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# Evaluating machine tool alternatives through modified TOPSIS and alpha-cut based fuzzy ANP

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ABSTRACT

The problem of machine tool selection among available alternatives has been critical issue for most companies in fast-growing markets for a long time. In the presence of many alternatives and selection criteria, the problem becomes a multiple-criteria decision making (MCDM) machine tool selection problem. Therefore, most companies have utilized various methods to successfully carry out this difficult and time-consuming process. In this work, both of the most used MCDM methods, the modified TOPSIS and the Analytical Network Process (ANP) are introduced to present a performance analysis on machine tool selection problem. The ANP method is used to determine the relative weights of a set of the evaluation criteria, as the modified TOPSIS method is utilized to rank competing machine tool alternatives in terms of their overall performance. Furthermore, in this paper, we use a fuzzy extension of ANP, a more general form of AHP, which uses uncertain human preferences as input information in the decision-making process, because AHP cannot accommodate the variety of interactions, dependencies and feedback between higher and lower level elements. Instead of using the classical eigenvector prioritization method in AHP, only employed in the prioritization stage of ANP, a fuzzy logic method providing more accuracy on judgments is applied. The resulting fuzzy ANP enhances the potential of the conventional ANP for dealing with imprecise and uncertain human comparison judgments. The proposed approach is also applied for a real-life case in a company.


## 1. Introduction and literature survey

A proper machine tool selection has been very important issue for manufacturing companies due to the fact that improperly selected machine tool can negatively affect the overall performance of a manufacturing system. In addition, the outputs of manufacturing system (i.e. the rate, quality and cost) mostly depend on what kinds of properly selected and implemented machines tools are used. On the other hand, the selection of a new machine tool is a time-consuming and difficult process requiring advanced knowledge and experience and experience deeply. So, the process can be hard task for engineers and managers, and also for machine tool manufacturer or vendor, to carry out. For a proper and effective evaluation, the decision-maker may need a large amount of data to be analyzed and many factors to be considered. The decision-maker should be an expert or at least be very familiar with the specifications of machine tool to select the most suitable among the others. However, a survey conducted by (Gerrard, 1988a) reveals that the role of engineering staff in authorization for final selection is 6%, the rest belongs to middle and upper management (94%). The author also indicated the need for a simplified and practical approach for the machine selection process. Evaluating machine tool alternatives is a multiple-criteria decision making (MCDM) problem in the presence of many quantitative and qualitative attributes. So, we selected analytic network process (ANP) method, because it has been widely used for selecting the best alternative among others.

In AHP, a hierarchy considers the distribution of a goal amongst the elements being compared, and judges which element has a greater influence on that goal. In reality, a holistic approach like ANP is needed if all criteria and alternatives involved are connected in a network system that accepts various dependencies. Several decision problems cannot be hierarchically structured because they involve the interactions and dependencies in higher or lower level elements. Not only does the importance of the criteria determine the importance of the alternatives as in AHP, but the importance of alternatives themselves also influences the importance of the criteria. In other words, ANP incorporates feedback and interdependent relationships among decision attributes and alternatives (Saaty, 1996). This provides a more accurate approach for modeling complex

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decision environment (Ayag and Özdemir, 2007; Chung et al., 2005). Another popular MCDM method used in this study is the TOPSIS (the Technique for Order Preference by Similarity to Ideal Solution) first developed by Hwang and Yoon (1981). TOPSIS bases on the concept that the best alternative should have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. Although its concept is rational and understandable, and the computation steps involved is uncomplicated, the inherent difficulty of assigning reliable subjective preferences to the criteria is worth of noting.

In addition, a decision maker's requirements on evaluating machine tool alternatives always contain ambiguity and multiplicity of meaning. Furthermore, it is also recognized that human assesment on qualitative attributes is always subjective and thus imprecise. Therefore, conventional ANP seems inadequate to capture decision maker's requirements explicitly. In order to model this kind of uncertainity in human preference, fuzzy sets could be incorporated with the pairwise comparison as an extension of ANP. The fuzzy ANP approach allows a more accurate description of the decision making process.

