channel estimation should be performed perfectly for a good initialization of the interference cancelation detector.

In this paper we consider an efficient iterative algorithm based on the Expectation-Maximization (EM) technique for multi-user data detection and channel estimation, jointly for uplink MC-CDMA systems in the presence of frequency selective fading channels. As will be seen shortly, a partial parallel interference cancelation (PIC) is incorporated into the resulting detection algorithm [4]. The work is an extension of [5] in which joint data detection and channel estimation of downlink DS-CDMA systems were considered based on an EM algorithm in the presence of flat Rayleigh channels. The channel estimation becomes more challenging for uplink systems since each channel between every user and the base station must be estimated rather that estimating a single channel in case of a down-link transmission.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; $(.)^*$, $(.)^T$ and $(.)^H$ denote the conjugate, transpose and conjugate transpose, respectively; $\|.\|$ denotes the Frobenius norm; $\mathbf{I}_L$ denotes the $L \times L$ identity matrix; $diag\{.\}$ denotes a diagonal matrix; and finally, $tr\{.\}$ denotes the trace of a matrix.

2. Signal Model

We consider a baseband MC-CDMA uplink system with P sub-carriers and K mobile users which are simultaneously active. For the kth user, each transmit symbol is modulated in the frequency domain by means of a $P \times 1$ specific spreading sequence $\mathbf{c}_k$. After transforming by a P-point IDFT and parallel-to-serial (P/S) conversion, a cyclic prefix (CP) is inserted of length equal to at least the channel memory (L). In this work, to simplify the notation, it is assumed that the spreading factor equals to the number of sub-carriers and all

users have the same spreading factor. Finally, the signal is transmitted through a multipath channel with impulse response

$$h_k(t) = \sum_{k=1}^{L} g_{k,l}\delta(t - \tau_{k,l}) \quad (1)$$

where L is the number of paths in the kth user's channel; g_{kl} and τ_{kl} are, respectively, the complex fading processes and the delay of the kth user's lth path. It is assumed that only the second-order statistics of the fading processes are known to the receiver. Also, fading can vary from symbol to symbol but remains constant over a symbol interval.

At the receiver, the received signal is first serial-to-parallel (S/P) converted, then CP is removed and DFT is then applied to the received discrete time signal to obtain the received vector expressed as

$$\mathbf{y}(m) = \sum_{k=1}^{K} b_k(m)\mathbf{C}_k\mathbf{F}\mathbf{h}_k + \mathbf{w}(m) \quad m = 1, 2, ..., M \quad (2)$$

where $b_k(m)$ denotes data sent by the user k within the mth symbol; $\mathbf{C}_k = \text{diag}(\mathbf{c}_k)$ with $\mathbf{c}_k = [c_{k1}, c_{k1}, ..., c_{kP}]^T$ where each chip, c_{ik}, takes values in the set $\{-\frac{1}{\sqrt{P}}, \frac{1}{\sqrt{P}}\}$ denoting the kth user's spreading code ; $\mathbf{F} \in \mathbb{C}^{P\times L}$ denotes the DFT matrix with the (k,l)th element given by $\frac{1}{\sqrt{P}}e^{-j2\pi kl/P}$; and $\mathbf{w}(m)$ is the $P \times 1$ zero-mean, i.i.d. Gaussian vector that models the additive noise in the P tones, with variance $\sigma^2/2$ per dimension.

Suppose M symbols are transmitted. We stack $\mathbf{y}(m)$ as $\mathbf{y}=[\mathbf{y}^T(1), ..., \mathbf{y}^T(M)]^T$. Then the received signal model can be written as

$$\mathbf{y} = \begin{bmatrix} b_1(1)\mathbf{C}_1\mathbf{F} & \cdots & b_K(1)\mathbf{C}_K\mathbf{F} \\ \vdots & \ddots & \vdots \\ b_1(M)\mathbf{C}_1\mathbf{F} & \cdots & b_K(M)\mathbf{C}_K\mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix} + \begin{bmatrix} \mathbf{w}(1) \\ \vdots \\ \mathbf{w}(M) \end{bmatrix} \quad (3)$$

and can be rewritten in more succinct form

$$\mathbf{y} = \mathbf{A}\mathbf{h} + \mathbf{w} \quad (4)$$

By using the assumption; $\mathbf{h}_k \sim N(0, \Sigma_{\mathbf{h}_k})$ where $\Sigma_{\mathbf{h}_k} = E[\mathbf{h}_k\mathbf{h}_k^\dagger]$, we have $\mathbf{h} \sim N(0, \Sigma_\mathbf{h})$ with $\Sigma_\mathbf{h} = \text{diag}[\Sigma_1, \cdots, \Sigma_K]$.

3. Joint Data Detection and Channel Estimation

The problem of interest is to derive an iterative algorithm based on the EM technique for data detection and channel estimation jointly employing the signal model given by (2). Since the EM method has been studied and applied to

$$-\beta_k \sum_{j=1, j\neq k}^{K} (b_j(m))^{[i]} E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{F} \mathbf{h}_j | \mathbf{y}, \mathbf{b}^{(i)}\}. \quad (18)$$

On the other hand, since $\mathbf{w} \sim N(0, \Sigma^2 \mathbf{I})$ and the prior pdf of $\mathbf{h}$ is chosen as $\mathbf{h} \sim N(0, \Sigma_{\mathbf{h}})$, we can write the conditional pdf's of $\mathbf{h}$ given $\mathbf{y}$ and $\mathbf{b}^{(i)}$ as

$$\begin{aligned} p(\mathbf{h}|\mathbf{y}, \mathbf{b}^{(i)}) &\sim p(\mathbf{y}|\mathbf{h}, \mathbf{b}^{(i)}) p(\mathbf{h}) \\ &\sim \exp\left[-\frac{1}{\sigma^2}(\mathbf{y} - \mathbf{A}\mathbf{h})^\dagger (\mathbf{y} - \mathbf{A}\mathbf{h}) - \mathbf{h}^\dagger \Sigma_{\mathbf{h}}^{-1} \mathbf{h}\right]. \quad (19) \end{aligned}$$

After some algebra it can be shown that

$$p(\mathbf{h}|\mathbf{y}, \mathbf{b}^{(i)}) \sim N(\mu_{\mathbf{h}}^{(i)}, \Sigma_{\mathbf{h}}^{(i)}) \quad (20)$$

where

$$\begin{aligned} \mu_{\mathbf{h}}^{(i)} &= \frac{1}{\sigma^2} \Sigma_{\mathbf{h}}^{(i)} \mathbf{A}^{(i)\dagger} \mathbf{y} \\ \Sigma_{\mathbf{h}}^{(i)} &= \left[\Sigma_{\mathbf{h}}{}^{-1} + \frac{1}{\sigma^2} (\mathbf{A}^{(i)})^\dagger \mathbf{A}^{(i)}\right]^{-1} \quad (21) \end{aligned}$$

and the matrix $\mathbf{A}$ is defined in (3)and (4).

Now let us compute the terms on the right hand side of Eq. (17). We calculate $E\{\mathbf{h}_k^\dagger \mathbf{h}_k | \mathbf{y}, \mathbf{b}^{(i)}\}$ as

$$(\|\mathbf{h}_k\|^2)^{[i]} \triangleq E\{\mathbf{h}_k^\dagger \mathbf{h}_k | \mathbf{y}, \mathbf{b}^{(i)}\} = \mathrm{tr}\left[\Sigma_{\mathbf{h}}^{(i)}[k,k] + \mu_h^{(i)}[k] \mu_h^{(i)}[k]^\dagger\right] \quad (22)$$

where $\Sigma_{\mathbf{h}}[i,j]$ denotes the (i,j)th element of the matrix $\Sigma_{\mathbf{h}}$ Second expectation in (18) can be computed as

$$(\mathbf{h}_k)^{[i]} \triangleq E\{\mathbf{h}_k | \mathbf{y}, \mathbf{b}^{(i)}\} = \mu_h^{(i)}[k]. \quad (23)$$

Finally, to calculate the last expectation $E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{F} \mathbf{h}_j | \mathbf{y}, \mathbf{b}^{(i)}\}$ in (18), define $\Psi_j \triangleq \mathbf{C}_j \mathbf{F}$, $\mathbf{s}_j \triangleq \Psi_j \mathbf{h}_j$. It then follows that

$$\Sigma_s^{(i)} \triangleq E[\mathbf{s}\mathbf{s}^\dagger | \mathbf{y}, \mathbf{b}^{(i)}] = E[\Psi \mathbf{h}\mathbf{h}^\dagger \Psi^\dagger | \mathbf{y}, \mathbf{b}^{(i)}] = \Psi \Sigma_{\mathbf{h}}^{(i)} \Psi^\dagger. \quad (24)$$

Therefore,

$$E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{F} \mathbf{h}_j | \mathbf{y}, \mathbf{b}^{(i)}\} = E[\mathbf{s}^\dagger \mathbf{s} | \mathbf{y}, \mathbf{b}^{(i)}] = \mathrm{tr}\left[\Sigma_s^{(i)}[k,j] + \mu_s^{(i)}[k] \mu_s^{(i)}[j]^\dagger\right] \quad (25)$$

where, $\mu_s^{(i)} \triangleq \Psi \mu_h^{(i)}$

Maximization-Step (M-Step): The second step to implement the EM algorithm is the *M-Step* where the parameter $\mathbf{b}$ is updated at the $(i+1)$th iteration according to

$$\mathbf{b}^{(i+1)} = \arg\max_{\mathbf{b}} Q(\mathbf{b}|\mathbf{b}_i) = \sum_{k=1}^{K} Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}). \quad (26)$$

M-Step can be performed by maximizing the terms $Q_k(\mathbf{b}_k|\mathbf{b}^{(i)})$ individually, as follows

$$\mathbf{b}_k^{(i+1)} = \arg\max_{\mathbf{b}_k} Q_k(\mathbf{b}_k|\mathbf{b}_i) \quad (27)$$

Moreover, when no coding is used, it follows from (27) that each component of $\mathbf{b}_k^{(i+1)}$ can be separately obtained by maximizing the corresponding summation in the right-hand expression, as follows

$$b_k^{(i+1)}(l) = \mathrm{sgn}\left[\Re\{(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]}\}\right]. \quad (28)$$

We finally have

$$\begin{aligned} b_k^{i+1}(l) &= \mathrm{sgn}\Bigg[\Re\Bigg\{\frac{1}{P} b_k^{[i]}(m) \|\mu_h^{(i)}[k]\|^2 (1-\beta_k) \\ &+ \beta_k (\mathbf{h}_k^{(i)\dagger} \Psi_k^T \left[\mathbf{y}(m) - \sum_{j=1, j\neq k}^{K} b_j^{[i]}(m) \Psi_j \mu_h^{(i)}[j]\right]\Bigg\}\Bigg]. \quad (29) \end{aligned}$$

As a conclusion, Equation (29) can be interpreted as joint channel estimation and data detection with partial interference cancelation. At each incretion step during data detection, the interference reduced signal is fed into a single user receiver consisting of a conventional coherent detector. As a result, a K-user optimization problem have been decomposed into K independent optimization problems which can e solved in a computationally feasible way. Finally we remark that this paper is an extension of the work presented by Fleury to the problem of joint channel estimation and data detection for the uplink multi-carrier CDMA systems operating in the presence of the frequency selective channels.

4. Simulations

In this section, performance of uplink MC-CDMA system based on proposed receiver is simulated for frequency selective channels. In simulations, we assume that all users are received the same power. The orthogonal Walsh sequences selected as spread code and processing gain equals to the number of subcarriers Nc = 16. The number of users is selected as U=8 and each user send frame ,that is composed of T preamble bits, and F data bits, over mobile fading channel. Wireless channels between mobiles antennas and receiver antenna are assumed to be complex gaussian channel of length L and it has distribution $N(0, Ch)$. For all simulations weight coefficients in (28) are chosen to be equal, i.e., $B_k = 1/U$

At the receiver, initial MMSE channel estimate is obtained by using T preamble bits while channel covariance matrix C_h is assumed to be know. Initial MMSE estimate of F data bits is computed from the observation of y while assuming we have estimated channel coefficients. We refer to this method for obtaining h and b as MMSE separate detection and estimation (MMSE-SDE). In the simulations, if the output of the (MMSE-SDE) is applied to paralel interference cancellation (PIC) receiver ,that we will compare with EM-JDE, is referred as Combined MMSE-PIC receiver. Moreover, we also simulated MMSE and Combined MMSE-PIC detectors for the perfect channel state information cases that are referred as CSI-MMSE and CSI-Combined MMSE-PIC respectively.

Fig. 1 compares the BER performance of the proposed EM-JDE approach with MMSE-SDE, Combined-MMSE-PIC, CSI-MMSE and CSI-Combined MMSE-PIC. For fair comparison we simulated Combined-MMSE-PIC, CSI-Combined MMSE-PIC and EM-JDE receivers for three of iterations. It is observed that the proposed EM-JDE outperforms the MMSE-SDE,Combined-MMSE-PIC as well as CSI-MMSE and approaches the CSI-Combined MMSE-PIC for higher Eb/No values.

To investigate initialization effect on the EM-JDE and Combined-MMSE-PIC, we also demonstrate BER performance of both algorithms for different preamble bits in Fig. 2. Lower preamble bits provide poor initial estimates, resulting in a more BER performance degradation in Combined-MMSE-PIC as compared to EM-JDE systems. On the other for higher preamble bits it was observed that performance difference between Combined-MMSE-PIC and EM-JDE decreases because of considerably good initial estimates of channel coefficients.

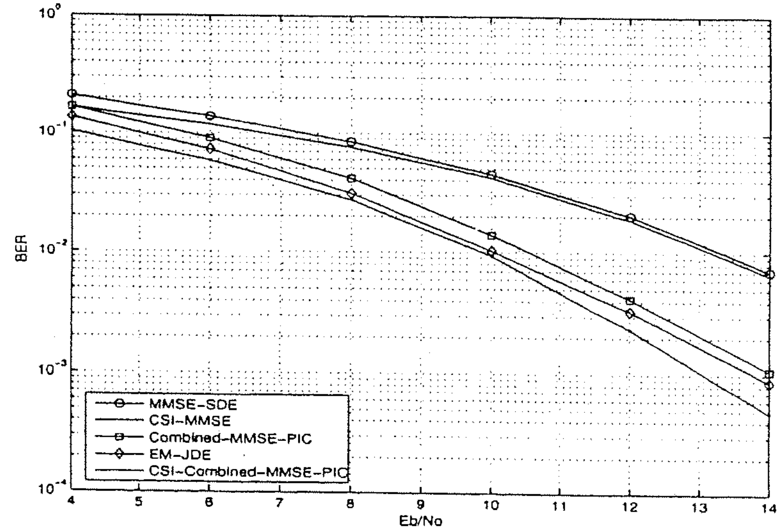


Figure 1. BER performance in frequency selective channels (T=4,F=40,L=8,U=8)

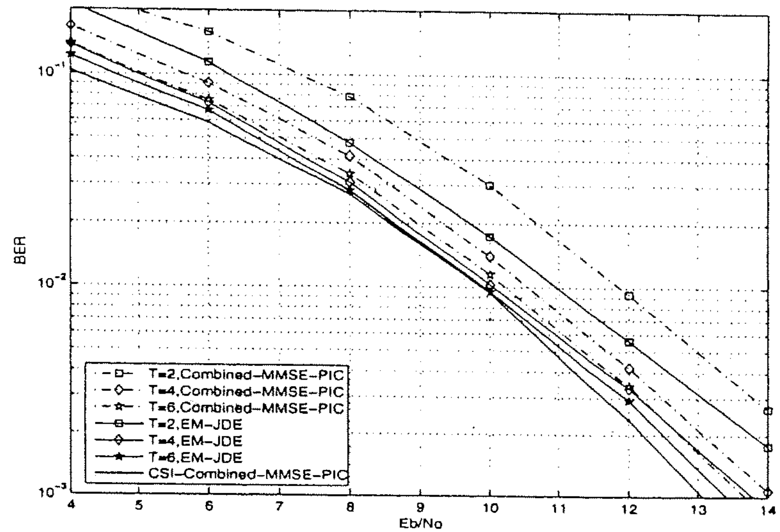


Figure 2. Behavior of the BER performance as a function of used preamble bits (F=40,L=8,U=8)

a number of problems in communications over the years, the details of the algorithm will not be presented in this paper. The reader is suggested to read [6] for a general exposition to EM algorithm and [8] for its application to the estimation problem related to the work herein. The suitable approach for applying the EM algorithm for the problem at hand is to decompose the received signal in (2) into the sum [7]

$$\mathrm{y}(m) = \sum_{k=1}^{K} \mathrm{x}_k(m), \quad m = 1, 2 \cdots, M \tag{5}$$

where

$$\mathrm{x}_k(m) \;=\; b_k(m)\mathrm{C}_k\mathrm{F}\mathrm{h}_k + \mathrm{w}_k(m). \tag{6}$$

$\mathrm{x}_k(m)$ represents the received signal component transmitted by the kth user through the channel with impulse response h_k. The Gaussian noise vector, $\mathrm{w}_k(m)$ ýn (6) represents the portion of $\mathrm{w}(m)$ in the decomposition defined by $\sum_{k=1}^{K} \mathrm{w}_k(m) = \mathrm{w}(m)$, whose variance is $N_0\beta_k$. The coefficients β_k determine the part of the noise power of $\mathrm{w}(m)$ assigned to $\mathrm{x}_k(m)$, satisfying $\sum_{k=1}^{K} \beta_k = 1, 0 \leq \beta_k \leq 1$.

The problem now is to estimate the transmitted symbols $\mathrm{b} = \{b_k(m)\}_{k=1 m=1}^{K,M}$ and the complex channel responses h_k for each user based on observed data y. In the EM algorithm, we view the observed data y as the "incomplete" data, and define the "complete" data as $\chi = \{(\mathrm{x}_1, \mathrm{h}_1), (\mathrm{x}_1, \mathrm{h}_1), \ldots, (\mathrm{x}_K, \mathrm{h}_K)\}$ where $\mathrm{x}_k = [\mathrm{x}_k(1), \ldots, \mathrm{x}_k(M)]^T$ for $k = 1, 2, \cdots, K$.

The EM algorithm: Given the complete data set as $\chi = \{(\mathrm{x}_1, \mathrm{h}_1), \ldots (\mathrm{x}_2, \mathrm{h}_2), \ldots, (\mathrm{x}_K, \mathrm{h}_K)\}$, the loglikelihood function of the parameter vector to be estimated b can be expressed as

$$\log p(\chi|\mathrm{b}) = \sum_{k=1}^{K} \log p(\mathrm{x}_k, \mathrm{h}_k|\mathrm{b}_k) \tag{7}$$

where

$$\log p(\mathrm{x}_k, \mathrm{h}_k|\mathrm{b}_k) = \log p(\mathrm{x}_k|\mathrm{b}_k, \mathrm{h}_k) + \log p(\mathrm{h}_k|\mathrm{b}_k) \tag{8}$$

and, $\mathrm{b}_k = [b_k(1), b_k(2), \cdots, b_k(M)]$. We neglect the $\log p(\mathrm{h}_k|\mathrm{b}_k)$ term in (7) since the data sequence b_k and h_k are independent of each other.

The first step to implement the EM algorithm, called the *Expectation Step (E-Step)*, is the computation of the average log-likelihood function. The conditional expectation is taken over χ given the observation y and that b equals its estimate calculated at ith iteration.

$$Q\left(\mathrm{b}|\mathrm{b}^{(i)}\right) = E\left\{\log p(\chi|\mathrm{b}|\mathrm{y}, \mathrm{b}^{(i)}\right\} \tag{9}$$

Taking into account the special form of $\log p(\chi|\mathrm{b})$ in (7), Eq. (8) can be decomposed as

$$Q\left(\mathrm{b}|\mathrm{b}^{(i)}\right) = \sum_{k=1}^{K} Q_k(\mathrm{b}_k|\mathrm{b}^{(i)}) \tag{10}$$

where

$$Q_k(\mathrm{b}_k|\mathrm{b}^{(i)}) \;=\; E\left\{\log p(\mathrm{x}_k, \mathrm{h}_k|\mathrm{b}_k)|\mathrm{y}, \mathrm{b}^{(i)}\right\} \tag{11}$$

$$= E\left\{\log p(\mathrm{x}_k|\mathrm{b}_k, \mathrm{h}_k)|\mathrm{y}, \mathrm{b}^{(i)}\right\}. \tag{12}$$

Note that (11) follows from (8).

Neglecting the terms independent of b_k, From (6), $\log p(\mathrm{x}_k|b_k, \mathrm{h}_k)$ can be calculated as

$$\log p(\mathrm{x}_k|\mathrm{b}_k, \mathrm{h}_k) \sim \sum_{m=1}^{M} \Re\{b_k(m)\mathrm{h}_k^\dagger \mathrm{F}^\dagger \mathrm{C}_k^T \mathrm{x}_k(m)\}. \tag{13}$$

Inserting (13) in (12), we have for $Q_k(\mathrm{b}_k|\mathrm{b}^{(i)})$

$$Q_k(\mathrm{b}_k|\mathrm{b}^{(i)}) = \sum_{m=1}^{M} \Re\{b_k(m)(\mathrm{h}_k^\dagger \mathrm{F}^\dagger \mathrm{C}_k^T \mathrm{x}_k(m))^{[i]}\} \tag{14}$$

where

$$(\mathrm{h}_k^\dagger \mathrm{F}^\dagger \mathrm{C}_k^T \mathrm{x}_k(m))^{[i]} \triangleq E\left\{\mathrm{h}_k^\dagger \mathrm{F}^\dagger \mathrm{C}_k^T \mathrm{x}_k(m)|\mathrm{y}, \mathrm{b}^{(i)}\right\} \tag{15}$$

$(\mathrm{h}_k^\dagger \mathrm{F}^\dagger \mathrm{C}_k^T \mathrm{x}_k(m))^{[i]}$ can be calculated by applying the conditional expectation rule as

$$\begin{aligned}(\mathrm{h}_k^\dagger \mathrm{F}^\dagger \mathrm{C}_k^T \mathrm{x}_k(m))^{[i]} \;&=\; E\{\mathrm{h}_k^\dagger E(\mathrm{F}^\dagger \mathrm{C}_k^T \mathrm{x}_k(m)|\mathrm{y}, \mathrm{b}^{(i)}, \mathrm{h})|\mathrm{y}, \mathrm{b}^{(i)}\} \\ &=\; E\{\mathrm{h}_k^\dagger \mathrm{F}^\dagger \mathrm{C}_k^T E(\mathrm{x}_k(m)|\mathrm{y}, \mathrm{b}^{(i)}, \mathrm{h})|\mathrm{y}, \mathrm{b}^{(i)}.\}\end{aligned} \tag{16}$$

The conditional distribution of $\mathrm{x}_k(m)$ given y, h and $\mathrm{b} = \mathrm{b}^{(i)}$ is Gaussian with mean

$$\begin{aligned}E(\mathrm{x}_k(m)|\mathrm{y}, \mathrm{b}^{(i)}, \mathrm{h}) \;&=\; (b_k(m))^{[i]}\mathrm{C}_k\mathrm{F}\mathrm{h}_k \\ &+\; \beta_k\left(\mathrm{y}(m) - \sum_{j=1}^{K}(b_j(m))^{[i]}\mathrm{C}_j\mathrm{F}\mathrm{h}_j\right)\end{aligned} \tag{17}$$

where $(b_k(m))^{[i]} \triangleq E(b_k(m)|\mathrm{y}, \mathrm{b}^{(i)}, \mathrm{h})$. Inserting (17) in (15) and using the properties $\mathrm{F}^\dagger\mathrm{F} = \mathrm{I}$ and $\mathrm{C}_k^T\mathrm{C}_k = \frac{1}{P}\mathrm{I}$. we can rewrite (15) as

$$(\mathrm{h}_k^\dagger \mathrm{F}^\dagger \mathrm{C}_k^T \mathrm{x}_k(m))^{[i]} = \frac{1}{P}(b_k(m))^{[i]} E\{\mathrm{h}_k^\dagger \mathrm{h}_k|\mathrm{y}, \mathrm{b}^{(i)}\} + \beta_k E\{\mathrm{h}_k^\dagger|\mathrm{y}, \mathrm{b}^{(i)}\}\mathrm{F}^\dagger \mathrm{C}_k^T \mathrm{y}(m)$$

References

[1] S. Hara and R. Prasad, "Overview of Multicarrier CDMA," *IEEE Communication Magazine*, pp. 126Ū133, December 1997.

[2] Z. Xie, R.T. Short and C.K.Rushfortli: "Family of suboptimum detectors for coherent multiuser communications." *IEEE Journal on Selected Areas in Communications*, vol. 8, no. 4, pp. 683-690, May 1990.

[3] V. Kuhn. "Combined MMSE-PIC in coded OFDM-CDMA systems." *IEEE GLOBECOM "01*, pp. 231-235, Nov. 2001.

[4] S. Iraji, T. Sipilia and J. Lilleberg, "Channel Estimation and Signal Detection for MC-CDMA in Multipath Fading Channels," *in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC)*, September 1993

[5] A. Kocian and B. H. Fleury. "EM-based joint data detection and channel estimation of DS-CDMA signals," *IEEE Trans. Commun.*, vol. 51, no. 10, pp. 1709Ū-1720, Oct. 2003.

[6] G.K. Kaleh and R. Valet. "Joint parameter estimation and symbol detection for linear or nonlinear unknown channels." *IEEE Trans. Commun.* vol. 42, No. 7, pp. 2406-2413. July 1994.

[7] M. Feder and E. Weinstein, "Parameter Estimation of superimposed signals using the EM algorithm,"*IEEE Tran. on Acoustic, Speech and Signal Processing*, Vol. 36, pp. 477-489, April 1988.

[8] H. Dogan, H. A. Cirpan and Erdal Panayirci, "Iterative Channel Estimation and Decoding of Turbo Coded SFBC-OFDM Systems," *IEEE Trans. Wireless Commun.*, accepted for publication in Oct. 2006.

ON THE PERFORMANCE OF MC-CDMA SYSTEMS WITH PARTIAL EQUALIZATION IN THE PRESENCE OF CHANNEL ESTIMATION ERRORS

Flavio Zabini
WiLab, Univ. of Bologna, Italy
fzabini@deis.unibo.it

Barbara M. Masini
WiLab, Univ. of Bologna, Italy
bmasini@deis.unibo.it

Andrea Conti
WiLab, ENDIF, Univ. of Ferrara
aconti@ieee.org

Abstract The paper investigates the performance of a multi-carrier code division multiple access (MC-CDMA) system adopting a partial equalization (PE) technique at the receiver. It has been recently shown that, in spite of the fact that the PE technique has the same complexity of the well known maximal ratio combining, orthogonality restoring combining and equal gain combining techniques, the PE significantly improves the performance in terms of bit error probability (BEP). Previous works considered PE with ideal channel state information (CSI) and evaluate the PE parameter which minimizes the BEP; here we relax this assumption considering the presence of CSI errors. In particular, we analytically derive the mean BEP for the downlink of a MC-CDMA system and the optimal PE parameter, when imperfect CSI is assumed. Numerical results show that the optimum choice of the PE with ideal CSI provides significant performance improvement also in the presence of CSI errors.

HW Communications

Home | About HW | Capabilities | Services | Events | News | Contact Us

Registration

Please download a Registration Form

Accomodation

Accommodation List

Programme

Download

Co Sponsors

Communications Research Network

IEEE

Rinicom Ltd

Ninth International Symposium on Communication Theory and Applications
ISCTA'07

16th - 20th July, 2007, Ambleside, Lake District, UK

Final Call for Papers

A major objective of the Symposium will be to pursue the progression from communication and information theory through to the implementation, evaluation performance of practical communication systems of various types. You are invited to submit original papers in the following and related areas:

Digital Transmission and Recording
Source and Channel Coding
Modulation, Detection, Channel Estimation
Channel Modelling, Synchronisation
Optical and Magnetic Recording

Special Topics in Channel Coding, Source Coding, Information Theory
Turbo Codes, Low Density Parity Check Codes
Source Coding and Data Compression
Privacy, Secrecy and Security
Multi-Functional Coding
Sequences and Arrays
Fountain, LT, etc. codes

Detection Techniques
Vector Detection, Multiuser Detection
Combined Equalisation, Decoding and Channel Estimation
Iterative (Turbo) Schemes

Broadband Techniques
Ultra Wideband
Fixed Wireless Access

Space-Time Techniques
Information-Theoretic Aspects, Channel Capacities
Space-Time-Coding, Signal Constellations
Spatial Spreading, Linear Dispersion Codes
MIMO Detection and Channel Estimation
MIMO Precoding, Writing on Dirty Paper
Time Reversal

Systems, Multiple Access, Protocols
Communication System Architectures
3G and Beyond-3G Wireless Communication Systems
Sensor Networks
Ad Hoc Networks
Multiple Access Techniques, Protocols
Multimedia Networking, wireless e-learning, e-commerce and e-health

Realisation
DSP for Communication Systems
MIMO Demonstrators
Complexity Considerations
Implementation of LDPC decoders

Fusion of Communication and Location
GNSS-based
Probabilistic algorithms
Applications

11:30-12:00 **Efficient Scalar Quantization for MIMO Spatial Multiplexing Receivers**
F. A. Monteiro and I. J. Wassell (University of Cambridge)

12:00-12:20 **Constructing Coset Codes with Optimal Same-Symbol Weight for Detecting Narrowband Interference in M-FSK Systems**
D.J.J. Versfeld, A.J.H. Vinck, J. N. Ridley and H.C. Ferreira (University of Johannesburg, South Africa)

12:20-12:40 **Joint Data Detection and Channel Estimation for Uplink MC-CDMA Systems with EM-Based algorithms**
Erdal Panayirci, Hakan Dogan, Hakan A.Cirpan, Bernard H. Fleury (Istanbul University and Aalborg University)

12:40-12:45 **ISCTA'07 CLOSING REMARKS**
Professor Michael Darnell

12:45-14:00 **LUNCH**
Dining Room, St. Martin's College

POSTER SESSIONS

POSTER I

Tuesday
17th July

Adaptive BCH Encoder/Decoder for DVB-S2 Applications Design and Implementation
Andrew Wells and Bahram Honary (Lancaster University)

Improved Error Protection using Adaptive PUM Codes
R. Wadekar, L. Fagoonee and b. Honary (Lancaster University)

Security and Coding Schemes over Powerline Communications
A. Falahati, V. Vakil and D. Abbasi Moghadam (Iran)

Iterative Soft-Decoding for Multilevel Two Dimensional Optical Storage Technology
V.Zarimpas, A. Moinian and B. Honary (Lancaster University)

OFDM Scheme in Impulsive Noise Channels
D. Abbasi Moghadam, A. Falahati and V.T. Vakili (Iran)

Joint Data Detection and Channel Estimation for Uplink MC-CDMA Systems with EM-Based algorithms

Erdal Panayırcı, Hakan Doğan, Hakan A. Çırpan, Bernard H. Fleury

Abstract—**This paper is concerned with joint multiuser detection and multichannel estimation (JDE) for uplink multicarrier code-division multiple-access (MC-CDMA) systems in the presence of frequency fading channel. The detection and estimation, implemented at the receiver, are based on a version of the expectation maximization (EM) algorithm and the space-alternating generalized the expectation-maximization (SAGE) which are very suitable for the the multicarrier signal formats. The EM-JDE receiver updates the data bit sequences in parallel, while the SAGE-JDE receiver reestimates them successively. The channel parameters are updated in parallel in both schemes. Application of the EM-based algorithm to the problem of iterative data detection and channel estimation leads to a receiver structure that also incorporates a partial interference cancelation. Computer simulations show that the proposed algorithm has excellent BER end estimation performance.**

I. Introduction

In conventional MC-CDMA systems, multiple access interference(MAI) mitigation is accomplished at the receiver using single-user or multi-user detection schemes [1]. However, multiuser detection scheme is known to increase the bandwidth efficiency of the system drastically, although its detection complexity grows exponentially with the number of users and the number of multipaths, which makes it infeasible to implement. Several suboptimal detection techniques have proposed in literature such as linear multiuser detection[2] and iterative cancelation of MAI, either in a successive or parallel way in the received signal prior to data detection [3]. Therefore, it is necessary to make excellent joint data and channel estimation for initialization of the interference cancelation detector. The work is an extension of [4] in which joint data detection and channel estimation of uplink DS-CDMA systems were considered based on an EM algorithm in the presence of flat Rayleigh channels. We extend their results for the uplink MC-CDMA systems with frequency selective channels. The channel estimation becomes more challenging for uplink systems since the each channel between every user and the base station must be estimated rather than estimating a single channel in case of a down-link transmission. In this paper, we apply the EM and the SAGE algorithms to the problem of joint multiuser data detection and channel estimation (JDE) of MC-CDMA signals transmitted through frequency selective channels. In this way, we obtain iterative methods of tractable complexity which smartly combine the two processes of data detection and channel estimation.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; $(.)^*$, $(.)^T$ and $(.)^H$ denote the conjugate, transpose and conjugate transpose, respectively; $\|.\|$ denotes the Frobenius norm; $\mathbf{I}_L$ denotes the $L \times L$ identity matrix; $diag\{.\}$ denotes a diagonal matrix; and finally, $tr\{.\}$ denotes the trace of a matrix.

This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 6th Framework Programme and the Research Fund of Istanbul University under Projects T-856/02062006. This work was also supported in part by the Turkish Scientific and Technical Research Institute (TUBITAK) under Grant 104E166.

Hakan Doğan and Hakan A. Çırpan are with the Department of Electrical and Electronics Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey, e-mail: {hdogan,hcirpan}@istanbul.edu.tr

Erdal Panayırcı is with the Department of Electronics Engineering, Kadir Has University, Cibali 34083, Istanbul, Turkey, e-mail: eepanay@khas.edu.tr

Bernard H. Fleury is with the Section Navigation and Communications, Department of Electronic Systems, Aalborg University Fredrik Bajers Vej 7A3 DK-9000 Aalborg, Denmark and Forschungszentrum Telekommunikation Wien (ftw.) Donau-City-Strasse 1 A-1220 Vienna, Austria e-mail: bfl@kom.auc.dk

II. Signal Model

We consider a baseband MC-CDMA uplink system with P sub-carriers and K mobile users which are simultaneously active. For the kth user, each transmit symbol is modulated in the frequency domain by means of a $P \times 1$ specific spreading sequence $\mathbf{c}_k$. After transforming by a P-point IDFT and parallel-to-serial (P/S) conversion, a cyclic prefix (CP) is inserted of length equal to at least the channel memory (L). In this work, to simplify the notation, it is assumed that the spreading factor equals to the number of sub-carriers and all users have the same spreading factor. Finally, the signal is transmitted through a multipath channel with impulse response

$$h_k(t) = \sum_{k=1}^{L} g_{k,l}\delta(t - \tau_{k,l}) \quad (1)$$

where L is the number of paths in the kth users channel; g_{kl} and τ_{kl} are, respectively, the complex fading processes and the delay of the kth users lth path. It is assumed that only the second-order statistics of the fading processes are known to the receiver. Also, fading can vary from symbol to symbol but remains constant over a symbol interval.

In the receiver, the received signal is first serial-to-parallel (S/P) converted, CP is removed and DFT is then applied to the received discrete time signal to obtain the received vector expressed as

$$\mathbf{y}(m)=\sum_{k=1}^{K} b_k(m)\mathbf{C}_k\mathbf{F}\mathbf{h}_k+\mathbf{w}(m), \quad m = 1, 2, ..., M \quad (2)$$

where $b_k(m)$ denotes data sent by the user k within the mth symbol time; $\mathbf{C}_k = \text{diag}(\mathbf{c}_k)$ with $\mathbf{c}_k = [c_{k1}, c_{k1}, ..., c_{kP}]^T$ where each chip, c_{ik}, takes values in the set $\{-\frac{1}{\sqrt{P}}, \frac{1}{\sqrt{P}}\}$ denoting the kth users spreading code ; $\mathbf{F} \in \mathbb{C}^{P\times L}$ denotes the DFT matrix with the (k,l)th element given by $\frac{1}{\sqrt{P}}e^{-j2\pi kl/P}$; and $\mathbf{w}(m)$ is the $P \times 1$ zero-mean, i.i.d. Gaussian vector that models the additive noise in the P tones, with variance $\sigma^2/2$ per dimension. Note that the channel impulse response $\mathbf{g}_k$ and the transmission power P_k are assumed to be combined as $\mathbf{h}_k = \sqrt{P_k}\mathbf{g}_k$, since they can not be separated from each other.

Suppose M symbols are transmitted. We stack $\mathbf{y}(m)$ as $\mathbf{y} = [\mathbf{y}^T(1), ..., \mathbf{y}^T(M)]^T$. Then the received signal model can be written as

$$\mathbf{y}=\begin{bmatrix} b_1(1)\mathbf{C}_1\mathbf{F} & \cdots & b_K(1)\mathbf{C}_K\mathbf{F} \\ \vdots & \ddots & \vdots \\ b_1(M)\mathbf{C}_1\mathbf{F} & \cdots & b_K(M)\mathbf{C}_K\mathbf{F} \end{bmatrix}\begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix}+\begin{bmatrix} \mathbf{w}(1) \\ \vdots \\ \mathbf{w}(M) \end{bmatrix} \quad (3)$$

and can be rewritten in more succinct form

$$\mathbf{y} = \mathbf{A}\mathbf{h} + \mathbf{w}. \quad (4)$$

By using the assumption; $\mathbf{h}_k \sim N(0, \Sigma_{\mathbf{h}_k})$ where $\Sigma_{\mathbf{h}_k} = E[\mathbf{h}_k\mathbf{h}_k^\dagger]$, we have $\mathbf{h} \sim N(0, \Sigma_{\mathbf{h}})$ with $\Sigma_{\mathbf{h}} = \text{diag}[\Sigma_{\mathbf{h}_1}, \cdots, \Sigma_{\mathbf{h}_K}]$. Note that due to the orthogonality property of spreading sequences, $\mathbf{C}^T\mathbf{C} = \mathbf{I}$.

III. Joint Data Detection and Channel Estimation with EM and SAGE Algorithms

A. EM-JDE Receiver

Let $\mathbf{b}$ denote a possibly vector-valued parameter to be estimated from some possibly vector-valued observation $\mathbf{y}$ with probability density $p(\mathbf{y}|\mathbf{b})$. The EM algorithm provides an iterative scheme to approach the ML estimate $\hat{\mathbf{b}} = \underset{\mathbf{b}}{argmax}p(\mathbf{y}|\mathbf{b})$ in cases where a direct computation of $\hat{\mathbf{b}}$ is prohibitive. The derivation of the EM algorithm relies on the concept of a hypothetical, so-called complete unobservable data χ which, if it could be observed, would ease the estimation of $\mathbf{b}$. The observed random variable $\mathbf{y}$ which is referred to as the incomplete data within the EM framework, is related to χ by a mapping $\chi \mapsto \mathbf{y}(\chi)$.

The suitable approach for applying the EM algorithm for the problem at hand is to decompose the received signal in 2 into the sum [5]

$$\mathbf{y}(m) = \sum_{k=1}^{K} \mathbf{x}_k(m), \quad m = 1, 2\cdots, M \quad (5)$$

where

$$\mathbf{x}_k(m) = b_k(m)\mathbf{C}_k\mathbf{F}\mathbf{h}_k + \mathbf{w}_k(m). \quad (6)$$

$\mathbf{x}_k(m)$ represents the received signal component transmitted by the kth user through the channel with impulse response $\mathbf{h}_k$. The Gaussian noise vector, $\mathbf{w}_k(m)$ in (6) represents the portion of $\mathbf{w}(m)$ in the decomposition defined by $\sum_{k=1}^{K}\mathbf{w}_k(m) = \mathbf{w}(m)$, whose variance is $N_0\beta_k$. The coefficients β_k determine the part of the noise power of $\mathbf{w}(m)$ assigned to $\mathbf{x}_k(m)$, satisfying $\sum_{k=1}^{K}\beta_k = 1$, $0 \le \beta_k \le 1$.

The problem now is to estimate the transmitted symbols $\mathbf{b} = \{b_k(m)\}_{k=1,m=1}^{K,M}$ and the complex channel responses $\mathbf{h}_k$ for each user based on observed data $\mathbf{y}$. In the EM algorithm, we view the observed data $\mathbf{y}$ as the incomplete data, and define the complete data as $\chi = \{(\mathbf{x}_1, \mathbf{h}_1), (\mathbf{x}_2, \mathbf{h}_2), ..., (\mathbf{x}_K, \mathbf{h}_K)\}$ where $\mathbf{x}_k = [\mathbf{x}_k(1), ..., \mathbf{x}_k(M)]^T$ for $k = 1, 2, \cdots, K$. Given the complete data set, the loglikelihood function of the parameter vector to be estimated $\mathbf{b}$ can be expressed as

$$\log p(\chi|\mathbf{b}) = \sum_{k=1}^{K} \log p(\mathbf{x}_k, \mathbf{h}_k|\mathbf{b}_k) \quad (7)$$

where

$$\log p(\mathbf{x}_k, \mathbf{h}_k|\mathbf{b}_k) = \log p(\mathbf{x}_k|\mathbf{b}_k, \mathbf{h}_k) + \log p(\mathbf{h}_k|\mathbf{b}_k) \quad (8)$$

and, $\mathbf{b}_k = [b_k(1), b_k(2), \cdots, b_k(M)]$. We neglect the $\log p(\mathbf{h}_k|\mathbf{b}_k)$ term in (8) since the data sequence $\mathbf{b}_k$ and $\mathbf{h}_k$ are independent of each other.

The first step to implement the EM algorithm, called the *Expectation Step (E-Step)*, is to compute the average log-likelihood function. The conditional expectation is taken over χ given the observation $\mathbf{y}$ and that $\mathbf{b}$ equals its estimate calculated at ith iteration.

$$Q\left(\mathbf{b}|\mathbf{b}^{(i)}\right) = E\left\{\log p(\chi|\mathbf{b}|\mathbf{y}, \mathbf{b}^{(i)}\right\} \quad (9)$$

Taking into account the special form of $\log p(\chi|\mathbf{b})$ in

(5), Eq. 9 can be decomposed as

$$Q\left(\mathbf{b}|\mathbf{b}^{(i)}\right) = \sum_{k=1}^{K} Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) \quad (10)$$

where

$$Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) = E\left\{\log p(\mathbf{x}_k, \mathbf{h}_k|\mathbf{b}_k)|\mathbf{y}, \mathbf{b}^{(i)}\right\} \quad (11)$$

$$= E\left\{\log p(\mathbf{x}_k|\mathbf{b}_k, \mathbf{h}_k)|\mathbf{y}, \mathbf{b}^{(i)}\right\}. \quad (12)$$

Note that (12) follows from (8).

