



Design of practical broadband matching networks with commensurate transmission lines

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ABSTRACT

To design broadband matching networks for microwave communication systems, commercially available computer aided design (CAD) tools are always preferred. But these tools need proper matching network topology and element values. Therefore, in this paper, a practical method is proposed to generate distributed-element matching networks with good initial element values. Then, the gain performance of the designed matching network can be optimized employing these tools. The utilization of the proposed method is illustrated by means of the given example. It is shown that proposed method provides very good initials for CAD tools.

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1. Introduction

In the design of high frequency communication systems, if the wavelength of the operation frequency is comparable with physical size of the lumped circuit elements, usage of distributed elements is inevitable. Therefore, at Radio Frequencies (RF), design of broadband matching networks with distributed elements or commensurate transmission lines have been considered as a vital problem for engineers [1].

Although analytic theory of broadband matching may be employed for simple problems [2,3], it is well known that this theory is inaccessible except for simple problems. Therefore, for practical applications, it is always preferable to utilize CAD tools, to design matching networks with distributed elements [4–6]. Matched system performance is optimized by all the commercially available CAD tools. At the end of this process, characteristic impedances and the delay lengths of the transmission lines are obtained. But performance optimization is highly nonlinear with respect to characteristic impedances and delay lengths, and requires proper initials [7]. Furthermore, selection of initial values is vital for successful optimization, since the convergence of the optimization depends on the selected initial values.

Therefore, in this paper, a well-established process is proposed, to design broadband matching networks with equal length or commensurate transmission lines. These lines are also called as unit elements (UEs).

2. Broadband matching problem

The broadband matching problem can be considered as the design of a lossless two-port network between a generator and complex load, in such a way that power transfer from the source to the load is maximized over a frequency band. The power transfer capability of the lossless matching network is best measured by means of the transducer power gain which can be defined as the ratio of power delivered to the load to the available power from the generator.

The matching problems can be grouped basically as single matching and double matching problems. In the single matching problems, the generator impedance is purely resistive and the load impedance is complex. On the other hand, if both terminating impedances are complex, then the problem is called as the double matching problem.

Let us consider the classical double matching problem depicted in Fig. 1. Transducer power gain (TPG) can be written in terms of the real and imaginary parts of the load impedance $Z_L = R_L + jX_L$ and those of the back-end impedance $Z_2 = R_2 + jX_2$, or in terms of the real and imaginary parts of the generator impedance $Z_G = R_G + jX_G$ and those of the front-end impedance $Z_1 = R_1 + jX_1$ of the matching network as follows:

$$TPG(\omega) = \frac{4R_\alpha R_\beta}{(R_\alpha + R_\beta)^2 + (X_\alpha + X_\beta)^2} \quad (1)$$

Here if $\alpha = 1$, $\beta = G$, and if $\alpha = 2$, $\beta = L$.

The objective in broadband matching problems is to design the lossless matching network in such a way that TPG given by (1) is maximized inside a frequency band. So the matching problem in this formalism can be regarded as the determination of a realizable

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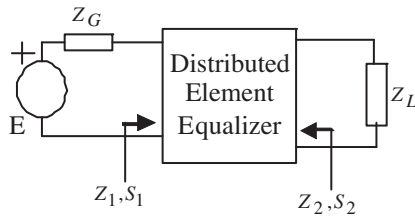


Fig. 1. Double matching arrangement.

impedance function Z_1 or Z_2 . Once Z_1 or Z_2 is obtained properly, the lossless matching network can be synthesized easily.

Real frequency line segment technique proposed by Carlin (RF-LST) is one of the best techniques to determine a realizable data set for Z_2 [8,9]. In this method, Z_2 is realized as a minimum reactance function and its real part $R_2(\omega)$ is resembled by line segments in such a way that $R_2(\omega) = \sum_{k=1}^m a_k(\omega)R_k$, passing through m -selected pairs designated by $\{R_k, \omega_k; k = 1, 2, \dots, m\}$. Here, break points (or break resistances) R_k are considered as the unknowns of the problem. Then, these points are obtained via nonlinear optimization of TPG.