Fuzzy set theory is a mathematical theory pioneered by Zadeh (1994), designed to model the vagueness or imprecision of human cognitive processes. This theory is basically a theory of classes with non-sharp boundaries. What is important to recognize is that any crisp theory can be made fuzzy by generalizing the concept of a set within that theory to the concept of a fuzzy set (Zadeh, 1994). Fuzzy set theory and fuzzy logic have been applied in a great variety of applications, as reviewed by several authors (Klir and Yuan, 1995) and (Zimmermann, 1996). Within the broad scope of the applications of fuzzy set theory, engineering design emerges as an important activity in today's organizations that has lacked tools that manage the great amount of imprecise information that is usually encountered.

In literature, we have been witnessed more studies which brings AHP and TOPSIS together to solve multiple-criteria decision making problems. Some of them recently published can be summarized as follows: aimed at improving the quality and effectiveness of decision-making in a new product introduction. They proposed a systematic decision process for selecting more rational new product ideas, and used fuzzy heuristic multi-attribute utility method for the identification of non-dominated new product candidates and a hierarchical fuzzy TOPSIS method for the selection of the best new product idea. Karpak and Topcu (2010) dealt with prioritizing measures of success and the antecedents for Turkish small to medium sized manufacturing enterprises. Işıklar and Büyüközkan (2007) used AHP and TOPSIS to evaluate mobile phone options in respect to the users' preferences order. Önüt and Soner (2007) also used AHP and fuzzy TOPSIS to solve the solid waste transshipment site selection problem. Lin et al. (2008) presented a framework that integrates the AHP and TOPSIS methods to assist designers in identifying customer requirements and design characteristics, and help achieve an effective evaluation of the final design solution. Tsaur et al. (2002) applied AHP in obtaining criteria weight and TOPSIS in ranking to evaluate of airline service quality.

In literature, to the best of our knowledge, we have not come across any work that both techniques, ANP and TOPSIS are used for machine tool selection. The reason could be due to the fact that ANP developed by Saaty is a newly introduced method to the world of the multiple criteria. But, there are several works that both techniques are used in various fields. Both of them are summarized as follows: proposed a hybrid model for supporting the vendor selection process in new task situations. They used both modified TOPSIS method to adopt in order to rank competing products in terms of their overall performances, and the ANP to yield the relative weights of the multiple evaluation criteria, which are obtained from the nominal group technique (NGT) with interdependence. In another work, (Petri Hallikainen et al., 2009) addressed the alignment between business processes and information technology in enterprise resource planning (ERP) implementation. Since the problem considered in the study involves organizational and technical issues that are connected to each other in networked manner, the analytic network process (ANP) methodology was selected for application.

In this paper, we utilize the fuzzy ANP and TOPSIS methods. The fuzzy ANP method is used to determine the relative weights of a set of the evaluation criteria, as the modified TOPSIS method is utilized to rank competing machine tool alternatives in terms of their overall performance. In this work, instead of using all the steps of the fuzzy ANP method to determine the final machine tool alternative, we integrated the fuzzy ANP with the modified TOPSIS to eliminate the time-consuming fuzzy calculations of the fuzzy ANP method. Only some steps of the fuzzy ANP method is used to weight the evaluation criteria required for the modified TOPSIS method to reflect the interdependences among them.

The proposed approach is also applied for a real-life case in a company which designs and manufacturers all kinds of cutting tools for national and international markets.

## 2. Proposed approach

In this paper, together with the modified TOPSIS, we propose a fuzzy ANP-based methodology using ANP of Saaty and fuzzy logic of (Zadeh, 1994) because; in the conventional ANP method, the evaluation of selection attributes is done using a nine-point scaling system, where a score of 1 represents equal importance between the two elements and a score of 9 indicates the extreme importance of one element, showing that each attribute is related with another. This scaling process is then converted to priority values to compare alternatives. In other words, the conventional ANP method does not take into account the vagueness and uncertainty on judgments of the decision-maker(s). Therefore, fuzzy logic is integrated with the Saaty's ANP to overcome the inability of ANP to handle the imprecision and subjectiveness in the pair wise comparison process. Fuzzy ANP-based methodology to machine tool selection problem is presented step-by-step below, as illustrated in Fig. 1.