Neglecting the terms independent of $\mathbf{b}_k$, from (6), $\log p(\mathbf{x}_k|b_k, \mathbf{h}_k)$ can be calculated as

$$\log p(\mathbf{x}_k|\mathbf{b}_k, \mathbf{h}_k) \sim \sum_{m=1}^{M} \Re\{b_k(m)\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{x}_k(m)\}. \quad (13)$$

Inserting (13) in (12), we have for $Q_k(\mathbf{b}_k|\mathbf{b}^{(i)})$

$$Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) = \sum_{m=1}^{M} \Re\{b_k(m)(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{x}_k(m))^{[i]}\} \quad (14)$$

where, adopting the notation used in *Fleury*,

$$(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{x}_k(m))^{[i]} \triangleq E\left\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{x}_k(m)|\mathbf{y},\mathbf{b}^{(i)}\right\}. \quad (15)$$

$(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{x}_k(m))^{[i]}$ can be calculated by applying the conditional expectation rule as

$$\begin{aligned} &= E\{\mathbf{h}_k^{\dagger}E(\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{x}_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h})|\mathbf{y}, \mathbf{b}^{(i)}\} \\ &= E\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T E(\mathbf{x}_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h})|\mathbf{y}, \mathbf{b}^{(i)}\}. \end{aligned} \quad (16)$$

The conditional distribution of $\mathbf{x}_k(m)$ given $\mathbf{y}$, $\mathbf{h}$ and $\mathbf{b} = \mathbf{b}^{(i)}$ is Gaussian with mean

$$E(\mathbf{x}_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h}) = (b_k(m))^{[i]}\mathbf{C}_k\mathbf{F}\mathbf{h}_k + \beta_k\left(\mathbf{y}(m) - \sum_{j=1}^{K}(b_j(m))^{[i]}\mathbf{C}_j\mathbf{F}\mathbf{h}_j\right) \quad (17)$$

where $(b_k(m))^{[i]} \triangleq E(b_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h})$. Inserting (16) in (17) and using the properties $\mathbf{F}^{\dagger}\mathbf{F} = \mathbf{I}$ and $\mathbf{C}^T\mathbf{C} = \mathbf{I}$ we can rewrite (17) as

$$\begin{aligned} (\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{x}_k(m))^{[i]} &= (b_k(m))^{[i]}E\{\mathbf{h}_k^{\dagger}\mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\} \\ &+\beta_k E\{\mathbf{h}_k^{\dagger}|\mathbf{y}, \mathbf{b}^{(i)}\}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{y}(m) \\ &-\beta_k \sum_{j=1, j\neq k}^{K} (b_j(m))^{[i]}E\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j|\mathbf{y}, \mathbf{b}^{(i)}\}. \end{aligned} \quad (18)$$

On the other hand, since $\mathbf{w} \sim N(0, \sigma^2\mathbf{I})$ and the prior pdf of $\mathbf{h}$ is chosen as $\mathbf{h} \sim N(0, \mathbf{\Sigma}_{\mathbf{h}}^0)$, we can write the conditional pdf's of $\mathbf{h}$ given $\mathbf{y}$ and $\mathbf{b}^{(i)}$ as

$$\begin{aligned} p(\mathbf{h}|\mathbf{y}, \mathbf{b}^{(i)}) &\sim p(\mathbf{y}|\mathbf{h}, \mathbf{b}^{(i)})p(\mathbf{h}) \\ &\sim \exp\left[-\frac{1}{\sigma^2}(\mathbf{y} - \mathbf{A}\mathbf{h})^{\dagger}(\mathbf{y} - \mathbf{A}\mathbf{h}) - \mathbf{h}^{\dagger}\mathbf{\Sigma}_{\mathbf{h}}^{-1}\mathbf{h}\right]. \end{aligned} \quad (19)$$

After some algebra it can be shown that

$$p(\mathbf{h}|\mathbf{y}, \mathbf{b}^{(i)}) \sim N(\boldsymbol{\mu}_{\mathbf{h}}^{(i)}, \mathbf{\Sigma}_{\mathbf{h}}^{(i)}) \quad (20)$$

where

$$\begin{aligned} \boldsymbol{\mu}_{\mathbf{h}}^{(i)} &= \frac{1}{\sigma^2}\mathbf{\Sigma}_{\mathbf{h}}^{(i)}\mathbf{A}^{(i)\dagger}\mathbf{y} \\ \mathbf{\Sigma}_{\mathbf{h}}^{(i)} &= \left[\mathbf{\Sigma}_{\mathbf{h}}^{(i)-1} + \frac{1}{\sigma^2}(\mathbf{A}^{(i)})^{\dagger}\mathbf{A}^{(i)}\right]^{-1} \end{aligned} \quad (21)$$

and the matrix $\mathbf{A}$ is defined in (3) and (4).

Now let us compute the terms on the right hand side of Eq. (16). Firstly, we compute $E\{\mathbf{h}_k^{\dagger}\mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\}$ as follows. From (20) we have

$$E\{\mathbf{h}\mathbf{h}^{\dagger}|\mathbf{y}, \mathbf{b}^{(i)}\} = \mathbf{\Sigma}_{\mathbf{h}}{}^{(i)} + \boldsymbol{\mu}_h^{(i)}\boldsymbol{\mu}_h^{(i)\dagger}. \quad (22)$$

For the kth element we get

$$E\{\mathbf{h}_k\mathbf{h}_k^{\dagger}|\mathbf{y}, \mathbf{b}^{(i)}\} = \mathbf{\Sigma}_{\mathbf{h}}^{(i)}[k,k] + \boldsymbol{\mu}_h^{(i)}[k]\boldsymbol{\mu}_h^{(i)}[k]^{\dagger}. \quad (23)$$

where $\mathbf{\Sigma}_{\mathbf{h}}[i,j]$ denotes the (i,j)th element of the matrix $\mathbf{\Sigma}_{\mathbf{h}}$. We can then calculate $E\{\mathbf{h}_k^{\dagger}\mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\}$ from (22) as

$$\begin{aligned} (\|\mathbf{h}_k\|^2)^{[i]} &\triangleq E\{\mathbf{h}_k^{\dagger}\mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\} \\ &= \operatorname{tr}\left[\mathbf{\Sigma}_{\mathbf{h}}^{(i)}[k,k] + \boldsymbol{\mu}_h^{(i)}[k]\boldsymbol{\mu}_h^{(i)}[k]^{\dagger}\right] \end{aligned} \quad (24)$$

Second expectation in (18) can be computed as

$$(\mathbf{h}_k)^{[i]} \triangleq E\{\mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\} = \boldsymbol{\mu}_h^{(i)}[k]. \quad (25)$$

Finally, to calculate the last expectation $E\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j|\mathbf{y}, \mathbf{b}^{(i)}\}$ in (18), define $\mathbf{\Psi}_j \triangleq \mathbf{C}_j\mathbf{F}$ and $\mathbf{s}_j \triangleq \mathbf{\Psi}_j\mathbf{h}_j$. It then follows that

$$\mathbf{s} = \mathbf{\Psi}\mathbf{h} = \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_K \end{bmatrix} = \begin{bmatrix} \mathbf{\Psi}_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{\Psi}_K \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix}, \quad (26)$$

$$\begin{aligned} \mathbf{\Sigma}_s^{(i)} &\triangleq E[\mathbf{s}\mathbf{s}^{\dagger}|\mathbf{y}, \mathbf{b}^{(i)}] \\ &= E[\mathbf{\Psi}\mathbf{h}\mathbf{h}^{\dagger}\mathbf{\Psi}^{\dagger}|\mathbf{y}, \mathbf{b}^{(i)}] = \mathbf{\Psi}\mathbf{\Sigma}_{\mathbf{h}}^{(i)}\mathbf{\Psi}^{\dagger}. \end{aligned} \quad (27)$$

Therefore,

$$\begin{aligned} E\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j|\mathbf{y}, \mathbf{b}^{(i)}\} &= E[\mathbf{s}^{\dagger}\mathbf{s}|\mathbf{y}, \mathbf{b}^{(i)}] \\ &= \operatorname{tr}\left[\mathbf{\Sigma}_s^{(i)}[k,j] + \boldsymbol{\mu}_s^{(i)}[k]\boldsymbol{\mu}_s^{(i)}[j]^{\dagger}\right] \end{aligned} \quad (28)$$

where, $\boldsymbol{\mu}_s^{(i)} \triangleq \mathbf{\Psi}\boldsymbol{\mu}_h^{(i)}$.

Maximization-Step (M-Step): The second step to implement the EM algorithm is the *M-Step* where the parameter $\mathbf{b}$ is updated at the $(i+1)$th iteration according to

$$\mathbf{b}^{(i+1)} = \arg\max_{\mathbf{b}} Q(\mathbf{b}|\mathbf{b}_i) = \sum_{k=1}^{K} Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}). \quad (29)$$

M-Step can be performed by maximizing the terms $Q_k(\mathbf{b}_k|\mathbf{b}^{(i)})$ individually in (29), as follows

$$\mathbf{b}_k^{(i+1)} = \arg\max_{\mathbf{b}_k} Q_k(\mathbf{b}_k|\mathbf{b}^i) \quad (30)$$

where from (14)

$$Q_k(\mathbf{b}_k|\mathbf{b}^i) = \sum_{m=1}^{M} b_k(m)\Re\{(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]}\}. \quad (31)$$

Moreover, when no coding is used, it follows from (31) that each component of $\mathbf{b}_k^{(i+1)}$ can be separately obtained by maximizing the corresponding summation in the right-hand expression, as follows

$$b_k^{(i+1)}(m) = \text{sgn}\left[\Re\{(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]}\}\right] \quad (32)$$

where we have previously obtained that

$$(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]} = b_k^{[i]}(m)(\|\mathbf{h}_k\|^2)^{[i]} \quad (33)$$

$$+\beta_k\left[(\mathbf{h}_k^\dagger)^{[i]}\mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{y}(m) - \sum_{j=1}^{K} b_j^{[i]}(m)(\mathbf{h}_k \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{F} \mathbf{h}_j)^{[i]}\right].$$

where, the quantities $(\|\mathbf{h}_k\|^2)^{[i]}, (\mathbf{h}_k^\dagger)^{[i]}$ and $(\mathbf{h}_k \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}^j \mathbf{F} \mathbf{h}_j)^{[i]}$ in (33) are given by (23), (25) and (28), respectively. Note that it was shown in [4] that for large observations frame M, $\text{tr}\left[\mathbf{\Sigma}_s^{(i)}[k,j]\right] \approx 0$ compared to $\text{tr}\left[\boldsymbol{\mu}_s^{[i]}[k]\boldsymbol{\mu}_s^{[i]}[j]^\dagger\right]$. That is,

$$\lim_{m\to\infty} (\mathbf{h}_k \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{h}_j)^{[i]} \approx \text{tr}\left[\boldsymbol{\mu}_s^{[i]}[k]\boldsymbol{\mu}_s^{[i]}[j]^\dagger\right]$$

$$= \boldsymbol{\mu}_s^{[i]}[k]^\dagger \boldsymbol{\mu}_s^{[i]}[j] \equiv (\mathbf{h}_k^\dagger)^{[i]}\mathbf{h}_k^{[i]} \quad (34)$$

Note that the identity on the right hand side of (34) follows from the facts that $\boldsymbol{\mu}_s^{(i)} \triangleq \mathbf{\Psi}\boldsymbol{\mu}_h^{(i)}$ and $\mathbf{\Psi}^\dagger\mathbf{\Psi} = \mathbf{I}$. Discarding the second term, we finally have

$$b_k^{i+1}(m) = \text{sgn}\left[\Re\left\{b_k^{[i]}(m)\|\boldsymbol{\mu}_h^{(i)}[k]\|^2(1-\beta_k)\right.\right.$$

$$\left.\left.+ \beta_k \mathbf{h}_k^{(i)\dagger}\mathbf{\Psi}_k^T\left[\mathbf{y}(m) - \sum_{j=1,j\neq k}^{K} b_j^{[i]}(m)\mathbf{\Psi}_j\boldsymbol{\mu}_h^{(i)}[j]\right]\right\}\right]. \quad (35)$$

As a conclusion, Equation (35) can be interpreted as joint channel estimation and data detection with partial interference cancelation. At each iteration step during data detection, the interference reduced signal is fed into a single user receiver consisting of a conventional coherent detector. As a result, a K-user optimization problem has been decomposed into K independent optimization problems which can be resolved in a computationally feasible way. Finally we remark that this paper is an extension of the work presented by Fleury to the problem of joint channel estimation and data detection for the uplink multicarrier CDMA systems operating in the presence of the frequency selective channels[4]. Fleury and Kocian investigates the same problem for the uplink Direct Sequence CDMA systems in the presence of flat fading channels.

B. SAGE-JDE Receiver

The SAGE algorithm proposed by Fessler et al. [9] is a twofold generalization of the EM algorithm. First, rather than updating all parameters simultaneously at iteration i, only a subset $\mathbf{b}_k$ of $\mathbf{b}$ indexed by k=k[i] is updated while keeping the parameters in the complement set $\mathbf{b}_{\bar{k}} \triangleq \mathbf{b}\backslash\mathbf{b}_k$ fixed; and second, the concept of the complete data χ is extended to that of the so-called hidden data χ_k to which the incomplete data $\mathbf{y}$ is related by means of a possibly nondeterministic mapping $\chi_k \mapsto \mathbf{y}(\chi_k)$. exhibiting some particular property. A hidden data space would be a complete data space for $\mathbf{b}_k$ in the EM framework, if $\mathbf{b}_{\bar{k}}$ were known [9]. The particular property of the mapping $\chi_k \mapsto \mathbf{y}(\chi_k)$ guarantees that the SAGE algorithm exhibits the monotonicity property, as well.

The convergence rate of the SAGE algorithm is usually higher than that of the EM algorithm, because the conditional Fisher information matrix of χ_k given $\mathbf{y}$ for each set of parameters $\mathbf{b}_k$ is likely smaller than that of the complete data χ, given $\mathbf{y}$ for the entire space $\mathbf{b}$. The SAGE algorithm, a generalized form of the EM algorithm [6], allows a more flexible optimization scheme and sometimes converges faster than the EM algorithm. Our main objective to estimate the transmitted symbols $\mathbf{b} = \{b_k(m)\}_{k=1,m=1}^{K,M}$ for each user k, based on observed data $\mathbf{y}$. The complex channel responses $\mathbf{h} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_K]^T$ are treated as nuisance parameters. In the SAGE algorithm, we view the observed data $\mathbf{y}$ as the incomplete data. At each iteration i, only the data sequence $\mathbf{b}_k = [b_k(1), b_k(2), \cdots, b_k(M)]$ of $\mathbf{b}$ indexed $k = k(i) = i \bmod K$ is updated while keeping the data sequences in the complement set $\mathbf{b}_{\bar{k}}$ fixed. $\mathbf{b}_{\bar{k}}$ is the vector obtained by canceling the components of $\mathbf{b}_k$ in $\mathbf{b}$. Then a natural choice for the so-called "hidden-data" set would be $\chi = (\mathbf{y}, \mathbf{h})$.

The SAGE algorithm is defined by the Expectation(E) and Maximization(M) steps as follow: At the ith iteration the E-step computes

$$Q_k\left(\mathbf{b}_k|\mathbf{b}^{(i)}\right) = E\left\{\log p(\chi|\mathbf{b}_k, \mathbf{b}_{\bar{k}}^{(i)}|\mathbf{y}, \mathbf{b}^{(i)}\right\} \quad (36)$$

In the M-step,only $\mathbf{b}_k$ is updated as

$$\mathbf{b}_k^{(i+1)} = \arg\max_{\mathbf{b}} Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) \quad (37)$$

$$\mathbf{b}_{\bar{k}}^{(i+1)} = \mathbf{b}_{\bar{k}}^{(i)} \quad (38)$$

Given the complete data set χ, the loglikelihood function of the parameter vector $\mathbf{b}$ to be estimated can be expressed as

$$\log p(\chi|\mathbf{b}) = \log p(\mathbf{y}, \mathbf{h}|\mathbf{b}), \quad (39)$$

From (2), the term $\log p(\mathbf{y}|\mathbf{b}, \mathbf{h})$ in (39) can be expressed as

$$\log p(\mathbf{y}|\mathbf{b}, \mathbf{h}) \sim \sum_{m=1}^{M} 2\Re\left\{\left(\sum_{j=1}^{K} b_j(m)\mathbf{C}_j\mathbf{F}\mathbf{h}_j\right)^{\dagger}\mathbf{y}(m)\right\}$$

$$-\|\left(\sum_{j=1}^{K} b_j(m)\mathbf{C}_j\mathbf{F}\mathbf{h}_j\right)^{\dagger}\mathbf{y}(m)\|^2. \quad (40)$$

Inserting (40) in (5), we have for $Q_k(\mathbf{b}_k|\mathbf{b}^{(i)})$

$$Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) = \sum_{m=1}^{M} b_k(m)\Re\{(\mathbf{h}_k^{[i]})^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{y}(m)$$

$$- \sum_{j=1, j\neq k}^{K} b_j(m)(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j)^{[i]}\} \quad (41)$$

The M-Step can be performed by maximizing each summand of the right-hand side expression individually in (41). After some algebra the final result is as follows.

$$b_k^{(i+1)}(l) = \text{sgn}[\Re\{[(\mathbf{h}_k^{\dagger})^{[i]}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{y}(m)$$

$$-\sum_{j=1}^{K} b_j^{[i]}(m)(\mathbf{h}_k\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j)^{[i]}]\}] \quad (42)$$

we finally have

$$b_k^{i+1}(m) = \text{sgn}[\Re\{\boldsymbol{\mu}_h^{(i)}[j]\boldsymbol{\Psi}_k^T[\mathbf{y}(m)$$

$$- \sum_{j=1, j\neq k}^{K} b_j^{[i]}(m)\boldsymbol{\Psi}_j\boldsymbol{\mu}_h^{(i)}[j]]\}] \quad (43)$$

According to (43), the tentative decisions of the bits are used to calculate an estimate of the MAI which is likely to be increasingly reliable with iteration i.

IV. SIMULATIONS

In this section, performance of uplink MC-CDMA system based on proposed receiver operating over frequency selective channels is investigated by computer simulations . In simulations, it is assumed that all the users receive the same average signal power. The orthogonal Walsh sequences are selected as spreading code and the processing gain equals to the number of subcarriers, $N_c = 16$. The number of users selected is $U = 16$ and each user sends frame over the mobile fading channel which is composed of T preamble bits, and F data bits. Wireless channels between mobiles antennas and receiver antenna are assumed to be complex gaussian channel of length L and it has distribution $N(0, \mathbf{C}_h)$.

In the the receiver, the initial MMSE channel estimate is obtained by using T preamble bits while channel covariance matrix $\mathbf{C}_h$ is assumed to be known. Initial MMSE estimate of F data bits is computed from the observation of $\mathbf{y}$ while assuming the channel coefficients have already been estimated. We refer to this method for obtaining $\mathbf{h}$ and $\mathbf{b}$ as MMSE separate detection and estimation (MMSE-SDE) scheme. If the output of the (MMSE-SDE) is applied to a parallel interference cancelation (PIC) receiver or the SAGE receiver, the resulting receiver structures are referred to the Combined MMSE-PIC and the SAGE-SDE, respectively. All of these schemes are based on separate detection and estimation methods and their performances are limited by the number of preamble bits, employed. Moreover, we also determined the performances of the MMSE-SDE, Combined MMSE-PIC and SAGE-SDE receivers for the perfect channel state information case that are referred to CSI-MMSE, CSI-Combined MMSE-PIC, CSI-SAGE-SDE respectively.

Fig.1 compares the BER performances of the proposed EM-JDE, SAGE-JDE approaches with the MMSE-SDE, Combined-MMSE-PIC, CSI-MMSE, CSI-Combined MMSE-PIC and the CSI-SAGE-SDE schemes. For fair comparison, we simulated Combined-MMSE-PIC, SAGE-SDE, EM-JDE, SAGE-JDE, CSI-Combined MMSE-PIC and CSI-SAGE-SDE receivers employing only 6 iterations. It is observed that the proposed JDE outperforms the all the SDE approaches while channel is unknown by the receiver. Moreover, it is observed that the performances of EM-JDE and SAGE-JDE approach the performances of CSI-Combined MMSE-PIC and CSI-SAGE-SDE receivers at higher Eb/No values.

In Figs. 2, the average BER performances of the EM-JDE and SAGE-JDE receivers are presented as a function of the number of iterations for the different number of preamble bits, chosen. It is concluded from these curves that the BER performance of the EM-based algorithms converge within 5-6 iterations. Also the cause of performance differences between these approaches is twofold. First, that short preamble lengths provide poor initial estimates, resulting in a more BER performance degradation in the EM-JDE as compared to the SAGE-JDE system. Secondly, the channel coefficients are updated only every stage in the EM-JDE scheme, rather

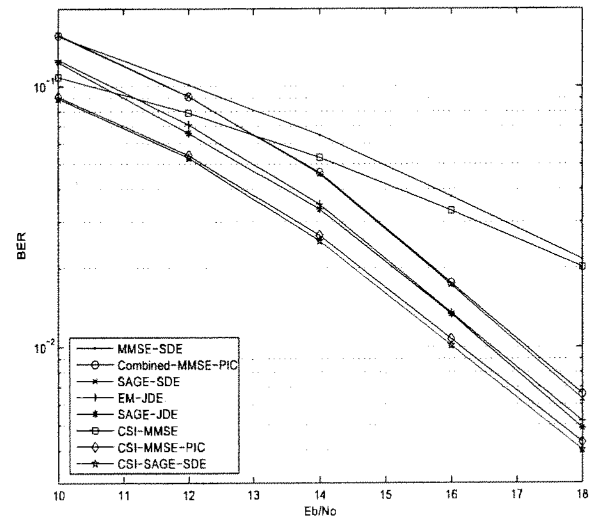


Fig. 1. BER performance in frequency selective channels (T=4,F=20,L=5,U=8)

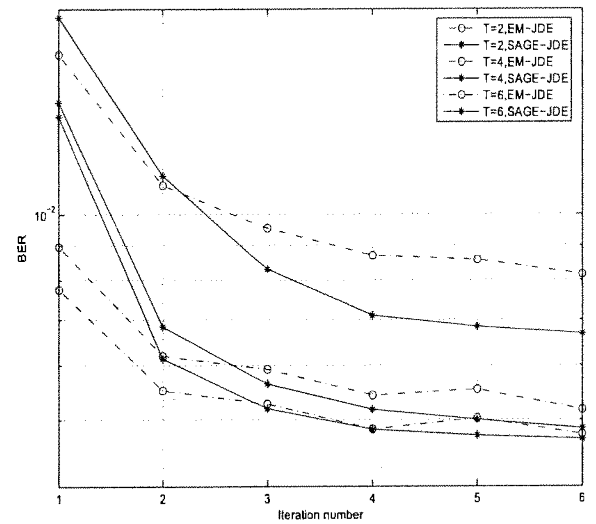


Fig. 2. Convergence of BER with respect to number of iterations (F=20,L=5,U=16)

that K times as performed in the SAGE-JDE receiver. However, for longer preamble lengths it is observed that the performance difference between EM-JDE and SAGE-JDE tends to decrease. On the other hand it was shown in [7], [8] that that for longer preamble lengths the combined MMSE-PIC receiver performance approaches to that of the EM-JDE and SAGE-JDE methods. As a result, we conclude that it is more advantages to adopt the SAGE based algorithms in receivers employing pilots of short lengths.

V. Conclusions

We presented two efficient iterative receiver structures of tractable complexity for joint multiuser detection and multichannel estimation (JDE) of direct-sequence code-division multiple-access signals. The schemes result from an application of EM and SAGE. algorithms, respectively. The EM-JDE receiver updates the data bit sequences in parallel, while the SAGE-JDE receiver reestimates them successively. The channel parameters are updated in parallel in both schemes. A closed form expression was derived for the data detection which incorporates the channel estimation as well as the partial interference cancelation steps in the algorithm. It was concluded that few pilot symbols were sufficient to initiated the EM and SAGE algorithms very effectively. A comparison with other previously known receiver structures was also made. In known channel, we observed that all schemes perform almost similarly. However, computer simulations demonstrated the effectiveness of the proposed algorithms in terms of BER performances when the channel needs to be estimated. We conclude that the robustness of the EM-JDE and SAGE-JDE which smartly combine the data detection and channel estimation in multiuser systems, unlike architectures where both process are implemented separately. On the other hand, we observed that EM is faster algorithms than the SAGE

References

[1] S. Hara and R. Prasad, "Overview of Multicarrier CDMA," IEEE Communication Magazine, pp. 126133, December 1997.

[2] Z. Xie. R.T. Short and C.K.Rushfort1i: "Family of suboptimum detectors for coherent multiuser communications." IEEE Journal on Selected Areas in Communications, vol. 8, no. 4, pp. 683-690, May 1990.

[3] V. Kuhn, "Combined MMSE-PIC in coded OFDM-CDMA systems," IEEE GLOBECOM 01, pp. 231-235, Nov. 2001.

[4] A. Kocian and B. H. Fleury, "EM-based joint data detection and channel estimation of DS-CDMA signals,"IEEE Trans. Commun., vol. 51, no. 10, pp. 17091720, Oct. 2003.

[5] M. Feder and E. Weinstein, "Parameter Estimation of superimposed signals using the EM algorithm,"*IEEE Tran. on Acoustic, Speech and Signal Processing*, Vol. 36, pp. 477-489, April 1988.

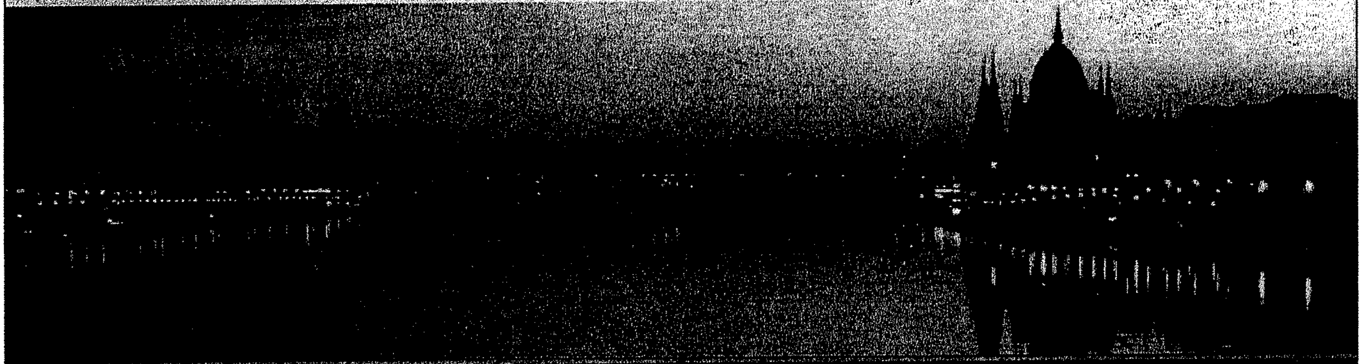
[6] X. Yongzhe and C.N. Georghiades, "Two EM-type channel estimation algorithms for OFDM with transmitter diversity," *IEEE Transactions on communications*, vol. 51, No. 1, pp. 106-115 January 2003.

[7] Erdal Panayirci, Hakan Dogan, Hakan A. Cirpan and Bernard H. Fleury,"Joint Data Detection and Channel Estimation for Uplink MC-CDMA Systems over Frequency Selective Channels", *6th International Workshop on Multi-Carrier Spread Spectrum (MC-SS 2007)*, May 07-09 2007, Herrsching, Germany.

[8] Erdal Panayirci, Hakan Dogan, Hakan A. Cirpan and Bernard H. Fleury, "An Efficient Joint Data Detection and Channel Estimation Technique for Uplink MC-CDMA Systems Based on SAGE Algorithm", *16th IST Mobile and Wireless Communications Summit*, Budapest, Hungary, July 1-5 2007.

[9] J. A. Fessler and A. O. Hero, "Space-alternating generalized expectation- maximization algorithm," *IEEE Trans. Signal Processing*, vol. 42, pp. 26642677, Oct. 1994.

16th IST Mobile & Wireless Communications Summit Budapest, Hungary 1-5 July 2007



Search by Author | Search by Title | Search by EU project

Budapest University of Technology and Economics

Information Society and Media

CONTENTS

Session B6

Back

6 matching records found.

Session	Author	Title	EU Project
B6.1	Bo Rong, Yi Qian, Kejie Lu	**Revenue and Fairness Guaranteed Downlink Adaptive Power Allocation in WIMAX Access Networks**	
B6.2	Mugdim Bublin, Dagmar Bosanska, Ondrej Hlinka, Peter Slanina	**A Self-Adaptive, Utility-based Scheduling for Wireless Cellular Networks**	WINNER
B6.3	Tarcisio F. Maciel, Anja Klein	**A resource allocation strategy for SDMA/OFDMA systems**	
B6.4	R. Müllner, C. F. Ball, K. Ivanov, H. Winkler, R. Perl, K. Kremnitzer	**Enhancing Packet Data Performance by dynamic Half-Rate Allocation of Speech Services**	
B6.5	Shahid Khattak, Gerhard Fettweis, Pradeep Thanneeru, Ratnajit Bhattacherjee	**SIR based Adaptive Subchannel Allocation in OFDMA Cellular System**	
B6.6	Erdal Panayırcı, Hakan Dogan, Hakan A. Cırpan, Bernard H. Fleury	**An Efficient Joint Data Detection and Channel Estimation Technique for Uplink MC-CDMA Systems Based on SAGE Algorithm**	NEWCOM

An Efficient Joint Data Detection and Channel Estimation Technique for Uplink MC-CDMA Systems Based on SAGE Algorithm

Erdal Panayırcı, Hakan Doğan, Hakan A. Çırpan, Bernard H. Fleury

Abstract

This paper is concerned with joint channel estimation and data detection for uplink multicarrier code-division multiple-access (MC-CDMA) systems in the presence of frequency fading channel. The detection and estimation algorithm, implemented at the receiver, is based on a version of the expectation maximization (EM) technique, called the spece-alternating-generalized-expectation-maximization (SAGE) algorithm which is very suitable for the the multicarrier signal formats. Application of the SAGE algorithm to the problem of iterative data detection and channel estimation leads to a receiver structure that also incorporates a partial interference cancelation. Computer simulations show that the proposed algorithm has excellent BER end estimation performance.

Index Terms: Joint data detection and channel estimation, MC-CDMA Systems, SAGE algorithm.

I. Introduction

In this paper we consider an efficient iterative algorithm based on the SAGE technique for multi-user data detection and channel estimation, jointly for uplink MC-CDMA systems in the presence of frequency selective fading channels. The SAGE algorithm is a broadly applicable approach to the iterative computation of parameters from intractable and high complexity likelihood functions. An EM approach proposed for the superimposed signals [1] is applied to the channel estimation for OFDM systems [2], [3] and compared with SAGE version in [4]. As will be seen shortly, a partial parallel interference cancelation (PIC) is incorporated into the resulting detection algorithm [5], [6]. The work is an extension of [7] in which joint data detection and channel estimation of uplink DS-CDMA systems were considered in the presence of flat Rayleigh channels. We extend their results for the uplink MC-CDMA systems with frequency selective channels. The channel estimation becomes more challenging for uplink systems since each channel between every user and the base station must be estimated rather than estimating a single channel in case of a downlink transmission.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; $(.)^*$, $(.)^T$ and $(.)^H$ denote the conjugate, transpose and conjugate transpose, respectively; $\|.\|$ denotes the Frobenius norm; $\mathbf{I}_L$ denotes the $L \times L$ identity matrix; $diag\{.\}$ denotes a diagonal matrix; and finally, $tr\{.\}$ denotes the trace of a matrix.

II. Signal Model

We consider a baseband MC-CDMA uplink system with P sub-carriers and K mobile users which are simultaneously active. For the kth user, each transmitted symbol is modulated in the frequency domain by means of a $P \times 1$ specific spreading sequence $\mathbf{c}_k$. After transforming by a P-point IDFT and parallel-to-serial (P/S) conversion, a cyclic prefix (CP) is inserted of length equal to at least the channel memory (L). In this work, to simplify the notation, it is assumed that the spreading factor equals to the number of sub-carriers and all users have the same spreading factor. Finally, the signal is transmitted through a multipath channel with impulse response

$$h_k(t) = \sum_{k=1}^{L} g_{k,l}\delta(t - \tau_{k,l}) \tag{1}$$

This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 6th Framework Programme and the Research Fund of Istanbul University under Projects UDP-921/09052007,T-856/02062006. This work was also supported in part by the Turkish Scientific and Technical Research Institute (TUBITAK) under Grant 104E166.

Hakan Doğan and Hakan A. Çırpan are with the Department of Electrical and Electronics Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey, e-mail: {hdogan,hcirpan}@istanbul.edu.tr

Erdal Panayırcı is with the Department of Electronics Enginering, Kadir Has University, Cibali 34083, Istanbul, Turkey, e-mail: eepanay@khas.edu.tr

Bernard H. Fleury is with the Section Navigation and Communications, Department of Electronic Systems, Aalborg University Fredrik Bajers Vej 7A3 DK-9000 Aalborg, Denmark and Forschungszentrum Telekommunikation Wien (ftw.) Donau-City-Strasse 1 A-1220 Vienna, Austria e-mail: bfl@kom.auc.dk

where L is the number of paths in the kth users channel; g_{kl} and τ_{kl} are, respectively, the complex fading processes and the delay of the kth users lth path. It is assumed that only the second-order statistics of the fading processes are known to the receiver. Also, fading can vary from symbol to symbol but remains constant over a symbol interval.

At the receiver, the received signal is first serial-to-parallel (S/P) converted, then CP is removed and DFT is then applied to the received discrete time signal to obtain the received vector expressed as

$$\mathbf{y}(m) = \sum_{k=1}^{K} b_k(m)\mathbf{C}_k\mathbf{F}\mathbf{h}_k + \mathbf{w}(m) \quad m = 1, 2, ..., M \tag{2}$$

where $b_k(m)$ denotes data sent by the user k within the mth symbol; $\mathbf{C}_k = \text{diag}(\mathbf{c}_k)$ with $\mathbf{c}_k = [c_{k1}, c_{k1}, ..., c_{kP}]^T$ where each chip, c_{ik}, takes values in the set $\{-\frac{1}{\sqrt{P}}, \frac{1}{\sqrt{P}}\}$ denoting the kth users spreading code ; $\mathbf{F} \in \mathbb{C}^{P\times L}$ denotes the DFT matrix with the (k,l)th element given by $\frac{1}{\sqrt{P}}e^{-j2\pi kl/P}$; and $\mathbf{w}(m)$ is the $P \times 1$ zero-mean, i.i.d. Gaussian vector that models the additive noise in the P tones, with variance $\sigma^2/2$ per dimension.

Suppose M symbols are transmitted. We stack $\mathbf{y}(m)$ as $\mathbf{y}=[\mathbf{y}^T(1), ..., \mathbf{y}^T(M)]^T$. Then the received signal model can be written as

$$\mathbf{y} = \begin{bmatrix} b_1(1)\mathbf{C}_1\mathbf{F} & \cdots & b_K(1)\mathbf{C}_K\mathbf{F} \\ \vdots & \ddots & \vdots \\ b_1(M)\mathbf{C}_1\mathbf{F} & \cdots & b_K(M)\mathbf{C}_K\mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix} + \begin{bmatrix} \mathbf{w}(1) \\ \vdots \\ \mathbf{w}(M) \end{bmatrix} \tag{3}$$

and can be rewritten in more succinct form

$$\mathbf{y} = \mathbf{A}\mathbf{h} + \mathbf{w} \tag{4}$$

By using the assumption; $\mathbf{h}_k \sim N(0, \mathbf{\Sigma}_{\mathbf{h}_k})$ where $\mathbf{\Sigma}_{\mathbf{h}_k} = E[\mathbf{h}_k\mathbf{h}_k^\dagger]$, we have $\mathbf{h} \sim N(0, \mathbf{\Sigma}_{\mathbf{h}})$ with $\mathbf{\Sigma}_{\mathbf{h}} = \text{diag}[\mathbf{\Sigma}_{\mathbf{h}_1}, \cdots, \mathbf{\Sigma}_{\mathbf{h}_K}]$.

III. Joint Data Detection and Channel Estimation

The problem of interest is to derive an iterative algorithm based on the SAGE algorithm for data detection and channel estimation jointly employing the signal model given by (2). The SAGE algorithm, a generalized form of the EM algorithm [4], allows a more flexible optimization scheme and sometimes converges faster than the EM algorithm. Since the SAGE method has been studied and applied to a number of problems in communications over the years, the details of the algorithm will not be presented in this paper. The reader is suggested to read [8] for a general exposition to SAGE algorithm and [7] for its application to the estimation problem related to the work herein.

Our main objective to estimate the transmitted symbols $\mathbf{b} = \{b_k(m)\}_{k=1,m=1}^{K,M}$ for each user k, based on observed data $\mathbf{y}$. The complex channel responses $\{\mathbf{h}_k\}$ are treated as nuisance parameters. In the SAGE algorithm, we view the observed data $\mathbf{y}$ as the incomplete data. At each iteration i, only the data sequence $\mathbf{b}_k = [b_k(1), b_k(2), \cdots, b_k(M)]$ of $\mathbf{b}$ indexed $k = k(i) = i \bmod K$ is updated while keeping the data sequences in the complement set $\mathbf{b}_{\bar{k}}$ fixed. $\mathbf{b}_{\bar{k}}$ is the vector obtained by canceling the components of $\mathbf{b}_k$ in $\mathbf{b}$. Then a natural choice for the so-called "hidden-data" set would be $\chi = \{(\mathbf{y}_1, \mathbf{h}_1), (\mathbf{y}, \mathbf{h})\}$.

The SAGE algorithm is defined by the Expectation(E) and Maximization(M) steps as follow:

At the ith iteration the E-step computes

$$Q_k\left(\mathbf{b}_k|\mathbf{b}^{(i)}\right) = E\left\{\log p(\chi|\mathbf{b}_k, \mathbf{b}_{\bar{k}}^{(i)}|\mathbf{y}, \mathbf{b}^{(i)}\right\} \tag{5}$$

In the M-step,only $\mathbf{b}_k$ is updated as

$$\mathbf{b}_k^{(i+1)} = \arg\max_{\mathbf{b}} Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) \tag{6}$$

$$\mathbf{b}_{\bar{k}}^{(i+1)} = \mathbf{b}_{\bar{k}}^{(i)} \tag{7}$$

Given the complete data set χ, the loglikelihood function of the parameter vector $\mathbf{b}$ to be estimated can be expressed as

$$\log p(\chi|\mathbf{b}) = \log p(\mathbf{y}, \mathbf{h}|\mathbf{b}), \tag{8}$$

where

$$\log p(\mathbf{y}, \mathbf{h}|\mathbf{b}) = \log p(\mathbf{y}|\mathbf{b}, \mathbf{h}) + \log p(\mathbf{h}|\mathbf{b}). \tag{9}$$

We neglect the $\log p(\mathbf{h}|\mathbf{b})$ term in (9) since the data sequence $\mathbf{b}$ and $\mathbf{h}$ are independent of each other.

A. Implementation of the Expectation Step *(E-Step)*

From (2), the term $\log p(\mathbf{y}|\mathbf{b},\mathbf{h})$ in (9) can be expressed as

$$\log p(\mathbf{y}|\mathbf{b},\mathbf{h}) \sim \sum_{m=1}^{M} 2\Re\left\{\left(\sum_{j=1}^{K} b_j(m)\mathbf{C}_j\mathbf{F}\mathbf{h}_j\right)^{\dagger}\mathbf{y}(m)\right\} - \|\left(\sum_{j=1}^{K} b_j(m)\mathbf{C}_j\mathbf{F}\mathbf{h}_j\right)^{\dagger}\mathbf{y}(m)\|^2. \quad (10)$$

Inserting (10) in (5), we have for $Q_k(\mathbf{b}_k|\mathbf{b}^{(i)})$

$$Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) = \sum_{m=1}^{M} b_k(m)\Re\left\{(\mathbf{h}_k^{[i]})^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{y}(m) - \sum_{j=1,j\neq k}^{K} b_j(m)(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j)^{[i]}\right\} \quad (11)$$

where, adopting the notation used in [7],

$$(\mathbf{h}_k)^{[i]} \triangleq E\left\{\mathbf{h}_k|\mathbf{y},\mathbf{b}^{(i)}\right\} \quad (12)$$

$$(\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j)^{[i]} \triangleq E\left\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j|\mathbf{y},\mathbf{b}^{(i)}\right\} \quad (13)$$

On the other hand, since $\mathbf{w} \sim N(0,\Sigma^2\mathbf{I})$ and the prior pdf of $\mathbf{h}$ is chosen as $\mathbf{h} \sim N(0,\Sigma_{\mathbf{h}})$, we can write the conditional pdf's of $\mathbf{h}$ given $\mathbf{y}$ and $\mathbf{b}^{(i)}$ as

$$\begin{aligned} p(\mathbf{h}|\mathbf{y},\mathbf{b}^{(i)}) &\sim p(\mathbf{y}|\mathbf{h},\mathbf{b}^{(i)})p(\mathbf{h}) \\ &\sim \exp\left[-\frac{1}{\sigma^2}(\mathbf{y}-\mathbf{A}\mathbf{h})^{\dagger}(\mathbf{y}-\mathbf{A}\mathbf{h}) - \mathbf{h}^{\dagger}\Sigma_{\mathbf{h}}^{-1}\mathbf{h}\right]. \end{aligned} \quad (14)$$

After some algebra it can be shown that

$$p(\mathbf{h}|\mathbf{y},\mathbf{b}^{(i)}) \sim N(\mu_{\mathbf{h}}^{(i)},\Sigma_{\mathbf{h}}^{(i)}) \quad (15)$$

where

$$\begin{aligned} \mu_{\mathbf{h}}^{(i)} &= \frac{1}{\sigma^2}\Sigma_{\mathbf{h}}^{(i)}\mathbf{A}^{(i)\dagger}\mathbf{y} \\ \Sigma_{\mathbf{h}}^{(i)} &= \left[\Sigma_{\mathbf{h}}^{-1} + \frac{1}{\sigma^2}(\mathbf{A}^{(i)})^{\dagger}\mathbf{A}^{(i)}\right]^{-1} \end{aligned} \quad (16)$$

and the matrix $\mathbf{A}$ is defined in (3) and (4).

The expectation in (12) can be computed as

$$(\mathbf{h}_k)^{[i]} \triangleq E\{\mathbf{h}_k|\mathbf{y},\mathbf{b}^{(i)}\} = \mu_h^{(i)}[k]. \quad (17)$$

To calculate the expectation $E\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j|\mathbf{y},\mathbf{b}^{(i)}\}$ in (13), define $\Psi_j \triangleq \mathbf{C}_j\mathbf{F}$ and $\mathbf{s}_j \triangleq \Psi_j\mathbf{h}_j$. It then follows that

$$\mathbf{s} = \Psi\mathbf{h} = \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_K \end{bmatrix} = \begin{bmatrix} \Psi_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Psi_K \end{bmatrix}\begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix}, \quad (18)$$

$$\Sigma_s^{(i)} \triangleq E[\mathbf{s}\mathbf{s}^{\dagger}|\mathbf{y},\mathbf{b}^{(i)}] = E[\Psi\mathbf{h}\mathbf{h}^{\dagger}\Psi^{\dagger}|\mathbf{y},\mathbf{b}^{(i)}] = \Psi\Sigma_{\mathbf{h}}^{(i)}\Psi^{\dagger}. \quad (19)$$

Therefore,

$$E\{\mathbf{h}_k^{\dagger}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j|\mathbf{y},\mathbf{b}^{(i)}\} = E[\mathbf{s}^{\dagger}\mathbf{s}|\mathbf{y},\mathbf{b}^{(i)}] = \mathrm{tr}\left[\Sigma_s^{(i)}[k,j] + \mu_s^{(i)}[k]\mu_s^{(i)}[j]^{\dagger}\right] \quad (20)$$

where, $\mu_s^{(i)} \triangleq \Psi \mu_h^{(i)}$.

B. Implementation of the Maximization-Step (M-Step)

The M-Step can be performed using eq.(6) and the final result is as follows

$$b_k^{(i+1)}(l) = \mathrm{sgn}\left[\Re\left\{\left[(\mathbf{h}_k^{\dagger})^{[i]}\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{y}(m) - \sum_{j=1}^{K} b_j^{[i]}(m)(\mathbf{h}_k\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j)^{[i]}\right]\right\}\right] \tag{21}$$

where, the quantities $(\mathbf{h}_k^{\dagger})^{[i]}$ and $(\mathbf{h}_k\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j)^{[i]}$ in (21) are given by (17)and (20), respectively. Note that for large observations frame M, $\mathrm{tr}\left[\Sigma_s^{(i)}[k,j]\right] \approx 0$ compared to $\mathrm{tr}\left[\mu_s^{[i]}[k]\mu_s^{[i]}[j]^{\dagger}\right]$, [8]. That is

$$\begin{aligned}\lim_{m\to\infty}(\mathbf{h}_k\mathbf{F}^{\dagger}\mathbf{C}_k^T\mathbf{C}_j\mathbf{h}_j)^{[i]} &\approx \mathrm{tr}\left[\mu_s^{[i]}[k]\mu_s^{[i]}[j]^{\dagger}\right] \\ &= \mu_s^{[i]}[k]^{\dagger}\mu_s^{[i]}[j] \equiv (\mathbf{h}_k^{\dagger})^{[i]}\mathbf{h}_k^{[i]}\end{aligned} \tag{22}$$

Note that the identity on the right hand side of (22) follows from the facts that $\mu_s^{(i)} \triangleq \Psi\mu_h^{(i)}$ and $\Psi_k^{\dagger}\Psi_k = \frac{1}{P}\mathbf{I}$. Discarding the second term, we finally have

$$b_k^{i+1}(m) = \mathrm{sgn}\left[\Re\left\{\mu_h^{(i)}[j]\Psi_k^T\left[\mathbf{y}(m) - \sum_{j=1,j\neq k}^{K} b_j^{[i]}(m)\Psi_j\mu_h^{(i)}[j]\right]\right\}\right] \tag{23}$$

As a conclusion, Equation (23) can be interpreted as joint channel estimation and data detection with partial interference cancelation. At each iteration step during data detection, the interference reduced signal is fed into a single user receiver consisting of a conventional coherent detector. As a result, a K-user optimization problem have been decomposed into K independent optimization problems which can be resolved in a computationally feasible way. Finally we remark that this paper is an extension of the work presented by [7] to the problem of joint channel estimation and data detection for the uplink multicarrier CDMA systems operating in the presence of the frequency selective channels. Fleury and Kocian [7] investigates the same problem for the uplink Direct Sequence CDMA systems in the presence of flat fading channels.