The imaginary part $X_2(\omega) = \sum_{k=1}^m b_k(\omega)R_k$ of Z_2 is also expressed by means of the same break points R_k . It is important to note that the coefficients $a_k(\omega)$ are known quantities and they are calculated in terms of the pre-selected break frequencies ω_k . The coefficients $b_k(\omega)$ are obtained by means of Hilbert transformation relation given for minimum reactance functions. If $H\{\cdot\}$ represents the Hilbert transformation operator, then $b_k(\omega) = H\{a_k(\omega)\}$.

In RF-LST, two independent approximation steps seem to be disadvantages of the method. Although it is possible to extend the method to solve double matching problems, the computational efficiency applies only for single matching problems.

The basic principle of the direct computational technique (DCT) is similar to that of the real frequency line segment technique [10]. In this method, the real part of the unknown matching network impedance R_2 is written as a real even rational function. Then the unknown coefficients of this function are optimized to get the best gain performance.

In DCT, the unknown coefficients of R_2 must be determined so that R_2 is a nonnegative even rational function, which in turn ensures the realizability of the resulting impedance function Z_2 . So in order to guarantee the realizability, an auxiliary polynomial is utilized for constructing an intrinsically nonnegative real part function R_2 . By the introduction of this polynomial, although the realizability is simply ensured, the computational effort and the nonlinearity of the transducer power gain with respect to the optimization parameters are increased.

In Fettweis's method, parametric representation of the positive real back-end driving point impedance Z_2 is utilized [11]. Namely, the positive real impedance Z_2 is expressed in a partial fraction expansion, and then the poles of Z_2 are optimized to get the best gain performance of the system in the frequency band.

The parametric method constitutes an efficient approach for solving single matching problems. The only problem is the initialization of the location of poles, which may be critical.

In all the methods explained briefly above, the lossless matching network is described in terms of a set of free parameters by means of back-end driving point impedance Z_2 . But, the matching problem can also be described by means of any other set of parameters. In the real frequency scattering approach which is referred to as the Simplified Real Frequency Technique (SRFT), the canonic polynomial representation of the scattering matrix is employed to describe the lossless matching network [12,13].

In another method proposed in [7,14], the back-end driving point impedance of the matching network Z_2 is modeled as a minimum reactance function, then, if necessary, a Foster impedance is connected in series.

As the result of the explanation above, it is desired to express the back-end impedance Z_2 of the matching network in terms of any set of free parameters. Then gain performance of the matching network is optimized via (1). But the determination of the back-end impedance expression is complicated. There is a very simple and obvious way to determine the back-end impedance Z_2 or front-end impedance Z_1 of the matching network. This is the crux of the proposed method.

In the proposed method, these driving point impedances (Z_2 or Z_1) are determined utilizing the scattering parameters of the lossless matching network, source and load reflection coefficients. So in the next section, canonic polynomial representation of a distributed-element two-port network is briefly summarized, and then rationale of the proposed method is given.

3. Canonic polynomial representation of a distributed element two-port network

Most of the design methods for microwave networks incorporate finite homogenous transmission lines of commensurable lengths as ideal UEs [15]. By commensurate, it must be understood that all line lengths in a network are multiples of the UE length. Richards has shown that the distributed-element networks composed of commensurate transmission lines (UEs) can be proceeded in analysis or synthesis as lumped element networks under the transformation

$$\lambda = \tanh p\tau,$$

where τ is the commensurate delay of the transmission lines, p is the usual complex frequency variable ($p = \sigma + j\omega$) and λ is the so called Richards variable, $\lambda = \Sigma + j\Omega$. Specifically, on the imaginary axis, the transformation takes the form $\lambda = j\Omega = j \tan \omega\tau$.