In the first step, a cross-functional team is set up for machine tool selection problem. This team is also responsible of generating a number of the possible machine too alternatives according to the needs of the company. If the number of alternatives is more than expected, it may not be effective for the proposed approach. In this case, the number of the alternatives is narrowed down using *Pareto optimality*.

After this, a set of evaluation criteria mainly expressing machine tool characteristics is determined. Next, as seen in Fig. 1, we construct two modules, one of which is the fuzzy ANP module that allows determining the relative weights of the evaluation criteria; another is modified TOPSIS method to rank the competing alternatives. Finally, the ultimate machine tool alternative is presented to the company's management for approval.

### 2.1. Making of fuzzy ANP-based calculations

In this study, in order to capture the vagueness, triangular fuzzy numbers, $\tilde{1}$–$\tilde{9}$, are used to represent subjective pair wise comparisons of selection process. Triangular fuzzy numbers (TFNs) show the participants' judgments or preferences among the options such as equally important, weakly more important, strongly more important, very strongly more important, and

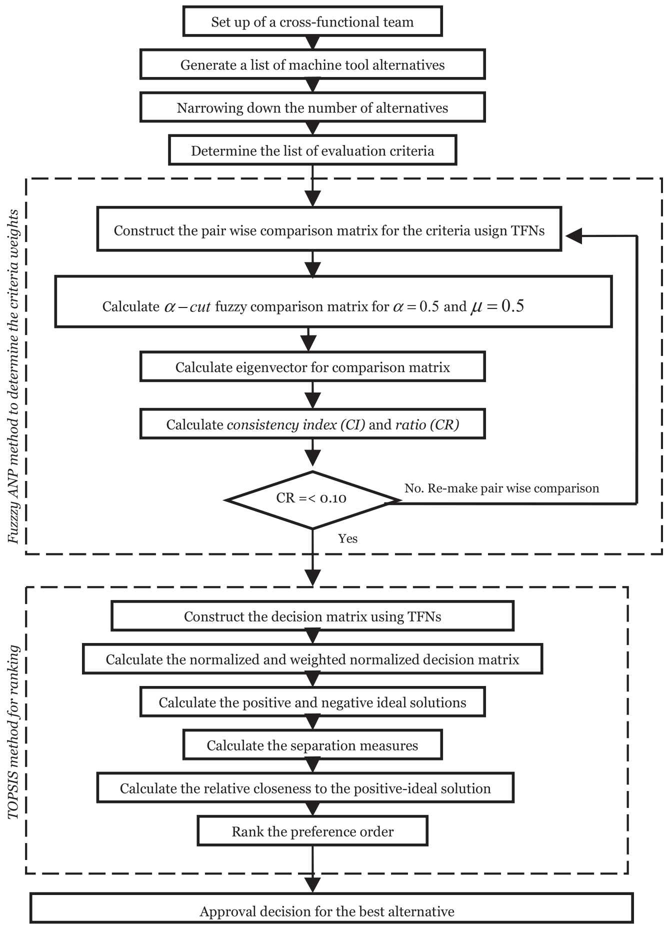


**Fig. 1.** Overall procedure for machine tool selection problem.

extremely more important preferred. $F=\{(x,\mu_{\tilde{M}}(x)),x\in R\}$ indicates a fuzzy set, where x takes it values on the real line, $R$: $-\infty < x < +\infty$ and $\mu_{\tilde{M}}(x)$ is a continuous mapping from $R$ to the closed interval [0,1]. The element $x$ in the set expresses the real values in the closed interval $[l,u]$, including mean ($m$) of each triangular fuzzy number. A triangular fuzzy number denoted as $\tilde{M}=[l,u]$ has the following triangular type membership function;

$$\mu_{\tilde{M}}(x)=\begin{cases} 0 & x<l \\ x-l/m-l & l\le x\le m \\ u-x/u-m & m\le x\le u \\ 0 & x>u \end{cases}$$