IV. Simulations

In this section, the performance of uplink MC-CDMA systems based on the proposed receiver is analyzed for frequency selective channels. In computer simulations, we assume that all users are received with the same power level. Orthogonal Walsh sequences selected as a spreading code and the processing gain is chosen equal to the number of subcarriers $P = 16$. The number of active users are selected as $K = 16$ and each user sends its data frame composed of T preamble bits, and F data bits, over mobile fading channel. Wireless channels between mobiles antennas and receiver antenna are assumed to be complex Gaussian channel of length L with $N(0, \Sigma_h)$.

In the receiver, the initial MMSE channel estimate is obtained by using T preamble bits while channel covariance matrix Σ_h is assumed to be known. Initial MMSE estimate of F data bits is computed from the observation vector $\mathbf{y}$ assuming that the channel coefficients were estimated by means of the pilot symbols. We refer to this method for obtaining $\mathbf{h}$ and $\mathbf{b}$ as the MMSE Separate Detection and Estimation (MMSE-SDE). In simulations, if the output of the the MMSE-SDE is applied to the parallel interference cancelation (PIC) receiver which is compared with the SAGE-JDE, it is referred to the Combined MMSE-PIC Receiver. Moreover, we also simulated both MMSE-SDE and Combined MMSE-PIC detectors referred as CSI-MMSE and CSI-Combined MMSE-PIC, respectively for the perfect channel state information cases.

Fig.1A compares the BER performance of the proposed SAGE-JDE approach with MMSE-SDE, Combined-MMSE-PIC, CSI-MMSE and CSI-Combined MMSE-PIC. For fair comparison we simulated Combined-MMSE-PIC, CSI-Combined MMSE-PIC and the EM-JDE receivers for three iterations. It is observed that the proposed EM-JDE outperforms MMSE-SDE, Combined-MMSE-PIC as well as CSI-MMSE and approaches the CSI-Combined MMSE-PIC.

To investigate initialization effect on the SAGE-JDE and Combined-MMSE-PIC, we also studied the BER performance of these algorithms for different preamble lengths presented in Fig.1B It is concluded that low preamble

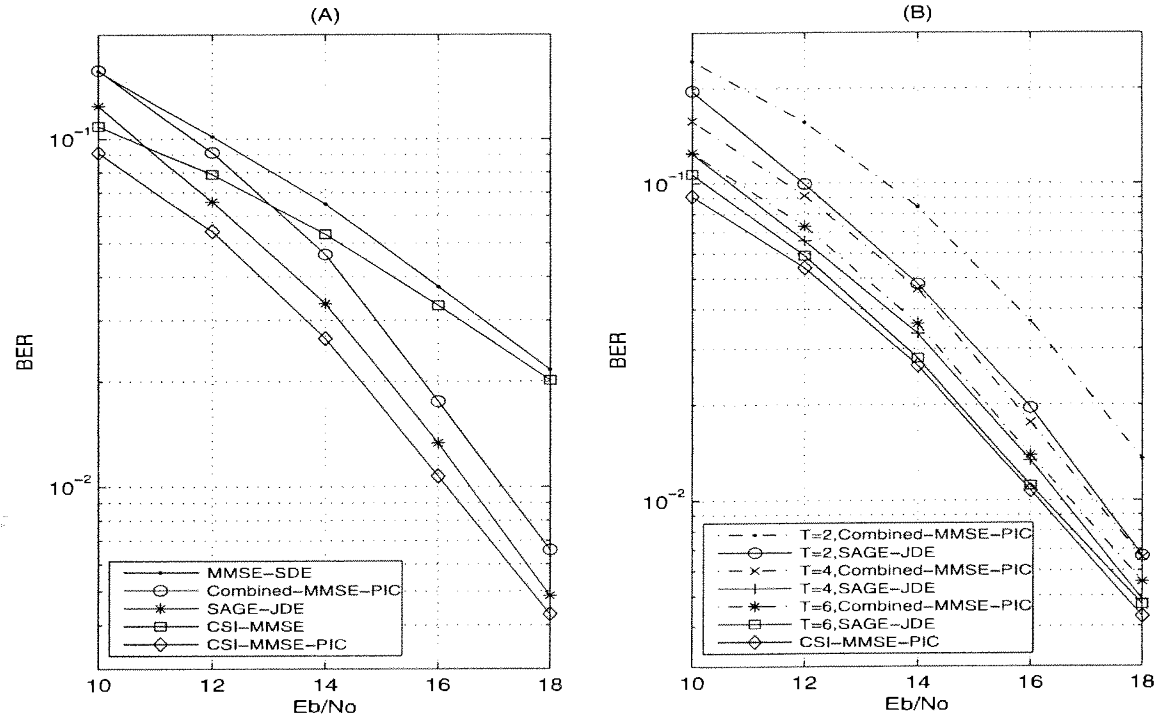


Fig. 1. (A) Behavior of the BER performance as a function of used preamble bits (F=20,T=4,L=5)(B)Behavior of the BER performance as a function of used preamble bits (F=20,L=5)

lengths provide poor initial estimates, resulting in a more BER performance degradation in the Combined-MMSE-PIC as compared to the SAGE-JDE system. On the other hand for higher preamble lengths it was observed that performance difference between Combined-MMSE-PIC and EM-JDE decreases because of considerably good initial estimates of channel coefficients.

V. Conclusions

The problem of joint data detection and channel estimation for uplink MC-CDMA systems operating in the presence of frequency selective fading channels was investigated in this work. We presented an iterative approach based on a version of the SAGE algorithm suitable for superimposed signals. A closed form expression was derived for the data detection which incorporates the channel estimation as well as the partial interference cancelation steps in the algorithm. It was concluded that few pilot symbols were sufficient to initiated the SAGE algorithm very effectively. Computer simulations were presented to demonstrate the effectiveness of the proposed algorithm in terms of BER performances.

References

[1] M. Feder and E. Weinstein, "Parameter Estimation of superimposed signals using the EM algorithm,"*IEEE Tran. on Acoustic, Speech and Signal Processing*, Vol. 36, pp. 477-489, April 1988.

[2] H.A Cirpan , E. Panayirci , H. Dogan, "Nondata-aided Channel Estimation for OFDM Systems with Space-Frequency Transmit Diversity"*IEEE Transactions on Vehicular Technology*, March 2006, vol. 55, Issue 2, pp. 449 - 457

[3] H. Dogan, H. A. Cirpan and Erdal Panayirci, "Iterative Channel Estimation and Decoding of Turbo Coded SFBC-OFDM Systems," *IEEE Trans. Wireless Commun.*, accepted for publication in Oct. 2006.

[4] X. Yongzhe and C.N. Georghiades, "Two EM-type channel estimation algorithms for OFDM with transmitter diversity," *IEEE Trans. on Commun.*, vol. 51, No. 1, pp. 106-115 January 2003.

[5] V. Kuhn, "Combined MMSE-PIC in coded OFDM-CDMA systems," *IEEE GLOBECOM 01*, pp. 231-235, Nov. 2001.

[6] S. Iraji, T. Sipila and J. Lilleberg, "Channel Estimation and Signal Detection for MC-CDMA in Multipath Fading Channels," *in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC)*, September 1993

[7] A. Kocian and B. H. Fleury, "EM-based joint data detection and channel estimation of DS-CDMA signals," *IEEE Trans. Commun.*, vol. 51, no. 10, pp. 1709-1720, Oct. 2003.

[8] J.A. Fessler, A.O. Hero, Space-alternating generalized expectation-maximization, Algorithm, *IEEE Trans. Signal Process.* 42 (10) pp. 2664-2677, October 1994

6th International Workshop on

Multi-Carrier Spread-Spectrum MC-SS 2007

Tentative Program

May 07-09, 2007, Herrsching, Germany

Monday, May 07

Room A	Room B
9:00-13:30: Registration	
9:00-12:00: Tutorial *on MC-CDMA* (Room A)	
13:30-15:00: Session 1, Plenary, *General Issues-I* (Room A+B)	
15:30-17:10: Session 2, Parallel *Channel Estimation-I* (Room A)	15:30-17:10: Session 3, Parallel *Multiple Antennas* (Room B)

Tuesday, May 08

Room A	Room B
9:00-10:40: Session 4, Parallel *Channel Estimation-II* (Room A)	9:00-10:40: Session 5, Parallel *Adaptive Transmission* (Room B)
11:10-12:30: Session 6, Plenary, *General Issues-II* (Room A+B)	
14:00-15:20: Session 7, Parallel *Interleave Division Multiple Access (IDMA)* (Room A)	14:00-15:20: Session 8, Parallel *Spectrum and Interference* (Room B)
15:45-17:00: Session 9, Parallel *Synchronization and Tracking* (Room A)	15:45-17:00: Session 10, Parallel *Multi-Cell and Multi-Link* (Room B)

Third Day, Wednesday, May 09, 2007

9:00 – 10:40 *Session 11 (Parallel Session)*

Title: Channel Estimation-III

Chairperson: Gordon L. Stuber, Georgia Institute of Technology

1) **Joint Data Detection and Channel Estimation for Uplink MC-CDMA Systems over Frequency Selective Channels**
E. Panayirci (Kadir Has University),
H. Dogan, H.A. Cirpan, (Istanbul University)
B.H. Fleury (Aalborg University)

2) **On the Performance of MC-CDMA Systems with Partial Equalization in the Presence of Channel Estimation Errors**
F. Zabini, B.M. Masini, (University of Bologna)
A. Conti (University of Ferrara)

3) **Kalman vs H_∞ Algorithms for MC-DS-CDMA Channel Estimation With or Without A Priori AR Modeling**
A. Jamoos, J. Grolleau, E. Grivel (University of Bordeaux 1),
H. Abdel-Nour (Al-Quds University)

4) **Cross-Coupled Rao-Blackwellized Particle and Kalman Filters for the Joint Symbol-Channel Estimation in MC-DS-CDMA Systems**
J. Grolleau, A. Giremus, E. Grivel (University of Bordeaux 1)

9:00 – 10:40 *Session 12 (Parallel Session)*

Title: Data Transmission

Chairperson: Stephan Sand, German Aerospace Center (DLR)

1) **RAKE Reception for Signature-Interleaved DS CDMA in Rayleigh Multipath Channel**
A. Dudkov (University of Turku)

2) **Accurate BER of MC-DS-CDMA over Rayleigh Fading Channels**
B. Smida (Harvard University),
L. Hanzo (University of Southampton),
S. Affes (Université du Québec)

3) **OFDM/OQAM for Spread-Spectrum Transmission**
C. Lele, P. Siohan, R. Legouable, M. Bellanger (France Telecom)

4) **A Multi-Carrier Downlink Transmitter for LEO Satellite**
R.M. Vitenberg (Wavetone Technologies Ltd.)

10:40 - 11:10 Coffee Break

Joint Data Detection and Channel Estimation for Uplink MC-CDMA Systems over Frequency Selective Channels

Erdal Panayırcı, Hakan Doğan, Hakan A. Çırpan,

Abstract

This paper is concerned with joint channel estimation and data detection for uplink multicarrier code-division multiple-access (MC-CDMA) systems in the presence of frequency fading channel. The detection and estimation algorithm, implemented at the receiver, is based on a version of the expectation maximization (EM) technique which is very suitable for the the multicarrier signal formats. Application of the EM-based algorithm to the problem of iterative data detection and channel estimation leads to a receiver structure that also incorporates a partial interference cancelation. Computer simulations show that the proposed algorithm has excellent BER end estimation performance.

Index Terms: Joint data detection and channel estimation, MC-CDMA Systems, EM algorithm, Superimposed signals

I. Introduction

MC-CDMA transmission through the frequency-selective fading channels causes the signal-to-noise ratio (SNR) degradation and the occurrence of the multiple-access interference (MAI). To eliminate or reduce the resulting performance degradations, equalization and detection of the received signal can be performed at the receiver based on the complete channel information. In conventional MC-CDMA systems, MAI mitigation is accomplished at the receiver using single-user or multi-user detection schemes [1]. However, multiuser detection scheme is known to increase the bandwidth efficiency of the system drastically, although its detection complexity grows exponentially with the number of users and the number of multipaths, which makes it infeasible to implement. Several suboptimal detection techniques have proposed in literature such as linear multiuser detection[2] and iterative cancelation of MAI, either in a successive or parallel way in the received signal prior to data detection [3]. Therefore data detection and channel estimation should be performed perfectly for a good initialization of the interference cancelation detector.

In this paper we consider an efficient iterative algorithm based on the Expectation-Maximization (EM) technique for multi-user data detection and channel estimation, jointly for uplink MC-CDMA systems in the presence of frequency selective fading channels. As will be seen shortly, a partial parallel interference cancelation (PIC) is incorporated into the resulting detection algorithm [4]. The work is an extension of [5] in which joint data detection and channel estimation of downlink DS-CDMA systems were considered based on an EM algorithm in the presence of flat Rayleigh channels. The channel estimation becomes more challenging for uplink systems since each channel between every user and the base station must be estimated rather that estimating a single channel in case of a down-link transmission.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; $(.)^*$, $(.)^T$ and $(.)^H$ denote the conjugate, transpose and conjugate transpose, respectively; $\|.\|$ denotes the Frobenius norm; $\mathbf{I}_L$ denotes the $L \times L$ identity matrix; $diag\{.\}$ denotes a diagonal matrix; and finally, $tr\{.\}$ denotes the trace of a matrix.

II. Signal Model

We consider a baseband MC-CDMA uplink system with P sub-carriers and K mobile users which are simultaneously active. For the kth user, each transmit symbol is modulated in the frequency domain by means of a $P \times 1$ specific spreading sequence $\mathbf{c}_k$. After transforming by a P-point IDFT and parallel-to-serial (P/S) conversion, a cyclic prefix (CP) is inserted of length equal to at least the channel memory (L). In this work, to simplify the notation, it is assumed that the spreading factor equals to the number of sub-carriers and all users have the same spreading factor. Finally, the signal is transmitted through a multipath channel with impulse response

$$h_k(t) = \sum_{k=1}^{L} g_{k,l}\delta(t - \tau_{k,l}) \quad (1)$$

where L is the number of paths in the kth users channel; g_{kl} and τ_{kl} are, respectively, the complex fading processes and the delay of the kth users lth path. It is assumed that only the second-order statistics of the fading processes are known to the receiver. Also, fading can vary from symbol to symbol but remains constant over a symbol interval.

At the receiver, the received signal is first serial-to-parallel (S/P) converted, then CP is removed and DFT is then applied to the received discrete time signal to obtain the received vector expressed as

$$\mathbf{y}(m) \;=\; \sum_{k=1}^{K} b_k(m)\mathbf{C}_k\mathbf{F}\mathbf{h}_k + \mathbf{w}(m) \;\; m = 1, 2, ..., M \quad (2)$$

This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 6th Framework Programme and the Research Fund of Istanbul University under Projects T-856/02062006. This work was also supported in part by the Turkish Scientific and Technical Research Institute(TUBITAK) under Grant 104E166.

Hakan Doğan and Hakan A. Çırpan are with the Department of Electrical and Electronics Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey, e-mail: {hdogan,hcirpan}@istanbul.edu.tr, tel :+90 (212) 4737070 ×17904 fax : +90 (212) 5911997.

Erdal Panayırcı is with the Department of Electronics Enginering, Kadir Has University, Cibali 34083, Istanbul, Turkey, e-mail: eepanay@khas.edu.tr, tel :+90 (212) 533 65 32 × 540 fax : +90 212 533 57 53

where $b_k(m)$ denotes data sent by the user k within the mth symbol; $\mathbf{C}_k = \text{diag}(\mathbf{c}_k)$ with $\mathbf{c}_k = [c_{k1}, c_{k1}, ..., c_{kP}]^T$ where each chip, c_{ik}, takes values in the set $\{-\frac{1}{\sqrt{P}}, \frac{1}{\sqrt{P}}\}$ denoting the kth users spreading code ; $\mathbf{F} \in \mathbb{C}^{P \times L}$ denotes the DFT matrix with the (k,l)th element given by $\frac{1}{\sqrt{P}} e^{-j2\pi kl/P}$; and $\mathbf{w}(m)$ is the $P \times 1$ zero-mean, i.i.d. Gaussian vector that models the additive noise in the P tones, with variance $\sigma^2/2$ per dimension.

Suppose M symbols are transmitted. We stack $\mathbf{y}(m)$ as $\mathbf{y}=[\mathbf{y}^T(1), ..., \mathbf{y}^T(M)]^T$. Then the received signal model can be written as

$$\mathbf{y} = \begin{bmatrix} b_1(1)\mathbf{C}_1\mathbf{F} & \cdots & b_K(1)\mathbf{C}_K\mathbf{F} \\ \vdots & \ddots & \vdots \\ b_1(M)\mathbf{C}_1\mathbf{F} & \cdots & b_K(M)\mathbf{C}_K\mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix} + \begin{bmatrix} \mathbf{w}(1) \\ \vdots \\ \mathbf{w}(M) \end{bmatrix} \tag{3}$$

and can be rewritten in more succinct form

$$\mathbf{y} = \mathbf{A}\mathbf{h} + \mathbf{w} \tag{4}$$

By using the assumption; $\mathbf{h}_k \sim N(0, \boldsymbol{\Sigma}_{\mathbf{h}_k})$ where $\boldsymbol{\Sigma}_{\mathbf{h}_k} = E[\mathbf{h}_k\mathbf{h}_k^\dagger]$, we have $\mathbf{h} \sim N(0, \boldsymbol{\Sigma}_{\mathbf{h}})$ with $\boldsymbol{\Sigma}_{\mathbf{h}} = \text{diag}[\boldsymbol{\Sigma}_1, \cdots, \boldsymbol{\Sigma}_K]$.

III. Joint Data Detection and Channel Estimation

The problem of interest is to derive an iterative algorithm based on the EM technique for data detection and channel estimation jointly employing the signal model given by (2). Since the EM method has been studied and applied to a number of problems in communications over the years, the details of the algorithm will not be presented in this paper. The reader is suggested to read [6] for a general exposition to EM algorithm and [7] for its application to the estimation problem related to the work herein. The suitable approach for applying the EM algorithm for the problem at hand is to decompose the received signal in (2) into the sum [sumerimposed signals]

$$\mathbf{y}(m) = \sum_{k=1}^{K} \mathbf{x}_k(m), \qquad m = 1, 2 \cdots, M \tag{5}$$

where

$$\mathbf{x}_k(m) \quad = \quad b_k(m)\mathbf{C}_k\mathbf{F}\mathbf{h}_k + \mathbf{w}_k(m). \tag{6}$$

$\mathbf{x}_k(m)$ represents the received signal component transmitted by the kth user through the channel with impulse response $\mathbf{h}_k$. The Gaussian noise vector, $\mathbf{w}_k(m)$ n (6) represents the portion of $\mathbf{w}(m)$ in the decomposition defined by $\sum_{k=1}^{K} \mathbf{w}_k(m) = \mathbf{w}(m)$, whose variance is $N_0\beta_k$. The coefficients β_k determine the part of the noise power of $\mathbf{w}(m)$ assigned to $\mathbf{x}_k(m)$, satisfying $\sum_{k=1}^{K} \beta_k = 1$, $0 \leq \beta_k \leq 1$.

The problem now is to estimate the transmitted symbols $\mathbf{b} = \{b_k(m)\}_{k=1,m=1}^{K,M}$ and the complex channel responses $\mathbf{h}_k$ for each user based on observed data $\mathbf{y}$. In the EM algorithm, we view the observed data $\mathbf{y}$ as the incomplete data, and define the complete data as $\chi = \{(\mathbf{x}_1, \mathbf{h}_1), (\mathbf{x}_1, \mathbf{h}_1), ..., (\mathbf{x}_K, \mathbf{h}_K)\}$ where $\mathbf{x}_k = [\mathbf{x}_k(1), ..., \mathbf{x}_k(M)]^T$ for $k = 1, 2, \cdots, K$.

The EM algorithm: Given the complete data set as $\chi = \{(\mathbf{x}_1, \mathbf{h}_1), (\mathbf{x}_1, \mathbf{h}_1), ..., (\mathbf{x}_K, \mathbf{h}_K)\}$, the loglikelihood function of the parameter vector to be estimated $\mathbf{b}$ can be expressed as

$$\log p(\chi|\mathbf{b}) = \sum_{k=1}^{K} \log p(\mathbf{x}_k, \mathbf{h}_k|\mathbf{b}_k) \tag{7}$$

where

$$\log p(\mathbf{x}_k, \mathbf{h}_k|\mathbf{b}_k) = \log p(\mathbf{x}_k|\mathbf{b}_k, \mathbf{h}_k) + \log p(\mathbf{h}_k|\mathbf{b}_k) \tag{8}$$

and, $\mathbf{b}_k = [b_k(1), b_k(2), \cdots, b_k(M)]$. We neglect the $\log p(\mathbf{h}_k|\mathbf{b}_k)$ term in (7) since the data sequence $\mathbf{b}_k$ and $\mathbf{h}_k$ are independent of each other.

The first step to implement the EM algorithm, called the *Expectation Step (E-Step)*, is the computation of the average log-likelihood function. The conditional expectation is taken over χ given the observation $\mathbf{y}$ and that $\mathbf{b}$ equals its estimate calculated at ith iteration.

$$Q\left(\mathbf{b}|\mathbf{b}^{(i)}\right) = E\left\{\log p(\chi|\mathbf{b}|\mathbf{y}, \mathbf{b}^{(i)}\right\} \tag{9}$$

Taking into account the special form of $\log p(\chi|\mathbf{b})$ in (7), Eq. (8) can be decomposed as

$$Q\left(\mathbf{b}|\mathbf{b}^{(i)}\right) = \sum_{k=1}^{K} Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) \tag{10}$$

where

$$Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) = E\left\{\log p(\mathbf{x}_k, \mathbf{h}_k|\mathbf{b}_k)|\mathbf{y}, \mathbf{b}^{(i)}\right\} \quad (11)$$

$$= E\left\{\log p(\mathbf{x}_k|\mathbf{b}_k, \mathbf{h}_k)|\mathbf{y}, \mathbf{b}^{(i)}\right\}. \quad (12)$$

Note that (11) follows from (8).

Neglecting the terms independent of $\mathbf{b}_k$, From (6), $\log p(\mathbf{x}_k|b_k, \mathbf{h}_k)$ can be calculated as

$$\log p(\mathbf{x}_k|\mathbf{b}_k, \mathbf{h}_k) \sim \sum_{m=1}^{M} \Re\{b_k(m)\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m)\}.. \quad (13)$$

Inserting (13) in (12), we have for $Q_k(\mathbf{b}_k|\mathbf{b}^{(i)})$

$$Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) = \sum_{m=1}^{M} \Re\{b_k(m)(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]}\} \quad (14)$$

where

$$(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]} \triangleq E\left\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m)|\mathbf{y}, \mathbf{b}^{(i)}\right\} \quad (15)$$

$(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]}$ can be calculated by applying the conditional expectation rule as

$$\begin{aligned}(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]} &= E\{\mathbf{h}_k^\dagger E(\mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h})|\mathbf{y}, \mathbf{b}^{(i)}\} \\ &= E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T E(\mathbf{x}_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h})|\mathbf{y}, \mathbf{b}^{(i)}.\} \end{aligned} \quad (16)$$

The conditional distribution of $\mathbf{x}_k(m)$ given $\mathbf{y}$, $\mathbf{h}$ and $\mathbf{b} = \mathbf{b}^{(i)}$ is Gaussian with mean

$$\begin{aligned}E(\mathbf{x}_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h}) &= (b_k(m))^{[i]}\mathbf{C}_k\mathbf{F}\mathbf{h}_k \\ &+ \beta_k\left(\mathbf{y}(m) - \sum_{j=1}^{K}(b_j(m))^{[i]}\mathbf{C}_j\mathbf{F}\mathbf{h}_j\right)\end{aligned} \quad (17)$$

where $(b_k(m))^{[i]} \triangleq E(b_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h})$. Inserting (17) in (16) and using the properties $\mathbf{F}^\dagger\mathbf{F} = \mathbf{I}$ and $\mathbf{C}_k^T\mathbf{C}_k = \frac{1}{P}\mathbf{I}$. we can rewrite (16) as

$$\begin{aligned}(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]} &= \frac{1}{P}(b_k(m))^{[i]}E\{\mathbf{h}_k^\dagger\mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\} + \beta_k E\{\mathbf{h}_k^\dagger|\mathbf{y}, \mathbf{b}^{(i)}\}\mathbf{F}^\dagger\mathbf{C}_k^T\mathbf{y}(m) \\ &- \beta_k \sum_{j=1, j\neq k}^{K} (b_j(m))^{[i]}E\{\mathbf{h}_k^\dagger\mathbf{F}^\dagger\mathbf{C}_k^T\mathbf{C}_j\mathbf{F}\mathbf{h}_j|\mathbf{y}, \mathbf{b}^{(i)}\}.\end{aligned} \quad (18)$$

On the other hand, since $\mathbf{w} \sim N(0, \Sigma^2\mathbf{I})$ and the prior pdf of $\mathbf{h}$ is chosen as $\mathbf{h} \sim N(0, \Sigma_\mathbf{h})$, we can write the conditional pdf's of $\mathbf{h}$ given $\mathbf{y}$ and $\mathbf{b}^{(i)}$ as

$$\begin{aligned}p(\mathbf{h}|\mathbf{y}, \mathbf{b}^{(i)}) &\sim p(\mathbf{y}|\mathbf{h}, \mathbf{b}^{(i)})p(\mathbf{h}) \\ &\sim \exp\left[-\frac{1}{\sigma^2}(\mathbf{y} - \mathbf{A}\mathbf{h})^\dagger(\mathbf{y} - \mathbf{A}\mathbf{h}) - \mathbf{h}^\dagger\Sigma_\mathbf{h}^{-1}\mathbf{h}\right].\end{aligned} \quad (19)$$

After some algebra it can be shown that

$$p(\mathbf{h}|\mathbf{y}, \mathbf{b}^{(i)}) \sim N(\mu_\mathbf{h}^{(i)}, \Sigma_\mathbf{h}^{(i)}) \quad (20)$$

where

$$\begin{aligned}\mu_\mathbf{h}^{(i)} &= \frac{1}{\sigma^2}\Sigma_\mathbf{h}^{(i)}\mathbf{A}^{(i)\dagger}\mathbf{y} \\ \Sigma_\mathbf{h}^{(i)} &= \left[\Sigma_\mathbf{h}{}^{-1} + \frac{1}{\sigma^2}(\mathbf{A}^{(i)})^\dagger\mathbf{A}^{(i)}\right]^{-1}\end{aligned} \quad (21)$$

and the matrix $\mathbf{A}$ is defined in (3)and (4).

Now let us compute the terms on the right hand side of Eq. (18). We calculate $E\{\mathbf{h}_k^\dagger\mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\}$ as

$$(\|\mathbf{h}_k\|^2)^{[i]} \triangleq E\{\mathbf{h}_k^\dagger\mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\} = \mathrm{tr}\left[\Sigma_\mathbf{h}^{(i)}[k, k] + \mu_h^{(i)}[k]\mu_h^{(i)}[k]^\dagger\right]. \quad (22)$$

where $\Sigma_\mathbf{h}[i, j]$ denotes the (i, j)th element of the matrix $\Sigma_\mathbf{h}$ Second expectation in (18) can be computed as

$$(\mathbf{h}_k)^{[i]} \triangleq E\{\mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\} = \mu_h^{(i)}[k]. \quad (23)$$

Finally, to calculate the last expectation $E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{F} \mathbf{h}_j | \mathbf{y}, \mathbf{b}^{(i)}\}$ in (18), define $\Psi_j \triangleq \mathbf{C}_j \mathbf{F}$, $\mathbf{s}_j \triangleq \Psi_j \mathbf{h}_j$. It then follows that

$$\Sigma_s^{(i)} \triangleq E[\mathbf{s}\mathbf{s}^\dagger | \mathbf{y}, \mathbf{b}^{(i)}] = E[\Psi \mathbf{h}\mathbf{h}^\dagger \Psi^\dagger | \mathbf{y}, \mathbf{b}^{(i)}] = \Psi \Sigma_h^{(i)} \Psi^\dagger. \tag{24}$$

Therefore,

$$E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{F} \mathbf{h}_j | \mathbf{y}, \mathbf{b}^{(i)}\} = E[\mathbf{s}^\dagger \mathbf{s} | \mathbf{y}, \mathbf{b}^{(i)}] = \mathrm{tr}\left[\Sigma_s^{(i)}[k,j] + \mu_s^{(i)}[k]\mu_s^{(i)}[j]^\dagger\right] \tag{25}$$

where, $\mu_s^{(i)} \triangleq \Psi \mu_h^{(i)}$

Maximization-Step (M-Step): The second step to implement the EM algorithm is the *M-Step* where the parameter $\mathbf{b}$ is updated at the $(i+1)$th iteration according to

$$\mathbf{b}^{(i+1)} = \arg\max_{\mathbf{b}} Q(\mathbf{b}|\mathbf{b}_i) = \sum_{k=1}^{K} Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}). \tag{26}$$

M-Step can be performed by maximizing the terms $Q_k(\mathbf{b}_k|\mathbf{b}^{(i)})$ individually, as follows

$$\mathbf{b}_k^{(i+1)} = \arg\max_{\mathbf{b}_k} Q_k(\mathbf{b}_k|\mathbf{b}_i) \tag{27}$$

Moreover, when no coding is used, it follows from (27) that each component of $\mathbf{b}_k^{(i+1)}$ can be separately obtained by maximizing the corresponding summation in the right-hand expression, as follows

$$b_k^{(i+1)}(l) = \mathrm{sgn}\left[\Re\{(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]}\}\right] \tag{28}$$

We finally have

$$\begin{aligned} b_k^{i+1}(l) &= \mathrm{sgn}\left[\Re\left\{\frac{1}{P} b_k^{[i]}(m)\|\mu_h^{(i)}[k]\|^2(1-\beta_k)\right.\right. \\ &+ \left.\left.\beta_k(\mathbf{h}_k^{(i)\dagger}\Psi_k^T\left[\mathbf{y}(m) - \sum_{j=1,j\neq k}^{K} b_j^{[i]}(m)\Psi_j\mu_h^{(i)}[j]\right]\right\}\right] \end{aligned} \tag{29}$$

As a conclusion, Equation (32) can be interpreted as joint channel estimation and data detection with partial interference cancelation. At each incretion step during data detection, the interference reduced signal is fed into a single user receiver consisting of a conventional coherent detector. As a result, a K-user optimization problem have been decomposed into K independent optimization problems which can e solved in a computationally feasible way. Finally we remark that this paper is an extension of the work presented by Fleury to the problem of joint channel estimation and data detection for the uplink multicarrier CDMA systems operating in the presence of the frequency selective channels.

IV. Simulations

In this section, the performance of uplink MC-CDMA systems based on the proposed receiver is analyzed for frequency selective channels. In computer simulations, we assume that all users are received with the same power level. Orthogonal Walsh sequences selected as a spreading code and the processing gain is chosen equal to the number of subcarriers $P = 16$. The number of active users are selected as $K = 8$ and each user sends its data frame composed of T preamble bits, and F data bits, over mobile fading channel. Wireless channels between mobiles antennas and receiver antenna are assumed to be complex Gaussian channel of length L with $N(0, \Sigma_h)$. For all simulations the weight coefficients in (29) are chosen to be equal, i.e., $\beta_k = 1/K$.

At receiver,the initial MMSE channel estimate is obtained by using T preamble bits while channel covariance matrix Σ_h is assumed to be known. Initial MMSE estimate of F data bits is computed from the observation vector $\mathbf{y}$ assuming that the channel coefficients were estimated bi means of the pilot symbols. We refer to this method for obtaining $\mathbf{h}$ and $\mathbf{b}$ as the MMSE Separate Detection and Estimation (MMSE-SDE). In simulations, if the output of the the MMSE-SDE is applied to the parallel interference cancelation (PIC) receiver which is compared with the EM-JDE, it is referred to the Combined MMSE-PIC Receiver. Moreover, we also simulated both MMSE-SDE and Combined MMSE-PIC detectors referred as CSI-MMSE and CSI-Combined MMSE-PIC, respectively for the perfect channel state information cases.

Fig.1 compares the BER performance of the proposed EM-JDE approach with MMSE-SDE, Combined-MMSE-PIC, CSI-MMSE and CSI-Combined MMSE-PIC. For fair comparison we simulated Combined-MMSE-PIC, CSI-Combined MMSE-PIC and the EM-JDE receivers for three iterations. It is observed that the proposed EM-JDE outperforms MMSE-SDE, Combined-MMSE-PIC as well as CSI-MMSE and approaches the CSI-Combined MMSE-PIC.

To investigate initialization effect on the EM-JDE and Combined-MMSE-PIC, we also studied the BER performance of these algorithms for different preamble lengths presented in Fig.2. It is concluded that low preamble lengths provide poor initial

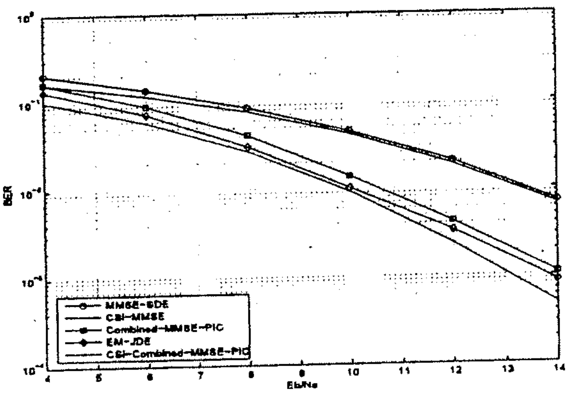


Fig. 1. BER performance in frequency selective channels (T=4,F=40,L=8,U=8)

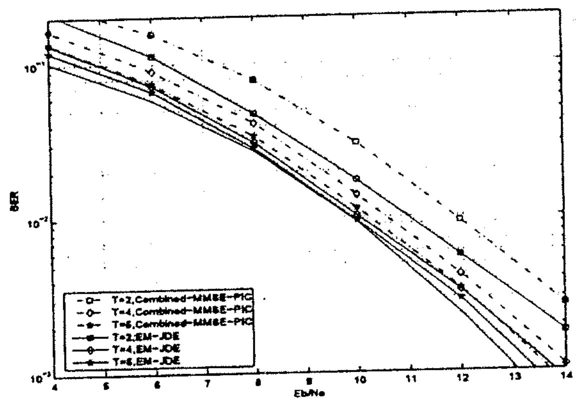


Fig. 2. Behavior of the BER performance as a function of used preamble bits (F=40,L=8,U=8)

estimates, resulting in a more BER performance degradation in the Combined-MMSE-PIC as compared to the EM-JDE system. On the other hand for higher preamble lengths it was observed that performance difference between Combined-MMSE-PIC and EM-JDE decreases because of considerably good initial estimates of channel coefficients.

V. Conclusions

The problem of joint data detection and channel estimation for uplink MC-CDMA systems operating in the presence of frequency selective fading channels was investigated in this work. We presented an iterative approach based on a version of the EM algorithm suitable for superimposed signals. A closed form expression was derived for the data detection which incorporates the channel estimation as well as the partial interference cancelation steps in the algorithm. It was conclude that few pilot symbols were sufficient to initiated the EM algorithm very effectively. Computer simulations were presented to demonstrate the effectiveness of the proposed algorithm in terms of BER performances.

References

[1] S. Hara and R. Prasad, "Overview of Multicarrier CDMA," *IEEE Communication Magazine*, pp. 126133, December 1997.

[2] Z. Xie. R.T. Short and C.K.Rushfortli: "Family of suboptimum detectors for coherent multiuser communications." *IEEE Journal on Selected Areas in Communications*, vol. 8, no. 4, pp. 683-690, May 1990.

[3] V. Kuhn, "Combined MMSE-PIC in coded OFDM-CDMA systems," *IEEE GLOBECOM 01*, pp. 231-235, Nov. 2001.

[4] S. Iraji, T. Sipila and J. Lilleberg, "Channel Estimation and Signal Detection for MC-CDMA in Multipath Fading Channels," *in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC)*, September 1993

[5] A. Kocian and B. H. Fleury, "EM-based joint data detection and channel estimation of DS-CDMA signals," *IEEE Trans. Commun.*, vol. 51, no. 10, pp. 1709-1720, Oct. 2003.

[6] G.K. Kaleh and R. Valet, "Joint parameter estimation and symbol detection for linear or nonlinear unknown channels," *IEEE Trans. Commun.* vol. 42, No. 7, pp. 2406-2413, July 1994.

[7] H. Dogan, H. A. Cirpan and Erdal Panayirci, "Iterative Channel Estimation and Decoding of Turbo Coded SFBC-OFDM Systems," *IEEE Trans. Wireless Commun.*, accepted for publication in Oct. 2006.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2007 proceedings.

EM-Based MAP Channel Estimation and Data Detection for Downlink MC-CDMA Systems

Hakan Doğan†, Erdal Panayırcı‡ and Hakan A. Çırpan†
†Department of Electrical and Electronics Engineering, Istanbul University, Avcilar 34850, Istanbul, Turkey
‡Department of Electronics Enginering, Kadir Has University, Cibali 34230, Istanbul, Turkey

***Abstract*—In this work, we propose a joint MAP channel estimation and data detection technique based on the Expectation Maximization (EM) method with paralel interference cancelation(PIC) for downlink multi- carrier (MC) code division multiple access (CDMA) systems in the presence of frequency selective channels. The EM algorithm derived estimates the complex channel parameters of each subcarriers iteratively and generates the soft information representing the data a posterior probabilities. The soft information is then employed in a PIC module to detect the symbols efficiently.**

Moreover, the MAP-EM approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. The performance of the proposed approach are studied in terms of bit-error rate (BER) and mean square error (MSE). Throughout the simulations, extensive comparisons with previous works in literature are performed, showing that the new scheme can offer superior performance.

I. INTRODUCTION

MC modulation technique, known also as OFDM (Orthogonal Frequency Division Multiplexing), has emerged as an attractive and powerful alternative to conventional modulation schemes in the recent past due to its various advantageous. The advantages of MC which lie behind such a success are robustness in the case of multipath fading, a very reduced system complexity due to equalization in the frequency domain and the capability of narrow-band interference rejection. OFDM has already been chosen as the transmission method for the European radio (DAB) and TV (DVB-T) standard and is used in xDSL systems as well. Supporting multiple users can be achieved in a variety of ways. One popular multiple access scheme is CDMA . CDMA makes use of spread spectrum modulation and distinct spreading codes to separate different users using the same channel It is well known that CDMA system has an ability to reduce user's signal power during transmission using a power control algorithm so that the user can communicate using a low level of transmitted signal which closes to noise power level. As a combination of MC and CDMA techniques, it combines the advantages of both MC and CDMA [1], [2], [3].

This work was supported by the Scientific and Technical Research Council of Turkey (TUBITAK) under Grant 104E166.The work was also supported by the NEWCOM Network of Excellence, a project funded by the European Commision's 6th Framework Programme and the Research Fund of the University of Istanbul through project T-856/02062006.

To evaluate the performance of these systems, ideal knowledge of transmission parameters is often assumed known. Iterative receivers for Coded MC-CDMA promise a significant performance gain compared to conventional non-iterative receivers by using Combined Minimum Mean Square Error-Parallel Interference Cancellation (Combined MMSE-PIC) detector[5] assuming perfectly known channel impulse response. However, the performance of MC-CDMA based transmission systems under realistic conditions critically depends on good estimate of the parameters, such as the channel parameters. In [4], different detection schemes were considered for Least square estimation case as well as perfect channel case.

The quality of multiple access interference (MAI), which can be improved by using channel estimation and data estimation of all active users, effects considerably the performance of PIC detector. Therefore, data and channel estimation performance is obtained in the initial stage has a significant relationship with the performance of PIC. So obviously it is necessary to make excellent joint data and channel estimation for initialization of PIC detector. Inspired by the conclusions in [5] and [4], we include channel estimation into the iterative receiver yields further improvements. We therefore consider iterative channel estimation techniques based on the Expectation-Maximization (EM) algorithm in this paper.

The EM algorithm is a broadly applicable approach to the iterative computation of parameters from intractable and high complexity likelihood functions. An EM approach proposed for the general estimation from superimposed signals [6] is applied to the channel estimation for OFDM systems and compared with SAGE version in [7]. For CDMA systems, Nelson and Poor [8] extend the EM and SAGE algorithms for detection, rather than for estimation of continues parameters. Moreover, EM based channel estimation algorithms were investigated in [9] and [10] for synchronous uplink DS-CDMA and asynchronous uplink DS-CDMA systems respectively. Unlike the EM approaches, we adopt a two-step detection procedure: (i) use the EM algorithm to estimate the channel (frequency domain estimation), and (ii) use the estimated channel to perform coherent detection [11], [12]. The major contribution of the paper is to obtain MAP-EM channel estimation algorithm. The proposed approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. The proposed EM-MAP receiver compared with the combined

MMSE-PIC receiver in the case of LS, LMMSE and perfect channel estimation[4].

II. DOWNLINK MC-CDMA

Transmission of MC-CDMA signals from the base station to mobile stations forms the downlink transmission. The Base station must detect all the signals while each mobile is related with its own signal. In the downlink applications, all the signals arriving from the base station to specific user propagate through the same channel. Therefore, channel estimation methods that is developed for OFDM systems can be appliciable for downlink application of MC-CDMA systems [11].

Let b^k's denote the QPSK modulated symbols that would be send for kth user within mobile cell $k = 1, ..., K$ where K is the number of mobile users which are simultaneously active. The base station spread the data b^k's over chips of length N_c by means of specific orthogonal spreading sequences, $\mathbf{c}^k = (c_1^k, c_2^k, ..., c_{N_c}^k)^T$ where each chip, c_i^k, takes values in the set $\{-\frac{1}{\sqrt{N_c}}, \frac{1}{\sqrt{N_c}}\}$. Then, the spreaded sequences of all users $\mathbf{c}^k b^k$ are summed together to form the input sequences of the OFDM block. After summation process, OFDM modulator block takes inverse discrete Fourier transform (IDFT) and inserts cyclic prefix (CP) of length equal to at least the channel memory (L). Pilot tones uniformly inserted in OFDM modulated data for the initial channel estimation [19]. In this work, to simplify the notation, it is assumed that the spreading factor equals to the number of sub-carriers and all users have the same spreading factor.

At the receiver, CP is removed and DFT is then applied to the received discrete time signal to obtain the received vector expressed as

$$\mathbf{R} = \mathcal{H}\mathbf{C}\mathbf{b} + \mathbf{W} \quad (1)$$

where $\mathbf{C} = [\mathbf{c}^1, ..., \mathbf{c}^K]$ is the $N_c \times K$ spreading code matrix, $\mathbf{b} = [b^1, ..., b^K]^T$ is the $K \times 1$ vector of the transmitted symbols by the K users. $\mathcal{H}$ is the $N_c \times N_c$ diagonal channel matrix whose elements representing the fading of the sub-carriers are modeled in the next section, $\mathbf{W}$ is the $N_c \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive noise in the N_c tones, with variance $\sigma^2/2$ per dimension. Note that due to the orthogonality property of spreading sequences, $\mathbf{C}^T\mathbf{C} = \mathbf{I}$.

In this study, our major focus lies on the development of MAP-EM channel estimation approach based on observation model (1). However, in the sequel we will first present the channel model based on KL basis expansions.