Referring to the double matching configuration shown in Fig. 1, the scattering parameters of the lossless matching network can be written in terms of three real polynomials by using the well known Belevitch representation as follows:

$$\begin{aligned} S_{11}(\lambda) &= \frac{h(\lambda)}{g(\lambda)}, & S_{12}(\lambda) &= \frac{\mu f(-\lambda)}{g(\lambda)}, \\ S_{21}(\lambda) &= \frac{f(\lambda)}{g(\lambda)}, & S_{22}(\lambda) &= -\frac{\mu h(-\lambda)}{g(\lambda)}, \end{aligned} \tag{2}$$

where g is a strictly Hurwitz polynomial, f is a real polynomial which is constructed on the transmission zeros of the matching network and μ is a unimodular constant ($\mu = \pm 1$). If the two-port is reciprocal, then the polynomial f is either even or odd and $\mu = f(-\lambda)/f(\lambda)$.

The polynomials $\{f, g, h\}$ are related by the Feldtkeller equation

$$g(\lambda)g(-\lambda) = h(\lambda)h(-\lambda) + f(\lambda)f(-\lambda). \tag{3}$$

It can be concluded from (3) that the Hurwitz polynomial $g(\lambda)$ is a function of $h(\lambda)$ and $f(\lambda)$. If the polynomials $f(\lambda)$ and $h(\lambda)$ are known, then the scattering parameters of the two-port network, and then the network itself can completely be defined.

In almost all practical applications, the designer has an idea about transmission zero locations of the matching network. Hence the polynomial $f(\lambda)$ is usually constructed by the designer. For practical problems, the designer may use the following form of $f(\lambda)$

$$f(\lambda) = f_0(\lambda)(1 - \lambda^2)^{n_\lambda/2} \tag{4}$$

where n_λ specifies the number of equal-length transmission lines in cascade, and $f_0(\lambda)$ is an arbitrary real polynomial. A powerful class of networks contains series or shunt stubs and equal-length

transmission lines only. Series-short stubs and shunt-open stubs produce transmission zeros at $\lambda = \infty$. Series-open stubs and shunt-short stubs produce transmission zeros at $\lambda = 0$. For such networks, the polynomial $f(\lambda)$ takes the more practical form

$$f(\lambda) = \lambda^k (1 - \lambda^2)^{n_\lambda/2} \quad (5)$$

where k is the total number of series-open and shunt-short stubs, and the difference $n - (n_\lambda + k)$ gives the number of series-short and shunt-open stubs. Here, n is the degree of the matching network, which is also the degree of the polynomial $g(\lambda)$ or $h(\lambda)$.

4. Fundamentals of the proposed method

Consider the double matching arrangement shown in Fig. 1. Input reflection coefficient of the matching network when its output port is terminated in Z_L can be expressed in terms of scattering parameters of the matching network as

$$S_1 = S_{11} + \frac{S_{12}S_{21}S_L}{1 - S_{22}S_L} \quad (6)$$

where S_L is the load reflection coefficient and expressed as

$$S_L = \frac{Z_L - 1}{Z_L + 1} \quad (7)$$

Similarly, output reflection coefficient of the matching network when its input port is terminated in Z_G can be written in terms of scattering parameters of the matching network as

$$S_2 = S_{22} + \frac{S_{12}S_{21}S_G}{1 - S_{22}S_G} \quad (8)$$

where S_G is the source reflection coefficient and expressed as

$$S_G = \frac{Z_G - 1}{Z_G + 1} \quad (9)$$

So the front-end and back-end driving point impedances of the matching network can be calculated via the following equations, respectively;

$$Z_1 = \frac{1 + S_1}{1 - S_1} \quad (10)$$

$$Z_2 = \frac{1 + S_2}{1 - S_2} \quad (11)$$

As the result, the following algorithm can be proposed to solve both single and double broadband matching problems with distributed elements.