If $x$ value is less than lower level of a fuzzy number ($l$), the function gets the value of 0 (zero), bigger than/equal lower level ($l$) and less than/equal to mean level ($m$), the function gets the value of $x-l/m-l$, and bigger than/equal mean level ($m$) and less than/equal to upper level ($u$), the function gets the value of $u-x/u-m$. Alternatively, the interval of confidence level $\alpha$, can be characterized using the triangular fuzzy number as follows

$$\forall\alpha\in[0,1]\quad \tilde{M}_\alpha=[m_l^\alpha,m_u^\alpha]=[(m-l)\alpha+l,-(u-m)\alpha+u]$$

where $\alpha$ value determined by the decision-maker or the expert, indicates his/her confidence over his/her judgments to calculate the confidence interval.

Some main operations for positive fuzzy numbers are described by the interval of confidence, by Kaufmann and Gupta (1988) as given below;

$$\forall m_l,m_u,n_l,n_u\in R^+,\tilde{M}_\alpha=[m_l^\alpha,m_u^\alpha],\tilde{N}_\alpha=[n_l^\alpha,n_u^\alpha],\alpha\in[0,1]$$

$$\tilde{M}_\alpha\oplus\tilde{N}_\alpha=[m_l^\alpha+n_l^\alpha,m_u^\alpha+n_u^\alpha],\tilde{M}_\alpha-\tilde{N}_\alpha=[m_l^\alpha-n_l^\alpha,m_u^\alpha-n_u^\alpha]$$

$$\tilde{M}_\alpha\otimes\tilde{N}_\alpha=[m_l^\alpha n_l^\alpha,m_u^\alpha n_u^\alpha],\tilde{M}_\alpha/\tilde{N}_\alpha=[m_l^\alpha/n_l^\alpha,m_u^\alpha/n_u^\alpha]$$

where, $\tilde{M}_\alpha=[m_l^\alpha,m_u^\alpha]$ and $\tilde{N}_\alpha=[n_l^\alpha,n_u^\alpha]$ are the triangular fuzzy numbers with their lower and upper level values, $m_l,m_u,n_l,n_u$ for confidence level $\alpha$.

The triangular fuzzy numbers, $\tilde{1}-\tilde{9}$, are utilized to improve the conventional nine-point scaling scheme. In order to take the imprecision of human qualitative assessments into consideration, the five triangular fuzzy numbers ($\tilde{1},\tilde{3},\tilde{5},\tilde{7},\tilde{9}$) are defined with the corresponding membership function.

Using triangular fuzzy numbers ($\tilde{1},\tilde{3},\tilde{5},\tilde{7},\tilde{9}$), the decision-maker(s) are asked to respond to a series of pair wise comparisons of the criteria. These are conducted with respect to their relevance importance towards the control criterion. In the case of interdependencies, components in the same level are viewed as controlling components for each other. Levels may also be interdependent. Through pair wise comparisons using triangular fuzzy numbers ($\tilde{1},\tilde{3},\tilde{5},\tilde{7},\tilde{9}$), the fuzzy judgment matrix $\tilde{A}$ ($\tilde{a}_{ij}^\alpha$) is constructed as given below;

$$\tilde{A}=\begin{bmatrix} 1 & \tilde{a}_{12}^\alpha & .. & .. & \tilde{a}_{1n}^\alpha \\ \tilde{a}_{12}^\alpha & 1 & .. & .. & \tilde{a}_{2n}^\alpha \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \tilde{a}_{n1}^\alpha & \tilde{a}_{n2}^\alpha & .. & .. & 1 \end{bmatrix}$$

where, $\tilde{a}_{ij}^\alpha=1$, if $i$ is equal to $j$, and $\tilde{a}_{ij}^\alpha=\tilde{1},\tilde{3},\tilde{5},\tilde{7},\tilde{9}$ or $\tilde{1}^{-1},\tilde{3}^{-1},\tilde{5}^{-1},\tilde{7}^{-1},\tilde{9}^{-1}$, $\tilde{3}^{-1}$, if $i$ is not equal to $j$