III. CHANNEL: BASIS EXPANSION MODEL

The fading channel between the transmit and the receive antenna is assumed to be frequency and time selective and the fading processes is assumed to be constant during each OFDM symbol. Let $\mathbf{H} = [H_1, H_2, ..., H_{N_c}]^T$ denote the correlated channel coefficients corresponding frequency response of the channel between the transmit and the receive antenna. The KL expansion methodology has been applied for efficient simulation of multipath fading channels [14]. Prompted by the general applicability of KL expansion, we consider in this paper the parameters of $\mathbf{H}$ to be expressed by a linear combination of orthonormal bases.

$$\mathbf{H} = \mathbf{\Psi}\mathbf{G} \quad (2)$$

where $\mathbf{\Psi} = [\psi_1, \psi_2, \cdots, \psi_{N_c}]$, ψ_i's are the orthonormal basis vectors, $\mathbf{G} = [G_1, \cdots, G_{N_c}]^T$, and G_i is the vector representing the weights of the expansion. By using different basis functions $\mathbf{\Psi}$, we can generate sets of coefficients with different properties. The autocorrelation matrix $\mathbf{C_H} = E\left[\mathbf{H}\mathbf{H}^\dagger\right]$ can be decomposed as

$$\mathbf{C_H} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^\dagger \quad (3)$$

where $\mathbf{\Lambda} = E\{\mathbf{G}\mathbf{G}^\dagger\}$ where $\mathbf{\Lambda}$ of is a diagonal matrix (i.e., the coefficients are uncorrelated)(3). Then Eq. (3) represents the *eigendecomposition* of $\mathbf{C_H}$. The fact that only the eigenvectors diagonalize $\mathbf{C_H}$ leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in the Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem in this study is equivalent to estimating the i.i.d. Gaussian vector $\mathbf{G}$, namely, the KL expansion coefficients.

IV. EM BASED MAP CHANNEL ESTIMATION

In MC-CDMA system, channel equalization is moved from the time domain to the frequency domain i.e. the channel frequency response is estimated. Although, it is possible to estimate the channel parameters from the time-domain channel model (channel impulse response), in our work, time domain approach introduces additional complexity mainly because the frequency domain channel parameters are required and directly employed in the detection process. Moreover, the frequency domain estimator presented in this paper was inspired by the conclusions in [15]-[16], where it has been shown that time domain channel estimators based on a Discrete Fourier Transform (DFT) approach for non sample-spaced channels cause aliased spectral leakage and result in an error floor. Furthermore, our proposed frequency domain iterative channel estimation technique employs the KL expansion which reduces the overall computational complexity significantly.

Equation (1) can be rewritten by using the channel expansion to find MAP estimate of $\hat{\mathbf{G}}$ as follows

$$\mathbf{R} = \mathrm{diag}(\mathbf{C}\mathbf{b})\mathbf{\Psi}\mathbf{G} + \mathbf{W} \quad (4)$$

The MAP estimate is then given by

$$\hat{\mathbf{G}} = \arg\max_{\mathbf{G}} p(\mathbf{G}|\mathbf{R}) \quad (5)$$

Direct maximization of (5) is mathematically intractable. However, the solution can be obtained easily by means of the iterative EM algorithm. A natural choice for the complete data for this problem is $\chi = \{\mathbf{R}, \mathbf{b}\}$. The vector to be estimated is $\mathbf{G}$, and the incomplete data is $\mathbf{R}$. The EM algorithm stated above is equivalent to determining the parameter set $\mathbf{G}$ that maximize the Kullback-Leibler information measure defined by

$$Q(\mathbf{G}|\mathbf{G}^{(q)}) = \sum_{\mathbf{b}} p(\mathbf{R}, \mathbf{b}, \mathbf{G}^{(q)}) \log p(\mathbf{R}, \mathbf{b}, \mathbf{G}) \quad (6)$$

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2007 proceedings.

where $\mathbf{G}^{(q)}$ is the estimation of $\mathbf{G}$ at the qth iteration. This algorithm inductively reestimate $\mathbf{G}$ so that a monotonic increase in the *a posteriori* conditional pdf in (5) is guaranteed.

Note that, the term $\log p(\mathbf{R}, \mathbf{b}, \mathbf{G})$ in (6) can be expressed as,

$$\log p(\mathbf{R,b,G}) = \log p(\mathbf{b} \mid \mathbf{G}) + \log p(\mathbf{R} \mid \mathbf{b,G}) + \log p(\mathbf{G}). \quad (7)$$

The first term on the right hand side of (7) is constant, since, the data sequence $\mathbf{b}$ and $\mathbf{G}$ are independent of each other and $\mathbf{b}$ have equal *a priori* probability. The joint probability density function of $\mathbf{G}$ is known by the receiver and can be expressed as

$$p(\mathbf{G}) \sim \exp(-\mathbf{G}^{\dagger}\mathbf{\Lambda}^{-1}\mathbf{G}). \quad (8)$$

Also, given the transmitted symbols $\mathbf{b}$ and the discrete channel representations $\mathbf{G}$ and taking into account the independence of the noise components, the conditional probability density function of the received signal $\mathbf{R}$ can be expressed as,

$$p(\mathbf{R}|\mathbf{b},\mathbf{G}) \sim \exp\left[-(\mathbf{R} - \mathrm{diag}(\mathbf{Cb})\mathbf{\Psi}\mathbf{G})^{\dagger}\mathbf{\Sigma}^{-1}(\mathbf{R} - \mathrm{diag}(\mathbf{Cb})\mathbf{\Psi}\mathbf{G})\right] \quad (9)$$

where $\mathbf{\Sigma}$ is an $N_c \times N_c$ diagonal matrix with $\mathbf{\Sigma}[k,k] = \sigma^2$, for $k = 1, 2, \cdots, N_c$

Taking derivatives in (6) with respect to $\mathbf{G}$ and equating the resulting equations to zero, we have

$$\sum_{b} p(\mathbf{R}, \mathbf{b}, \mathbf{G}^{(q)})(\mathbf{\Psi}^{\dagger}\mathrm{diag}(\mathbf{b}^{\dagger}\mathbf{C}^{T})\mathbf{\Sigma}^{-1}(\mathbf{R} - \mathrm{diag}(\mathbf{Cb})\mathbf{\Psi}\mathbf{G}) - \mathbf{\Lambda}^{-1}\mathbf{G}) = 0 \quad (10)$$

Note that $p(\mathbf{R}, \mathbf{b}, \mathbf{G}^{(q)})$ may be replaced by $p(\mathbf{b} \mid \mathbf{R}, \mathbf{G}^{(q)})$ without violating the equalities in (12). Solving Eq.(12) for $\mathbf{G}$, after taking average over $\mathbf{b}$, the final expression of reestimate of $\hat{\mathbf{G}}^{(q+1)}$ can be obtained as follows:

$$\hat{\mathbf{G}}^{(q+1)} = (\mathbf{T}^{\dagger^{(q)}}\mathbf{T}^{(q)} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{T}^{\dagger^{(q)}}\mathbf{R} \quad (11)$$

where

$$\mathbf{T}^{(q)} = \mathrm{diag}(\mathbf{C}\mathbf{\Gamma}^{(q)})\mathbf{\Psi},$$

and $\mathbf{\Gamma}^{(q)} = [\Gamma^{(q)}(1), \Gamma^{(q)}(2), ..., \Gamma^{(q)}(K)]$ represents the *a posteriori probabilities* of the data symbols at the qth iteration step where

$$\Gamma^{(q)}(k) = \sum_{b \in S_k} bP(b^k = b|\mathbf{R}, \mathbf{G}^{(q)}). \quad (12)$$

$\mathbf{\Gamma}^{(q)}$ can be computed for QPSK signaling following the assumptions $\hat{\mathbf{b}} = 0$ $\mathbf{C}_b = \mathbf{I}$ [12], [13]:

$$\mathbf{\Gamma}^{(q)} = \frac{1}{\sqrt{2}}\tanh\left[\frac{\sqrt{2}}{\sigma^2}Re(\hat{\mathbf{Z}}^{(q)})\right] + \frac{j}{\sqrt{2}}\tanh\left[\frac{\sqrt{2}}{\sigma^2}Im(\hat{\mathbf{Z}}^{(q)})\right] \quad (13)$$

where

$$\hat{\mathbf{Z}}^{(q)} = \mathbf{C}^{T}(\hat{\mathcal{H}}^{\dagger^{(q)}}\hat{\mathcal{H}}^{(q)} + \sigma^2\mathbf{I})^{-1}\hat{\mathcal{H}}^{\dagger^{(q)}}\mathbf{R}.$$

Finally, the data $\mathbf{b}$ transmitted by each user can be estimated at the qth iteration step as

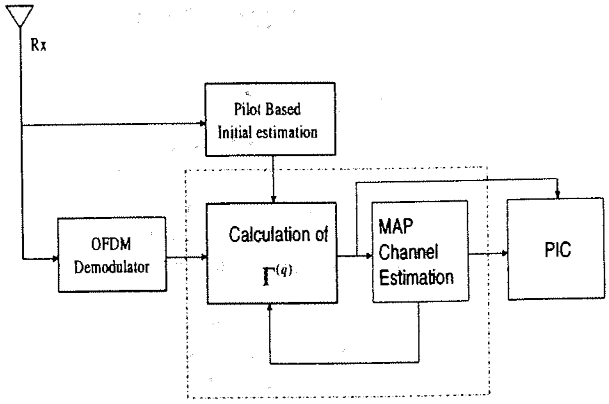


Fig. 1. Receiver structure for MC-CDMA systems.

$$\hat{\mathbf{b}}^{(q)} = \frac{1}{\sqrt{2}}csign(\mathbf{\Gamma}^{(q)}) \quad (14)$$

where "csign" is defined as $\mathrm{csign}(a + jb) = \mathrm{sign}(a) + j\mathrm{sign}(b)$.

Truncation property: The truncated basis vector $\mathbf{G}$ can be formed by selecting r orthonormal basis vectors among all basis vectors that satisfy $\mathbf{C_H}\mathbf{\Psi} = \mathbf{\Psi}\mathbf{\Lambda}$. The optimal solution that yields the smallest average mean-squared truncation error $\frac{1}{N_c}E[\epsilon_r^{\dagger}\epsilon_r]$ is the one expanded with the orthonormal basis vectors associated with the first largest r eigenvalues as given by

$$\frac{1}{N_c - r}E[\epsilon_r^{\dagger}\epsilon_r] = \frac{1}{N_c - r}\sum_{i=r}^{N_c}\lambda_i \quad (15)$$

where $\epsilon_r = \mathbf{G}_l - \mathbf{G}_{l,r}$. For the problem at hand, truncation property of the KL expansion results in a low-rank approximation as well. Thus, a rank-r approximation of $\mathbf{\Lambda}$ can be defined as $\mathbf{\Lambda}_r = diag\{\lambda_1, \lambda_2, \cdots, \lambda_r\}$ by ignoring the trailing $N_c - r$ variances $\{\lambda_l\}_{l=r}^{N_c}$, since they are very small compared to the leading r variances $\{\lambda_l\}_{l=1}^{r}$. Actually, the pattern of eigenvalues for $\mathbf{\Lambda}$ typically splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious. For instance, if the number of parameters in the expansion include dominant eigenvalues, it is possible to obtain a good approximation with a relatively small number of KL coefficients.

Complexity: Based on the approach presented in [17], the traditional LMMSE estimation for $\mathbf{H}$ can be easily expressed as

$$\hat{\mathbf{H}} = \mathbf{C_H}\underbrace{\left[\mathbf{C_H} + \mathbf{\Sigma}(\mathrm{diag}(\mathbf{Cb})^{\dagger}\mathrm{diag}(\mathbf{Cb}))^{-1}\right]^{-1}}_{"O(N_c^3)" computational\ complexity}[\mathrm{diag}(\mathbf{Cb})]^{-1}\mathbf{R}, \quad (16)$$

Since $\left[\mathbf{C_H} + \mathbf{\Sigma}(\mathrm{diag}(\mathbf{Cb})^{\dagger}\mathrm{diag}(\mathbf{Cb}))^{-1}\right]^{-1}$ change with data symbols, its inverse can not be pre-computed and has high computational complexity due to required large-scale matrix inversion [1]. Moreover, the error caused by the small fluctuations in $\mathbf{C_H}$ and $\mathbf{\Sigma}$ have an amplified effect on the channel

[1]The computational complexity of an $N_c \times N_c$ matrix inversion, using Gaussian elimination is $O(N_c^3)$.

estimation due to the matrix inversion. Furthermore, this effect becomes more severe as the dimension of the matrix, to be inverted, increases [18]. Therefore, the KL based approach is need to avoid large-scale matrix inversion. Using (2) and (11), iterative estimate of the $\mathbf{H}$ with KL expansion can be obtained as

$$\hat{\mathbf{H}}^{(q+1)} = \mathbf{\Psi}(\mathbf{T}^{\dagger^{(q)}}\mathbf{T}^{(q)} + \Sigma\Lambda^{-1})^{-1}\mathbf{T}^{\dagger^{(q)}}\mathbf{R}. \quad (17)$$

However, in this form, complexity of channel estimate is greater from traditional LMMSE estimate. Therefore, to reduce the complexity of the estimator firstly we rewrite equation

$$\hat{\mathbf{H}}^{(q+1)} = \mathbf{\Psi}\Lambda(\Lambda\mathbf{T}^{\dagger^{(q)}}\mathbf{T}^{(q)}\Lambda + \Sigma\Lambda)^{-1}\Lambda\mathbf{T}^{\dagger^{(q)}}\mathbf{R} \quad (18)$$

and we proceed with the low-rank approximations by considering only r column vectors of $\mathbf{\Psi}$ and $\mathbf{T}$ corresponding to the r largest eigenvalues of Λ, yielding

$$\hat{\mathbf{H}}^{(q+1)} = \mathbf{\Psi}_r\Lambda_r \underbrace{(\Lambda_r\mathbf{T}_r^{\dagger^{(q)}}\mathbf{T}_r^{(q)}\Lambda_r + \Sigma_r\Lambda_r)^{-1}}_{''O(r^3)'' computational complexity} \Lambda_r\mathbf{T}_r^{\dagger^{(q)}}\mathbf{R} \quad (19)$$

where $\mathbf{\Sigma}_r$ in (19) is a $r \times r$ diagonal matrix whose elements are equal to σ^2, $\mathbf{\Psi}_r$ and $\mathbf{T}_r$ is an $N_c \times r$ matrix which can be formed by omitting the last $N_c - r$ columns of $\mathbf{\Psi}$ and $\mathbf{T}$ respectively. Equation (19) can be rearrange as folows:

$$\hat{\mathbf{H}}^{(q+1)} = \mathbf{\Psi}_r(\mathbf{T}_r^{\dagger^{(q)}}\mathbf{T}_r^{(q)} + \Sigma_r\Lambda_r^{-1})^{-1}\mathbf{T}_r^{\dagger^{(q)}}\mathbf{R}. \quad (20)$$

If the number of parameters in the expansion include dominant eigenvalues (Rank=r), it is possible to obtain an excellent approximation with a relatively small number of KL coefficients. Furthermore, the advantage of this algorithm, besides its simple implementation, is that the channel estimation is instantaneous, since the signal and the pilot are Orthogonal Code Division Multiplexed (OCDM) and they are distorted at the same time.

V. Paralel Interface Cancellation (PIC)

Estimated complex QPSK vector $\hat{\mathbf{b}}$ obtained by eq. (14) passed to a PIC module after last iteration. In this module, the calculation of all interfering signals for user k can be written as

$$\mathbf{R}_{int}^{j} = \hat{\mathcal{H}}\mathbf{C}\hat{\mathbf{b}} \quad \text{for} \quad \hat{b}^k = 0. \quad (21)$$

Interfering signals for user j subtracted from the received signal $\mathbf{R}$ then passed to the single user detector. Finally, the PIC detector for jth user can be written as

$$b_{pic}^k = (\mathbf{c}^k)^T[\hat{\mathcal{H}}^\dagger(\mathbf{R} - \mathbf{R}_{int}^k)] \quad \text{for} \quad k = 1, ..., K \quad (22)$$

For the last iteration detected symbols for QPSK modulation are

$$\hat{b}_{pic}^k = \frac{1}{\sqrt{2}}\text{csign}(b_{pic}^k) \quad \text{for} \quad k = 1, ..., K \quad (23)$$

Initialization: Given the received signal $\mathbf{R}$, the EM algorithm starts with an initial value $\mathbf{G}^{(0)}$ of the unknown channel parameters $\mathbf{G}$. Corresponding to pilot symbols, we focus on a under-sampled signal model and employ the linear minimum mean-square error (LMMSE) estimate to obtain under-sampled channel parameters. Then the complete initial channel gains can easily be determined using an interpolation technique, i.e., Lagrange interpolation algorithm. Finally, the initial values of $\mathbf{G}_\mu^{(0)}$ are used in the iterative EM algorithm to avoid divergence. The details of the initialization process is presented in [11], [17].

VI. Simulations

In this section, performance of MC-CDMA system based on proposed receivers is simulated for frequency selective channels. In simulation, we assume that all users receive the same power. The orthogonal Gold Sequence code selected as spread code and processing gain equals to the number of subcarriers. The assumption of a full load system is made through simulations Fig.[1-2], i.e. the number of active users K, is equal to the length of the spreading code $Nc = 128$. We assume that the *rms* value of the multipath width is $\tau_{rms} = 1$ sample for the power-delay profile. With the τ_{rms} value chosen and to avoid ISI, the guard interval duration is chosen to be equal to 4 sample[17].

The performance merits of proposed structure over other candidates is confirmed by corroborating simulations. Fig. 2 compares the MSE performance of the EM-MAP channel estimation approach with a widely used LS and LMMSE pilot symbol assisted modulation (PSAM)schemes [14], as well as all-pilot estimation for MC-CDMA systems. Pilot Insertion Rate was chosen as (PIR) =1:8. That is one pilot is inserted for every 8 data symbols. It is observed that the proposed EM-MAP significantly outperforms the LS as well as LMMSE techniques and approaches the allpilot estimation case for higher Eb/No values. Moreover, the BER performance of the proposed system also studied for different detection schemes in Fig.3. It is shown that BER performance of the proposed receiver structure is much better that the combined MMSE-PIC receiver in the case of LS, LMMSE while approaches the performance of the all-pilot and perfect channel estimation cases.

The optimal truncation property of KL expansion minimizes the amount of information required to represent the statistically dependent data. Thus, this property can further reduce computational load on the channel estimation algorithm. The optimal truncation property of the KL expansion is exploited in Fig.4 and BER performances are given versus the number used KL expansion coefficients for 12 dB. If the number of parameters in the expansion include dominant eigenvalues (Rank=8), it is possible to obtain an excellent approximation with a relatively small number of KL coefficients.

VII. Conclusions

In this work we have presented an efficient EM-MAP receiver structure for downlink MC-CDMA systems. This algorithm performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing MPSK modulation scheme with additive Gaussian noise. Simulation studies have indicated that the MSE and BER performance of the proposed algorithm well over the

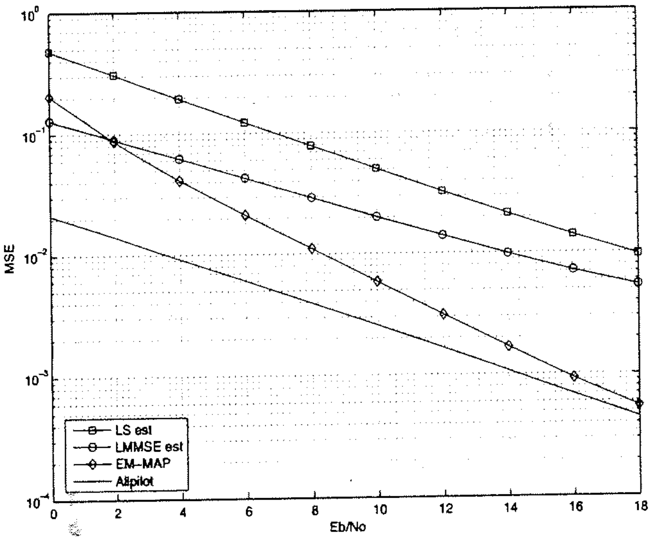


Fig. 2. Comparison of different channel estimation algorithms (MSE)

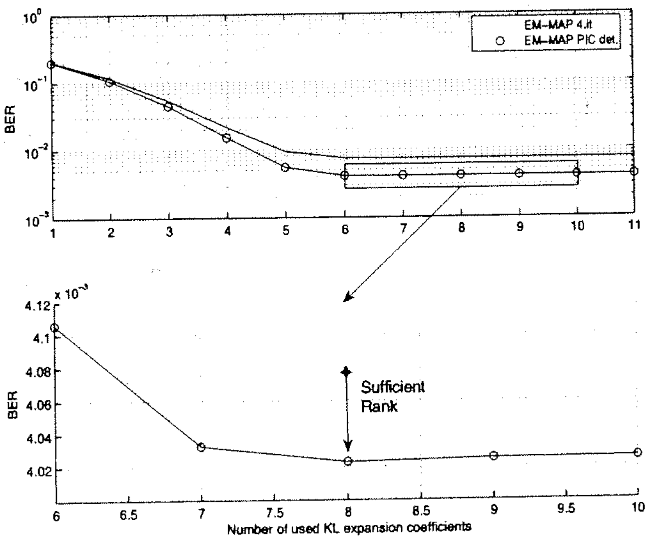


Fig. 4. Optimal truncation property of the KL expansion

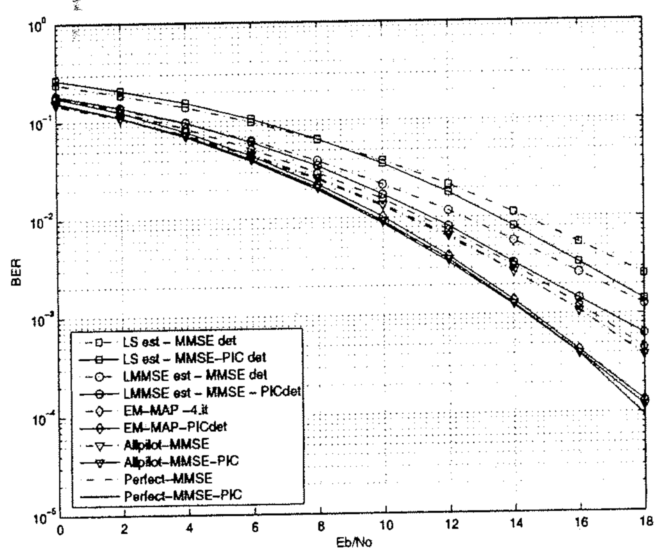


Fig. 3. BER Performances of receiver structures for full load system

conventional algorithms and approaches the perfect estimation case by iterative improvement.

REFERENCES

[1] N. Yee, J.-P. Linnarz, and G. Fettweis, "Multi-carrier CDMA in indoor wireless radio networks," in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC 93), pp. 109-113, September 1993.

[2] K. Fazel and L. Papke, "On the performance of convolutionally- coded CDMA/OFDM for mobile communication system," in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC 93), pp. 468-472, September 1993.

[3] S. Hara and R. Prasad, Overview of Multicarrier CDMA, IEEE Communication Magazine, pp. 126133, December 1997.

[4] S. Iraji, T. Sipila and J. Lilleberg, "Channel Estimation and Signal Detection for MC-CDMA in Multipath Fading Channels," in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC 93), September 7-10

[5] V. Kuhn, "Combined MMSE-PIC in coded OFDM-CDMA systems," IEEE GLOBECOM 01, pp. 231-235, Nov. 2001.

[6] M. Feder and E. Weinstein, "Parameter Estimation of superimposed signals using the EM algorithm,"*IEEE Tran. on Acoustic, Speech and Signal Processing*, Vol. 36, pp. 477-489, April 1988.

[7] X. Yongzhe and C.N. Georghiades, "Two EM-type channel estimation algorithms for OFDM with transmitter diversity," *IEEE Transactions on communications*, vol. 51, No. 1, pp. 106-115 January 2003.

[8] Nelson LB, "Poor HV Iterative multiuser receivers for CDMA channels: An EM-based approach,"*IEEE Trans. Commun.*44 (12): 1700-1710 Dec 1996

[9] A. Kocian and B. H. Fleury, "EM-based joint data detection and channel estimation of DS-CDMA signals,"IEEE Trans. Commun., vol. 51, no. 10, pp. 17091720, Oct. 2003.

[10] A. A. DAmico, U. Mengali, and M. Morelli, "Channel estimation for the uplink of a DS-CDMA system," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 11321137, Nov. 2003.

[11] H.A Çirpan , E. Panayirci , H. Dogan, "Nondata-aided Channel Estimation for OFDM Systems with Space-Frequency Transmit Diversity"*IEEE Trans. Vehicular Technology*, vol. 55, no.2, pp.449 - 457, March 2006.

[12] H. Dogan, H. A. Çirpan and Erdal Panayirci, "Iterative Channel Estimation and Decoding of Turbo Coded SFBC-OFDM Systems," *IEEE Trans. Wireless Commun.*, accepted for publication in Oct. 2006.

[13] S. Kay, Fundamentals of Statistical Signal Processing: Estimation. Theory. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[14] K. Yip and T. Ng, "Karhunen-Loeve expansion of the WSSUS channel output and its application to efficient simulation," *IEEE J. Selected Areas Commun.*, vol. 15, no 4, pp. 640-646, May 1997.

[15] Y. Baoguo, C. Zhigang, and K. Letaief, "Analysis of low-complexity windowed DFT-based MMSE channel estimator for OFDM systems," *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 19771987, Nov. 2001.

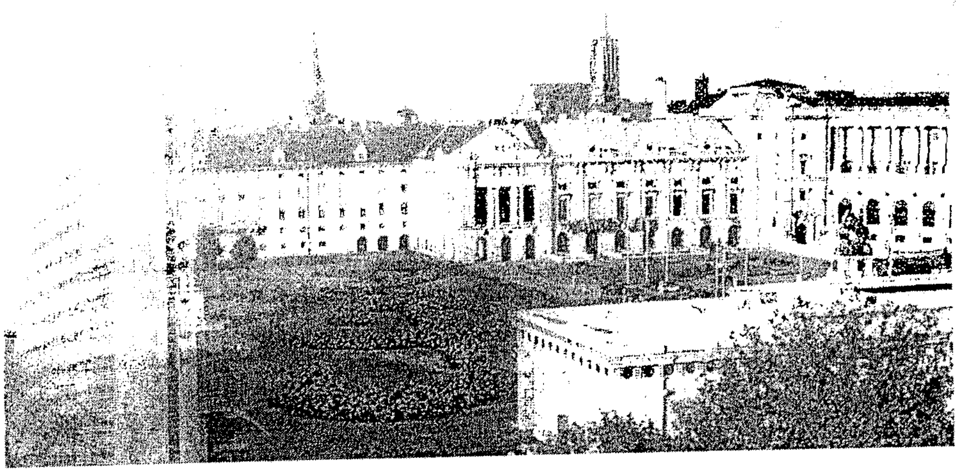
[16] O. Edfors, M. Sandell, J. J. van de Beek, S. K. Wilson, and P. O. Borjesson, "Analysis of DFT-based channel estimation for OFDM," *Wireless Personal Commun.*, vol. 12, no. 1, pp. 5570, Jan. 2000.

[17] O. Edfors, M. Sandell, J.-J. van de Beek, S.K. Wilson, P.O. Borjesson, "OFDM channel estimation by singular value decomposition," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 931-936, July 1998.

[18] Jie Zhu; Wookwon Lee, "A low-complexity channel estimator for OFDM systems in multipath fading channels," *In proceedings of IEEE-PIMRC 2004.*, vol. 3, pp.1978 - 1982, 5-8 Sept. 2004

[19] Coleri, S.; Ergen, M.; Puri, A.; Bahai, A., "Channel estimation techniques based on pilot arrangement in OFDM systems," *IEEE Trans. on Broadcasting*, Volume 48, Issue 3, pp. 223 - 229, Sept. 2002.

NEWCOM-ACoRN Joint Workshop 2006



Welcome Messages
Committees
Hosting Institutions
Sessions and Papers
BibTeX Database

Hosting Institutions

The **ARC Communications Research Network (ACoRN)** is a network of researchers and research students. ACoRN aims to stimulate creativity, innovation and breakthrough science, leading to technological advancement in telecommunications. ACoRN is made up of Australian Universities and Industry that are involved in telecommunications research and also has strong collaborative links with international groups, such as NEWCOM.

The **Telecommunications Research Center Vienna (ftw.)** is the joint research center of leading players in telecom business and science in Austria. Round 60 researchers from all over the world conduct research into telecommunication's technologies for tomorrow. Each project is defined through dialog between academia and industry and selected on the grounds of their industrial relevance.

With about 40 scientific staff members, the **Institute of Communications and Radio-Frequency Engineering** is one of the largest departments of the Faculty of Electrical Engineering and Information Technology at Vienna University of Technology. The department's research and development efforts emphasize fields of great practical relevance, including mobile communications, optical communications, radio-frequency engineering, rapid prototyping, and signal processing.

NEWCOM (Network of Excellence in Wireless COMmunications) is an European network that links in a cooperative way a large number of leading research groups addressing the Strategic Objective „Mobile and wireless systems beyond 3G", a frontier research area of the Priority Thematic Area of IST.

The **Vienna University of Technology** was founded in 1815 as Imperial-Royal Polytechnical Institute and was the first University of Technology within present-day German-speaking Europe. Today the university finds high international and domestic recognition in teaching as well as in research and is a highly esteemed partner of innovation oriented enterprises.

OFDM (Thursday, 2b)

Nonlinear mitigation for two carrier transmission using regenerative satellite payloads
Craig Burnet

* Iterative Channel Estimation and Decoding of Turbo/Convolutionally Coded STBC-OFDM Systems
Hakan Dogan; Hakan Cirpan; Erdal Panayirci

Partial transmit sequences method for reduction of PAPR in real-valued OFDM systems
Ping Wu; Guangyue Lu; Catharina Carlemalm-Logothetis

On timing synchronization approaches for OFDM systems with receive antenna diversity
Shuang Tian; Jean Armstrong

Error Control Coding (Thursday, 3a)

High Data Rate Trellis Coded Modulation
Yi Hong

Erasure Correcting LDPC Codes for Channels with Packet Losses
Sarah Johnson

Improved Sum-Min decoding of irregular LDPC codes using nonlinear post-processing
Gottfried Lechner; Jossy Sayir

Analysis and Design of Concatenated Codes
Fredrik Brännström; Alexandre Graell i Amat; Lars K. Rasmussen

Wireless Signal Processing (Thursday, 3b)

Symmetric Radial Basis Function Network Equaliser
Sheng Chen; Andreas Wolfgang; Sergio Benedetto; Pierre Duhamel; Lajos Hanzo

Design of compact components for ultra wideband communication front ends
Marek Bialkowski; Amin Abbosh; Hing Kan

Design and Simulation of NLOS High Data Rate mm-Wave WLANs
Ian Holland; André Pollok; William Cowley

ITERATIVE CHANNEL ESTIMATION AND DECODING OF TURBO/CONVOLUTIONALLY CODED STBC-OFDM SYSTEMS

Hakan Doğan[†], *Hakan A. Çırpan*[†] *and Erdal Panayırcı*[‡]

[†]Istanbul University, Department of Electrical and Electronics Engineering, Avcilar 34320, Istanbul, Turkey
[‡]Bilkent University, Department of Electrical and Electronics Engineering, Bilkent 06800, Ankara, Turkey

ABSTRACT

In this paper, we propose EM-based turbo receiver structure for STBC-OFDM systems in unknown wireless dispersive channels, with outer channel coding. The algorithm iteratively executes joint channel estimation and turbo decoding for STBC-OFDM systems. It is clear that good channel codes are more sensitive to the poorly estimated channel. With high correlation between the coded bits, a well designed channel code is more sensitive to channel estimation errors which might cause severe error propagation in the decoding process. To understand the behavior of different channel encoders, we therefore consider both turbo and convolutionally coded systems. Moreover, the performance of the proposed turbo receiver structure is studied for different pilot insertion rates and doppler frequencies.

1. INTRODUCTION

Iterative procedure based on Expectation-Maximization (EM) algorithm was also applied to channel estimation problem in the context of space-time block-coding (STBC) [1, 2] as well as transmit diversity OFDM systems with or without outer channel coding (e.g. convolutional code or Turbo code) [3, 4, 5]. In [3], maximum a *posteriori* (MAP) EM based iterative receivers for STBC-OFDM systems with Turbo code are proposed directly to detect transmitted symbols, assuming that fading processes remain constant across several OFDM words contained in one STBC code-word. An EM approach proposed for the general estimation from superimposed signals [6] is applied to the channel estimation for transmit diversity OFDM systems with outer channel code (convolutional code) and compared with SAGE version in [4]. Moreover in [5], a modified version of [4] is proposed for STBC-OFDM and space-frequency block-coding (SFBC)-OFDM systems.

Unlike the EM approaches used in [3, 4, 5], a non-data-aided EM-MAP channel estimation algorithm was proposed for SFBC-OFDM sytems assuming channel varies for every OFDM word [9]. This structure is also generalized for STBC/SFBC-OFDM systems with mismatch performances [10]. In this paper, we propose EM-based turbo receiver structures for turbo/convolutionally coded STBC-OFDM systems by using EM-MAP channel estimator.

The main contribution of the paper mainly comes from the fact that the channel parameter estimation technique proposed in our paper is for the STBC-OFDM transmitter diversity systems with outer channel coding. The estimation algorithm performs an iterative estimation of the fading channel parameters in frequency domain according to the maximum a posteriori criterion (MAP) as opposed to the ML approaches adopted in many publications appeared in the literature. Furthermore, our approach is based on a novel representation of the fading channel by means of the Karhunen-Loeve (KL) expansion and the application of this expansion to the turbo receiver structures for STBC-OFDM systems. Note that, KL orthogonal expansion together with space-time coded system based on the Alamouti orthogonal design enable us to estimate the channel in a very simple way without taking inverse of large dimensional matrices, yielding a computationally efficient iterative analytical expressions [9]. Moreover, optimal truncation property of the KL expansion is exploited in our paper resulting in a further reduction in computational load on the channel estimation algorithm. In order to explore the performance of the proposed turbo receivers, we first investigate the effect of a pilot spacing on the turbo receiver performance by considering average MSE as well as bit-error-rate (BER). We also analyze the sensitivity of turbo receiver structures on channel estimation errors.

This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 6th Framework Programme and the Research Fund of Istanbul University under Project T-856/02062006. This work was also supported in part by the Turkish Scientific and Technical Research Institute(TUBITAK) under Grant 104E166

2. CHANNEL ENCODED STBC-OFDM TRANSMITTER

As illustrated in Fig.1, the binary information bits $\{\mathbf{b}_i\}$, $1 \leq i \leq N_c$ of random data are encoded by an outer-channel-code, resulting in a BPSK-coded symbol stream $\{\mathbf{C}\}$. The coded symbols are then interleaved by a random permutation results in a stream of independent symbols $\{\mathbf{X}\}$of length $2N_c$.

The coded symbol vector $\mathbf{X}$ can therefore be expressed as $\mathbf{X} = [\mathbf{X_0}, \mathbf{X_1}, \cdots, \mathbf{X_{2N_c-1}}]^{\mathbf{T}}$. Resorting subchannel grouping, $\mathbf{X}$ is coded into two vectors $\mathbf{X(2n)}$ and $\mathbf{X(2n+1)}$ by the space-time encoder as

$$\mathbf{X(2n)} = [X_0, X_2, \cdots, X_{2N_c-4}, X_{2N_c-2}]^T,$$
$$\mathbf{X(2n+1)} = [X_1, X_3, \cdots, X_{2N_c-3}, X_{2N_c-1}]^T, \quad (1)$$

2.1 STBC-OFDM Systems

In order to compensate for the reduced data rate of turbo codes, some space-time codes having data rates greater than one could be employed. However it is well known from literature that the Alamouti antenna modulation configuration is the only scheme which retain orthogonality and full rate when the for complex-valued data as well as the low complexity. As will be seen shortly,orthogonality property is essential and required condition for the channel estimation algorithm in our paper. Moreover, orthogonality structure of Alamouti allows decoupling of the channel and reduces the equalizer complexity. Furthermore, the Alamouti's schemes has been adopted in several wireless standards such as WCDMA and CDMA2000. In this paper, we consider the transmit diversity scheme with 2 transmit and 1 receive antennas to provide a diversity of order 2. The wireless channel is assumed to be a quasi-static so that path gains are constant over a frame of L_{frame} and vary from one frame to another. The channel frequency response for the kth tone corresponding to the ith

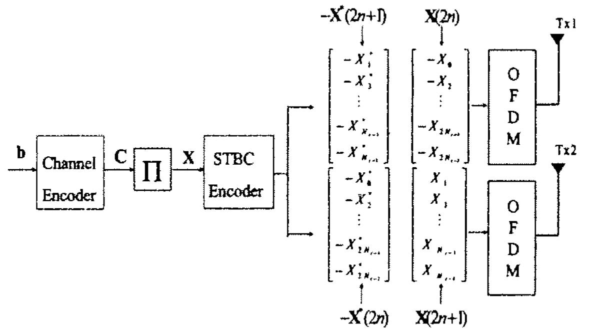


Figure 1: Transmitter structure for STBC-OFDM with outer channel coding

transmitter antenna and the receiver antenna is defined to be channel attenuations $H_l(k)$,

$$H_l(k) = H(k/NT_s), k = 0, 1, \cdots, N_c - 1 \quad (2)$$

where $H_l(.)$ is the frequency response of the channel $h_l(t)$ between the lth transmitter and the receiver.

Since the Alamouti's two branch transmit diversity scheme with one receiver is employed here, each pair of the two consecutive received signal can be expressed in vector form as

$$\begin{aligned} \mathbf{R}(2n) &= \mathcal{X}(2n)\mathbf{H}_1(2n) \\ &+ \mathcal{X}(2n+1)\mathbf{H}_2(2n) + \mathbf{W}(2n) \quad (3) \\ \mathbf{R}(2n+1) &= -\mathcal{X}^*(2n+1)\mathbf{H}_1(2n+1) \\ &+ \mathcal{X}^*(2n)\mathbf{H}_2(2n+1) + \mathbf{W}(2n+1) \end{aligned}$$

where $\mathbf{R}(2n) = [R_0, R_2, \cdots, R_{2N_c-2}]^T$ and $\mathbf{R}(2n+1) = [R_1, R_3, \cdots, R_{2N_c-1}]^T$ and $\mathcal{X}(2n)$ and $\mathcal{X}(2n+1)$ are an $N_c \times N_c$ diagonal matrices whose elements $\mathbf{X}(2n)$ and $\mathbf{X}(2n+1)$ respectively. Finally, $\mathbf{W}(2n)$ and $\mathbf{W}(2n+1)$ are an $N_c \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive noise in the N_c tones and $\mathbf{H}_l = [H_l(0), \cdots, H_l(N_c-1)]^T$ denotes the channel attenuations for the N tones between the lth transmitter and the receiver.

Equation (3) shows that the information symbols $\mathcal{X}(2n)$ and $\mathcal{X}(2n+1)$ are transmitted twice in two consecutive time intervals through two different channels. To simplify the problem, we assume that the complex channel gains remain constant over the duration of one ST-OFDM code word, i.e., $\mathbf{H}_1(2n) \approx \mathbf{H}_1(2n+1)$ and $\mathbf{H}_2(2n) \approx \mathbf{H}_2(2n+1)$. In order to estimate the channels and decode $\mathbf{X}$ with the embedded diversity gain through the repeated transmission, for each n, we define, $\mathbf{R} = [\mathbf{R}^T(2n) \ \ \mathbf{R}^T(2n+1)]^T$ and write (3) into a matrix form

$$\mathbf{R} = \mathcal{X}\ \mathbf{H} + \mathbf{W} \quad (4)$$

where $\mathbf{H} = [\mathbf{H}_1^T \ \ \mathbf{H}_2^T]^T$, $\mathbf{W} = [\mathbf{W}^T(2n) \ \ \mathbf{W}^T(2n+1)]^T$ and

$$\mathcal{X} = \begin{bmatrix} \mathcal{X}(2n) & \mathcal{X}(2n+1) \\ -\mathcal{X}^*(2n+1) & \mathcal{X}^*(2n) \end{bmatrix}. \quad (5)$$

Obviously, channel estimation is very essential for decoding STBC-OFDM with outer channel encoder. In this paper, a novel channel estimation algorithm is presented by representing the discrete multipath channel based on the Karhunen-Loeve (KL) orthogonal representation and make use of the Expectation Maximization technique.

3. CHANNEL: BASIS EXPANSION MODEL

Dispersive fading channels are modeled widely by the block fading channel model[8]. According to this model, the channel is assumed to remain constant over a block of a given size and successive blocks may be correlated or independent. This is an approximate model that would be applied to some of the practical communication systems such as OFDM, frequency-hopped spread-spectrum (FHSS) and time-division multiple access (TDMA).

In this paper, it is assumed that the channel is frequency selective during each OFDM symbol [7] and exhibits time selectivity over the OFDM symbols according to Doppler frequency. We consider the Alamouti transmitter diversity coding scheme, employed in an OFDM system utilizing N_c subcarrier per antenna transmissions. Note that N_c is chosen as an even integer.

Prompted by the general applicability of KL expansion, we consider in this paper the parameters of $\mathbf{H}_l$ to be given by a linear combination of orthonormal bases where $\mathbf{H}_l$ is the channel frequency response between the lth transmit antenna and the receive antenna. An orthonormal expansion of the vector $\mathbf{H}_l$ involves expressing the $\mathbf{H}_l$ as a linear combination of the orthonormal basis vectors as $\mathbf{H}_l = \mathbf{\Psi}\mathbf{G}_l$ where $\mathbf{\Psi} = [\psi_0, \psi_1, \cdots, \psi_{N_c-1}]$, ψ_i's are the orthonormal basis vectors, $\mathbf{G}_l = [G_{l,1}, \cdots, G_{l,N_c-1}]^T$, and $G_{l,i}$ is the weights of the expansion. By using different basis functions $\mathbf{\Psi}$, we can generate sets of coefficients with different properties. The autocorrelation matrix $\mathbf{C}_{\mathbf{H}_l} = E\left[\mathbf{H}_l\mathbf{H}_l^\dagger\right]$ can be decomposed as

$$\mathbf{C}_{\mathbf{H}_l} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^\dagger \quad (6)$$

where $\mathbf{\Lambda} = E\{\mathbf{G}_l\mathbf{G}_l^\dagger\}$. The KL expansion is one where $\mathbf{\Lambda}$ of (6) is a diagonal matrix (i.e., the coefficients are uncorrelated). If $\mathbf{\Lambda}$ is diagonal, then (6) must be *eigendecomposition* of $\mathbf{C}_{\mathbf{H}_l}$. The fact that only the eigenvectors diagonalize $\mathbf{C}_{\mathbf{H}_l}$ leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in the Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem in this application equivalent to estimating the i.i.d. Gaussian vector $\mathbf{G}_l$ KL expansion coefficients.

The channels between transmitter and receiver in this paper are assumed to be doubly-selective where, $\mathbf{H}_l(n)$'s have exponentially decaying power delay profiles, described by $\theta(\tau_l) = C\exp(-\tau_l/\tau_{rms})$. The delays τ_l are uniformly and independently distributed over the length of the cyclic prefix. τ_{rms} determines the decay of the power-delay profile and C is the normalizing constant.

For the channel model, normalized discrete channel-correlations for different subcarriers and blocks of this channel model were presented in [7] as follows,

$$r_2(k,k') = \frac{1-\exp\left[-L\left(\frac{1}{\tau_{rms}} + 2\pi j(k-k')/N_c\right)\right]}{\tau_{rms}(1-\exp(-L/\tau_{rms}))\left(\frac{1}{\tau_{rms}} + j2\pi(k-k')/N_c\right)} \quad (7)$$

$$r_1(n,n') = J_0(2\pi(n-n')f_dT_s) \quad (8)$$

where, (k,k') denotes different subcarriers, L is the cyclix prefix, N_c is the total number of subcarriers. Also in (6) (n,n') denotes the discrete times for the different OFDM symbols, $J_0(.)$ is the zeroth-order Bessel function of the first kind and f_d is the Doppler frequency.

4. TURBO RECEIVER

In recent years, inspired by the development of turbo coding, various types of iterative channel estimation, detection and decoding schemes have been proposed in the literature. These approaches have shown that iterative receivers can offer significant performance improvements over the nonitерative counterparts. Iterative receiver structure for coded STBC-OFDM system is illustrated in Fig. 2 [12]. which uses three submodules to carry on iterative process: Channel estimation, STBC-OFDM decoding and the MAP outer channel code decoding.