5. Proposed algorithm

Inputs:

- $Z_{L(measured)} = R_{L(measured)} + jX_{L(measured)}$, $Z_{G(measured)} = R_{G(measured)} + jX_{G(measured)}$: Measured load and generator impedance data, respectively.
- $\omega_{i(measured)}$: Measurement frequencies, $\omega_{i(measured)} = 2\pi f_{i(measured)}$.
- f_{norm} : Normalization frequency.
- R_{norm} : Impedance normalization number in ohms.
- $h_0, h_1, h_2, \dots, h_n$: Initial real coefficients of the polynomial $h(\lambda)$. Here n is the degree of the polynomial which is equal to the number of distributed elements in the matching network. The coefficients can be initialized as ± 1 in an ad hoc manner, or the approach explained in [16] can be followed.
- $f(\lambda)$: A polynomial constructed on the transmission zeros of the matching network. A practical form is given in (5).
- δ_c : The stopping criteria of the sum of the square errors.

Outputs:

- Analytic form of the input reflection coefficient of the lossless matching network, $S_{11}(\lambda) = h(\lambda)/g(\lambda)$. It is noted that this algorithm determines the coefficients of the polynomials $h(\lambda)$ and $g(\lambda)$, which in turn optimizes the gain performance of the system.
- Circuit topology of the lossless matching network with element values: The circuit topology and element values are obtained as the result of the synthesis of $S_{11}(\lambda)$. Synthesis is accomplished by extracting poles at 0 and ∞ , corresponding to stubs, while equal-length transmission lines are extracted by employing Richards extraction method [17] or the method given in [18]. Alternatively, the synthesis can be carried out in a more general fashion using the cascade decomposition technique by Fettweis, which is based on the factorization of scattering transfer matrices [19]. As a result, $S_{11}(\lambda)$ is synthesized as a lossless two-port which is the desired matching network.

Computational steps:

Step 1: Normalize the measured frequencies with respect to f_{norm} and set all the normalized angular frequencies

$$\omega_i = f_{i(measured)}/f_{norm}$$

Normalize the measured load and generator impedances with respect to impedance normalization number R_{norm} ;

$$R_L = R_{L(measured)}/R_{norm}, \quad X_L = X_{L(measured)}/R_{norm}, \\ R_G = R_{G(measured)}/R_{norm}, \quad X_G = X_{G(measured)}/R_{norm} \quad \text{over the entire frequency band.}$$

Step 2: Calculate corresponding values of Richards variable via $\lambda_i = j\Omega_i = j \tan \omega_i \tau$. The delay τ can be obtained as usual from the length l of the distributed-element and the phase velocity c : $\tau = l/c$. If l is chosen a fraction $1/K$ of the wavelength $\Lambda = c/f_m$ (where f_m is the maximum normalized frequency in the frequency band, $\omega_m = 2\pi f_m$), it follows that $\tau = 2\pi/K\omega_m$. To provide a safe limit for the end of the stop band, a scaled frequency $\gamma\omega_m$, $\gamma > 1$, may be advantageous [17]. Then $\tau = 2\pi/K\gamma\omega_m$.

Step 3: Obtain the strictly Hurwitz polynomial $g(\lambda)$ from (3). Then calculate scattering parameters via (2).

Step 4: Calculate load and source reflection coefficients S_L and S_G via (7) and (9), respectively.

Step 5: Calculate input and output reflection coefficients S_1 and S_2 via (6) and (8), respectively.

Step 6: Calculate input and output impedances Z_1 and Z_2 via (10) and (11), respectively.

Step 7: Calculate transducer power gain via (1).

Step 8: Calculate the error via $\epsilon(\omega) = 1 - TPG(\omega)$, then $\delta = \sum |\epsilon(\omega)|^2$.

Step 9: If δ is acceptable ($\delta \leq \delta_c$), stop the algorithm and synthesize $S_{11}(\lambda)$. Otherwise, change the initialized coefficients of the polynomial $h(\lambda)$ via any optimization routine and return to step 3.

6. Example

In this section, a double-matching example is presented to design a practical broadband matching network. The normalized source and load impedance data are given in Table 1. It should be noted that the given source data can easily modeled as a capacitor $C_G = 4$ in series with a resistor $R_G = 1$ (i.e. $R + C$ type of impedance), and the load data as a capacitor $C_L = 4$ in parallel with a resistance $R_L = 1$ (i.e. $R//C$ type of impedance). Since the given impedance data are normalized, there is no need a normalization step. The same example is solved here via SRFT.