For solving fuzzy eigenvalue: A fuzzy eigenvalue,$\lambda^{\sim}$, is a fuzzy number solution to

$$\tilde{A}\tilde{x}=\tilde{\lambda}\tilde{x} \tag{1}$$

where $\tilde{\lambda}_{\max}$ is the largest eigenvalue of $\tilde{A}$. Saaty (1981) provides several algorithms for approximating $\tilde{x}$.

where $\tilde{A}$ is $n\times n$ fuzzy matrix containing fuzzy numbers $\tilde{a}_{ij}$ and $\tilde{x}$is a non-zero $n\times 1$, fuzzy vector containing fuzzy number $\tilde{x}_i$. To perform fuzzy multiplications and additions using the interval arithmetic and $\alpha-cut$, the equation $\tilde{A}\tilde{x}=\tilde{\lambda}\tilde{x}$ is equivalent to

$$[a_{i1l}^\alpha x_{1l}^\alpha,a_{i1u}^\alpha x_{1u}^\alpha]\oplus\ldots\oplus[a_{inl}^\alpha x_{nl}^\alpha,a_{inu}^\alpha x_{nu}^\alpha]=[\lambda x_{il}^\alpha,\lambda x_{iu}^\alpha]$$

where,

$$\tilde{A}=[\tilde{a}_{ij}],\tilde{x}_i,=(\tilde{x}_1,\ldots,\tilde{x}_n),\ \tilde{a}_{ij}^\alpha=[a_{ijl}^\alpha,a_{iju}^\alpha],\tilde{x}_i^\alpha=[x_{il}^\alpha,x_{iu}^\alpha],\tilde{\lambda}^\alpha=[\lambda_l^\alpha,\lambda_u^\alpha] \tag{2}$$

for $0<\alpha\le 1$ and all $i$, $j$, where $i$=1, 2 … $n$, and $j$=1, 2 … $n$

$a_{ijl}^\alpha$; the lower value ($l$) of a triangular fuzzy number at $i$. line and $j$. column of the fuzzy judgment matrix, $\tilde{A}$ for a given $\alpha$ value
$a_{iju}^\alpha$; the upper value ($u$) of a triangular fuzzy number at $i$. line and $j$. column of the fuzzy judgment matrix, $\tilde{A}$ for a given $\alpha$ value
$x_{il}^\alpha$; the lower value ($l$) at $i$. line of the fuzzy vector for a given $\alpha$ value
$x_{iu}^\alpha$; the lower value ($u$) at $i$. line of the fuzzy vector for a given $\alpha$ value

$\alpha-cut$ is known to incorporate the experts or decision-maker(s) confidence over his/her preference or the judgments. Degree of satisfaction for the judgment matrix $\tilde{A}$ is estimated by the index of optimism $\mu$. The larger value of index $\mu$ indicates the higher degree of optimism. The index of optimism is a linear convex combination (Lee, 1999) defined as;

$$\tilde{a}_{ij}^\alpha=\mu a_{iju}^\alpha+(1-\mu)a_{ijl}^\alpha,\forall\mu\in[0,1] \tag{3}$$

While $\alpha$ is fixed, the following matrix can be obtained after setting the index of optimism, $\mu$, in order to estimate the degree of satisfaction. Both of them are defined in the range [0,1] by decision-makers.

$$\tilde{A}=\begin{bmatrix} 1 & \tilde{a}_{12}^{\alpha} & .. & .. & \tilde{a}_{1n}^{\alpha} \\ \tilde{a}_{12}^{\alpha} & 1 & .. & .. & \tilde{a}_{2n}^{\alpha} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \tilde{a}_{n1}^{\alpha} & \tilde{a}_{n2}^{\alpha} & .. & .. & 1 \end{bmatrix}$$

The eigenvector is calculated by fixing the $\mu$ value and identifying the maximal eigenvalue.