4.1 EM-Based MAP Channel Estimation

The MAP criterion is used in the fading channel as seen at the FFT output of the OFDM receiver since the joint probability density function of the random variables are known by the receiver and can be expressed as

$$p(\mathbf{G}_l) \sim \exp(-\mathbf{G}_l^H \mathbf{\Lambda}^{-1} \mathbf{G}_l), \quad l = 1, 2. \tag{9}$$

Given the transmitted signals $\mathbf{X}$ as coded according to Alamouti's scheme and the discrete channel representations $\mathbf{G} = [\mathbf{G}_1^T, \mathbf{G}_2^T]^T$ and taking into account the independence of the noise components, the conditional probability density function of the received signal $\mathbf{R}$ can be expressed as,

$$p(\mathbf{R}|\mathcal{X}, \mathbf{G}) \sim \exp\left[-(\mathbf{R} - \mathcal{X}\widetilde{\mathbf{\Psi}}\mathbf{G})^H \mathbf{\Sigma}^{-1} (\mathbf{R} - \mathcal{X}\widetilde{\mathbf{\Psi}}\mathbf{G})\right] \tag{10}$$

where $\mathbf{\Sigma}$ is an $2N_c \times 2N_c$ diagonal matrix with $\mathbf{\Sigma}[k,k] = \sigma^2$, for $k = 0, 1, \cdots, 2N_c - 1$ and

$$\widetilde{\mathbf{\Psi}} = \begin{bmatrix} \mathbf{\Psi} & 0 \\ 0 & \mathbf{\Psi} \end{bmatrix}. \tag{11}$$

After long algebraic manipulations, EM-MAP estimate $\mathbf{G}_l^{(q+1)}$ $(l = 1, 2)$ can be obtained as follows [9]:

$$\begin{aligned} \mathbf{G}_1^{(q+1)} &= (\mathbf{I} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1} \mathbf{\Psi}^\dagger \left[\hat{\mathcal{X}}^{\dagger(q)}(2n)\mathbf{R}(2n)\right. \\ &- \left.\hat{\mathcal{X}}^{(q)}(2n+1)\mathbf{R}(2n+1)\right] \\ \mathbf{G}_2^{(q+1)} &= (\mathbf{I} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1} \mathbf{\Psi}^\dagger \left[\hat{\mathcal{X}}^{\dagger(q)}(2n+1)\mathbf{R}(2n)\right. \\ &+ \left.\hat{\mathcal{X}}^{(q)}(2n)\mathbf{R}(2n+1)\right] \end{aligned} \tag{12}$$

where it can be easily seen that

$$(\mathbf{I} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1} = diag[(1 + \sigma^2/\lambda_0)^{-1}, \cdots, (1 + \sigma^2/\lambda_{N_c - 1})^{-1}]$$

and $\hat{\mathcal{X}}^q(2n)$ and $\hat{\mathcal{X}}^q(2n+1)$ in (12) is an $N_c \times N_c$ dimensional diagonal matrix representing the *a posteriori* probabilities of the data symbols at the qth iteration step.

Truncation property: The truncated basis vector $\mathbf{G}_{l,r}$ can be formed by selecting r orthonormal basis vectors among all basis vectors that satisfy $\mathbf{C}_{\mathbf{H}_l}\mathbf{\Psi} = \mathbf{\Psi}\mathbf{\Lambda}$. The optimal solution that yields the smallest average mean-squared truncation error $\frac{1}{N_c/2} E[\epsilon_r^\dagger \epsilon_r]$ is the one expanded with the orthonormal basis vectors associated with the first largest r eigenvalues as given by

$$\frac{1}{N_c/2 - r} E[\epsilon_r^\dagger \epsilon_r] = \frac{1}{N_c/2 - r} \sum_{i=r}^{N_c/2-1} \lambda_i \tag{13}$$

where $\epsilon_r = \mathbf{G}_l - \mathbf{G}_{l,r}$. For the problem at hand, truncation property of the KL expansion results in a low-rank approximation as well. Thus, a rank-r approximation to $\mathbf{\Lambda}_r$ is defined as $\mathbf{\Lambda}_r = diag\{\lambda_0, \lambda_1, \cdots, \lambda_{r-1}, 0, \cdots, 0\}$. Since the trailing $N_c/2 - r$ variances $\{\lambda_l\}_{l=r}^{N_c/2-1}$ are very small compared to the leading r variances $\{\lambda_l\}_{l=0}^{r-1}$, they are set to zero to produce the approximation. Actually, the pattern of eigenvalues for $\mathbf{\Lambda}$ typically splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious. The optimal truncated KL (rank-r) estimates of (12) can easily be obtained by replacing $\mathbf{\Lambda}$ with $\mathbf{\Lambda}_r$.

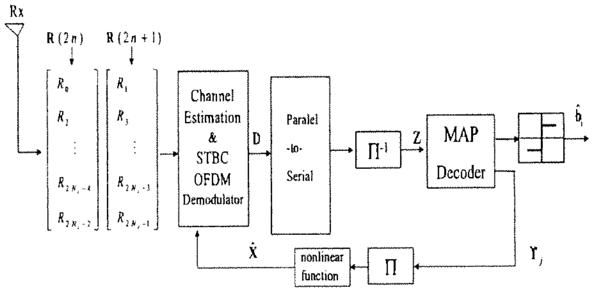


Figure 2: Receiver structure STBC-OFDM with outer channel coding

4.2 Iterative Equalization and Decoding

We now consider the STBC-OFDM decoding algorithm and the MAP outer channel code decoding to complete the description of the Turbo receiver. Since the channel vectors or equivalently expansion coefficients are estimated through EM based iterative approach, it is possible to decode $\mathbf{R}$ with diversity gains by a simple matrix multiplication. Before dealing with how we resolve decoding, let us first re-express received signal model (3) as

$$\tilde{\mathbf{R}} = \mathcal{H}\tilde{\mathbf{X}} + \tilde{\mathbf{W}} \tag{14}$$

where $\tilde{\mathbf{R}} = [\mathbf{R}^T(n), \mathbf{R}^\dagger(2n+1)]^T$, $\tilde{\mathbf{X}} = [\mathbf{X}^T(n), \mathbf{X}^T(2n+1)]^T$, $\tilde{\mathbf{W}} = [\mathbf{W}^T(n), \mathbf{W}^\dagger(2n+1)]^T$ and

$$\mathcal{H} = \begin{bmatrix} \mathcal{H}_1(2n) & \mathcal{H}_2(2n) \\ \mathcal{H}_2^\dagger(2n+1) & -\mathcal{H}_1^\dagger(2n+1) \end{bmatrix} \tag{15}$$

where $\mathcal{H}_l(2n)$ and $\mathcal{H}_l(2n+1)$ $l = 1, 2$ are $N_c \times N_c$ diagonal matrices

Depending on complexity versus performance tradeoffs, any linear equalizer can be applied to retrieve $\tilde{\mathbf{X}}$ from (14). In this paper, we consider linear equalizer where the parameters are updated using the MMSE criterion. We define the linear MMSE estimate $\mathbf{D}$ of $\tilde{\mathbf{X}}$ given the observation $\tilde{\mathbf{R}}$ by

$$\begin{aligned} \mathbf{D} &= \bar{\tilde{\mathbf{X}}} + \mathbf{C}_{\tilde{\mathbf{X}}}\mathcal{H}^\dagger(\mathcal{H}^\dagger\mathbf{C}_{\tilde{\mathbf{X}}}\mathcal{H} + \mathbf{C}_{\tilde{\mathbf{W}}})^{-1} \\ &\times (\tilde{\mathbf{R}} - \mathcal{H}\bar{\tilde{\mathbf{X}}}) \end{aligned} \tag{16}$$

where $\bar{\tilde{\mathbf{X}}}$, $\mathbf{C}_{\tilde{\mathbf{X}}}$ and $\mathbf{C}_{\tilde{\mathbf{W}}}$ are the mean of $\tilde{\mathbf{X}}$, the covariance matrix of $\tilde{\mathbf{X}}$ and the covariance matrix of $\tilde{\mathbf{W}}$ respectively.

With a scaled unitary matrix $\mathcal{H}$ and approximately constant complex channel gains with $\mathcal{H}_1^2(2n) + \mathcal{H}_2^2(2n) \approx 1$ assumptions, we can simplify $\mathcal{H}^\dagger\mathcal{H}$ as

$$\mathcal{H}^\dagger\mathcal{H} = \mathbf{I}_{2N_c \times 2N_c} \tag{17}$$

where $\mathbf{I}_{2N_c \times 2N_c}$ is the $2N_c \times 2N_c$ identity matrix. Moreover, following the assumptions used in [13], $\bar{\tilde{\mathbf{X}}} = \mathbf{0}$, $\mathbf{C}_{\tilde{\mathbf{X}}} = \mathbf{I}$, (16) becomes

$$\mathbf{D} = (\mathbf{I} + \sigma_n^2\mathbf{I})^{-1}\mathcal{H}^\dagger\tilde{\mathbf{R}}. \tag{18}$$

If we set $\mathbf{C}_{\tilde{\mathbf{W}}} = \mathbf{0}$ in (16), further simplified form of linear equalizer called zero forcing (ZF) equalizer is obtained resulting in

$$\mathbf{D} = \mathcal{H}^\dagger\tilde{\mathbf{R}} = \mathcal{H}^\dagger\mathcal{H}\tilde{\mathbf{X}} + \eta \tag{19}$$

where $\eta = \mathcal{H}^\dagger \tilde{\mathbf{W}}$.

Since we propose a Turbo receiver structure for STBC-OFDM systems in the paper, it consists of an iterative MAP-EM channel estimation algorithm, STBC decoder and a soft MAP outer-channel-code decoder. As shown in Fig.2, first EM based channel estimator computes channel gains according to pilot symbols as described in the initialization step. Output of the estimator is used in the STBC demodulator (19). Next, the equalized symbol sequence $\{\mathbf{D}\}$ is passed through a channel deinterleaver, resulting deinterleaved equalized symbols sequence $\{\mathbf{Z}\}$. Finally, $\{\mathbf{Z}\}$ is applied to a MAP decoder by deinterleaved estimated channel gains. In the MAP decoder submodule, log likelihood ratio's (LLRs) of a posteriori probabilities on the coded and uncoded bits are yielded. In the next iteration, LLRs of coded bits $\{\Upsilon_j\}$ are reinterleaved and passed through a nonlinearity. Output of the nonlinearity computes soft value estimation of $\mathbf{X}$ as $\hat{\mathbf{X}}$ in Fig2.

$\hat{\mathbf{X}}$ is used as $\hat{\mathcal{X}}^{(q)}(2n)$ and $\hat{\mathcal{X}}^{(q)}(2n+1)$ in (12) for next iteration. Thus, the MAP-EM channel estimator iteratively generates the channel estimates by taking the received signals from receiver antennas and interleaved soft value of LLRs which are computed by the outer channel code decoder in the previous iteration. Then, STBC-OFDM decoder takes channel estimates together with the received signals and computes equalized symbol sequence for next turbo iteration. Iterative operation is fulfilled between these three submodules. In all simulations, three iterations are employed.

5. SIMULATIONS

In this section, computer simulations are carried out to evaluate performances of the proposed turbo receiver structure for STBC-OFDM systems. We consider both turbo and convolutionally coded STBC-OFDM systems. In case of Turbo Encoder, two identical recursive systematic convolutional component codes (RSC) with generator $(1{,}5_8/7_8)$ concatenated in parallel via a pseudorandom interleaver form the encoder [14]. For the convolutionally coded system, a $(5_8, 7_8)$ code with rate 1/2 code was used.

The scenario for STBC-OFDM simulation study consists of BPSK modulation format. The system has a 2.28 MHz bandwidth (for the pulse roll-off factor $\alpha = 0.2$) and is divided into $N_c = 512$ tones with a total period Ts of 136 μs, of which 1.052 μs constitute the cyclic prefix (L=4). The data rate is 1.9 Mbit/s. We assume that the rms value of the multipath width is $\tau_{rms} = 1$ sample (0.263 μs) for the power-delay profile. In order to choose good initial values for the unknown channel parameters, the N_{PS} data symbols $\{X_k(n)\}$ for $k \in S_{PS}$, in each OFDM frame are inserted as pilot symbols known by the receiver. The details of the initialization process is presented in [7].

Fig. 3 compares the BER performance of the EM-MAP channel estimation approach with a EM-ML [4] and widely used LMMSE pilot symbol assisted modulation (LMMSE-PSAM) channel estimation techniques [11]for turbo coded STBC-OFDM systems in case of pilot insertion rate (PIR)=(1:8). It can be seen that the proposed EM-MAP significantly outperforms to EM-ML as well as PSAM techniques. Moreover, It performs comparable performance to perfect channel case. On the other hand, it should be noted that EM-MAP and LMMSE outperforms EM-ML but require the knowledge of channel statistics. Similar results was given in figure Fig.4 as MSE performance of Channel estimator with the modified Cramer Rao Bound (MCRB) [9].

The sensitivity of a good channel code to imperfect channel estimation is obvious. With high dependence between the coded bits, a good channel code is also more sensitive to channel estimation which might cause severe error propagation in the decoding process. Therefore, in Fig.5 we investigate Channel estimation algorithm performances for convolutionally coded STBC-OFDM systems. We demonstrate EM-MAP channel estimation advantage just well over EM-ML techniques but not LMMSE technique for convolutionally coded systems.

Moreover, In Fig.6 and Fig.7 sensitivity to channel parameter estimation errors is investigated for Turbo/Convolutionally coded STBC-OFDM systems as a function of PIRs (1:8,1:16,1:32) for $f_d = 50$ and $f_d = 200$ respectively. More performance degradation in turbo coded systems compared to convolutionally encoded systems is observed since lower pilot insertion rates provide poor initial estimates.

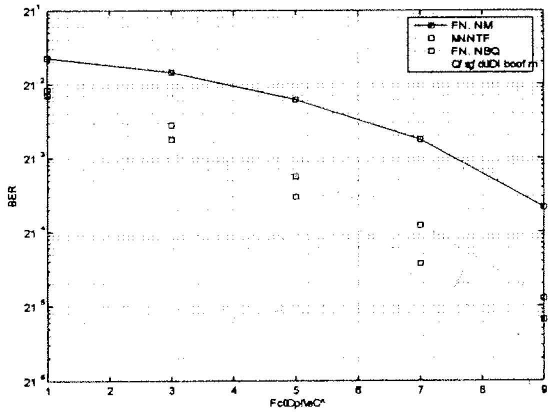


Figure 3: BER performance of channel estimators for Turbo coded STBC-OFDM systems ($f_d = 50$, PIR=1:8)

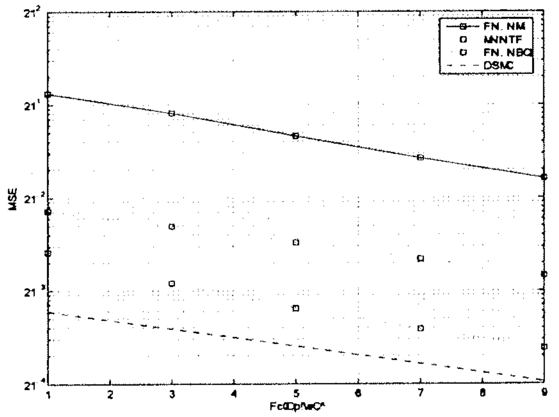


Figure 4: MSE performance of channel estimators for Turbo coded STBC-OFDM systems ($f_d = 50$, PIR=1:8)

6. CONCLUSION

In this paper, a low-complexity EM based MAP channel estimation algorithm based on a novel representation of the fading channel by means of the Karhunen-Loeve (KL) expansion has been proposed for Turbo/Convolutionally coded STBC-OFDM systems. It was demonstrated that the proposed EM-MAP significantly outperforms to EM-ML as well as PSAM techniques while approaches Perfect Channel situation for Turbo coded STBC-OFDM systems. Moreover,

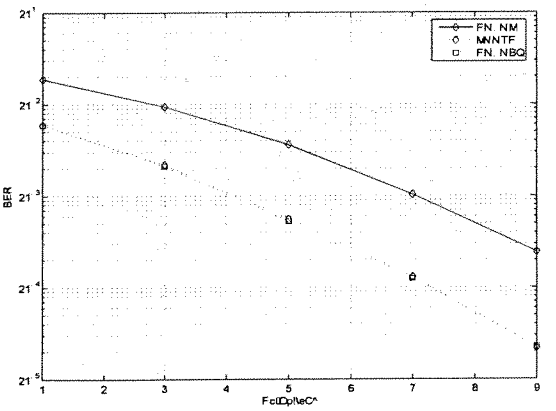


Figure 5: BER performance of channel estimators for Convolutionally coded STBC-OFDM systems (f_d = 50, PIR=1:8)

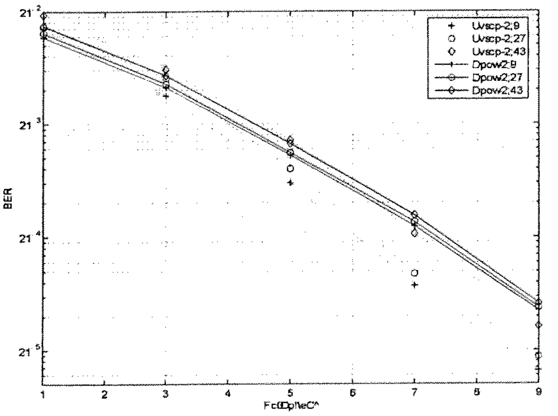


Figure 6: BER performance of the EM-MAP algorithm for Turbo/Convolutionally coded STBC-OFDM systems as a function of PIRs (f_d = 50)

sensitivity to channel estimation errors of turbo receivers are investigated for turbo coded and convolutionally coded STBC-OFDM systems. As a final remark, it has been shown that receiver with turbo codes perform well over convolutional coded receiver structures assuming channel estimation performance is sufficient.

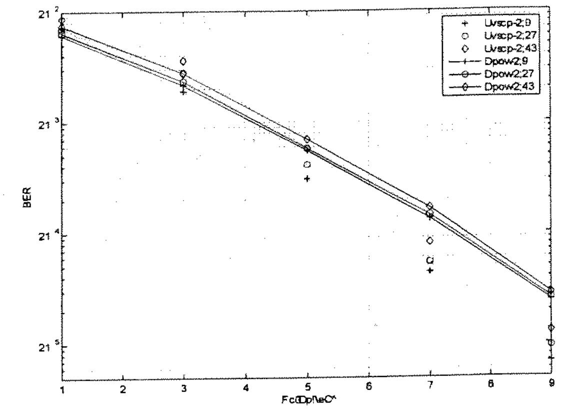


Figure 7: BER performance of the EM-MAP algorithm for Turbo/Convolutionally coded STBC-OFDM systems as a function of PIRs (f_d = 200)

REFERENCES

[1] Y. Li, C.N. Georghiades, and G. Huang, "Iterative maximum likelihood sequence estimation for space-time coded systems," *IEEE Trans. on Commun.*, vol. 49, No. 6, pp. 948-951, June 2001.

[2] C. Cozzo and B.L. Hughes, "Joint channel estimation and data detection in space-time communications," *IEEE Trans. on Commun.*, Vol 51, No. 8, pp 1266-1270., August 2003.

[3] B. Lu, X. Wang and Y. (G) Li, "Iterative receivers for space-time block-coded ofdm systems in dispersive fading channels," *IEEE Trans.on Wireless Commun.*, vol. 1, No. 2, pp. 213-225, April 2002.

[4] X. Yongzhe and C.N. Georghiades, "Two EM-type channel estimation algorithms for OFDM with transmitter diversity," *IEEE Trans. on Commun.*, vol. 51, No. 1, pp. 106-115, January 2003.

[5] X. Ma, H. Koyabashi and S. C. Schwarts, "An EM-based channel estimation for space-time and space-frequency block coded OFDM," *In proceedings of ICASSP '03*, Volume: 4 , pp. 389 - 392, April 2003.

[6] M. Feder and E. Weinstein, "Parameter Estimation of superimposed signals using the EM algorithm," *IEEE Tran. on Acoustic, Speech and Signal Processing*, Vol. 36, pp. 477-489, April 1988.

[7] O. Edfors, M. Sandell, J.-J. van de Beek, S.K. Wilson, P.O. Borjesson, "OFDM channel estimation by singular value decomposition," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 931-936, July 1998.

[8] R. J. McEliece and W. E. Stark, Channels with block interference, IEEE Trans. Inform. Theory, vol. 30, no. R. G. Gallager, Low Density Purity Check Codes, MIT Press, 1963. 1, pp. 44-53, Jan. 1984.

[9] H.A Cirpan , E. Panayirci , H. Dogan, "Nondata-aided Channel Estimation for OFDM Systems with Space-Frequency Transmit Diversity" *IEEE Transactions on Vehicular Technology*, Volume 55, Issue 2, Page(s):449 - 457, March 2006.

[10] Cirpan HA, Panayirci E, Dogan H, "Iterative channel estimation approach for space-time/frequency coded OFDM systems with transmitter diversity," *Eur. Trans. Telecommun.*, 15 (3): 235-248 May-Jun 2004.

[11] M. Morelli and U. Mengali, "A comparison of pilot-aided channel estimation methods for OFDM systems," *IEEE Trans. Signal Processing*, vol. 49, pp. 3065 3073, Dec. 2001.

[12] Valenti, M.C.; Woerner, B.D.,"Iterative channel estimation and decoding of pilot symbol assisted turbo codes over flat-fading channels," *IEEE J. Select. Areas Commun.*, Volume: 19 , Issue: 9 , pp. 1697 - 1705, Sept. 2001.

[13] M. Tuchler, R. Koetter, and A. C. Singer, "Turbo equalization: principles and new results," *IEEE Trans. Commun.*, vol. 50, pp. 754767, May 2002.

[14] J.P. Woodard, L. Hanzo, "Comparative study of turbo decoding techniques: an overview," *Vehicular Technology, IEEE Transactions on*, Volume: 49 , Issue: 6 , pp. 2208 - 2233, Nov. 2000

EUSIPCO 2006

European Signal Processing Conference, September 4 - 8 2006, Florence, Italy

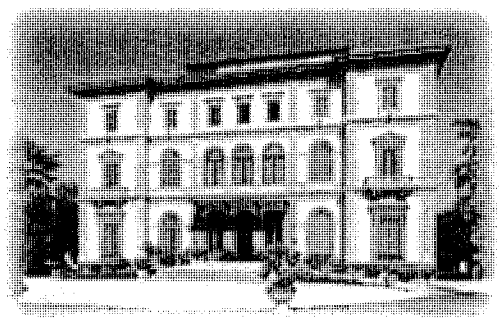
DATES AND PLACES
TUTORIALS
SPECIAL SESSIONS
ORGANIZING COMMITTEE
TECH. PROGRAM COMMITTEE
PLENARY LECTURES
GENERAL CHAIRMAN'S ADDRESS
TECH. CHAIRMEN'S ADDRESS
SPONSORS
REVIEWERS
TECH. PROGRAM AT-A-GLANCE

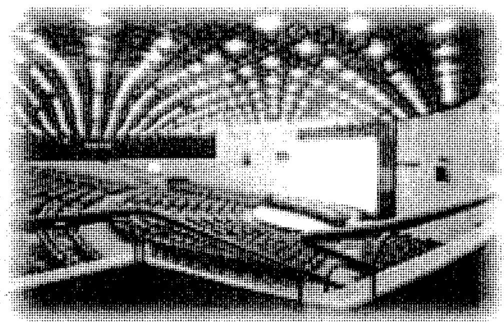
SESSION INDEX
AUTHOR INDEX

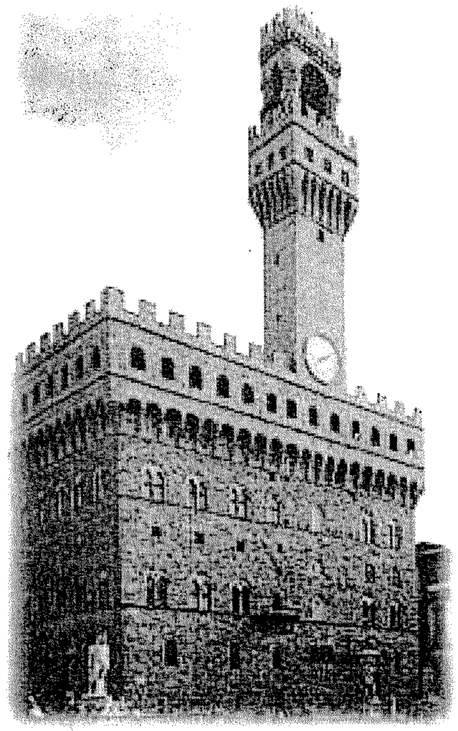
FULL-TEXT SEARCH

HOME

The 2006 European Signal Processing Conference is the 14th in series conference promoted by EURASIP, the European Association for Signal, Speech and Image Processing and organized by the University of Pisa.







EURASIP European Association for Signal Processing

UNIVERSITÀ DI PISA

INCOR-DGMP

BLIND PHASE NOISE ESTIMATION AND DATA DETECTION BASED ON SMC TECHNIQUE AND UNSCENTED FILTERING

Erdal Panayırcı[†], *Hakan A. Çırpan*[‡], *Marc Moeneclaey*[§], *and Nele Noels*[§]

[†]Department of Electronics Engineering, Kadir Has University, Cibali 34230, Istanbul, Turkey
[‡]Department of Electrical Engineering, Istanbul University, Avcilar 34320, Istanbul, Turkey
[§]TELIN/DIGCOM Department, Ghent University, B9000 Gent, Belgium

ABSTRACT

In this paper, a computationally efficient algorithm is presented for tracing phase noise with linear drift and blind data detection jointly, based on a sequential Monte Carlo(SMC) method. Tracing of phase noise is achieved by Kalman filter and the nonlinearity of the observation process is taken care of by unscented filter rather that using extended Kalman technique. On the other hand,SMC method treats the transmitted symbols as " missing data" and draw samples sequentially of them based on the observed signal samples up to time t. This way, the Bayesian estimates of the phase noise and the incoming data are obtained through these samples, sequentially drawn, together with their importance weights. The proposed receiver structure is seen to be ideally suited for high-speed parallel implementation using VLSI technology.

1. INTRODUCTION

The problem of carrier phase synchronization is of great importance in coherent digital communication systems. A considerable amount of research has been carried out for data detection in the presence of the time-varying phase noise as well as the fixed phase offset [1]. Estimating the phase offset and detecting the data jointly by maximum likelihood (ML) technique does not seem to be analytically tractable. Even if the likelihood function can be evaluated offline, however, it is invariably a nonlinear function of the parameter to be estimated, which makes the maximization step (which must be performed in real-time) computationally infeasible. A number of suboptimal algorithms have thus been proposed, most of which employ a two-stage receiver structure with a phase noise estimation stage followed by the data detection [2]. Phase synchronization is typically implemented by a decision directed(or data aided) or non-decision directed (or non-data aided). Decision directed schemes depend on availability of reliably detected symbol for obtaining the phase estimate, and therefore, they usually require transmission of pilot or training data. However, in applications where bandwidth is the most precious resource, training data can significantly reduce the overall system capacity. Thus blind or non-data aided techniques become an attractive alternative [3, 4].

Unlike data-aided techniques, non-data-aided methods do not require knowledge of the transmitted data, and instead, they exploit statistics of digital transmitted signal. ML estimation techniques can also be used in non-decision-directed methods if the symbols transmitted are treated as random variables with known statistics so that the likelihood function can be averaged over the data sequence received. Unfortunately, except for few simple cases, this averaging process is mathematically impracticable and it can be obtained only by some approximations which are valid only either at high or low SNR values [5].

On the other hand, in order to provide an implementable solution, recently there have been a substantial amount of work on iterative formulation of the parameter estimation problem based on the Expectation-Maximization (EM) technique [6]. It is known that the EM algorithm derives iteratively and converges to the true ML estimation of these unknown parameters. The main drawbacks of this approach are that the algorithm is sensitive to the initial starting values chosen for the parameters, it does not necessarily converge to the global extremum and the convergence can be slow. Furthermore, in situation where the posterior distribution must be constantly updated with arrival of the new data with missing parts, EM algorithm can be highly inefficient, because the whole iteration process must be redone with additional data. The sequential Monte Carlo(SMC) methodology [7] that has emerged in the field of statistics and engineering has shown great promise to solve such problems. This technique can approximate the optimal solution directly without compromising the system model. Additionally, the decision made at time t does not depend on any decisions made previously, and thus, no error is propagated in their implementation. More importantly, the SMC yields a fully blind algorithm and allows for both Gaussian and non-Gaussian ambient noise as well as high-speed parallel implementations. Furthermore, the tracking the time-varying phase noise and the data detection are naturally integrated. The algorithm is self-adaptive and no training/pilot symbols or decision feedback are needed [8].

The main objective of this paper is to investigate the use of the SMC method to the problem of jointly detecting the data and tracking the phase noise with linear drift in the presence of white Gaussian noise. The algorithm is based on

This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 6th Framework Programme. This work was also supported in part by the Turkish Scientific and Technical Research Institute(TUBITAK) under Grant 104E166 and the Research Fund of the University of Istanbul under Project: UDP-732/05052006.

a Bayesian formulation. The tracking of phase noise is implemented by a Kalman filter algorithm. Rather than applying the extended Kalman algorithm, an *unscented filter* technique is employed to handle nonlinearity of the observation process. On the other hand, the basic idea SMC method is to treat the transmitted symbols as "missing data" and to sequentially draw samples of them based on the current observation and computing appropriate importance sampling weights. The technique does not require iterations and updating with new data can be done cheaply.

2. SYSTEM DESCRIPTION

We consider a digital communication system in the presence of random phase noise and the additive Gaussian noise. The input binary information bit d_t are encoded using some channel code, resulting in a code bit stream b_t. The code bits are passed to a symbol mapper, yielding complex data symbols s_t, which take values from a finite alphabet set $A = \{a_1, a_2, \cdots, a_{|A|}\}$, where $|A|$ represents the cardinality of the set A. Each data symbol is then transmitted through a channel whose input-output relationship is given by

$$y_t = s_t e^{i\theta_t} + n_t, \qquad t = 0, 1, \cdots \tag{1}$$

where y_t, s_t, θ_t, are the received signal, the transmitted symbols and the phase noise, respectively, and n_t the additive complex Gaussian noise with mean zero and the variance $\sigma_n^2 = E[|n_t|^2]$. The phase noise process θ_t at tth sampling instant is defined as a Wiener process determined as

$$\begin{aligned} \theta_t &= \theta_{t-1} + u_t, \qquad t = 1, 2, \cdots \\ \theta_0 &\sim \text{uniform}(-\pi, +\pi) \end{aligned} \tag{2}$$

where $\{u_t\}$ is a sequence of independent and identically distributed (i.i.d.) zero-mean random variables with variance equal to σ_u^2. Note that as Wiener phase noise is the accumulation of white noise, its variance increase linearly with t. It is assumed that u_t and n_t are independent.

Our main objective is to solve the problem of online detection of the symbols s_t and estimation of the phase noise θ_t, completely blindly, based on the received signals up to time t, $\{y_i\}_{i=0}^t$. Defining the vectors, $\boldsymbol{S}_t = [s_0, s_1, \cdots, s_t]^T$, $\boldsymbol{Y}_t = [y_0, y_1, \cdots y_t]^T$, $\boldsymbol{\theta}_t = [\theta_0, \theta_1, \cdots, \theta_t]^T$, the the problem may be formulated by making Bayesian inference with respect to the posterior distribution

$$\begin{aligned} p(\boldsymbol{\theta}_t, \boldsymbol{S}_t|\boldsymbol{Y}_t) &\propto p(\theta_0)p(\boldsymbol{S}_t)p(y_0|\theta_0, s_0)\prod_{j=1}^{t} p(\theta_j|\theta_{j-1})p(y_j|\theta_j, s_j) \\ &\propto p(\theta_0)p(\boldsymbol{S}_t)\exp\left(-\frac{1}{\sigma^2}|y_0 - s_0e^{j\theta_0}|^2\right) \\ &\times \prod_{j=1}^{t}\exp\left(-\frac{1}{\sigma_u^2}(\theta_j - \theta_{j-1})^2 - \frac{1}{\sigma^2}|y_j - s_je^{j\theta_j}|^2\right). \end{aligned}$$

Although this joint distribution can be written out explicitly up to a normalizing constant, the computation of the corresponding marginal joint distributions $p(s_t, \theta_t|\boldsymbol{Y}_t)$, necessary for online joint symbol detection and phase noise estimation involve very high dimensional integration. Therefore, the task is mathematically infeasible in practice. The Gibbs samples [9] is a Monte Carlo method for overcoming this difficulty. However it is not an adaptive procedure and has difficulty dealing with sequentially observed data. With new data coming the whole computation must be repeated to incorporate new information. we now present an adaptive blind algorithm for the joint symbol detection and the phase noise estimation which is based on a Bayesian formulation of the problem called Sequential Monte Carlo(SMC) method first developed by [9].

Consider the case of uncoded system, where the symbols are assumed to independent and identically distributed, i.e.,

$$P(s_t = a_i|\boldsymbol{S}_{t-1}) = P(s_t = a_i), a_i \in A \ . \tag{3}$$

For simplicity the symbols are chosen from a QPSK constellation.When no prior information about the symbols is available, the symbols are assumed to take each possible value in A with equal probability, i.e., $P(s_t = a_i) = 1/|A|$. Since we are interested in jointly estimating the symbol s_t and the phase noise θ_t, at time t based on the observation $\boldsymbol{Y}_t$, the Bayes solution requires the posterior distribution

$$p(s_t, \theta_t|\boldsymbol{Y}_t) = \int p(\theta_t|\boldsymbol{Y}_t, \boldsymbol{S}_t)p(\boldsymbol{S}_t|\boldsymbol{Y}_t)d\boldsymbol{S}_{t-1}. \tag{4}$$

Note that with a given $\boldsymbol{S}_t$, the nonlinear (Kalman filter) model (1), (2) can be solved using the extended Kalman filter (EKF) technique [10] by linearizing the observation equation (1) [8]. The EKF is the most widely used estimation algorithm for nonlinear systems. However, the long past experience has shown that it is difficult to implement, difficult to tune, and only reliable for systems that are almost linear on the time scale of the updates. Many of these difficulties are mainly due to the linearization process inherent in the EKF technique. To overcome this limitation, the *unscented filtering (UF)* technique was developed as a method of propagate mean and covariance information through nonlinear transformation. It was shown that it is more accurate, easier to implement and uses the same order of calculation as linearization. Therefore, we apply in this paper the UF technique for solving the Kalman filtering part of our problem. However, before the details of the Kalman algorithm is given, the UF technique is explained briefly in the following section.

3. THE UNSCENTED FILTERING

The Unscented Filtering is a technique for calculating the mean and covariance of a random variable which undergoes a nonlinear transformation. The details of technique can be found in [14] and is summarized as follows: Suppose that $\boldsymbol{x}$ is an $n \times 1$ dimensional random vector with mean $\boldsymbol{\mu_x}$ and covariance $\mathbf{P_{xx}}$. A second random variable , $\boldsymbol{y}$ is related to $\boldsymbol{x}$ through the nonlinear function

$$\boldsymbol{y} = \psi(\boldsymbol{x}).$$

The mean $\boldsymbol{\mu_y}$ and covariance $\mathbf{P_{yy}}$ of $\boldsymbol{y}$ can be calculated as follows: The n-dimensional random vector $\boldsymbol{x}$ with mean $\boldsymbol{\mu_x}$ and covariance $\mathbf{P_{xx}}$ is approximated by $2n+1$ points χ_i, called the *sigma-points*, and the weights W_i given by

$$\chi_0 = \boldsymbol{\mu_x}, \qquad W_0 = \kappa/(n+\kappa)$$

$$\begin{aligned} \mathcal{X}_i &= \mu_{\boldsymbol{x}} + \left(\sqrt{(n+\kappa)\mathbf{P}_{\boldsymbol{xx}}}\right)_i, & W_i = 1/2(n+\kappa) \\ \mathcal{X}_{i+n} &= \mu_{\boldsymbol{x}} - \left(\sqrt{(n+\kappa)\mathbf{P}_{\boldsymbol{xx}}}\right)_i, & W_{i+n} = 1/2(n+\kappa) \end{aligned}$$

where $\kappa \in \Re$ and $(\sqrt{(n+\kappa)\mathbf{P}_{\boldsymbol{xx}}})_i$ represents the ith row or column of the matrix square root of $(n+\kappa)\mathbf{P}_{\boldsymbol{xx}}$ and W_i is the weight which is associated with the ith point. The transformation procedure is as follows:

1. Obtain the set of transformed sigma-points,

$$\mathcal{Y}_i = \psi(\mathcal{X}_i), \qquad i = 0, 1, \cdots, 2n$$

2. Compute the mean given by the weighted average of the transformed points

$$\mu_{\boldsymbol{y}} = \sum_{i=0}^{2n} W_i \mathcal{Y}_i$$

3. Compute the covariance by the weighted outer product of the transform points,

$$\mathbf{P}_{\boldsymbol{yy}} = \sum_{i=0}^{2n} W_i (\mathcal{Y}_i - \mu_{\boldsymbol{y}})(\mathcal{Y}_i - \mu_{\boldsymbol{y}})^\dagger$$

The detailed properties of the algorithms can be found in [14]. Note that κ in the algorithm provides an extra degree of freedom to adjust the higher order moments of the approximation and can be used to to reduce the overall prediction errors. It was shown in [14] that when $\boldsymbol{x}$ is Gaussian, a useful choice of the κ is $\kappa = n - 3$.

4. KALMAN FILTERING BASED ON UNSCENTED TRANSFORMATION

The phase noise process (1) is a Gaussian process. Hence,

$$p(\theta_t | \boldsymbol{S}_t, \boldsymbol{Y}_t) \sim N(\mu_{\theta_t}(\boldsymbol{S}_t), \sigma^2_{\theta_t}(\boldsymbol{S}_t)), \tag{5}$$

where the mean $\mu_{\theta_t}(\boldsymbol{S}_t)$ and the variance $\sigma^2_{\theta_t}(\boldsymbol{S}_t)$ can be obtained as follows. Denoting $\widehat{\theta}_{t|t-1}$ as the estimator of θ_t based on the observations $\boldsymbol{Y}_{t-1} = (y_0, y_1 \cdots, y_{t-1})$ and

$$\mu_{\theta_t}(\boldsymbol{S}_t) \triangleq \widehat{\theta}_{t|t} \quad \text{and} \quad \sigma^2_{\theta_t}(\boldsymbol{S}_t) \triangleq M_{t|t}, \tag{6}$$

$\widehat{\theta}_{t|t}$ and $M_{t|t}$ can be calculated recursively by using the Kalman Technique [10, page 449-452] along with the unscented transformation, given $\boldsymbol{S}_t$ as:

$$\begin{aligned} \widehat{\theta}_{t|t} &= \widehat{\theta}_{t|t-1} + K_t \nu & (7) \\ M_{t|t} &= M_{t|t-1} - K_t M^{\nu\nu}_{t|t-1} K^*_t & (8) \\ \nu &= y_t - \widehat{y}_{t|t-1} & (9) \\ K_t &= M^{\theta\nu}_{t|t-1} (M^{\nu\nu}_{t|t-1})^{-1}, & (10) \end{aligned}$$

where

$$\begin{aligned} M^{\nu\nu}_{t|t-1} &= E\left[(y_t - \widehat{y}_{t|t-1})(y_t - \widehat{y}_{t|t-1})^* | \boldsymbol{Y}_{t-1}\right] \\ M^{\theta\nu}_{t|t-1} &= E\left[(\theta_t - \widehat{\theta}_{t|t-1})(y_t - \widehat{y}_{t|t-1})^* | \boldsymbol{Y}_{t-1}.\right] \end{aligned}$$

In order to implement the Kalman filter algorithm given above, one needs to compute

- Prediction of the new state of the phase noise $\widehat{\theta}_{t|t-1}$ and its variance $M_{t|t-1}$
- Prediction of the expected observation $\widehat{y}_{t|t-1}$ and the innovation variance $M^{\nu\nu}_{t|t-1}$
- Prediction of the cross correlation $M^{\theta\nu}_{t|t-1}$.

Note that, since the state equation (2) is linear we can easily obtain $\widehat{\theta}_{t|t-1}$ and $M_{t|t-1}$ as follows:

$$\begin{aligned} \widehat{\theta}_{t|t-1} &= \widehat{\theta}_{t-1|t-1} & (11) \\ M_{t|t-1} &= M_{t|t-1} + \sigma^2_u. & (12) \end{aligned}$$

Furthermore, it can be easily shown from (1) and (2) that

$$\begin{aligned} \widehat{y}_{t|t-1} &= E\{\exp(j\theta_t) | \boldsymbol{Y}_{t-1}\} & (13) \\ M^{\nu\nu}_{t|t-1} &= 1 + \sigma^2_w - |\widehat{y}_{t|t-1}|^2 & (14) \\ M^{\theta\nu}_{t|t-1} &= E\{\theta_t \exp(-j\theta_t) | \boldsymbol{Y}_{t-1}\} - \widehat{y}^*_{t|t-1}\widehat{\theta}_{t|t-1}. & (15) \end{aligned}$$

The expectations above can be computed by the unscented filtering technique as follows: Since the θ_t's are one-dimensional and Gaussian, three sigma-points would be sufficient to implement the algorithm. The three sigma points and the corresponding weights are chosen according to the general formulation explained in Section 3 as

$$\begin{aligned} \Theta^{(0)}_{t|t-1} &= \widehat{\theta}_{t|t-1}, & W_0 = \kappa/(1+\kappa) \\ \Theta^{(1)}_{t|t-1} &= \widehat{\theta}_{t|t-1} + \sqrt{(1+\kappa)M_{t|t-1}}, & W_1 = 1/2(1+\kappa) \\ \Theta^{(2)}_{t|t-1} &= \widehat{\theta}_{t|t-1} - \sqrt{(1+\kappa)M_{t|t-1}}, & W_2 = 1/2(1+\kappa). \end{aligned}$$

Note that since θ_t is Gaussian, and $n = 3$, the value of $\kappa = 0$ as pointed out in Section 3. Therefore $W_0 = 0$ and the only following two sigma-points are sufficient to implement the algorithm.

$$\begin{aligned} \Theta^{(1)}_{t|t-1} &= \widehat{\theta}_{t|t-1} + \sqrt{M_{t|t-1}}, & W_1 = 1/2 \quad (16) \\ \Theta^{(2)}_{t|t-1} &= \widehat{\theta}_{t|t-1} - \sqrt{M_{t|t-1}}, & W_2 = 1/2 \quad (17) \end{aligned}$$

We now summarize the algorithm to compute the expectations (13) and (15).