In the design, $K = 8$, $\gamma = 1.3$, and $\omega_m = 1$ are chosen, eventually leading to a normalized value of $\tau = 0.6042$.

In the matching network, it is not desired to have a transformer, so the least degree coefficient of the polynomial $h(\lambda)$ must

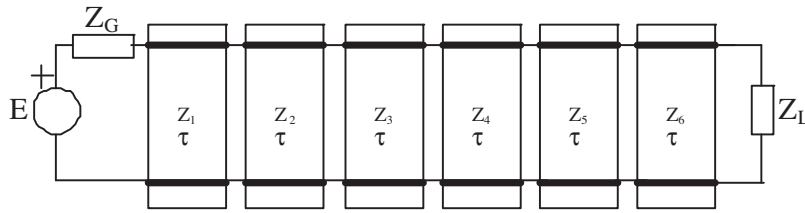


Fig. 2. Designed distributed-element double matching network; proposed: $Z_1=1.3114, Z_2=1.5137, Z_3=0.33667, Z_4=1.8733, Z_5=1.18295, Z_6=1.2459, \tau=0.6042$; SRFT: $Z_1=1.3117, Z_2=1.5067, Z_3=0.33697, Z_4=1.8708, Z_5=1.18295, Z_6=1.2453, \tau=0.6042$ (normalized).

be restricted, i.e. $h_0=0$ [20]. Then the polynomial $h(\lambda)$ is initialized as $h(\lambda) = -\lambda^6 + \lambda^5 - \lambda^4 + \lambda^3 - \lambda^2 + \lambda$ in an ad hoc manner. Also the polynomial $f(\lambda)$ is selected as $f(\lambda) = (1 - \lambda^2)^3$. So in the matching network there will be six cascaded unit elements only. In the example, α and β are selected as $\alpha=1, \beta=G$. So front-end driving point impedance Z_1 and source impedance Z_G are used in the TPG expression in Step 7. Then after running the proposed algorithm, the following scattering parameter of the matching network is obtained

$$S_{11}(\lambda) = \frac{h(\lambda)}{g(\lambda)}$$

where

$$h(\lambda) = -21.8564\lambda^6 + 39.7070\lambda^5 + 16.2816\lambda^4 + 0.3535\lambda^3 + 9.4704\lambda^2 - 2.3660\lambda,$$

$$g(\lambda) = 21.8792\lambda^6 + 80.5973\lambda^5 + 96.0175\lambda^4 + 67.2069\lambda^3 + 33.1840\lambda^2 + 8.8298\lambda + 1.$$

After synthesizing the obtained scattering parameter, the matching network seen in Fig. 2 is obtained.

As it is seen from Fig. 3, initial performance of the matched system looks very good. However, it can be further improved via optimization utilizing the commercially available design tool called Microwave Office of Applied Wave Research Inc. (AWR) [4]. Thus, the final normalized elements values are given as $Z_1=0.7517, Z_2=1.709, Z_3=0.2315, Z_4=1.512, Z_5=0.1459, Z_6=1.145$. For comparison purpose, both initial and the optimized performances of the matched system and the performance obtained via SRFT are depicted in Fig. 3.

In Fig. 4, transducer power gain curves are zoomed. The curves obtained via the proposed method and SRFT are very close to each other, nearly the same. The algorithm is implemented via Matlab. The elapsed time for this example is 84.1322 s. It is 84.6948 s via SRFT. Also total squared error ($\delta = \sum |\epsilon(\omega)|^2$) which is calculated at Step 8 is 1.7189. It is 1.7194 for SRFT. There is a very small

Table 1
Given normalized load and source impedance data.