1. Select the two sigma points $\Theta^{(0)}_{t|t-1}, \Theta^{(1)}_{t|t-1}$ according to (16) and (17).
2. for $i = 1, 2$, compute the transform sets

$$\begin{aligned} \mathcal{Y}^{(i)} &= \exp(j\Theta^{(i)}_{t|t-1}) \\ \mathcal{Z}^{(i)} &= \Theta^{(i)}_{t|t-1} \exp(-j\Theta^{(i)}_{t|t-1}) \end{aligned}$$

3. Compute

$$\begin{aligned} \widehat{y}_{t|t-1} &= \sum_{i=1}^{2} W_i \mathcal{Y}^{(i)} \\ M^{\theta\nu}_{t|t-1} &= \sum_{i=1}^{2} W_i \mathcal{Z}^{(i)} - \widehat{y}^*_{t|t-1}\widehat{\theta}_{t|t-1} \\ M^{\nu\nu}_{t|t-1} &= 1 + \sigma^2_w - |\widehat{y}_{t|t-1}|^2. \end{aligned}$$

5. SMC TECHNIQUE FOR BLIND DETECTION AND ESTIMATION

We can now make timely estimates of θ_t and detection of s_t based on the currently available observation $\boldsymbol{Y}_t$, up to time t, blindly, as follows. With the Bayes theorem, we realize that the optimal solution to this problem is

$$\begin{aligned}\widehat{\theta_t} &= E\{\theta_t|\boldsymbol{Y}_t\} = \int \theta_t p(\theta_t|\boldsymbol{Y}_t)d\theta_t \\ &= \int_{\boldsymbol{S}_t} \underbrace{\left[\int_{\theta_t} \theta_t p(\theta_t|\boldsymbol{S}_t, \boldsymbol{Y}_t)d\theta_t\right]}_{\mu_{\theta_t}(\boldsymbol{S}_t)} p(\boldsymbol{S}_t|\boldsymbol{Y}_t), d\boldsymbol{S}_t.\end{aligned} \tag{18}$$

It then follows that

$$\widehat{\theta_t} = E\{\theta_t|\boldsymbol{Y}_t\} = \int_{\boldsymbol{S}_t} \mu_{\theta_t}(\boldsymbol{S}_t)p(\boldsymbol{S}_t|\boldsymbol{Y}_t)d\boldsymbol{S}_t \ . \tag{19}$$

Similarly, the data can be detected by the hard decisions on the symbol s_t by

$$\widehat{s_t} = \arg\max_{a_i \in A} P(s_t = a_i|\boldsymbol{Y}_t) \tag{20}$$

where

$$P(s_t = a_i|\boldsymbol{Y}_t) = E\{1(s_t = a_i)|\boldsymbol{Y}_t\} \ . \tag{21}$$

$1\{.\}$ in (21) is an indicator function defined as

$$1(s_t = a_i)\begin{cases} 1 & \text{if } s_t = a_i \\ 0 & \text{otherwise.}\end{cases}$$

In most cases, an exact evaluations of the expectations (19) and (21) are analytically intractable. SMC technique can provide us an alternative way for the required computation. Specifically, following the notation adopted in [11], if we can draw m independent random samples $\{\boldsymbol{S}_t^{(j)}\}_{j=1}^m$ from the distribution $p(\boldsymbol{S}_t|\boldsymbol{Y}_t)$, then we can approximate the quantities of interest $E\{\theta|\boldsymbol{Y}_t\}$ and $E\{1(s_t = a_i)|\boldsymbol{Y}_t\}$ in (11) and (13), respectively, by

$$E\{\theta|\boldsymbol{Y}_t\} \cong \frac{1}{m}\sum_{j=1}^{m} \mu_{\theta_t}(\boldsymbol{S}_t^{(j)}) \tag{22}$$

$$E\{1(s_t = a_i)|\boldsymbol{Y}_t\} \cong \frac{1}{m}\sum_{j=1}^{m} 1(s_t^{(j)} = a_i) \tag{23}$$

But, usually drawing samples from $p(\boldsymbol{S}_t|\boldsymbol{Y}_t)$ directly is usually difficult. Instead, sample generation from some *trial distribution* may be easier. In this case, the idea of *importance sampling* can be used. Suppose a set of random samples $\{\boldsymbol{S}_t^{(j)}\}_{j=1}^m$ is generated from a trial distribution $q(\boldsymbol{S}_t|\boldsymbol{Y}_t)$, which

- is strictly positive, $q(.|.) > 0$, and
- has the domain as $p(.|.)$.

By associating the weight

$$w_t^{(j)} = \frac{p(\boldsymbol{S}_t^{(j)}|\boldsymbol{Y}_t)}{q(\boldsymbol{S}_t^{(j)}|\boldsymbol{Y}_t)} \tag{24}$$

to the samples, the quantities of interest, $E\{1(s_t = a_i)|\boldsymbol{Y}_t\}$ and $E\{\theta_t|\boldsymbol{S}_t\}$ can be approximated as follows.

$$\begin{aligned} E\{\theta|y_t\} &\cong \frac{1}{W_t}\sum_{j=1}^{m} \mu_t(\boldsymbol{S}_t^{(j)})w_t^{(j)} \\ E\{1(s_t = a_i)|\boldsymbol{Y}_t\} &\cong \frac{1}{W_t}\sum_{j=1}^{m} 1(s_t^{(j)} = a_i)w_t^{(j)}, \ i = 1, 2, .., |A| \end{aligned} \tag{25}$$

with $W_t = \sum w_t^{(j)}$. The pair $(\boldsymbol{S}_t^{(j)}, w_t^{(j)}), j = 1, 2, \cdots, m$ is called a properly weighted sample with respect to distribution $p(\boldsymbol{S}_t|\boldsymbol{Y}_t)$. Note that the samples $\boldsymbol{S}_t^{(j)}$ can be drawn from the distribution $q(\boldsymbol{S}_t|\boldsymbol{Y}_t)$ sequentially as follows. We can choose $q(.)$ to satisfy

$$q(\boldsymbol{S}_{t-1}|\boldsymbol{Y}_t) = q(\boldsymbol{S}_{t-1}|\boldsymbol{Y}_{t-1}).$$

Then, it can be easily shown that

$$q(\boldsymbol{S}_t|\boldsymbol{Y}_t) = q(s_t|\boldsymbol{Y}_t, \boldsymbol{S}_{t-1})q(\boldsymbol{S}_{t-1}|\boldsymbol{Y}_{t-1}),$$

and one can draw samples $s_t^{(j)}$ from a trial distribution $q(s_t|\boldsymbol{Y}_t, \boldsymbol{S}_{t-1}^{(j)})$ and let $\boldsymbol{S}_t^{(j)} = (s_t^{(j)}, \boldsymbol{S}_{t-1}^{(j)})$ for $t = 0, 1, \cdots$. Specifically, it was shown in [11] that a suitable choice for the trial distribution is of the form:

$$q(s_t|\boldsymbol{Y}_t, \boldsymbol{S}_{t-1}^{(j)}) = p(s_t|\boldsymbol{Y}_t, \boldsymbol{S}_{t-1}^{(j)}) \ . \tag{26}$$

For this trial distribution, it is shown in [11] that the importance weight is updated according to

$$w_t^{(j)} = w_{t-1}^{(j)}p(y_t|\boldsymbol{Y}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}), \quad t = 0, 1, \cdots \tag{27}$$

The predictive distribution in (27) is given by

$$\begin{aligned} p(y_t|\boldsymbol{Y}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) &= \sum_{a_i \in A} p(y_t|\boldsymbol{Y}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t = a_i)P(s_t = a_i|\boldsymbol{Y}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) \\ &= \sum_{a_i \in A} p(y_t|\boldsymbol{Y}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t = a_i)P(s_t = a_i) \end{aligned} \tag{28}$$

where (28) holds because s_t is independent of $\boldsymbol{S}_{t-1}$ and $\boldsymbol{Y}_{t-1}$. Furthermore, it can be shown from the state and observation equations in (1) and (2),respectively, that

$$p(y_t|\boldsymbol{Y}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t = a_i) \sim N(\mu_{y_t}^{(j)}(i), \sigma_{y_t}^{2(j)}(i)) \tag{29}$$

with mean and variance given by

$$\begin{aligned} \mu_{y_t}^{(j)}(i) &= E\{y_t|\boldsymbol{Y}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t = a_i\} \\ &= a_i(H_t\mu_{\theta_{t-1}}^{(j)} + Q_t) \end{aligned} \tag{30}$$

$$\begin{aligned} \sigma_{y_t}^{2(j)}(i) &= \mathrm{Var}\{y_t|\boldsymbol{Y}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t = a_i\} \\ &= \sigma_{\theta_{t-1}}^{2(j)} + \sigma_n^2 + \sigma_p^2 \end{aligned} \tag{31}$$

where the quantities $\mu_{\theta_t^{(j)}}$ and $\sigma_{\theta_t^{2(j)}}$ in (30)and (31) can be computed recursively for the Extended Kalman equations given in (7-8). The trial distribution in (30) can be computed as follows:

$$\begin{aligned} p(s_t = a_i|\boldsymbol{Y}_t, \boldsymbol{S}_{t-1}^{(j)}) &= p(y_t|\boldsymbol{Y}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}, s_t = a_i) \\ &\quad \times P(s_t = a_i|\boldsymbol{Y}_{t-1}, \boldsymbol{S}_{t-1}^{(j)}) \\ &\triangleq \xi_{t,i}^{(j)} \end{aligned} \tag{32}$$

where it follows from (2) that

$$\xi_{t,i}^{(j)} = \frac{1}{\pi\sigma_{y_t}^{2(j)}(i)} \exp\left(-\frac{||y_t - \mu_{y_t}^{(j)}(i)||^2}{\sigma_{y_t}^{2(j)}(i)}\right) P(s_t = a_i). \quad (33)$$

We now summarize the SMC blind data detection and phase noise estimation algorithm as follows:

Step 1- *Initialization:*

- Initialize the Kalman filter: Choose the initial mean and the variance of the estimated θ_t as the mean and the variance of a uniform distribution defined on $-\pi, +\pi)$ as

$$\begin{aligned} \mu_{\theta_0}^{(j)} &= \widehat{\theta}_{0|0}^{(j)} = 0 \\ \sigma_{\theta_0}^{2(j)} &= M_{0|0}^{(j)} = \pi^2/12, \quad j = 1, 2, \cdots, m. \end{aligned} \quad (34)$$

- Initialize the importance weights: All importance weights are initialized as $w_0^{(j)} = 1, j = 1, 2, \cdots, m$. Since the data symbols are are assumed to be independent, initial symbols are not needed be generated.

Step 2- *Compute* $\xi_{t,i}^{(j)}$*:*

For each $a_i \in A$ compute the $\mu_{y_t}^{(j)}(i), \sigma_{y_t}^{2(j)}(i)$ and $\xi_{t,i}^{(j)}$ from (30), (31) and (33),respectively.

Step 3- *Draw samples* $s_t^j, j = 1, 2, \cdots, m$

Draw $s_t^{(j)}$ from the set A with probabilities

$$P(s_t^{(j)} = a_i) \propto \xi_{t,i}^{(j)}, \quad a_i \in A. \quad (35)$$

Append $s_t^{(j)}$ to $S_{t-1}^{(j)}$ to obtain $S_t^{(j)}$.

Step 4- *Compute the importance weights:*

$$w_t^{(j)} = w_{t-1}^{(j)} \sum_{a_i \in A} \xi_{t,i}^{(j)}.$$

Step 5-*Detect the symbol* s_t*:*

Detect the symbol s_t from (23).

Step 6-*Update the a posteriori mean and variance of the phase noise:*

If the samples drawn up to time t is S_t in Step 2, set

$$\begin{aligned} \mu_{\theta_t}(S_t^{(j)}) &\triangleq \mu_{\theta_t}^{(j)} = \widehat{\theta}_{t|t}^{(j)} \\ \sigma_{\theta_t}^{2(j)}(S_t^{(j)}) &\triangleq \sigma_{\theta_t}^{2(j)} = M_{t|t}^{(j)} \quad j = 1, 2, \cdots, m. \end{aligned}$$

and update according to the Kalman equations (7), (8).

Step 7- *Do the restamping as described in Section 6.*

6. RESAMPLING METHOD

A major problem in the practical implementation of the SMC method described so far is that after a few iteration most of the importance weights have negligible values that is $w_t^{(j)} \approx 0$. A relatively small weight implies that the sample is drawn far from the main body of the posterior distribution and has a small contribution in the final estimation. Such sample is said to be ineffective. The SMC algorithm becomes ineffective if there are too many ineffective samples. The common solution to this problem is *resampling.* Restampling is a an algorithmic step that stochastically eliminates those samples with small weights. Basically, the resampling method takes the samples, to be generated sequentially $\Xi_t = \{S_t^{(j)}, \mu_{\theta_t}^{(j)}, \sigma_{\theta_t}^{2(j)}\}_{j=1}^m$ with corresponding weights $\{w_t^{(j)}\}_{j=1}^m$ as an input and generates a new set of samples $\widetilde{\Xi}_t = \{\widetilde{S}_t^{(j)}, \widetilde{\mu}_{\theta_t}^{(j)}, \widetilde{\sigma}_{\theta_t}^{2(j)}\}_{j=1}^m$ with equal weights, i.e $\{w_t^{(j)} = 1/m\}_{j=1}^m$, assuming they are normalized to $\sum_{j=1}^m w_t^{(j)} = 1$. A simple procedure to achieve this goal is, for each $j = 1, 2, \cdots, m$, to choose $(\widetilde{S}_t^{(j)}, \widetilde{\mu}_{\theta_t}^{(j)}, \widetilde{\sigma}_{\theta_t}^{2(j)}) = (S_t^{(j)}, \mu_{\theta_t}^{(i)}, \sigma_{\theta_t}^{2(i)})$ with probability $w_t^{(i)}$.

In this paper, a resampling technique suggested by [12] is employed. This technique forms a new set of weighted samples $\widetilde{\Xi}_t = \{\widetilde{S}_t^{(j)}, \widetilde{\mu}_{\theta_t}^{(j)}, \widetilde{\sigma}_{\theta_t}^{2(j)}\}_{j=1}^m$ according to the following algorithm. (assume that $\sum_{j=1}^m w_t^j = m$)

1. For $j = 1, 2, \cdots, m$, retain $\ell_j = w_t^j$ copies of the samples $(S_t^{(j)}, \mu_{\theta_t}^{(i)}, \sigma_{\theta_t}^{2(i)})$. Denote $L_r = m - \sum_{j=1}^m \ell_j$.
2. Obtain L_r i.i.d. draws from the original sample set $\{(S_t^{(j)}, \mu_{\theta_t}^{(i)}, \sigma_{\theta_t}^{2(i)})\}_{j=1}^m$, with probabilities proportional to $(w_t^j - \ell_j), j = 1, 2, \cdots, m$.
3. Assign equal weights, that is, set $w_t^j = 1$, for each new sample.

It is shown in [12] that the samples drawn by the above procedure are properly weighted with respect to $p(S_t|Y_t)$, provided that m is sufficiently large. Note that resampling at every time step is not needed in general. In one way the resampling can be done every k_0 recursions where k_0 is a prefixed resampling interval. On the other hand, the resampling can be carried out whenever the effective sample size, approximated as

$$\widehat{N}_{eff} = \frac{1}{\sum_{j=1}^m (w_t^j)^2} \leq m \quad (36)$$

goes below a certain threshold, typically a fraction of m. Intuitively, $\widehat{N}_{eff}$ reflects the equivalent size of i.i.d samples from the true posterior densities of interest for the set of m weighted ones. It is suggested in [13] that resampling should be performed when $\widehat{N}_{eff} < m/10$. Alternatively, one can conduct the first approach to conduct resampling at every fixed-length time interval say every five steps.

7. SIMULATION RESULTS

In this section, we provide some computer simulation examples to demonstrate the performance of the proposed SMC and UF approach for joint blind phase noise estimation and data detection. The phase process is modelled by AR process driven by a white Gaussian noise with $\sigma_u^2 = 0.0314$. It is assumed BPSK modulation is employed. In order to demonstrate the performance of the adaptive SMC approach, we first present the performance (in terms of the phase error $\phi(k) = \theta_t - \widehat{\theta}_t$) during one simulation run for different initial phase errors $\phi(k) = 0, \pi/4, \pi/2, 3\pi/4, \pi$. The phase error for several values of $\phi(0)$ at $SNR = 10dB$ is shown in Fig. 1.

The performance of the proposed algorithm is further exploited by the evaluation of average BER over observed block for different SNRs and different intial phase errors. The uncoded average BER performance of this adaptive approach is plotted in Fig. 2.

Our simulations indicate that

- as the initial phase error $\phi(0)$ approaches π, the probability that the phase error converges to the dual equilibrium point becomes very high
- as the initial phase error $\phi(0)$ approaches π, the BER increases, for $\phi(0) = \pi/2$, the BER is almost equal to 1 (due to ambiguity).

8. CONCLUSIONS

We have developed a new adaptive Bayesian approach for blind phase noise estimation and data detection based on sequential Monte Carlo methodology. The optimal solutions to joint symbol detection and phase noise tracking problem is computationally prohibitive to implement by conventional methods. Thus the proposed sequential approach offers an novel and powerful approach to tackling this problem at a reasonable computational cost.

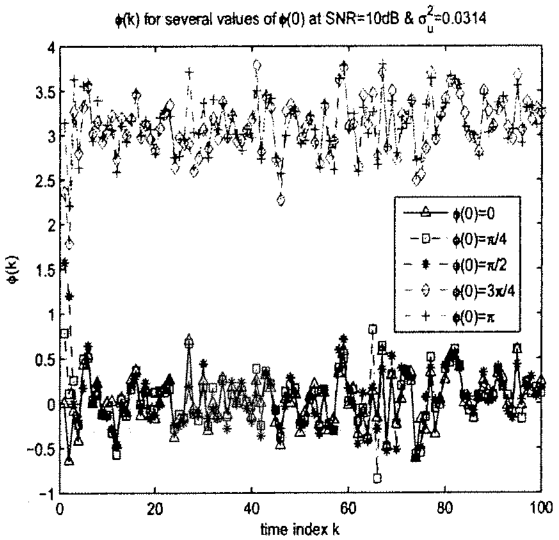


Figure 1: Tracking performance for different initializations at SNR=10dB

REFERENCES

[1] H. Meyr, M. Moeneclaey and S.A. Fechtel, *Digital Communications Receivers*, John Wiley, New York, 1998.

[2] C. N. Georghiades and J.C. Han, "Optimum decoding of trellis coded modulation in the presence of phase noise", *Proc. 1990 Int. Symp. on Information Theory and Its Applications (ISITA'90)* Hawaii, Nov. 1990.

[3] A. T. Huq, E. Panayirci and C. N. Georghiades, " ML NDA carrier phase recovery for OFDM systems with M-PSK signaling" , *Proc. IEEE Int. Conf. on Communications (ICC'99)*, June 6-10, 1999,Vancouver, Canada.

[4] E. Panayırcı and C. N. Georghiades," Non-data-aided ML carrier frequency and phase synchronization in OFDM

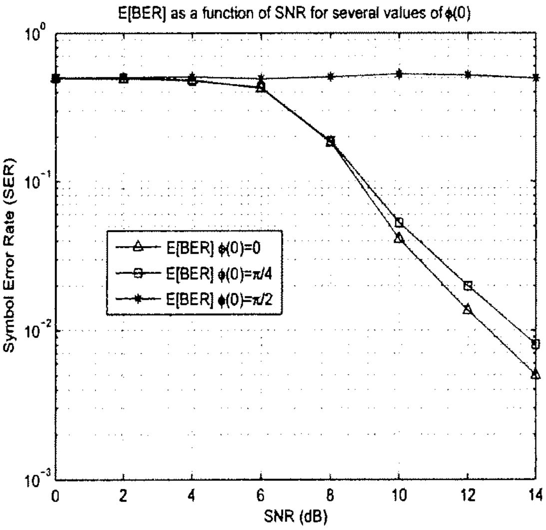


Figure 2: BER performance

systems", *European Transactions on Telecommunications (ETT)*, 2000.

[5] U. Mengali, A.L. D'Andrea, *Synchronization Techniques for Digital Receivers*, Plenum Press, New York 1997.

[6] A. P. Dempster, N. M. Laird and D. B. Rubin,"Maximum likelihood from incomplete data via the EM algorithm",*Ann. Roy. Stat. Soc.*, pp. 1-38, December 1977.

[7] M. Pitt and N. Shephard," Filtering via simulation: auxiliry particle filter",*J. Amer. Statist. Soc. B* , no.63, 2001.

[8] E. Panayirci, H. A. Cirpan and M. Moeneclaey,A Sequential Monte Carlo Method for Blind Phase Noise Estimation and Data Detection, 13th European Signal Processsing Conference, EUSIPCO 2005, Sep. 4-8 2005, Antalya, Turkiye.

[9] S. Geman and D. Geman," Stochastic relaxation, Gibbs distribution and the Bayesian restoration of images",*IEEE Trans. on Pattern Analysis nad Machine Intelligence* vol. 6 ,pp. 721-741, 1984.

[10] S. M. Kay, *Fundamentals of Statistical Signal Processing*, Upper Saddle River, NJ: Prentice Hall, 1993.

[11] Z. Yang and X. Wang, " A sequential Monte Carlo blind receiver for OFDM systems in frequency-selective fading channels",*IEEE Trans. Signal Proc.*,vol. 50, no. 2, pp.271-280, Feb. 2003.

[12] R. Chen, X. Wang and J. S. Liu, " Adaptive joint detection and decoding in flat-fading channels via mixture Kalman filering", *IEEE Trans.Information Theo.*,vol. 46, no. 6, pp.2070-2094, Sept. 2000.

[13] J. S. Liu and R. Chen,"Blind deconvolution via sequential imputations", *J. Amer. Statist. Assoc.*, vol. 90, pp. 567-576, 1995.

[14] S. J. Julier and J.K. Uhlmann,"Unscented filtering and nonlinear estimation", *Proc. of the IEEE*, vol. 92, no.3, pp,401-422, 2004.

EM based MAP iterative channel estimation for turbo coded SFBC-OFDM systems

Hakan Doğan[1], Hakan A. Çırpan[1], Erdal Panayırcı[2],

[1] Department of Electrical-Electronics Engineering, Istanbul University, 34320 Avcilar, Istanbul, Turkey, e-mail: {hdogan,hcirpan}@istanbul.edu.tr

[2] Department of Electrical-Electronics Engineering, Bilkent University, 06800 Bilkent, Ankara, Turkey e-mail: eepanay@bilkent.edu.tr

Abstract

We consider the design of turbo receiver structures for space-frequency block coded (SFBC-) OFDM systems in unknown frequency selective fading channels. The Turbo receiver structures for SFBC-OFDM systems under consideration consists of an iterative MAP Expectation/Maximization (EM) channel estimation algorithm, soft MMSE SFBC decoder and a soft MAP outer-channel-code decoder. MAP-EM employs iterative channel estimation and it improves receiver performance by re-estimating the channel after each decoder iteration. Moreover, the MAP-EM approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. The performance of the proposed approaches are studied in terms of mean square error and bit-error rate. Through experimental results, the effect of a pilot spacing on the channel estimator performance and sensitivity of turbo receiver structures on channel estimation error are studied.

1 Introduction

The continued increase in demand for all types of services further necessitates the need for higher capacity and data rates. In this context, emerging technology that improves the wireless systems spectrum efficiency is error control coding. Recent trends in coding favor parallel and/or serially concatenated coding and probabilistic soft-decision iterative (turbo-style) decoding. Such codes are able to exhibit near-Shannon-limit performance with reasonable complexities in many cases and are of significant interest for communications applications that require moderate error rates. In addition to outher channel code transmit diversity OFDM system can be applied to further improve the receiver performance. We therefore consider the combination of turbo codes with the transmit diversity OFDM systems. Especially we address the design of iterative channel estimation approach for an transmit diversity OFDM systems employing an outer channel code.

Channel estimation for transmit diversity OFDM systems has attracted much attention with pioneering works by Li [1], [2]. However, most of the early work on channel estimation for transmit diversity OFDM systems focused on uncoded systems. Since most practical systems use error control coding, more recent work has addressed coded transmit diversity OFDM systems. Among many other techniques, an iterative procedures based on Expectation-Maximization (EM) algorithm was also applied to channel estimation problem in the context of space-time block-coding (STBC) [3], [4] as well as transmit diversity OFDM systems with or without outer channel coding (e.g. convolutional code or Turbo code) [6], [7], [8], [9]. In [6], maximum a *posteriori* (MAP) EM based iterative receivers for STBC-OFDM systems with Turbo code are proposed to directly detect transmitted symbols under the assumption that fading processes remain constant across several OFDM words contained in one STBC code-word. An EM approach proposed for the general estimation from superimposed signals [5] is applied to the channel estimation for transmit diversity OFDM systems with outer channel code (convolutional code) and compared with SAGE version in [8]. Moreover in [9], a modified version of [8] is proposed for STBC-OFDM and space-frequency block-coding (SFBC)-OFDM systems. Unlike the EM approaches treated in [6], [7], [8], [9], we propose Turbo receiver based on MAP-EM channel estimation algorithm for SFBC-OFDM systems employing outer channel coding.

The Turbo receiver scheme under consideration employs iterative channel estimation and it improves receiver performance by re-estimating the channel after each decoder iteration. Moreover, the proposed approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. Orthogonality of the SFBC system based on the Alamouti orthogonal design as well as the Karhunen-Loeve expansion of the multipath channel result in a simple iterative EM based channel

This research has been conducted within the NEWCOM Network of Excellence in Wireless Communications funded through the EC 6th Framework Programme. This work was also supported in part by the Turkish Scientific and Technical Research Institute (TUBITAK) under Grant 104E166.

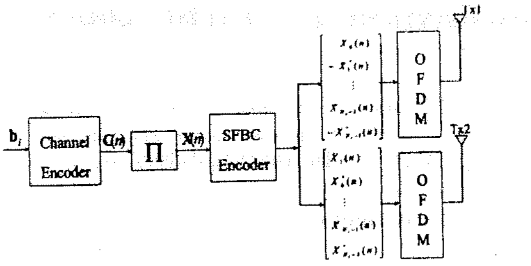


Fig. 1. Transmitter structure for turbo coded SFBC-OFDM systems.

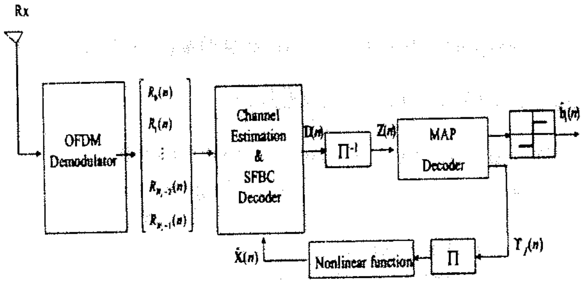


Fig. 2. Turbo Receiver Structure for SFBC-OFDM systems.

estimation algorithm in frequency domain.

2 Turbo Coded SFBC-OFDM

We consider a SFBC-OFDM system with outer channel coding. Turbo code is applied in addition to the SFBC to further improve the error performance of the SFBC-OFDM system. A block diagram of a turbo coded two-branch SFBC-OFDM system is shown in Fig. 1.

2.1 Turbo Encoder

Turbo codes are a class of powerful error correction codes that enable reliable communications with power efficiencies close to the theoretical Shannon channel capacity limit. In particular, a turbo code is formed from the parallel or serial concatenation of codes separated by an interleaver.

As illustrated in Fig. 1, the binary information bits $\{b_i\}$, $1 \leq i \leq N_c/2$ of random data are encoded by an outer-channel-code using a Turbo encoder, resulting in a BPSK-coded symbol stream $\{C(n)\}$. The coded symbols are then interleaved by a random permutation results in a stream of independent symbols. A code-bit interleaver reduces probability of error bursts and removes correlation in coded symbol stream. The coded, interleaved symbols yield an independent symbol stream (of length $N_c/2$) is designated $\{\mathbf{X}(n)\}$. Finally, the modulated BPSK symbols are encoded by an SFBC encoder and transmitted from 2 transmit antennas at a particular OFDM subcarriers.

In general, Turbo codes are low-rate codes which require considerable bandwidth expansion for high rate data transmission. In order to improve spectral efficiency, it is necessary to combine turbo codes with a bandwidth efficient transmit diversity systems. Thus combinations of implicit (turbo coding) and external (i.e. multiple transmit antenna) diversity can be used to improve the performance of the communication system in fading environments.

2.2 SFBC-OFDM Encoder

In this paper, we consider a transmitter diversity OFDM scheme in conjunction with inner channel coding. The use of OFDM in transmitter diversity systems also offers the possibility of coding in a form of space-frequency OFDM [12]. In [12], under the assumption that the channel responses are known or can be estimated accurately at the receiver, it was shown that the SFBC-OFDM system has the same performance as a previously reported STBC-OFDM scheme in slow fading environments but shows better performance in the more difficult fast fading environments. Also, since, SFBC-OFDM transmitter diversity scheme performs the decoding within one OFDM block, it only requires half of the decoder memory needed for the STBC-OFDM system of the same block size. Similarly, the decoder latency for SFBC-OFDM is also half that of the STBC-OFDM implementation. In SFBC-OFDM systems, the set of OFDM subchannels is first divided into groups of subchannels. This subchannel grouping with appropriate system parameters does preserve diversity gain while simplifying not only the code construction but decoding algorithm significantly as well [12]. Following to notation of [12], let $N_c \times 1$ as a turbo coded, interleaved and BPSK modulated be denoted as a vector $\mathbf{X}(n) = [X(nN_c), X(nN_c+1), \cdots, X(nN_c+N_c-1)]^T$, where $X_k(n)$ denote the kth forward polyphase component of the serial data symbols, i.e, $X_k(n) = X(nN_c+k)$ for $k = 0, \cdots, N_c-1$. Polyphase component $X_k(n)$ can also be viewed as the coded symbol to be transmitted on the kth tone during the block instant n. The coded symbol vector $\mathbf{X}(n)$ can therefore be expressed as $\mathbf{X}(n) = [X_0(n), X_1(n), \cdots, X_{N_c-1}]^T$. Resorting subchannel grouping, $\mathbf{X}(n)$ is coded into two vectors $\mathbf{X_e}(n)$ and $\mathbf{X_o}(n)$ by the space-frequency encoder as

$$\begin{aligned} \mathbf{X_e}(n) &= [X_0(n), X_2(n), \cdots, X_{N_c-4}(n), X_{N_c-2}(n)]^T, \\ \mathbf{X_o}(n) &= [X_1(n), X_3(n), \cdots, X_{N_c-3}(n), X_{N_c-1}(n)]^T, \end{aligned} \quad (1)$$

where $\mathbf{X_e}(n)$ and $\mathbf{X_o}(n)$ actually corresponds to even and odd polyphase component vectors of $\mathbf{X}(n)$. Then the space-frequency block code transmission matrix may be represented by,

$$\begin{matrix} & frequency \rightarrow \\ space \downarrow & \begin{bmatrix} \mathbf{X_e}(n) & -\mathbf{X_o}^*(n) \\ \mathbf{X_o}(n) & \mathbf{X_e}^*(n) \end{bmatrix} \end{matrix} \quad (2)$$

where * stands for complex conjugation.

3 Channel: Basis Expansion Model

In wireless mobile communications, channel variations evolve in a progressive fashion and hence fit in some evolution models. It appears that a basis expansion approach would be natural way of modelling the channel variation. A convenient choice for basis expansion of random processes is Karhunen-Loeve (KL) series. Moreover, the KL expansion methodology has been also used for efficient simulation of multipath fading environments [10]. Prompted by the general applicability of KL expansion, we consider in this paper the parameters of $\mathbf{H}_\mu(n)$ to be given by a linear combination of orthonormal bases where $\mathbf{H}_\mu(n)$ is the channel frequency response between the μth transmit antenna and the receive antenna at the nth time slot.

An orthonormal expansion of the vector $\mathbf{H}_\mu(n)$ involves expressing the $\mathbf{H}_\mu(n)$ as a linear combination of the orthonormal basis vectors as $\mathbf{H}_\mu(n) = \mathbf{\Psi}\mathbf{G}_\mu(n)$ where $\mathbf{\Psi} = [\psi_0, \psi_1, \cdots, \psi_{N_c-1}]$, ψ_i's are the orthonormal basis vectors, $\mathbf{G}_\mu(n) = [G_{\mu,1}(n), \cdots, G_{\mu,N_c-1}(n)]^T$, and $G_{\mu,i}(n)$ is the weights of the expansion. By using different basis functions $\mathbf{\Psi}$, we can generate sets of coefficients with different properties. The autocorrelation matrix $\mathbf{C}_{\mathbf{H}_\mu} = E\left[\mathbf{H}_\mu \mathbf{H}_\mu^\dagger\right]$ can be decomposed as

$$\mathbf{C}_{\mathbf{H}_\mu} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^\dagger \tag{3}$$

where $\mathbf{\Lambda} = E\{\mathbf{G}_\mu \mathbf{G}_\mu^\dagger\}$. The KL expansion is one where $\mathbf{\Lambda}$ of (3) is a diagonal matrix (i.e., the coefficients are uncorrelated). If $\mathbf{\Lambda}$ is diagonal, then (3) must be *eigendecomposition* of $\mathbf{C}_{\mathbf{H}_\mu}$. The fact that only the eigenvectors diagonalize $\mathbf{C}_{\mathbf{H}_\mu}$ leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in the Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem in this application equivalent to estimating the i.i.d. Gaussian vector $\mathbf{G}_\mu$ KL expansion coefficients.

4 Received Signal Model

At the receiver, the signals are received from receiver antenna. After matched filtering and symbol rate sampling, the discrete Fourier transform is then applied to the received discrete time signal to obtain $\mathbf{R}(n)$. If the received signal sequence is also parsed in even and odd blocks of N_c tones, $\mathbf{R}_e(n) = [R_0(n), R_2(n), \cdots, R_{N_c-2}(n)]^T$ and $\mathbf{R}_o(n) = [R_1(n), R_3(n), \cdots, R_{N_c-1}(n)]^T$, the received signal can be expressed in vector form as

$$\begin{aligned}\mathbf{R}_e(n) &= \mathcal{X}_e(n)\mathbf{H}_{1,e}(n) + \mathcal{X}_o(n)\mathbf{H}_{2,e}(n) + \mathbf{W}_e(n) \quad (4)\\ \mathbf{R}_o(n) &= -\mathcal{X}_o^\dagger(n)\mathbf{H}_{1,o}(n) + \mathcal{X}_e^\dagger(n)\mathbf{H}_{2,o}(n) + \mathbf{W}_o(n)\end{aligned}$$

where $\mathcal{X}_e(n)$ and $\mathcal{X}_o(n)$ are $N_c/2 \times N_c/2$ diagonal matrices whose elements are $\mathbf{X_e}(n)$ and $\mathbf{X_o}(n)$ respectively and † denotes conjugate transpose. $\mathbf{H}_{\mu,e}(n) = [H_{\mu,0}(n), H_{\mu,2}(n), \cdots, H_{\mu,N_c-2}(n)]^T$ and $\mathbf{H}_{\mu,o}(n) = [H_{\mu,1}(n), H_{\mu,3}(n) \cdots, H_{\mu,N_c-1}(n)]^T$ are $N_c/2$ length vectors denoting the even and odd component vectors of the channel attenuations between the μth transmitter and the receiver. Finally, $\mathbf{W}_e(n)$ and $\mathbf{W}_o(n)$ are an $N_c/2 \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive noise in the N_c tones, with variance $\sigma^2/2$ per dimension.

Equation (4) shows that the information symbols $\mathcal{X}_e(n)$ and $\mathcal{X}_o(n)$ are transmitted twice in two consecutive adjacent subchannel groups through two different channels. In order to estimate the channels and decode $\mathcal{X}$ with the embedded diversity gain through the repeated transmission, for each n, we can write the following from (4):

$$\begin{bmatrix}\mathbf{R}_e(n)\\ \mathbf{R}_o(n)\end{bmatrix} = \begin{bmatrix}\mathcal{X}_e(n) & \mathcal{X}_o(n)\\ -\mathcal{X}_o^\dagger(n) & \mathcal{X}_e^\dagger(n)\end{bmatrix}\begin{bmatrix}\mathbf{H}_{1,e}(n)\\ \mathbf{H}_{2,e}(n)\end{bmatrix} + \begin{bmatrix}\mathbf{W}_e(n)\\ \mathbf{W}_o(n)\end{bmatrix} \tag{5}$$

where the complex channel gains between adjacent subcarriers are assumed to be approximately constant, i.e., $\mathbf{H}_{1,e}(n) \approx \mathbf{H}_{1,o}(n)$ and $\mathbf{H}_{2,e}(n) \approx \mathbf{H}_{2,o}(n)$. The effect of this assumption allows us to omit dependence of $\mathbf{H}_{1,e}(n)$ and $\mathbf{H}_{2,e}(n)$ on even channel components. Using (5) and dropping subscript "$_e$", we have

$$\begin{bmatrix}\mathbf{R}_e(n)\\ \mathbf{R}_o(n)\end{bmatrix} = \begin{bmatrix}\mathcal{X}_e(n) & \mathcal{X}_o(n)\\ -\mathcal{X}_o^\dagger(n) & \mathcal{X}_e^\dagger(n)\end{bmatrix}\begin{bmatrix}\mathbf{H}_1(n)\\ \mathbf{H}_2(n)\end{bmatrix} + \begin{bmatrix}\mathbf{W}_e(n)\\ \mathbf{W}_o(n)\end{bmatrix} \tag{6}$$

or in more succinct form

$$\mathbf{R}(n) = \mathcal{X}(n)\mathbf{H}(n) + \mathbf{W}(n) . \tag{7}$$

5 Iterative Channel Estimation

In recent years, inspired by the development of turbo coding, various types of iterative channel estimation, detection and decoding schemes have been proposed in the literature. These proposed approaches have shown that iterative receivers can offer significant performance improvements over the noniterative counterparts. In this section, we therefore consider EM based MAP iterative KL coefficient estimation for turbo coded SFBC-OFDM systems.

The prior probability density function (PDF) of the KL expansion coefficient r.v.'s of the fading channel can be expressed as $p(\mathbf{G}) \sim \exp(-\mathbf{G}^\dagger \widetilde{\mathbf{\Lambda}}^{-1}\mathbf{G})$ where $\mathbf{G} = [\mathbf{G}_1^T, \mathbf{G}_2^T]^T$, $\widetilde{\mathbf{\Lambda}} = \mathrm{diag}(\mathbf{\Lambda}\ \ \mathbf{\Lambda})$, $\widetilde{\mathbf{\Sigma}}$ is an $N_c \times N_c$ diagonal matrix with $\widetilde{\mathbf{\Sigma}}[k,k] = \sigma^2$, for $k = 0, 1, \cdots, N_c - 1$ and $\widetilde{\mathbf{\Psi}} = \mathrm{diag}(\mathbf{\Psi}\ \ \mathbf{\Psi})$. In the MAP estimation approach we choose $\widehat{\mathbf{G}}$ to maximize the posterior PDF or

$$\widehat{\mathbf{G}}_{\mathrm{MAP}} = \arg\max_{\mathbf{G}} p(\mathbf{G}|\mathbf{R}) . \tag{8}$$

To find MAP estimator, we must equivalently maximize $p(\mathbf{R}|\mathbf{G})p(\mathbf{G})$. Hence, the MAP estimator (8) is equivalently takes the form

$$\widehat{\mathbf{G}}_{\mathrm{MAP}} = \arg\max_{\mathbf{G}} \left[\ln p(\mathbf{R}|\mathbf{G}) + \ln p(\mathbf{G})\right] . \tag{9}$$

Given the transmitted signals $\mathcal{X}$ as coded according to transmit diversity scheme and the discrete channel orthonormal series expansion representation coefficients $\mathbf{G}$ and taking into account the independence of the noise components, the conditional PDF of the received signal $\mathbf{R}$ can be expressed as,

$$p(\mathbf{R}|\mathcal{X},\mathbf{G}) \sim \exp\left[-(\mathbf{R}-\mathcal{X}\tilde{\mathbf{\Psi}}\mathbf{G})^{\dagger}\tilde{\mathbf{\Sigma}}^{-1}(\mathbf{R}-\mathcal{X}\tilde{\mathbf{\Psi}}\mathbf{G})\right]. \quad (10)$$

Obtaining MAP estimate of $\mathbf{G}$ from (10) is a complicated optimization problem and does not yield to a closed form solution. Fortunately, the solution can be obtained easily by means of the iterative EM algorithm. The basic idea of the EM algorithm is to solve estimation problems iteratively according to Expectation and Maximization steps respectively. Basically, this algorithm inductively reestimate $\mathbf{G}$ so that a monotonic increase in the *a posteriori* conditional pdf in (10) is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q(\mathbf{G}|\mathbf{G}^{(q)}) = \sum_{\mathcal{X}} p(\mathbf{R},\mathcal{X},\mathbf{G}^{(q)}) \log p(\mathbf{R},\mathcal{X},\mathbf{G}) \quad (11)$$

where $\mathbf{G}^{(q)}$ is the estimation of $\mathbf{G}$ at the qth iteration.

Note that $p(\mathbf{R},\mathcal{X},\mathbf{G}) \sim p(\mathbf{R}|\mathcal{X},\mathbf{G})p(\mathbf{G})$ since the data symbols $\mathcal{X} = \{X_k(n)\}$ are assumed to be i.i.d. distributed and the fact that they are independent of $\mathbf{G}$. Therefore, (11) can be evaluated by means of the expressions $p(\mathbf{G})$ and (10).

Given the received signal $\mathbf{R}$, the EM algorithm starts with an initial value $\mathbf{G}^0$ of the unknown channel parameters [11]. The $(q+1)$th estimate of $\mathbf{G}$ is obtained by the maximization step described by

$$\mathbf{G}^{(q+1)} = \arg\max_{\mathbf{G}} Q(\mathbf{G}|\mathbf{G}^{(q)}). \quad (12)$$

The expression of the reestimate $\mathbf{G}_{\mu}^{(q+1)}$ $(\mu = 1,2)$ for SFBC-OFDM can be obtained as follows:

$$\mathbf{G}_2^{(q+1)} = (\mathbf{I}+\mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{\Psi}^{\dagger}\left[\hat{\mathcal{X}}_e^{\dagger(q)}\mathbf{R}_e(n) - \hat{\mathcal{X}}_o^{(q)}\mathbf{R}_o(n)\right]$$
$$\mathbf{G}_2^{(q+1)} = (\mathbf{I}+\mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{\Psi}^{\dagger}\left[\hat{\mathcal{X}}_o^{\dagger(q)}\mathbf{R}_e(n) + \hat{\mathcal{X}}_e^{(q)}\mathbf{R}_o(n)\right] \quad (13)$$

where it can be easily seen that $(\mathbf{I}+\mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1} = \mathrm{diag}([(1+\sigma^2\lambda_0)^{-1}, \cdots, (1+\sigma^2\lambda_{\frac{N_c}{2}-1})^{-1}])$ and $\hat{\mathcal{X}}_e^{(q)}$, $\hat{\mathcal{X}}_o^{(q)}$ in (13) are an $\frac{N_c}{2} \times \frac{N_c}{2}$ dimensional diagonal matrix representing a *a posteriori* probabilities of the data symbols which are computed at the qth iteration. If only a few expansion coefficients is employed to reduce the complexity of the proposed estimator, then the MSE between channel parameters becomes large. However, if the number of parameters in the expansion is increased to include dominant eigenvalues, we are able to obtain a good approximation with a relatively small number of KL coefficients. For instance, by replacing 256×256 diagonal $\mathbf{\Lambda}$ in (13) with 8×8 diagonal $\mathbf{\Lambda}_r$ decreases computational complexity enormously. This enables feasibility of the proposed receiver.

6 Iterative Equalization and Decoding

We now consider the SFBC-OFDM decoding algorithm and the MAP outer channel code decoding to complete the description of the Turbo receiver.

6.1 SFBC-OFDM Decoding Algorithm

Since the channel vectors or equivalently expansion coefficients are estimated through EM based iterative approach, it is possible to decode $\mathbf{R}$ with diversity gains by a simple matrix multiplication. Before dealing with how we resolve decoding, let us first re-express received signal model (4). as follows (4) to write $\tilde{\mathbf{R}}(n) = [\mathbf{R}_e^T(n), \mathbf{R}_o^{\dagger}(n)]^T$ as

$$\tilde{\mathbf{R}}(n) = \mathcal{H}(n)\tilde{\mathbf{X}}(n) + \tilde{\mathbf{W}}(n) \quad (14)$$

where $\tilde{\mathbf{X}}(n) = [\mathbf{X}_e^T(n), \mathbf{X}_o^T(n)]^T$, $\tilde{\mathbf{W}}_k(n) = [\mathbf{W}_e^T(n), \mathbf{W}_o^{\dagger}(n)]^T$ and

$$\mathcal{H}(n) = \begin{bmatrix} \mathcal{H}_{1,e}(n) & \mathcal{H}_{2,e}(n) \\ \mathcal{H}_{2,o}^{\dagger}(n) & -\mathcal{H}_{1,o}^{\dagger}(n) \end{bmatrix} \quad (15)$$

where $\mathcal{H}_{\mu,e}(n)$ and $\mathcal{H}_{\mu,o}(n)$ $\mu = 1,2$ are $N_c/2 \times N_c/2$ diagonal matrices whose elements are $\mathbf{H}_{\mu,e}(n) = [H_{\mu,0}(n), H_{\mu,2}(n), \cdots, H_{\mu,N_c-2}(n)]^T$ and $\mathbf{H}_{\mu,o}(n) = [H_{\mu,1}(n), H_{\mu,3}(n) \cdots, H_{\mu,N_c-1}(n)]^T$ respectively.

Depending on complexity versus performance tradeoffs, any linear equalizer can be applied to retrieve $\tilde{\mathbf{X}}(n)$ from (14). In this paper, we consider linear equalizer where the parameters are updated using the MMSE criterion. We define the linear MMSE estimate $\mathbf{D}(n)$ of $\tilde{\mathbf{X}}(n)$ given the observation $\tilde{\mathbf{R}}(n)$ by

$$\begin{aligned} \mathbf{D}(n) &= \bar{\tilde{\mathbf{X}}} + \mathbf{C}_{\tilde{\mathbf{X}}}\mathcal{H}(n)^{\dagger}(\mathcal{H}(n)^{\dagger}\mathbf{C}_{\tilde{\mathbf{X}}}\mathcal{H}(n) + \mathbf{C}_{\tilde{\mathbf{W}}})^{-1} \\ &\times (\tilde{\mathbf{R}}(n) - \mathcal{H}(n)\bar{\tilde{\mathbf{X}}}(n)) \end{aligned} \quad (16)$$

where $\bar{\tilde{\mathbf{X}}}$, $\mathbf{C}_{\tilde{\mathbf{X}}}$ and $\mathbf{C}_{\tilde{\mathbf{W}}}$ are the mean of $\tilde{\mathbf{X}}(n)$, the covariance matrix of $\tilde{\mathbf{X}}(n)$ and the covariance matrix of $\tilde{\mathbf{W}}(n)$ respectively.

With a scaled unitary matrix $\mathcal{H}(n)$ and approximately constant complex channel gains with $\mathcal{H}_{1,e}^2 + \mathcal{H}_{2,e}^2 \approx 1$ assumptions, we can simplify $\mathcal{H}^{\dagger}(n)\mathcal{H}(n)$ as

$$\mathcal{H}(n)^{\dagger}\mathcal{H}(n) = \begin{bmatrix} \mathcal{H}_{1,e}^2 + \mathcal{H}_{2,e}^2 & 0 \\ 0 & \mathcal{H}_{1,e}^2 + \mathcal{H}_{2,e}^2 \end{bmatrix} = \mathbf{I}_{N \times N} \quad (17)$$

where $\mathbf{I}_{N \times N}$ is the $N \times N$ identity matrix. Moreover, following the assumptions used in [13], $\bar{\tilde{\mathbf{X}}} = 0$, $\mathbf{C}_{\tilde{\mathbf{X}}} = \mathbf{I}$, (16) becomes

$$\mathbf{D}(n) = (\mathbf{I} + \sigma_n^2\mathbf{I})^{-1}\mathcal{H}^{\dagger}\tilde{\mathbf{R}}(n). \quad (18)$$

If we set $\mathbf{C}_{\tilde{\mathbf{W}}} = 0$ in (16), further simplified form of linear equalizer called zero forcing (ZF) equalizer is obtained resulting in

$$\mathbf{D}(n) = \mathcal{H}^{\dagger}(n)\tilde{\mathbf{R}}(n) = \mathcal{H}^{\dagger}(n)\mathcal{H}(n)\tilde{\mathbf{X}}(n) + \eta(n) \quad (19)$$

where $\eta(n) = \mathcal{H}^{\dagger}(n)\tilde{\mathbf{W}}(n)$.

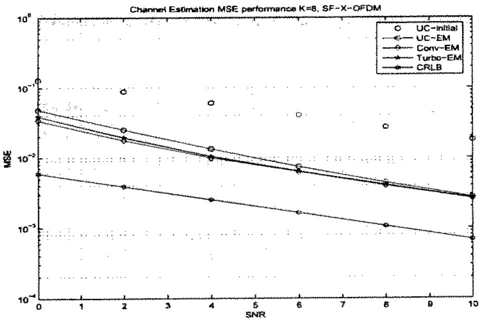


Fig. 3. MSE performance of SF-X-OFDM turbo receivers, Pilot insertion rate=8

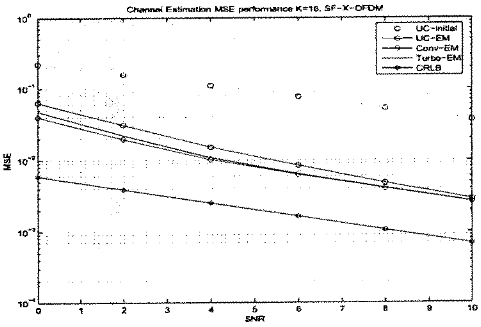


Fig. 4. MSE performance of SF-X-OFDM turbo receivers, Pilot insertion rate=16

Since we propose a Turbo receiver structure for SFBC OFDM systems in the paper, it consists of an iterative MAP-EM channel estimation algorithm, SFBC decoder and a soft MAP outer-channel-code decoder. As shown in Fig.2, first EM based channel estimator computes channel gains according to pilot symbols as described in the initialization step. Output of the estimator is used in the SFBC demodulator (19). Next, the equalized symbol sequence $\{\mathbf{D}(n)\}$ is passed through a channel deinterleaver, resulting deinterleaved equalized symbols sequence $\{\mathbf{Z}(n)\}$. Finally, $\{\mathbf{Z}(\mathrm{n})\}$ is applied to a MAP decoder by deinterleaved estimated channel gains. In the MAP decoder submodule, log likelihood ratio's (LLRs) of a posteriori probabilities on the coded and uncoded bits are yielded. In the next iteration, LLRs of coded bits $\{\Upsilon_j\}$ are reinterleaved and passed through a nonlinearity.25 Output of the nonlinearity computes soft value estimation of $\mathbf{X}(n)$ as $\hat{\mathbf{X}}(n)$ in Fig2.

$\hat{\mathbf{X}}(n)$ is used as $\hat{\mathcal{X}}_e^{(q)}$ and $\hat{\mathcal{X}}_o^{(q)}$ in (13) for next iteration. Thus, the MAP-EM channel estimator iteratively generates the channel estimates by taking the received signals from receiver antennas and interleaved soft value of LLRs which are computed by the outer channel code decoder in the previous iteration. Then, SFBC decoder takes channel estimates together with the received signals and computes equalized symbol sequence for next turbo iteration. Iterative operation is fulfilled between these three submodules.

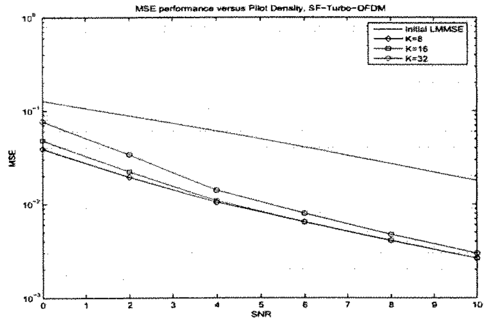


Fig. 5. MSE performance of SF-Turbo-OFDM as a function of Pilot Density

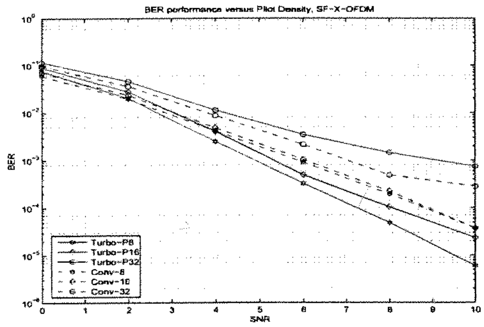


Fig. 6. BER performance of the EM algorithms as a function of Pilot Density

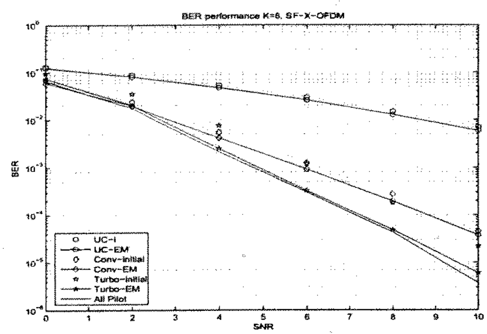


Fig. 7. BER performance of the turbo receiver structures as a function of average Eb/No, Pilot insertion rate=8

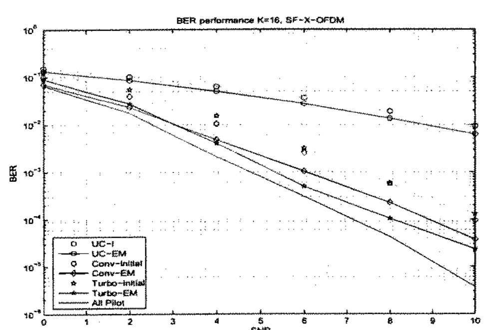


Fig. 8. BER performance of the turbo receiver structures as a function of average Eb/No, Pilot insertion rate=16

7 Simulations

In this section, computer simulations are carried out to evaluate performances of the proposed turbo receiver structure for SFBC-OFDM systems. We consider both turbo and convolutionally coded SFBC-OFDM systems. In case of Turbo Encoder, two identical recursive systematic convolutional component codes (RSC) with generator $(1,5_8/7_8)$ concatenated in parallel via a pseudorandom interleaver form the encoder [14]. For the convolutionally coded system, a $(5_8,7_8)$ code with rate 1/2 code was used.

The scenario for SFBC-OFDM simulation study consists of BPSK modulation format and the number of transmitting and receiving antennas are chosen as T=2 and R=1 respectively. The system has a 2.28 MHz bandwidth (for the pulse roll-off factor $\alpha = 0.2$) and is divided into $N_c = 512$ tones with a total period Ts of 136 μs, of which 1.052 μs constitute the cyclic prefix (L=4). The data rate is 1.9 Mbit/s. We assume that the rms value of the multipath width is $\tau_{rms} = 1$ sample (0.263 μs) for the power-delay profile. In all simulations, 3 iterations for the whole receiver process are used.

Fig. 3 and Fig 4. compare the channel estimation MSE performance of EM-based Turbo receiver structures for SFBC-OFDM systems with different pilot densities respectively [11]. Slightly better MSE performance is observed in convolutional codes compared to turbo code due to punctured structure of turbo codes. Moreover, the effect of pilot densities on the channel estimation MSE performance is presented in Fig. 5 for Turbo coded SFBC-OFDM systems. Although slight MSE performance difference for chosen different pilot densities is observed in Fig. 5, its effect on BER performance is more obvious especially for high SNR values in Fig.6.

The sensitivity of a good channel code to imperfect channel estimation is obvious. With high dependence between the coded bits, a good channel code is also more sensitive to channel estimation which might cause severe error propagation in the decoding process. Therefore, it is expected that BER performance of turbo coded structure degrades more than convolutionally coded structure. Thus, low pilot densities providing poor initial estimates result in more BER performance loss in turbo coded systems compared to convolutionally encoded systems. This effect is demonstrated in Fig.6.

BER performances of the proposed systems are presented together with initial values for pilot densities $K = 8$ and $K = 16$ in Fig.7 and Fig.8 respectively. All pilot cases for turbo coding are included as well. Due to its structure, turbo decoding evidently provides more reliable priori information for iterative processing compared to convolutional decoding. Therefore, in Fig.7, BER performance improvement observed due to iterative turbo coding structure is 1.3 dB for $Pe = 10^{-4}$, meanwhile it is 0.3 dB for convolutionally coded systems.

Although overall performance degrades by decreasing pilot density, iterative decoding provides better performance improvements on each iteration for lower pilot densities (e.g. $K = 16$). From simulation studies, it is observed that iterative performance gain is increased for K=16 as 2.3 dB and 1 dB in Fig. 8 for turbo and convolutionally coded systems respectively. As previously mentioned, turbo coding is more sensitive to channel estimation performance than convolutional coding, iterative performance gain increment for turbo coded structure is therefore lower than convolutionally coded systems.

References

[1] Y. (G.) Li, L. J. Cimini, N. Seshadri and S. Ariyavistakul, "Channel estimation for OFDM systems with transmitter diversity in mobile wireless channels," *IEEE Journal Selected Areas on Communications*, vol. 17, no.3, pp. 461-471, July 1999.

[2] Y. (G.) Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," *IEEE Transactions on Wireless Communications*, vol. 1, No. 1, pp. 67-75, January 2002. No. 8, pp. August 2003.

[3] Y. Li, C.N. Georghiades, and G. Huang, "Iterative maximum likelihood sequence estimation for space-time coded systems," *IEEE Transactions on communications*, vol. 49, No. 6, pp. 948-951 June 2001.

[4] C. Cozzo and B.L. Hughes, "Joint channel estimation and data detection in space-time communications," *IEEE Transactions on Communications*, Vol 51, No. 8, pp. August 2003.

[5] M. Feder and E. Weinstein, "Parameter Estimation of superimposed signals using the EM algorithm," *IEEE Tran. on Acoustic, Speech and Signal Processing*, Vol. 36, pp. 477-489, April 1988.

[6] B. Lu, X. Wang and Y. (G) Li, "Iterative receivers for space-time block-coded ofdm systems in dispersive fading channels," *IEEE Transactions on Wireless Communications*, vol. 1, No. 2, pp. 213-225, April 2002.

[7] B. Lu, X. Wang and K.R. Narayanan, "LDPC-based space-time coded OFDM over correlated fading channels," *IEEE Transactions on Communications*, Vol 50, No. 1, pp 74-88. January 2002.

[8] X. Yongzhe and C.N. Georghiades, "Two EM-type channel estimation algorithms for OFDM with transmitter diversity," *IEEE Transactions on communications*, vol. 51, No. 1, pp. 106-115 January 2003.

[9] X. Ma, H. Koyabashi and S. C. Schwarts, "An EM-based channel estimation for space-time and space-frequency block coded ofdm", *In proceedings of ICASSP '03*, Volume: 4, April 6-10, 2003 Page(s): IV-389 - IV-392.

[10] K. Yip and T. Ng, "Karhunen-Loeve Expansion of the WSSUS Channel Output and its Application to Efficient Simulation," *IEEE Journal on Selected Areas in Communications*, vol. 15, no 4, pp.640-646, May 1997.

[11] Cirpan HA, Panayirci E, Dogan H, "Non-Data-Aided Channel Estimation for OFDM Systems with Space-Frequency Transmit Diversity " [accepted for publication in]*Vehicular Technology, IEEE Transactions on*, Aug. 2005

[12] K.F. Lee and D.B. Williams,"A Space-Frequency Transmitter Diversity Technique for OFDM Systems", *in Proc. IEEE Globecom*, San Francisco, CA, November 2000, pp. 1473-1477.

[13] M. Tuchler, R. Koetter, and A. C. Singer, Turbo equalization: Principles and new results, IEEE Trans. Commun., vol. 50, pp. 754767, May 2002.

[14] Woodard, J.P.; Hanzo, L.; "Comparative study of turbo decoding techniques: an overview ",*Vehicular Technology, IEEE Transactions on*, Volume: 49 , Issue: 6 , Nov. 2000 Pages:2208 - 2233

Aşağı Link MC-CDMA Sistemlerinde Kullanılan PIC Alıcının EM-MAP Tabanlı Olarak İlklendirilmesi
EM-MAP Based Initializaition of PIC Receiver for Downlink MC-CDMA Systems

Hakan Doğan†, *Hakan A. Çırpan*† *and Erdal Panayırcı*‡

†İstanbul Üniversitesi Elektrik-Elektronik Mühendisliği Bölümü 34320 Avcılar, İstanbul
‡Kadir Has Üniversitesi Elektronik Mühendisliği Bölümü 34320 Cibali, İstanbul
{hdogan,hcirpan}@istanbul.edu.tr, eepanay@khas.edu.tr

Özetçe

Bu çalışmada[1], frekans seçici kanallar üzerinden çalışan aşağı link MC-CDMA sistemleri için alıcıda kanal bilgisinin olmaması durumunda, alıcı tasarımı problemine yaklaşımda bulunulmuştur. Alıcıdaki paralel karışım engelleyicinin (PIC) düzgün çalışması için gerekli birleşik MAP kanal kestirimi ve bilgi sezimi işleminin "Beklenti-Enbüyükleme (EM)" algoritması tabanlı olarak yapılması öngörülmüştür. Ayrıca, algoritmanın karmaşıklığının azaltılması için kanal değişimleri rastgele bir süreç olarak modellenip Karhunen-Loeve (KL) dik seri açılımı uygulanmıştır. Yapılan bilgisayar benzetimlerinde önerilen algoritmanın başarımı daha önce önerilen algoritmaların başarımlarıyla karşılaştırılmış ve KL açılımı sayesinde algoritmanın karmaşıklığının azaltılabileceği gösterilmiştir.

Abstract

The quality of multiple access interference (MAI), which can be improved by using channel estimation and data estimation of all active users, effects considerably the performance of PIC detector. Therefore, data and channel estimation performance is obtained in the initial stage has a significant relationship with the performance of PIC. So obviously it is necessary to make excellent joint data and channel estimation for initialization of PIC detector. In this work, to initiate PIC module efficiently a joint MAP channel estimation and data detection technique based on the Expectation Maximization (EM) method has been proposed.

Moreover, the MAP-EM approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion. The performance of the proposed approach are studied in terms of bit-error rate (BER). Throughout the simulations, extensive comparisons with previous works in literature are performed, showing that the new scheme can offer superior performance.

[1]Bu çalışma, TÜBİTAK Araştırma Projesi (104E166) ve İstanbul Üniversitesi Bilimsel Araştırma Projeleri Birimi tarafından desteklenmektedir (T-856/02062006). Bu çalışmanın bir kısmı "*IEEE Wireless Communications and Networking Conference (WCNC-2007)*" isimli konferansta sunulacaktır.

1-4244-0720-6/07/$20.00 ©2007 IEEE

1. Giriş

OFDM ve CDMA sistemlerinin kombinasyonunu öngören MC-CDMA sistemleri her iki sistemin avatajlarını kullanabilmek için 1993 yılında önerilmiştir [1]. İlk dönemde bu sistemlerin başarımı kanal parametrelerinin alıcıda tamamen bilinmesi durumunda incelenmiştir. Yapılan çalışmalarda, MC-CDMA sistemlerinde bilgi sezimi için en küçük karesel kestirimci ve PIC sistemlerinin birleştirilmesini öngören (Tümleşik MMSE-PIC) iteratif yöntemlerin iteratif olmayan yöntemlerden üstünlüğü gösterilmiştir [2].

Kanal bilgisinin alıcıda bilinmemesi durumunda ise, mevcut önerilen bilgi sezim yöntemlerinin başarımları *Iraji* tarafindan incelenmiştir[3]. Pilot tonlar yardımıyla yapılan en küçük karesel kanal kestiriminin (LS) başarımı, kanalın tamamiyle bilinmesi durumundaki başarımlarla karşılaştırılmış ve PIC alıcının başarımının sistemin ilklendirilmesinde kullanılan kanal kestirim ve bilgi sezimi işlemiyle oldukça ilgili olduğu görülmüştür. *Kuhn ve Iraji*'nin yaptığı bu çalışmalardan kanal kestirim işleminin iteratif bir biçimde alıcı yapısına katılması gerektiği görülmüştür. Bu sebeble, PIC alıcının ilklendirilmesi işleminde, genel sinyal modeline ait birleşik bilgi ve kanal kestirim işleminin gerekliliği ortaya çıkmıştır. EM algoritması, yüksek karmaşıklı ve çözümü oldukça zor olan olasılık fonksyonlarının en büyüklemesi için önerilen iteratif bir yöntemdir. Karıştırılmış sinyallerin genel kestirimi için önerilen yapı OFDM sistemlerinde yapılacak kanal kestirimi için önerilmiş SAGE biçimi ile karşılaştırılmıştır[4]. Ayrıca turbo kodlanmış uzay-frekans blok kodlamalı OFDM sistemleri için MAP kanal kestirimide bu yöntemle gerçekleştirilmiştir [5].

EM algoritması CDMA sistemleri için *Nelson ve Poor* tarafından sürekli zaman kanal parametrelerinin kestiriminden ziyade bilgi sezimi işleminin gerçekleştirilesi önerilmiş ve genelleştirilmiş yapısı olan SAGE biçimi ile karşılaştırılmıştır[6]. Bu çalışmada ise yapılan çalışmalardan farklı olarak MC-CDMA sistemleri için önerilmiş PIC alıcı yapısının ilklendirilmesi için frekans spektrumunda çalışan EM tabanlı MAP kanal kestirimci önerilmiştir.

2. Aşağı link MC-CDMA

Baz istasyonundan kullanıcılara bilgi akışını sağlayan iletişim yönü aşağı link olarak tanımlanmaktadır. Sistemdeki her bir kullanıcı baz istasyonundan gelen sinyal dizisi içinden kendine ait bilgi dizisini çözmek zorundadırki bu çözüm işleminde alıcı baz istasyonu ve kullanıcı arasındaki kanalı kestirmek yanında diğer kullanıcılara ait gönderilen sembolleride bilmelidir.

Mobil hücre içerisinde k. kullanıcıya ait gönderilecek MPSK modüleli sembol b^k ile gösterilmek üzere, toplam aktif kullanıcı sayısının K adet olduğu varsayılmaktadır. Baz istasyonu, N_c boyunda birimdik yayıcı seri yardımıyla b^k sembolünü yaymaktadır. Her bir kullanıcıya ait birimdik yayıcı seri $\mathbf{c}^k = (c_1^k, c_2^k, ..., c_{N_c}^k)^T$ vektörü ile gösterilmek üzere, vektörün her bir elemanı c_i^k, $\{-\frac{1}{\sqrt{N_c}}, \frac{1}{\sqrt{N_c}}\}$ değerlerini almaktadır. Bütün kullanıcılara ait yayılmış semboller toplanarak ters ayrık Fourier dönüşümü (IDFT) alınır ve daha sonra gönderilecek sinyale kanalda meydana gelebilecek gecikmeden fazla olacak şekilde çevirimli ön ek eklenmektedir. Yapılacak kanal kestiriminin başlangıç değerlerinin elde edilmesi için OFDM bloğu içine eşit aralıklarla pilot tonlar eklenir. Bu çalışmada, notasyon ve sinyal modelini basitleştirmek için kullanıcılara ait yayma miktarlarının aynı olduğu ve OFDM sisteminde kullanılan alt band sayısına eşit olduğu varsayılmıştır.

Gönderilen işaret, frekans seçici kanal üzerinden alıcıya geldiğinde, işarete ait önek kaldırılarak ayrık Fourier dönüşümü (DFT) uygulanır. Yapılan bu işlemlerden sonra, kod yayma matrisi $N_c \times K$ boyutlu $\mathbf{C} = [\mathbf{c}^1, ..., \mathbf{c}^K]$ ile, K adet kullanıcıya ait iletilecek sembolleri gösteren $\mathbf{b} = [b^1, ..., b^K]^T$ vektörü $K \times 1$ boyutlu olmak üzere elde edilen sinyal modeli

$$\mathbf{R} = \mathcal{H}\mathbf{C}\mathbf{b} + \mathbf{W} \qquad (1)$$

şeklinde ifade edilebilir. Burada $\mathcal{H}$ elemanları her bir altbanda ait karmaşık sönümleme katsayılarını ifade eden $N_c \times N_c$ boyutlu köşegen kanal matrisi, $\mathbf{W}$ ise kanalda eklenen toplamsal gürültüyü ifade eden $N_c \times 1$ boyutlu sıfır ortalamalı ve boyut başına $\sigma^2/2$ değişintiye sahip Gauss dağılımlı vektörü ifade etmektedir. Yayma dizilerinin dikliği ise $\mathbf{C}^T\mathbf{C} = \mathbf{I}$ olarak yazılabilir. Bu çalışmada gözlemlenen sinyal modeli (1)'e göre MAP-EM kanal kestirimi önerilecektir. MAP-EM kanal kestirimi aynı zamanda bilgi sezimi işleminin iteratif olarak iyileşmesini sağlamaktadır [5].

3. Kanal Frekans Cevabının KL Açılımı

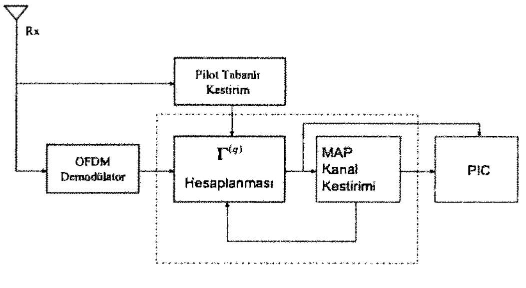
Alıcı ve verici anten arasındaki sönümlemeli kanalın her bir OFDM sembolü için değişmediği, diğer zaman aralığında ise değiştiği kabul edilmiştir. Kanalın frekans cevabı frekans seçici olarak modellenmiş ve ilişkili altbandlara ait frekans cevabını gösteren $\mathbf{H} = [H_1, H_2, ..., H_{N_c}]^T$ vektörü KL açılımı yardımıyla modellenerek birimdik taban fonksiyonlarının doğrusal kombinasyonu sayesinde ifade edilebilmektedir. Birim dik taban fonksiyonları ψ_i ile ve bu açılıma ait ağırlık katsayılarında G_i ile gösterilmek üzere, $\mathbf{\Psi} = [\psi_1, \psi_2, \cdots, \psi_{N_c}]$ ve $\mathbf{G} = [G_1, \cdots, G_{N_c}]^T$ vektörleri tanımlanarak alıcı ve verici arasındaki mobil iletişim kanalı

$$\mathbf{H} = \mathbf{\Psi}\mathbf{G} \qquad (2)$$

şeklinde yazılabilir. Farklı $\mathbf{\Psi}$'ların (taban fonksiyonlarının) kullanılması sonucu farklı özelliklere sahip kanallar modellenebilir. Kanala ait özilinti matrisi $\mathbf{C_H} = E\left[\mathbf{H}\mathbf{H}^\dagger\right]$ olmak üzere

$$\mathbf{C_H} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^\dagger \qquad (3)$$

şeklinde öz değer ve öz vektörlerine ayrıştırılabilir. Burada $\mathbf{\Lambda}$ ağırlık katsayılarının değişintilerini gösteren köşegen matristir ve $\mathbf{\Lambda} = E\{\mathbf{G}\mathbf{G}^\dagger\}$ şeklinde ifade edilmektedir(3).



Şekil 1: MC-CDMA sistemleri için Alıcı Yapısı.

4. EM tabanlı MAP kanal kestirimi

MC-CDMA sistemlerinde frekans domeninde gerçekleştirilebilen kanal denkleştirme işlemi kanalın frekans cevabına ihtiyaç duymaktadır. Zaman izgesinde kanal kestirim işlemi gerçekleştirilebilse de, ayrık Fourier dönüşümü (DFT) tabanlı zaman izgesinde yapılan kanal kestirimcilerin örneklenmeyen uzaylı kanallar için izge sızıntısına sebep olduğu ve hata düzlemi oluşturduğu bilinmektedir[7]. Diğer taraftan kanalın frekans cevabının denkleştirme işleminde direk kullanılması zaman domeninde yapılan kestirimin tekrardan frekans domenine geçirilmesini zorunlu kılmaktadır. KL açılımı yardımıyla (1) denklemi

$$\mathbf{R} = \mathrm{diag}(\mathbf{C}\mathbf{b})\mathbf{\Psi}\mathbf{G} + \mathbf{W} \qquad (4)$$

şeklinde tekrar yazılabilir. (4) denkleminde G'ye ait MAP kestirim işlemi

$$\widehat{\mathbf{G}} = \arg\max_{\mathbf{G}} p(\mathbf{G}|\mathbf{R}) \qquad (5)$$

şeklinde yazılabilir. (5) denkleminin enbüyükleme işleminin matematiksel olarak çözümlenmesi oldukça zordur. EM algoritması yardımıyla çözüm iteratif olarak gerçekleştirilebilir. Çözümlenmesi gereken bu problemde *tam* olan bilgi $\chi = \{\mathbf{R}, \mathbf{b}\}$, kestirilecek vektör $\mathbf{G}$ ve *eksik* (tam olmayan)bilgi ise $\mathbf{R}$ şeklinde tanımlanmaktadır. Bu durumda $\mathbf{G}$ ye ait kestirim işlemi

$$Q(\mathbf{G}|\mathbf{G}^{(q)}) = \sum_{\mathbf{b}} p(\mathbf{R}, \mathbf{b}, \mathbf{G}^{(q)}) \log p(\mathbf{R}, \mathbf{b}, \mathbf{G}) \qquad (6)$$

Kullback-Leibler denkleminin iteratif olarak en büyükleme problemine dönüşmektedir. Burada $\mathbf{G}^{(q)}$ ifadesi, $\mathbf{G}$'nin q. adımdaki kestirim işlemini göstermektedir. (5) denkleminde verilen koşullu olasılık fonksyonunun en büyükleme işlemindeki monotik artış sağlanabilir. (6) denklemindeki $\log p(\mathbf{R}, \mathbf{b}, \mathbf{G})$ ifadesi, *Bayesian* kuralı yardımıyla,

$$\log p(\mathbf{R},\mathbf{b},\mathbf{G}) = \log p(\mathbf{b}\,|\,\mathbf{G}) + \log p(\mathbf{R}\,|\,\mathbf{b},\mathbf{G}) + \log p(\mathbf{G}). \qquad (7)$$

şeklinde ifade edilebilir. (7) denklemindeki soldan ilk terim, $\mathbf{b}$ ve $\mathbf{G}$ vektörlerinin bağımsız olması ve $\mathbf{b}$'lerin eşit olasılıklı olmasından dolayı en büyükleme işlemini etkilememektedir. Son

2. Aşağı link MC-CDMA

Baz istasyonundan kullanıcılara bilgi akışını sağlayan iletişim yönü aşağı link olarak tanımlanmaktadır. Sistemdeki her bir kullanıcı baz istasyonundan gelen sinyal dizisi içinden kendine ait bilgi dizisini çözmek zorundadırki bu çözüm işleminde alıcı baz istasyonu ve kullanıcı arasındaki kanalı kestirmek yanında diğer kullanıcılara ait gönderilen sembolleride bilmelidir.

Mobil hücre içerisinde k. kullanıcıya ait gönderilecek MPSK modüleli sembol b^k ile gösterilmek üzere, toplam aktif kullanıcı sayısının K adet olduğu varsayılmaktadır. Baz istasyonu, N_c boyunda birimdik yayıcı seri yardımıyla b^k sembolünü yaymaktadır. Her bir kullanıcıya ait birimdik yayıcı seri $\mathbf{c}^k = (c_1^k, c_2^k, ..., c_{N_c}^k)^T$ vektörü ile gösterilmek üzere, vektörün her bir elemanı c_i^k, $\{-\frac{1}{\sqrt{N_c}}, \frac{1}{\sqrt{N_c}}\}$ değerlerini almaktadır. Bütün kullanıcılara ait yayılmış semboller toplanarak ters ayrık Fourier dönüşümü (IDFT) alınır ve daha sonra gönderilecek sinyale kanalda meydana gelebilecek gecikmeden fazla olacak şekilde çevirimli ön ek eklenmektedir. Yapılacak kanal kestiriminin başlangıç değerlerinin elde edilmesi için OFDM bloğu içine eşit aralıklarla pilot tonlar eklenir. Bu çalışmada, notasyon ve sinyal modelini basitleştirmek için kullanıcılara ait yayma miktarlarının aynı olduğu ve OFDM sisteminde kullanılan alt band sayısına eşit olduğu varsayılmıştır.

Gönderilen işaret, frekans seçici kanal üzerinden alıcıya geldiğinde, işarete ait önek kaldırılarak ayrık Fourier dönüşümü (DFT) uygulanır. Yapılan bu işlemlerden sonra, kod yayma matrisi $N_c \times K$ boyutlu $\mathbf{C} = [\mathbf{c}^1, ..., \mathbf{c}^K]$ ile, K adet kullanıcıya ait iletilecek sembolleri gösteren $\mathbf{b} = [b^1, ..., b^K]^T$ vektörü $K \times 1$ boyutlu olmak üzere elde edilen sinyal modeli

$$\mathbf{R} = \mathcal{H}\mathbf{C}\mathbf{b} + \mathbf{W} \qquad (1)$$

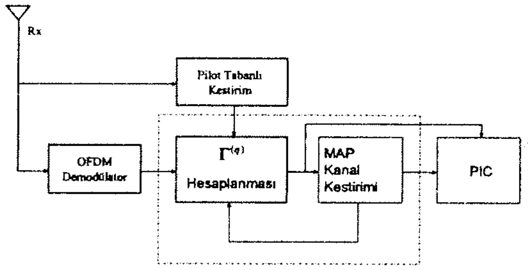
şeklinde ifade edilebilir. Burada $\mathcal{H}$ elemanları her bir altbanda ait karmaşık sönümleme katsayılarını ifade eden $N_c \times N_c$ boyutlu köşegen kanal matrisi, $\mathbf{W}$ ise kanalda eklenen toplamsal gürültüyü ifade eden $N_c \times 1$ boyutlu sıfır ortalamalı ve boyut başına $\sigma^2/2$ değişintiye sahip Gauss dağılımlı vektörü ifade etmektedir. Yayma dizilerinin dikliği ise $\mathbf{C}^T\mathbf{C} = \mathbf{I}$ olarak yazılabilir. Bu çalışmada gözlemlenen sinyal modeli (1)'e göre MAP-EM kanal kestirimi önerilecektir. MAP-EM kanal kestirimi aynı zamanda bilgi sezimi işleminin iteratif olrak iyileşmesini sağlamaktadır [5].

3. Kanal Frekans Cevabının KL Açılımı

Alıcı ve verici anten arasındaki sönümlemeli kanalın her bir OFDM sembolü için değişmediği, diğer zaman aralığında ise değiştiği kabul edilmiştir. Kanalın frekans cevabı frekans seçici olarak modellenmiş ve ilişkili altbandlara ait frekans cevabını gösteren $\mathbf{H} = [H_1, H_2, ..., H_{N_c}]^T$ vektörü KL açılımı yardımıyla modellenerek birimdik taban fonksiyonlarının doğrusal kombinasyonu sayesinde ifade edilebilmektedir. Birim dik taban fonksiyonları ψ_i ile ve bu açılıma ait ağırlık katsayılarıda G_i ile gösterilmek üzere, $\Psi = [\psi_1, \psi_2, \cdots, \psi_{N_c}]$ ve $\mathbf{G} = [G_1, \cdots, G_{N_c}]^T$ vektörleri tanımlanarak alıcı ve verici arasındaki mobil iletişim kanalı

$$\mathbf{H} = \Psi\mathbf{G} \qquad (2)$$

şeklinde yazılabilir. Farklı Ψ'ların (taban fonksiyonlarının) kullanılması sonucu farklı özelliklere sahip kanallar modellenebilir. Kanala ait özilinti matrisi $\mathbf{C_H} = E\left[\mathbf{H}\mathbf{H}^\dagger\right]$ olmak



Şekil 1: MC-CDMA sistemleri için Alıcı Yapısı.

üzere

$$\mathbf{C_H} = \Psi\Lambda\Psi^\dagger \qquad (3)$$

şeklinde öz değer ve öz vektörlerine ayrıştırılabilir. Burada Λ ağırlık katsayılarının değişintilerini gösteren köşegen matristir ve $\Lambda = E\{\mathbf{G}\mathbf{G}^\dagger\}$ şeklinde ifade edilmektedir(3).

4. EM tabanlı MAP kanal kestirimi

MC-CDMA sistemlerinde frekans domeninde gerçekleştirilebilen kanal denkleştirme işlemi kanalın frekans cevabına ihtiyaç duymaktadır. Zaman izgesinde kanal kestirim işlemi gerçekleştirilebilse de, ayrık Fourier dönüşümü (DFT) tabanlı zaman izgesinde yapılan kanal kestirimcilerin örneklenmeyen uzaylı kanallar için izge sızıntısına sebep olduğu ve hata düzlemi oluşturduğu bilinmektedir[7]. Diğer taraftan kanalın frekans cevabının denkleştirme işleminde direk kullanılması zaman domeninde yapılan kestirimin tekrardan frekans domenine geçirilmesini zorunlu kılmaktadır. KL açılımı yardımıyla (1) denklemi

$$\mathbf{R} = \text{diag}(\mathbf{C}\mathbf{b})\Psi\mathbf{G} + \mathbf{W} \qquad (4)$$

şeklinde tekrar yazılabilir. (4) denkleminde G'ye ait MAP kestirim işlemi

$$\widehat{\mathbf{G}} = \arg\max_{\mathbf{G}} p(\mathbf{G}|\mathbf{R}) \qquad (5)$$

şeklinde yazılabilir. (5) denkleminin enbüyükleme işleminin matematiksel olarak çözümlenmesi oldukça zordur. EM algoritması yardımıyla çözüm iteratif olarak gerçekleştirilebilir. Çözümlenmesi gereken bu problemde *tam* olan bilgi $\chi = \{\mathbf{R}, \mathbf{b}\}$, kestirilecek vektör $\mathbf{G}$ ve *eksik* (tam olmayan)bilgi ise $\mathbf{R}$ şeklinde tanımlanmaktadır. Bu durumda G ye ait kestirim işlemi

$$Q(\mathbf{G}|\mathbf{G}^{(q)}) = \sum_{\mathbf{b}} p(\mathbf{R}, \mathbf{b}, \mathbf{G}^{(q)}) \log p(\mathbf{R}, \mathbf{b}, \mathbf{G}) \qquad (6)$$

Kullback-Leibler denkleminin iteratif olarak en büyükleme problemine dönüşmektedir. Burada $\mathbf{G}^{(q)}$ ifadesi, G'nin q. adımdaki kestirim işlemini göstermektedir. (5) denkleminde verilen koşullu olasılık fonksyonunun en büyükleme işlemindeki monotik artış sağlanabilir. (6) denklemindeki $\log p(\mathbf{R}, \mathbf{b}, \mathbf{G})$ ifadesi, *Bayesian* kuralı yardımıyla,

$$\log p(\mathbf{R},\mathbf{b},\mathbf{G}) = \log p(\mathbf{b}\,|\,\mathbf{G}) + \log p(\mathbf{R}\,|\,\mathbf{b},\mathbf{G}) + \log p(\mathbf{G}). \qquad (7)$$

şeklinde ifade edilebilir. (7) denklemindeki soldan ilk terim, b ve G vektörlerinin bağımsız olması ve b'lerin eşit olasılıklı olmasından dolayı en büyükleme işlemini etkilememektedir. Son terim G'nin alıcıdaki birleşik olasılık yoğunluk işlevi

$$p(\mathbf{G}) \sim \exp(-\mathbf{G}^{\dagger}\mathbf{\Lambda}^{-1}\mathbf{G}). \quad (8)$$

şeklinde ifade edilebilir. Denklemdeki ikinci terim ise, iletilen semboller $\mathbf{b}$, ayrık kanal tanımlanması $\mathbf{G}$'ler ve bağımsız gürültü bileşenleri göz önüne alınırsa, alıcıda gözlemlenen $\mathbf{R}$ sinyaline ait koşullu olasılık yoğunluk işlevi

$$p(\mathbf{R}|\mathbf{b},\mathbf{G}) \sim \exp\left[-(\mathbf{R}-\text{diag}(\mathbf{Cb})\mathbf{\Psi G})^{\dagger}\mathbf{\Sigma}^{-1}(\mathbf{R}-\text{diag}(\mathbf{Cb})\mathbf{\Psi G})\right] \quad (9)$$

şeklinde yazılabilir. $\mathbf{\Sigma}$ burada $N_c \times N_c$ boyutlu ve her bir elemanı $\mathbf{\Sigma}[k,k] = \sigma^2$ $k = 1, 2, \cdots, N_c$ olmak üzere kanalda eklenen gürültünün değişintilerini ifade etmektedir. (8) ve (9) denklemleri yardımıyla, (6) denkleminin $\mathbf{G}$'ye göre türev alınarak sıfıra eşitlenmesi sonucu

$$\sum_{b} p(\mathbf{R},\mathbf{b},\mathbf{G}^{(q)})(\mathbf{\Psi}^{\dagger}\text{diag}(\mathbf{b}^{\dagger}\mathbf{C}^{T})\mathbf{\Sigma}^{-1}(\mathbf{R}-\text{diag}(\mathbf{Cb})\mathbf{\Psi G})-\mathbf{\Lambda}^{-1}\mathbf{G}) = 0 \quad (10)$$

denklemi elde edilir. (10) denkleminde, denklem eşitliği bozulmayacak şekilde, $p(\mathbf{R}, \mathbf{b}, \mathbf{G}^{(q)})$ ifadesi yerine $p(\mathbf{b} \mid \mathbf{R}, \mathbf{G}^{(q)})$ ifadesi konulabilir. $\mathbf{b}$'ler üzerinden ortalaması alınan (10) denkleminin $q+1$ iterasyon için $\mathbf{G}$'ye ait çözümünü ifade eden $\hat{\mathbf{G}}^{(q+1)}$, $\mathbf{\Gamma}^{(q)} = [\Gamma^{(q)}(1), \Gamma^{(q)}(2), ..., \Gamma^{(q)}(K)]$ vektörü q. adımda bilgi sembollerine ait sonsal olasılıkları göstermek üzere

$$\hat{\mathbf{G}}^{(q+1)} = (\mathbf{T}^{\dagger^{(q)}}\mathbf{T}^{(q)} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{T}^{\dagger^{(q)}}\mathbf{R} \quad (11)$$

$$\mathbf{T}^{(q)} = \text{diag}(\mathbf{C}\mathbf{\Gamma}^{(q)})\mathbf{\Psi}, \quad (12)$$

şeklinde ifade edilebilir. $\mathbf{\Gamma}^{(q)}$ vektörünün elemanları olan $\Gamma^{(q)}(k)$

$$\Gamma^{(q)}(k) = \sum_{b \in S_k} bP(b^k = b|\mathbf{R}, \mathbf{G}^{(q)}). \quad (13)$$

şeklinde ifade edilebilir. Gönderilen sembol dizisi için $\hat{\mathbf{b}} = 0$ $\mathbf{C}_b = \mathbf{I}$ [5] varsayımları yapılarak $\mathbf{\Gamma}^{(q)}$ ifadesi QPSK modüleli sinyaller için

$$\hat{\mathbf{Z}}^{(q)} = \mathbf{C}^{T}(\hat{\mathcal{H}}^{\dagger^{(q)}}\hat{\mathcal{H}}^{(q)} + \sigma^2\mathbf{I})^{-1}\hat{\mathcal{H}}^{\dagger^{(q)}}\mathbf{R}.$$

olmak üzere

$$\mathbf{\Gamma}^{(q)} = \frac{1}{\sqrt{2}}\tanh\left[\frac{\sqrt{2}}{\sigma^2}Re(\hat{\mathbf{Z}}^{(q)})\right] + \frac{j}{\sqrt{2}}\tanh\left[\frac{\sqrt{2}}{\sigma^2}Im(\hat{\mathbf{Z}}^{(q)})\right] \quad (14)$$

hesaplanabilir. Son olarak q. adımda herbir kullanıcılara ait giden sembol dizisini ifade eden $\mathbf{b}$'vektörü, "csign" ifadesi $\text{csign}(a+jb) = \text{sign}(a) + j\text{sign}(b)$ şeklinde tanımlanmak üzere

$$\hat{\mathbf{b}}^{(q)} = \frac{1}{\sqrt{2}}csign(\mathbf{\Gamma}^{(q)}) \quad (15)$$

şeklinde hesaplanabilir.