ω	R_L	X_L	R_G	X_G
0.1	0.86	-0.34	1.00	-2.2500
0.2	0.60	-0.49	1.00	-1.2500
0.3	0.41	-0.49	1.00	-0.8333
0.4	0.28	-0.45	1.00	-0.6250
0.5	0.20	-0.40	1.00	-0.5000
0.6	0.14	-0.35	1.00	-0.4167
0.7	0.11	-0.32	1.00	-0.3571
0.8	0.09	-0.28	1.00	-0.3125
0.9	0.07	-0.26	1.00	-0.2778
1.0	0.06	-0.23	1.00	-0.2500

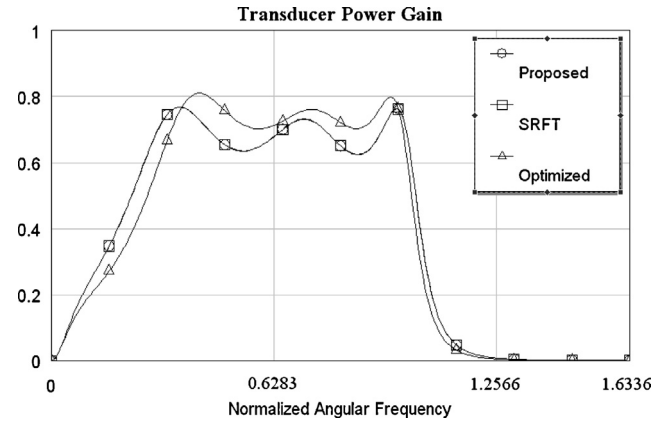


Fig. 3. Performance of the matched system designed with distributed elements.

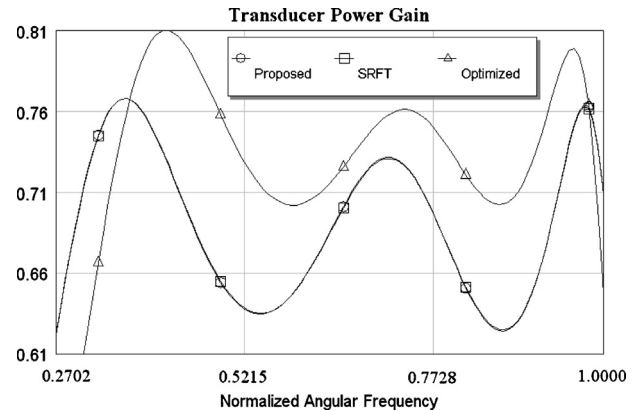


Fig. 4. Closer examination of the gain performances.

difference which can be negligible. Consequently, it can be said that the proposed method and SRFT nearly have the same performance.

7. Conclusion

Design of practical broadband matching networks is one of the important problems of microwave engineers. In this regard, commercially available computer-aided design tools are utilized. Once the matching network topology and proper initial element values are obtained, these tools are excellent to optimize system performance by working on the initialized element values. So initial element values become very vital, since the system performance is highly nonlinear in terms of the element values of the matching network. Therefore, in this paper, an initialization method is proposed to construct lossless broadband matching networks with distributed elements.

In the proposed method, the back-end or front-end driving point impedance of the matching network is determined in terms of the scattering parameters of the matching network, source and load

reflection coefficients. Then this impedance and one of the termination impedances (Z_G or Z_L) are used to calculate the transducer power gain of the system. Scattering parameters of the matching network are optimized to get the best gain performance.

Finally, it is synthesized as a lossless two-port resulting the desired matching network topology with initial element values. Eventually, the actual performance of the matched system is improved by means of a commercially available CAD tool.

Basic advantages of the proposed method can be summarized as follows: The transmission zeros of the matching network can be controlled directly by the choice of the polynomial $f(\lambda)$. Since the transducer power gain is quadratically dependent on the optimization parameters, the problem reduces to that of a quadratic optimization. So the numerical convergence of the method is excellent. Also the proposed method is applicable to both single and double matching problems.

An example is presented to construct broadband matching network with distributed elements. It is shown that the proposed method generates very good initials to further improve the matched system performance by working on the element values. Therefore, it is expected that the proposed algorithm is used as a front-end for the commercially available CAD tools to design practical broadband matching networks for microwave communication systems.

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