İşlemsel Karmaşıklık: Kanal kestirimi $\mathbf{H}$ için önerilen geleneksel LMMSE kestirimi [8]

$$\hat{\mathbf{H}} = \mathbf{C_H}\underbrace{\left[\mathbf{C_H} + \mathbf{\Sigma}(\text{diag}(\mathbf{Cb})^{\dagger}\text{diag}(\mathbf{Cb}))^{-1}\right]^{-1}}_{''O(N_c^3)''islemsel \quad karmasıklık}[\text{diag}(\mathbf{Cb})]^{-1}\mathbf{R},$$

olarak hesaplanabilir. (18) denklemindeki $\left[\mathbf{C_H} + \mathbf{\Sigma}(\text{diag}(\mathbf{Cb})^{\dagger}\text{diag}(\mathbf{Cb}))^{-1}\right]^{-1}$ ifadesi, gönderilen bilgi sembollerine bağlı olduğundan önceden hesaplanamaz ve alıcıdaki hesaplama işlemi matris tersi alınması gerekliliğinden dolayı, işlemsel karmaşıklığı oldukça fazladır. [2] Ayrıca $\mathbf{C_H}$ ve $\mathbf{\Sigma}$ değerlerinde yapılabilecek küçük hatalar, matris ters alma işleminden dolayı kanal kestiriminde büyük değişimlere neden olabilmektedir. Dahası bu etki, tersi alınan matrisin boyutuyla orantılı olarak artmaktadır [9]. Bu açıdan KL açılımlı yaklaşım, büyük ölçekli matris alma işleminden kurtulmak için önerilmiştir. (2) ve (11) denklemleri ile $\mathbf{H}$ ait iteratif kestirim işlemi

$$\hat{\mathbf{H}}^{(q+1)} = \mathbf{\Psi}(\mathbf{T}^{\dagger^{(q)}}\mathbf{T}^{(q)} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{T}^{\dagger^{(q)}}\mathbf{R}. \quad (16)$$

şeklinde ifade edilebilir. Ancak elde edilen bu form, OFDM sistemlerindeki gibi direk ters alma işleminden kurtulabilmeyi sağlayamamaktadır ve mevcut LMMSE kestiriminde elde edilecek işlemsel karmaşıklıktan da az değildir[?]. Bu açıdan elde edilen denklem tekrardan

$$\hat{\mathbf{H}}^{(q+1)} = \mathbf{\Psi\Lambda}(\mathbf{\Lambda T}^{\dagger^{(q)}}\mathbf{T}^{(q)}\mathbf{\Lambda} + \mathbf{\Sigma\Lambda})^{-1}\mathbf{\Lambda T}^{\dagger^{(q)}}\mathbf{R} \quad (17)$$

şeklinde yazılıp $\mathbf{\Lambda}$ matrisinin en büyük r adet elemanına ($\mathbf{\Lambda}_r = diag\{\lambda_1, \lambda_2, \cdots, \lambda_r\}$) karşılık gelen $\mathbf{\Psi}$ ve $\mathbf{T}$ matrislerinin sütun elemanları alınarak oluşturulmuş $N_c \times r$ boyutlu $\mathbf{\Psi}_r$ and $\mathbf{T}_r$ matrisleri yardımıyla düşük ranklı yaklaşım

$$\hat{\mathbf{H}}^{(q+1)} = \mathbf{\Psi}_r\mathbf{\Lambda}_r\underbrace{(\mathbf{\Lambda}_r\mathbf{T}_r^{\dagger^{(q)}}\mathbf{T}_r^{(q)}\mathbf{\Lambda}_r + \mathbf{\Sigma}_r\mathbf{\Lambda}_r)^{-1}}_{''O(r^3)''islemselkarmasiklik}\mathbf{\Lambda}_r\mathbf{T}_r^{\dagger^{(q)}}\mathbf{R} \quad (18)$$

şeklinde hesaplanabilir. Burada $\mathbf{\Sigma}_r$ elemanları σ^2 eşit olan $r \times r$ boyutlu köşegen matristir. Bu durumda (18) denklemi

$$\hat{\mathbf{H}}^{(q+1)} = \mathbf{\Psi}_r(\mathbf{T}_r^{\dagger^{(q)}}\mathbf{T}_r^{(q)} + \mathbf{\Sigma}_r\mathbf{\Lambda}_r^{-1})^{-1}\mathbf{T}_r^{\dagger^{(q)}}\mathbf{R}, \quad (19)$$

şeklinde yazılabilir. Açılımdaki etkin öz değerlerin kullanımıyla elde edilen (18) denklemine göre, kanal kestiriminde tersi alınması gerekli olan matrisin boyutu oldukça küçülmektedir. Ayrıca gönderilen her bir OFDM sembolünde pilot tonlarla birlikte bozulan OFDM örnekleri sayesinde algoritma kanaldaki değişimlere çabuk cevap verebilme özelliğine sahiptir.

5. Paralel Karışım Engelleyici (PIC)

Gönderilen sembollerin kestirimini ifade eden $\hat{\mathbf{b}}$ vektörü PIC alıcı bloğuna girer. Bu blokta k. kullanıcı hariç diğer kullanıcılara ait sinyallerin toplamı

$$\mathbf{R}_{int}^{k} = \hat{\mathcal{H}}\mathbf{C}\hat{\mathbf{b}} \quad \text{for} \quad \hat{b}^k = 0. \quad (20)$$

şeklinde yazılabilir. k. kullanıcıya göre karışım sinyallerini ifade eden $\mathbf{R}_{int}^{k}$ vektörü, gözlemlenen $\mathbf{R}$ sinyalinden çıkartılarak tek kullanıcı sezicisine girerse k. kullanıcıya ait PIC alıcısı, b_{pic}^{k} değerlerini

$$b_{pic}^{k} = (\mathbf{c}^k)^T[\hat{\mathcal{H}}^{\dagger}(\mathbf{R} - \mathbf{R}_{int}^{k})] \quad \text{for} \quad k = 1, ..., K \quad (21)$$

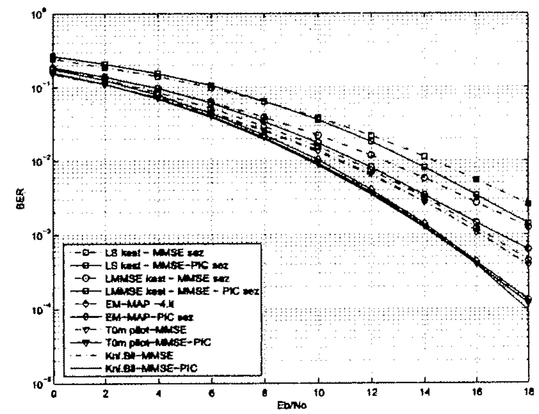
şeklinde hesaplayabilir. Son iterasyonda QPSK modülasyonu için sezilen semboller

$$\hat{b}_{pic}^{k} = \frac{1}{\sqrt{2}}\text{csign}(b_{pic}^{k}) \quad \text{for} \quad k = 1, ..., K \quad (22)$$

şeklinde bulunabilir.

---

[2] $N_c \times N_c$ boyutlu bir matrisin ters alma işlemi Gaussian eliminasyon yöntemi ile $O(N_c^3)$ bir karmaşıklığa sahiptir.



Şekil 2: Farklı kanal kestirim algoritmalarının kullanılması sonucu çeşitli alıcı yapılarının BER başarımları

6. Bilgisayar Benzetimleri

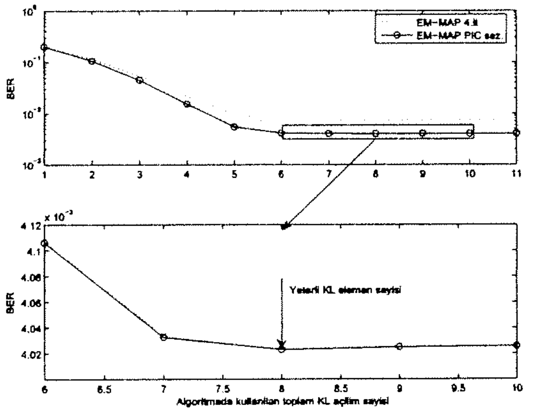
Frekans seçici kanallar için bilgisayar benzetimi yapılan MC-CDMA sistemleri için herbir kullanıcının eşit miktarda güç aldığı varsayılmıştır. Kodları arasında diklik özelliği bulunan *Gold* dizisi, boyutu kanalda kullanılan altband sayısına eşit olacak şekilde her bir kullanıcı için tanımlanmıştır. Şekil 2-3 için sistemin tamamen kullanıldığı yani toplam kullanıcı sayısının kullanılan yayma kodunun uzunluğuna eşit olduğu varsayılarak sistemin benzetimi yapılmıştır ($K = N_c$). Ayrıca algoritmanın ilklendirilmesi için, sekiz OFDM örnek aralığında bir adet pilot ton kullanılmıştır (PIR)= 1:8.

Şekil 2'de, EM-MAP tabanlı alıcı yapısının BER başarımı, daha önce önerilen LS ve LMMSE kanal kestirimlerinin kullanıldığı MMSE ve tümleşik MMSE-PIC alıcı yapılarının BER başarımlarıyla karşılaştırılmıştır. Yapılan bilgisayar benzetimlerinde, EM-MAP kanal kestirimine dayalı alıcı yapısının LS ve LMMSE kanal kestirimci tabanlı diğer alıcı yapılarından üstünlüğü gözlemlenmiştir. Ayrıca EM-MAP alıcının kanalın alıcıda tamamen bilinmesi (Knl.Bil) ve gönderilen OFDM sembolünün tamamen pilot tonlardan oluşması durumunda (Tüm Pilot) yapılan kanal kestirim başarımına (tüm pilot) yakınsadığı görülmüştür.

KL açılımında kullanılan katsayı kesme özelliği istatistiksel bağımlı olan kanal parametrelerini ifade etmek için gerekli olan bilgi miktarını en küçükleyebilir. KL açılımında verilen düşük ranklı kanal kestirim yaklaşımı, Şekil 3'te 12 dB için incelenmiştir. Kullanılan KL katsayısına göre EM-MAP alıcı yapısının BER başarımında benzetimi yapılmış kanal yapısı için, 8 adet KL katsayısının yeterli olduğu şeklin detaylı verilen bölümünde görülmüştür. Yapılan yaklaşımla, 128 × 128'lık matris ters alma işlemi yerine 8 × 8 'lik matris ters alma işleminin yeterli olabileceği görülmüştür.

7. Sonuç

Bu çalışmada, PIC alıcı yapısının başlangıç değerlerini bulmak için EM-MAP tabanlı alıcı yapısı önerilmiştir. Önerilen alıcı yapısı, daha önce önerilen LS, LMMSE tabanlı kanal kestirimci yapılarıyla karşılaştırılarak EM-MAP algoritmasının üstünlüğü gösterilmiştir. Algoritmanın, kanalın alıcıda tamamen bilindiği durumdaki başarımına yaklaştığı gözlemlenmiştir. Ayrıca, algoritmanın karmaşıklığının azaltılması için kanal değişimleri



Şekil 3: Kullanılan KL katsayısına Algoritmanın BER başarımı

rastgele bir süreç olarak modellenip Karhunen-Loeve (KL) dik seri açılımı uygulanmıştır. Yapılan KL açılımı sayesinde alınması gerekli olan matris ters alma işleminin boyutunun küçültülebildiği gösterilmiştir.

8. Kaynakça

[1] S. Hara and R. Prasad, "Overview of Multicarrier CDMA," *IEEE Communication Magazine*, pp. 126-133, December 1997.

[2] V. Kuhn, "Combined MMSE-PIC in coded OFDM-CDMA systems," *IEEE GLOBECOM "01* , pp. 231-235, Nov. 2001.

[3] S. Iraji, T. Sipil¨a and J. Lilleberg, "Channel Estimation and Signal Detection for MC-CDMA in Multipath Fading Channels," *in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC 93)*, September 7-10

[4] X. Yongzhe and C.N. Georghiades, "Two EM-type channel estimation algorithms for OFDM with transmitter diversity," *IEEE Transactions on communications*, vol. 51, No. 1, pp. 106-115 January 2003.

[5] H. Dogan, H. A. Çirpan and Erdal Panayirci, "Iterative Channel Estimation and Decoding of Turbo Coded SFBC-OFDM Systems," *IEEE Trans. Wireless Commun.*, accepted for publication in Oct. 2006.

[6] Nelson LB, Poor HV "Iterative multiuser receivers for CDMA channels: An EM-based approach,"*IEEE Trans. Commun.*44 (12): 1700-1710 Dec 1996

[7] Y. Baoguo, C. Zhigang, and K. Letaief, "Analysis of low-complexity windowed DFT-based MMSE channel estimator for OFDM systems," *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 1977-1987, Nov. 2001.

[8] O. Edfors, M. Sandell, J.-J. van de Beek, S.K. Wilson, P.O. Borjesson, "OFDM channel estimation by singular value decomposition," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 931-936, July 1998.

[9] Jie Zhu; Wookwon Lee, "A low-complexity channel estimator for OFDM systems in multipath fading channels," *In proceedings of IEEE-PIMRC 2004.*, vol. 3, pp.1978 - 1982, 5-8 Sept. 2004

Frekans Seçici Kanallarda Çalışan Yukarı Link MC-CDMA Sistemleri için EM Tabanlı Birleşik Bilgi Sezim ve Kanal Kestirim Yöntemi
EM-based Joint Data Detection and Channel Estimation for Uplink MC-CDMA Systems over Frequency Selective Channels

Hakan Doğan[1], Erdal Panayırcı[2], Hakan A. Çırpan[1], Bernard H. Fleury [3]

[1]İstanbul Üniversitesi Elektrik-Elektronik Mühendisliği Bölümü
34320 Avcılar, İstanbul, Türkiye email: {hdogan,hcirpan}@istanbul.edu.tr
[2]Kadir Has Üniversitesi Elektronik Mühendisliği Bölümü
34320 Cibali, İstanbul, Türkiye email: epanayirci@khas.edu.tr
[3]Aalborg University, Elektronik Sistemler Bölümü, Haberleşme ve Hareket Kısmı
Fredrik Bajers, Vej 7A3 DK-9000 Aalborg, Danimarka email: bfl@kom.auc.dk

Özetçe

Bu çalışmada, frekans seçici kanallarda çalışan yukarı link MC-CDMA sistemleri için birleşik kanal kestirimi ve bilgi sezimi işlemini gerçekleştirebilecek alıcı tasarımı problemi üzerinde çalışılmıştır. Beklenti-Enbüyükleme (EM) algoritması, her bir kullanıcıya ait bilgi sezimi ve kanal kestirim işleminin ortak olasılık fonksyonunu en büyükleme işlemi için kullanılmış ve problemin kapalı formda çözümü gerçekleştirilmiştir. Önerilen EM tabanlı alıcının BER başarımı literatürde daha önce önerilen alıcı yapılarıyla karşılaştırılmış ve önerilen alıcı yapısının başarım üstünlüğü gösterilmiştir.

Abstract

This paper is concerned with joint channel estimation and data detection for uplink multicarrier code-division multiple-access (MC-CDMA) systems in the presence of frequency fading channel. The detection and estimation algorithm, implemented at the receiver, is based on a version of the expectation maximization (EM) technique which is very suitable for the multicarrier signal formats. Application of the EM-based algorithm to the problem of iterative data detection and channel estimation leads to a receiver structure that also incorporates a partial interference cancelation. Computer simulations show that the proposed algorithm has excellent BER end estimation performance.

1. Giriş

Geleneksel MC-CDMA sistemlerinde kullanıcılar arası karışımın (MAI) engellenmesi tekli kullanıcı veya çoklu kullanıcı sezimi kullanılarak sağlanmaktadır[1]. Çoklu kullanıcı sezim işlemi geleneksel RAKE alıcılarına göre bant genişliğini etkili kullanmakla birlikte alıcı yapısının karmaşıklığı sistemdeki aktif kullanıcı ve iletişimin gerçekleştiği kanala ait çoklu yolun sayısıyla birlikte üssel olarak artmaktadır. Literatürde, doğrusal çoklu kullanıcı sezimi ve iteratif olarak MAI yok edilmesi için optimum olmayan sezim teknikleri önerilmiştir[2],[3].

Bu çalışmalar genel olarak aşağı link ve yukarı link olarak iki ana grupta toplanmıştır. Örneğin, aşağı link MC-CDMA sistemleri için data kestiriminde parçalı paralel karışım engelleyici önerilmiştir[4]. Yukarı link MC-CDMA sistemleri içinse MAI engellenmesi kanalın alıcıda bilindiği varsayılarak çoklu kullanıcı sezim yöntemiyle gerçekleştirilmiştir. Ancak bu yöntemlerden farklı olarak *Kocian* çalışmasında [5], Rayleigh kanallar üzerinden çalışan yukarı link DS-CDMA sistemleri için EM tabanlı birleşik bilgi sezimi ve kanal kestirimi işlemi gerçekleştiren alıcı tasarımını ortaya koymuştur.

Diğer taraftan birleşik bilgi sezimi ve kanal kestirimi işlemi frekans seçici yukarı link sistemlerinde daha karmaşık bir hale gelmektedir. Literatürde bu kanal modelleri için yapılacak bilgi sezimi ve kanal kestirimi işlemini ayrı ayrı gerçekleştiren alıcı tasarımları yapılmıştır. Bu çalışmada, *Kocian'nın* düz sönümlemeli DS-CDMA üzerine önerdiği EM algoritması, frekans seçici kanallar üzerinden çalışan yukarı link MC-CDMA sistemlerinde çoklu kullanıcı bilgi sezimi ve kanal kestirimi işlemini gerçekleştirebilmek için genişletilmiştir.

Notasyon: Makalede vektörler geniş küçük harflerle ifade edilirken matrisler büyük geniş harflerle ifade edilmiştir. Ayrıca $(.)^*$, $(.)^T$ ve $(.)^H$,$\|.\|$ ise sırasıyla konjuge, devrik, Hermityan ve Frobenius normunu ifade etmektedir. $\mathbf{I}_L$ ifadesi $L \times L$ boyutlu birim matrisi ifade ederken, $diag\{.\}$ ifadesi köşegen matrisi, $tr\{.\}$ ise matrisin izini vermektedir.

2. Sinyal Modeli

Temel bandda çalışan P adet taşıyıcıya sahip yukarı link MC-CDMA sistemi için K adet aktif kullanıcının olduğu varsayılmıştır. Sistemdeki k. kullanıcının ileteceği sembol frekans domeninde $P \times 1$ boyutlu yayıcı dizi olan $\mathbf{c}_k$ ile yayılmaktadır. Yayıcı kodla yayılan iletilecek sembol, P noktalı IDFT işleminden sonra paralelden seriye çevrilir ve yayılmış diziye kanalda meydana gelebilecek gecikmeden daha fazla olacak şekilde çevirimli önek eklenir. Bu çalışmada, notasyon ve sinyal modelini basitleştirmek için her bir kullanıcının göndereceği sembolleri yayan yayıcı kodun uzunluklarının aynı ve sistemde kullanılan alt band sayısına eşit olduğu varsayılmıştır. Vericiden çıkan k. kullanıcıya ait sinyal L adet çoklu yola sahip kanal üzerinden iletilmektedir. Kanal katsayılarının bir sembol boyunca değişmediği ancak sembolden sembole değişmekte ve alıcıda kanala ait ikinci dereceden istatistiksel özelliklerin bilindiği varsayılmaktadır.

Alıcıdan alınan sinyal, ilk olarak seriden paralele çevirilir ve önek kaldırılarak DFT işlemi uygulanır. $b_k(m)$, k. kullanıcıya ait m. sembolü, $\mathbf{F} \in \mathbb{C}^{P\times L}$ olmak üzere (k,l). elemanı $\frac{1}{\sqrt{P}}e^{-j2\pi kl/P}$ şeklinde tanımlanan DFT matrisini göstermek üzere, sinyal modeli

$$\mathbf{y}(m)=\sum_{k=1}^{K} b_k(m)\mathbf{C}_k\mathbf{F}\mathbf{h}_k + \mathbf{w}(m) \quad m = 1,2,...,M \tag{1}$$

şeklinde yazılabilir. Burada $\mathbf{w}(m)$, $P\times 1$ boyutlu sıfır ortalamalı ve boyut başına $\sigma^2/2$ değişintiye sahip gauss dağılımlı kanalda eklenen toplamsal gürültüyü ifade eden vektörü, $\mathbf{C}_k = \text{diag}(\mathbf{c}_k)$ ise k. kullanıcıya ait yayma kodunu gösteren elemanları $\{-\frac{1}{\sqrt{P}}, \frac{1}{\sqrt{P}}\}$ değerlerini alan $\mathbf{c}_k = [c_{k1}, c_{k1}, ..., c_{kP}]^T$ şeklinde tanımlanmıştır. Toplamda K adet kullanıcının M adet sembolü göndermek üzere gözlemlenen $\mathbf{y}(m)$ sinyalini vektör formunda $\mathbf{y}=[\mathbf{y}^T(1),...,\mathbf{y}^T(M)]^T$ yazarak alınan sinyal modeli

$$\mathbf{y}=\begin{bmatrix} b_1(1)\mathbf{C}_1\mathbf{F} & \cdots & b_K(1)\mathbf{C}_K\mathbf{F} \\ \vdots & \ddots & \vdots \\ b_1(M)\mathbf{C}_1\mathbf{F} & \cdots & b_K(M)\mathbf{C}_K\mathbf{F} \end{bmatrix}\begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix} + \begin{bmatrix} \mathbf{w}(1) \\ \vdots \\ \mathbf{w}(M) \end{bmatrix} \tag{2}$$

şeklinde yazılabilir. Bu denklem daha basit formda

$$\mathbf{y} = \mathbf{A}\mathbf{h} + \mathbf{w} \tag{3}$$

olarak yazılabilir. Burada $\Sigma_{\mathbf{h}_k} = E[\mathbf{h}_k\mathbf{h}_k^\dagger]$ olmak üzere her bir kanalın dağılımı $\mathbf{h}_k \sim N(0, \Sigma_{\mathbf{h}_k})$ şeklinde modellenmiş ve (3) denklemindeki bütün kanalları ifade eden $\mathbf{h}$ vektörüne ait dağılım ise $\Sigma_{\mathbf{h}} = \text{diag}[\Sigma_1, \cdots, \Sigma_K]$ olmak üzere $\mathbf{h} \sim N(0, \Sigma_{\mathbf{h}})$ şeklinde yazılabilmektedir.

3. Tümleşik Bilgi Sezimi ve Kanal Kestirimi

Bu çalışmada, (1) denkleminde verilen sinyal modeli için tümleşik bilgi sezimi ve kanal kestirimini gerçekleştirecek EM tabanlı iteratif algoritmanın çıkarımı yapılmıştır. (1) denklemindeki karıştırılmış sinyallerin toplamını ifade eden sinyal modelininin çözümlenmesi EM algoritmasıyla mümkündür[6]. Toplamsal olarak elde edilen $\mathbf{y}(m)$ sinyalinin çözümlenmesi

$$\mathbf{y}(m) = \sum_{k=1}^{K} \mathbf{x}_k(m), \quad m = 1,2\cdots,M \tag{4}$$

$$\mathbf{x}_k(m) = b_k(m)\mathbf{C}_k\mathbf{F}\mathbf{h}_k + \mathbf{w}_k(m). \tag{5}$$

şeklinde yazılabilir. Burada $\mathbf{x}_k(m)$, gözlemlenen sinyalde k. kullanıcının birim basamak cevabı $\mathbf{h}_k$ ile ifade edilmiş kanal üzerinden gönderdiği sembole ait parçasını göstermektedir. $\mathbf{w}_k(m)$, sisteme etki eden $\mathbf{w}(m)$ gürültüsünün k. kullanıcıya ait kısmını göstermekte olup herbir kullanıcaya düşen gürültülerin toplam değişintileri $N_0\beta_k$ olmak üzere $\sum_{k=1}^{K}\mathbf{w}_k(m) = \mathbf{w}(m)$ şeklinde tanımlanmıştır. $N_0\beta_k$ değerleri, $\sum_{k=1}^{K}\beta_k = 1$, $0 \leq \beta_k \leq 1$ koşullarını sağlamak üzere $\mathbf{x}_k(m)$ sinyali için belirlenmiş gürültü gücünün parçasını belirlemektedir.

Bu durumda çözümlenmesi gereken problem, gözlemlenen $\mathbf{y}$ bilgisinden her bir kullanıcının sinyalini gönderdiği $\mathbf{h}_k$ kanalının ve iletilen sembol dizisinin $\mathbf{b} = \{b_k(m)\}_{k=1,m=1}^{K,M}$ bulunmasıdır. EM algoritmasında gözlemlenen $\mathbf{y}$ dizisi *"eksik"* bilgi olarak, $\mathbf{x}_k = [\mathbf{x}_k(1),...,\mathbf{x}_k(M)]^T$ $k = 1,2,\cdots,K$ ise *"tüm"* bilgi olmak üzere, $\chi = \{(\mathbf{x}_1,\mathbf{h}_1),(\mathbf{x}_1,\mathbf{h}_1),...,(\mathbf{x}_K,\mathbf{h}_K)\}$ şeklinde tanımlanmıştır.

EM algoritması: Verilen *tüm* bilgiden $\chi = \{(\mathbf{x}_1,\mathbf{h}_1),(\mathbf{x}_1,\mathbf{h}_1),...,(\mathbf{x}_K,\mathbf{h}_K)\}$, $\mathbf{b}$'nin kestirimi için koşullu olasılık fonksiyonu

$$\log p(\chi|\mathbf{b}) = \sum_{k=1}^{K} \log p(\mathbf{x}_k, \mathbf{h}_k|\mathbf{b}_k) \tag{6}$$

şeklinde tanımlanmıştır. Burada $\mathbf{b}_k = [b_k(1),b_k(2),\cdots,b_k(M)]$ olmak üzere $p(\mathbf{x}_k,\mathbf{h}_k|\mathbf{b}_k)$ ifadesi Bayesian kuralından

$$\log p(\mathbf{x}_k,\mathbf{h}_k|\mathbf{b}_k) = \log p(\mathbf{x}_k|\mathbf{b}_k,\mathbf{h}_k) + \log p(\mathbf{h}_k|\mathbf{b}_k) \tag{7}$$

olarak yazılabilir. Gönderilen bilgi dizisi $\mathbf{b}_k$ ve kanal cevabı $\mathbf{h}_k$ birbirinden bağımsız olduğu için (7) denklemindeki $\log p(\mathbf{h}_k|\mathbf{b}_k)$ ihmal edilebilir. Bu durumda EM algoritmasında yapılması gereken ilk işlem *"beklenen değer bulma"* işlemi olup logaritmik-olasılık fonksiyonun ortalamasının alınmasıdır. Gözlemlenmiş $\mathbf{y}$'den χ 'e ait koşullu beklenti hesabı yapılırsa i. iterasyon için $\mathbf{b}$'nin değeri

$$Q\left(\mathbf{b}|\mathbf{b}^{(i)}\right) = E\left\{\log p(\chi|\mathbf{b}|\mathbf{y},\mathbf{b}^{(i)}\right\} \tag{8}$$

şeklinde yazılabilir. (7) denklemindeki $\log p(\chi|\mathbf{b})$ ifadesinin özel formunu dikkate alarak (8) denklemi

$$Q\left(\mathbf{b}|\mathbf{b}^{(i)}\right) = \sum_{k=1}^{K} Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) \tag{9}$$

şeklinde ayrıştırılabilmektedir. Buradaki $Q_k(\mathbf{b}_k|\mathbf{b}^{(i)})$ ifadesi Bayesian kuralı ve (7) denklemi göz önüne alınarak

$$Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) = E\left\{\log p(\mathbf{x}_k|\mathbf{b}_k,\mathbf{h}_k)|\mathbf{y},\mathbf{b}^{(i)}\right\} \tag{10}$$

şeklinde tekrar yazılabilir. (5) denkleminde mevcut $\mathbf{b}_k$ dan bağımsız terimler ihmal edilerek $\log p(\mathbf{x}_k|b_k,\mathbf{h}_k)$ ifadesi

$$\log p(\mathbf{x}_k|\mathbf{b}_k,\mathbf{h}_k) \sim \sum_{m=1}^{M} \Re\{b_k(m)\mathbf{h}_k^\dagger\mathbf{F}^\dagger\mathbf{C}_k^T\mathbf{x}_k(m)\}. \tag{11}$$

olarak yazılabilir. (11) denklemindeki ifade, (10) denkleminde yerine koyulursa

$$(\mathbf{h}_k^\dagger\mathbf{F}^\dagger\mathbf{C}_k^T\mathbf{x}_k(m))^{[i]} \triangleq E\left\{\mathbf{h}_k^\dagger\mathbf{F}^\dagger\mathbf{C}_k^T\mathbf{x}_k(m)|\mathbf{y},\mathbf{b}^{(i)}\right\} \tag{12}$$

olmak üzere, $Q_k(\mathbf{b}_k|\mathbf{b}^{(i)})$ ifadesi

$$Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}) = \sum_{m=1}^{M} \Re\{b_k(m)(\mathbf{h}_k^\dagger\mathbf{F}^\dagger\mathbf{C}_k^T\mathbf{x}_k(m))^{[i]}\} \tag{13}$$

şeklinde yazılabilir. (12) denklemindeki şartlı beklenti kuralının uygulanması durumunda

$$(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]} = E\{\mathbf{h}_k^\dagger E(\mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h})|\mathbf{y}, \mathbf{b}^{(i)}\}$$

$$= E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T E(\mathbf{x}_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h})|\mathbf{y}, \mathbf{b}^{(i)}\} \quad (14)$$

denklemi elde edilir. Verilen $\mathbf{y}$, $\mathbf{h}$ ve $\mathbf{b} = \mathbf{b}^{(i)}$ için $\mathbf{x}_k(m)$'e ait koşullu dağılım Gauss olmaktadır ve $(b_k(m))^{[i]} \triangleq E(b_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h})$ olmak üzere

$$E(\mathbf{x}_k(m)|\mathbf{y}, \mathbf{b}^{(i)}, \mathbf{h}) = (b_k(m))^{[i]} \mathbf{C}_k \mathbf{F} \mathbf{h}_k \quad (15)$$
$$+ \beta_k \left(\mathbf{y}(m) - \sum_{j=1}^{K} (b_j(m))^{[i]} \mathbf{C}_j \mathbf{F} \mathbf{h}_j \right)$$

şeklinde hesaplanabilir. (15) denklemindeki ifadeler, (14) denkleminde yerine koyulursa $\mathbf{F}^\dagger \mathbf{F} = \mathbf{I}$ and $\mathbf{C}_k^T \mathbf{C}_k = \frac{1}{P}\mathbf{I}$ olmak üzere, (14) denklemi tekrardan

$$(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]} = \frac{1}{P}(b_k(m))^{[i]} E\{\mathbf{h}_k^\dagger \mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\}$$
$$+\beta_k E\{\mathbf{h}_k^\dagger|\mathbf{y}, \mathbf{b}^{(i)}\} \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{y}(m)$$
$$-\beta_k \sum_{j=1, j\neq k}^{K} (b_j(m))^{[i]} E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{F} \mathbf{h}_j|\mathbf{y}, \mathbf{b}^{(i)}\}. \quad (16)$$

olarak yazılabilir. Sistemdeki gürültünün dağılımının $\mathbf{w} \sim N(0, \Sigma^2 \mathbf{I})$, $\mathbf{h}$'ın öncül pdf'i ise $\mathbf{h} \sim N(0, \Sigma_{\mathbf{h}})$ olduğu için, verilmiş $\mathbf{y}$ ve $\mathbf{b}^{(i)}$'e göre $\mathbf{h}$'ın koşullu pdf'i

$$p(\mathbf{h}|\mathbf{y}, \mathbf{b}^{(i)}) \sim p(\mathbf{y}|\mathbf{h}, \mathbf{b}^{(i)}) p(\mathbf{h})$$
$$\sim \exp\left[-\frac{1}{\sigma^2}(\mathbf{y} - \mathbf{A}\mathbf{h})^\dagger (\mathbf{y} - \mathbf{A}\mathbf{h}) - \mathbf{h}^\dagger \Sigma_{\mathbf{h}}^{-1} \mathbf{h} \right]. (17)$$

şeklinde yazılabilir. Koşullu pdf $p(\mathbf{h}|\mathbf{y}, \mathbf{b}^{(i)})$'in

$$\mu_{\mathbf{h}}^{(i)} = \frac{1}{\sigma^2} \Sigma_{\mathbf{h}}^{(i)} \mathbf{A}^{(i)\dagger} \mathbf{y}, \quad \Sigma_{\mathbf{h}}^{(i)} = \left[\Sigma_{\mathbf{h}}^{-1} + \frac{1}{\sigma^2} (\mathbf{A}^{(i)})^\dagger \mathbf{A}^{(i)} \right]^{-1} \quad (18)$$

olmak üzere Gauss dağılımına sahip olduğu gösterilebilir. (16) denklemindeki sağdan ilk terimdeki $E\{\mathbf{h}_k^\dagger \mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\}$ ifadesi, $\Sigma_{\mathbf{h}}[i,j]$, $\Sigma_{\mathbf{h}}$ matrisinin (i,j). elemanını göstermek üzere

$$(\|\mathbf{h}_k\|^2)^{[i]} \triangleq E\{\mathbf{h}_k^\dagger \mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\} = \mathrm{tr}\left[\Sigma_{\mathbf{h}}^{(i)}[k,k] + \mu_h^{(i)}[k] \mu_h^{(i)}[k]^\dagger \right] \quad (19)$$

şeklinde hesaplanabilir. (16) denklemindeki ikinci beklenti hesabı

$$(\mathbf{h}_k)^{[i]} \triangleq E\{\mathbf{h}_k|\mathbf{y}, \mathbf{b}^{(i)}\} = \mu_h^{(i)}[k]. \quad (20)$$

şeklinde bulunabilir. (16) denklemindeki son terim olan $E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{F} \mathbf{h}_j|\mathbf{y}, \mathbf{b}^{(i)}\}$ ifadesinin hesaplaması için $\Psi_j \triangleq \mathbf{C}_j \mathbf{F}$, $\mathbf{s}_j \triangleq \Psi_j \mathbf{h}_j$ tanımlamaları yapılırsa

$$\Sigma_s^{(i)} \triangleq E[\mathbf{s}\mathbf{s}^\dagger|\mathbf{y}, \mathbf{b}^{(i)}] = E[\Psi \mathbf{h}\mathbf{h}^\dagger \Psi^\dagger|\mathbf{y}, \mathbf{b}^{(i)}] = \Psi \Sigma_{\mathbf{h}}^{(i)} \Psi^\dagger. \quad (21)$$

şeklinde ifade edilebilir. Buradan (21) denklemi, $\mu_s^{(i)} \triangleq \Psi \mu_h^{(i)}$ olmak üzere

$$E\{\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{C}_j \mathbf{F} \mathbf{h}_j|\mathbf{y}, \mathbf{b}^{(i)}\} = E[\mathbf{s}^\dagger \mathbf{s}|\mathbf{y}, \mathbf{b}^{(i)}]$$
$$= \mathrm{tr}\left[\Sigma_s^{(i)}[k,j] + \mu_s^{(i)}[k] \mu_s^{(i)}[j]^\dagger \right] \quad (22)$$

olarak bulununur.

En Büyükleme Adımı (M-Adımı): EM algoritmasını gerçekleştirebilmek için ikinci adımı oluşturan *M-Adımı*'na göre $\mathbf{b}$ ifadesi $(i+1)$. iterasyonda

$$\mathbf{b}^{(i+1)} = \arg\max_{\mathbf{b}} Q(\mathbf{b}|\mathbf{b}_i) = \sum_{k=1}^{K} Q_k(\mathbf{b}_k|\mathbf{b}^{(i)}). \quad (23)$$

olarak yazılabilir. (23) denklemindeki her bir $Q_k(\mathbf{b}_k|\mathbf{b}^{(i)})$ ifadesi için en büyüklenme işlemi sistemde kodlama kullanılmamışsa ayrı ayrı

$$b_k^{(i+1)}(l) = \mathrm{sgn}\left[\Re\{(\mathbf{h}_k^\dagger \mathbf{F}^\dagger \mathbf{C}_k^T \mathbf{x}_k(m))^{[i]}\} \right] \quad (24)$$

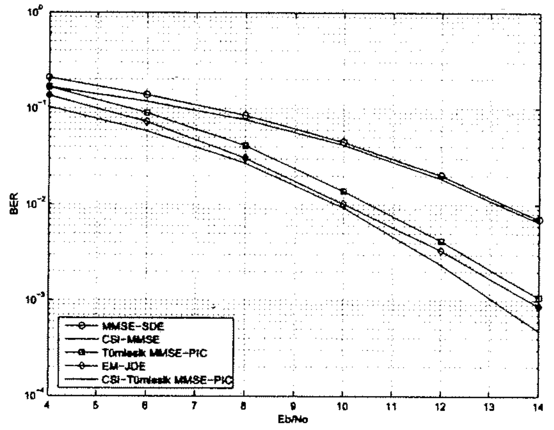
şeklinde gerçeklenebilir. EM algoritması sonucunda kullanıcılara ait gönderilen bilgi bit dizisi kestirimi

$$b_k^{i+1}(l) = \mathrm{sgn}\left[\Re\left\{ \frac{1}{P} b_k^{[i]}(m) \|\mu_h^{(i)}[k]\|^2 (1 - \beta_k) \right.\right.$$
$$\left.\left. + \beta_k (\mathbf{h}_k^{(i)\dagger} \Psi_k^T \left[\mathbf{y}(m) - \sum_{j=1, j\neq k}^{K} b_j^{[i]}(m) \Psi_j \mu_h^{(i)}[j] \right] \right\} \right] \quad (25)$$

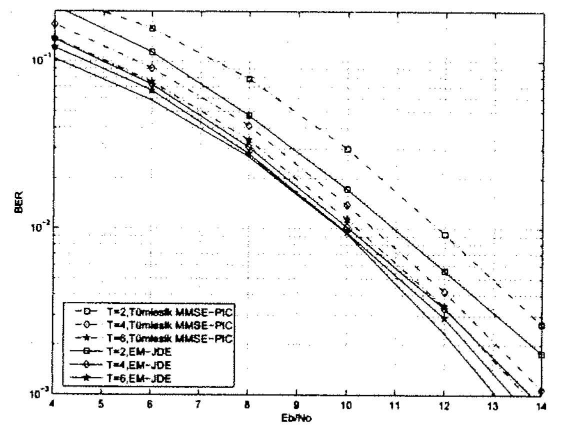
şeklinde hesaplanabilir. Sonuçta (25) denklemi, birleşik kanal kestirimi ve parçalı karışım yoketmeli bilgi sezimi olarak düşünülebilir. Alıcı yapısındaki her bir bilgi sezimi için gelen sinyalden, ilgili kullanıcıdan hariç, diğer kullanıcılara ait sinyallerin karışımı azaltılarak tek kullanıcı için önerilmiş alıcı yapısına uygulanmaktadır. Sonuçta K kullanıcılı optimizasyon problemi, çözüm işlemi daha kolay olan K adet birbirinden bağımsız optimizasyon problemine dönüşmektedir.

4. Bilgisayar Benzetimleri

Bu bölümde, frekans seçici kanallar üzerinden çalışan yukarı link MC-CDMA için önerilen alıcı yapısının başarımı, her bir kullanıcının aynı seviyede güç miktarı aldığı varsayılarak incelenmiştir. Her bir kullanıcı için, kodları arasında diklik özelliği bulunan Walsh dizisi, boyutu kanalda kullanılan altbant sayısına eşit olacak şekilde tanımlanmıştır: $P = 16$. Sistemde aktif kullanıcı sayısı $K = 8$ seçilmiş ve her bir kullanıcının gönderdiği bilgi çercevesi T adet pilot, F adet bilgi sembolünden oluşmaktadır. Verici antenler ve alıcı anten arasındaki kablosuz haberleşme kanallarının L uzunluğunda karmaşık katsayılardan oluşan $N(0, \Sigma_h)$ dağılımlı olduğu varsayılmaktadır. Kanal özilişki matrisi olan Σ_h'ın alıcıda bilindiği varsayılmıştır. Literatürde daha önce önerilen alıcı yapılarına göre; gelen bilgi çervesindeki T adet pilot sembolü kullanılarak MMSE kanal kestirimini gerçekleştiren alıcı, bulduğu bu kanal katsayıları yardımıyla gelen çerçevedeki F adet bilgi sembolünün MMSE kestirimini gerçekleştirmektedir. $\mathbf{h}$ ve $\mathbf{b}$'nin ayrı iki adet MMSE kestirimci yardımıyla bulunduğu için bu alıcı yapısı MMSE-SDE (separate detection and estimation) olarak tanımlanmıştır. Ayrıca MMSE-SDE'nin çıkışının PIC alıcısına uygulanması durumunda, alıcıya tümleşik MMSE-PIC denilmiştir. Birleşik bilgi sezimi ve kanal kestirim işlemini gerçekleştiren EM-JDE (Joint Detection and Estimation) algoritmasının başarımları MMSE-SDE ve Tümleşik-MMSE-PIC alıcı yapılarıyla karşılaştırılmıştır. Kanala ait bilgilerinin alıcıda tamamen bilinmesi durumunu gösteren CSI (Channel state information) durumları



Şekil 1: Frekans Seçici Kanallarda Alıcı Yapılarının BER Başarımları (T= 4,F= 40,L= 8,U= 8)



Şekil 2: Kullanılan önek uzunluklarına göre BER başarımının incelenmesi (F= 40,L= 8,U= 8)

da CSI-MMSE ve CSI-Tümleşik MMSE-PIC olarak ayrıca incelenmiştir. Kanalın tamamen bilinmesi durumunda gelen bilgi dizisinin en iyi kestirimi yapılarak PIC yapısına uygulanacağı için CSI-Tümleşik MMSE-PIC eğrisi bize mevcut iletişim şartları için alt-sınırı vermektedir.

Şekil 1'de EM-JDE'nin BER başarımı yukarıda bahsedilen MMSE-SDE, Tümleşik-MMSE-PIC, CSI-MMSE ve CSI-Tümleşik MMSE-PIC sistemleri ile karşılaştırılmıştır. Adaletli bir karşılaştırma için Tümleşik-MMSE-PIC, CSI-Tümleşik MMSE-PIC ve EM-JDE alıcı yapıları üç iterasyon için çalıştırılmıştır. Şekil 1'de EM-JDE alıcısının başarımı MMSE-SDE, Tümleşik-MMSE-PIC, CSI-MMSE alıcılarının başarımlarından daha iyi olduğu ve CSI-Tümleşik MMSE-PIC durumundaki başarıma yaklaştığı gözlemlenmiştir.

Şekil 2'de kullanılan farklı pilot sembol sayılarına göre yapılan ilklendirme işleminin EM-JDE ve Tümleşik MMSE-PIC alıcı yapılarının başarımlarına olan etkisi incelenmiştir. Yapılan bilgisayar benzetimlerinde az sayıda pilot kullanımının yapılan ilklendirme işlemini etkileyerek sistemin başarımını azalttığı, ancak başarımdaki bu düşüşün Tümleşik-MMSE-PIC 'de EM-JDE'ye göre daha fazla olduğu görülmüştür. Diğer taraftan yapılan ilklendirme işlemi için kullanılan pilot sembollerin sayısının artırılması durumda yapılacak kestirimin oldukça iyileşmesinden dolayı Tümleşik-MMSE-PIC ve EM-JDE arasındaki başarım farkının azaldığı gözlemlenmiştir.

5. Sonuç

Bu çalışmada, frekans seçici kanallarda çalışan yukarı link MC-CDMA sistemleri için birleşik bilgi sezimi ve kanal kestirim işlemi problemine yaklaşımda bulunulmuştur. *Kocian*'nın düz sönümlemeli Rayleigh kanallar üzerinden çalışan yukarı link DS-CDMA sistemler için önerdiği EM tabanlı alıcı yapısı genelleştirilerek probleme uygulanmış ve kullanıcılara ait bilgi dizilerinin sezimi için parçalı karışım engelleyici ve kanal kestirimci kullanılarak kapalı formda çözüm elde edilmiştir. Yapılan bilgisayar benzetimlerinde kullanılan az sayıdaki pilotun EM algoritmasını ilklendirmek için yeterli olduğu ve daha önce önerilen MMSE-SDE, Tümleşik-MMSE-PIC alıcı yapılarından üstünlüğü gösterilmiştir. İterasyonla birlikte yapılan kestirim işlemlerinin düzeltildiği ve kanallara ait bilgilerin alıcıda tamamen bilinmesi durumundaki Tümleşik MMSE-PIC alıcı yapısının başarımına yaklaştığı gözlemlenmiştir.

6. Kaynakça

[1] S. Hara and R. Prasad, "Overview of Multicarrier CDMA," *IEEE Communication Magazine*, pp. 126-133, December 1997.

[2] Z. Xie, R.T. Short and C.K.Rushfortli: "Family of suboptimum detectors for coherent multiuser communications." *IEEE Journal on Selected Areas in Communications*, vol. 8, no. 4, pp. 683-690, May 1990.

[3] V. Kuhn, "Combined MMSE-PIC in coded OFDM-CDMA systems," *IEEE GLOBECOM '01*, pp. 231-235, Nov. 2001.

[4] S. Iraji, T. Sipil¨a and J. Lilleberg, "Channel Estimation and Signal Detection for MC-CDMA in Multipath Fading Channels," *in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC)*, September 1993

[5] A. Kocian and B. H. Fleury, "EM-based joint data detection and channel estimation of DS-CDMA signals," *IEEE Trans. Commun.*, vol. 51, no. 10, pp. 1709-1720, Oct. 2003.

[6] M. Feder and E. Weinstein, "Parameter Estimation of superimposed signals using the EM algorithm,"*IEEE Tran. on Acoustic, Speech and Signal Processing*, Vol. 36, pp. 477-489, April 1988.

7. Teşekkür

Bu çalışma, TÜBİTAK Araştırma Projesi (104E166) ve İstanbul Üniversitesi Bilimsel Araştırma Projeleri Birimi tarafından desteklenmektedir (T-856/02062006). Bu çalışmanın bir kısmı, "*6th International Workshop on Multi-Carrier Spread Spectrum (MC-SS 2007)* " isimli konferansta sunulacaktır.