After defuzzification of each pair wise matrix, the consistency ratio (CR) for each matrix is calculated. The deviations from consistency are expressed by the following equation consistency index, and the measure of inconsistency is called the consistency index (CI);

$$CI=\lambda_{\max}-n/n-1. \tag{4}$$

The consistency ratio (CR) is used to estimate directly the consistency of pair wise comparisons. The CR is computed by dividing the CI by a value obtained from a table of Random Consistency Index (RI);

$$CR=\frac{CI}{RI}. \tag{5}$$

If the CR is less than 0.10, the comparisons are acceptable, otherwise not. RI is the average index for randomly generated weights (Saaty, 1981).

In order to reflect the interdependencies in the network, a set of pair wise comparison matrices are constructed for each criterion and their consistency ratios are calculated. These matrices are used to identify the relative impacts of the criteria interdependent relationships. The normalized principal eigenvectors for the matrices are calculated and shown as column component in the manipulated matrix, $S$, where zeroes are assigned in the matrix if there is no relationship between the related criteria.

Finally, we can obtain the priorities of the criteria by synthesizing the results of previous calculations as follows

$$w_{criteria}=S{*}w^T, \tag{6}$$

where $S$ is the manipulated matrix and $w$ is the weight line vector of the criteria.

Moreover, the calculations steps used to construct Tables 2 and 3 should be thought as the ANP part of the study in order to determine the weights of the evaluation criteria.

### 2.2. Ranking machine tool alternatives through the modified TOPSIS

In the previous section, we have defined and shown how to calculate the importance weights of the evaluation criteria using the fuzzy ANP method. And now, it is time to apply the modified TOPSIS approach to rank the alternatives. On the other hand, all the steps of the fuzzy ANP would have been applied to rank the alternatives, if we have had a small number of criteria and alternatives. But, in this study, to keep the number of pairwise comparisons made by a decision-maker below a reasonable threshold, we only used the fuzzy ANP to determine the relative weights of the evaluation criteria, and then the modified TOPSIS to achieve the final ranking result. For example, if there are n criteria and m alternatives, then to run a full fuzzy ANP solution there are $3\times n\times m\times(m-1)/2$ pairwise comparisons remaining to be performed.

The basic TOPSIS technique consists of the following steps (Triantaphyllou, 2000):

*Step* 1. Construct the normalized decision matrix: The method evaluates the following decision matrix, $D$, which refers to $m$ alternatives that are evaluated in terms of $n$ criteria.

$$D=\begin{bmatrix} x_{11} & x_{12} & x_{13} & . & . & x_{1n} \\ x_{21} & x_{22} & x_{23} & . & . & x_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_{m1} & x_{m2} & x_{m3} & . & . & x_{mn} \end{bmatrix}$$

Then, it converts the various criteria dimensions into non-dimensional criteria. An element $r_{ij}$ of the normalized decision matrix, $R$ is thus calculated as follows

$$r_{ij}=\frac{x_{ij}}{\sqrt{\sum_{j=1}^{n}x_{ij}^2}}, \tag{7}$$

where $i$ and $j$ are indices to represent each element ($r_{ij}$) in the matrix $R$, and $i, j=1, 2, \ldots, n$ ($n$ is the size of the matrix).

*Step* 2. Construct the weighted normalized decision matrix: A set of weights $W=(w_1,w_2,w_3, \ldots, w_n)$, where $\sum w_i=1$ is defined by the decision-maker is next used with the decision matrix to generate the weighted normalized matrix, $V(v_{ij}=w_j r_{ij})$ as follows

$$V=\begin{bmatrix} w_1x_{11} & w_2x_{12} & w_3x_{13} & . & . & w_nx_{1n} \\ w_1x_{21} & w_2x_{22} & w_3x_{23} & . & . & w_nx_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ w_1x_{m1} & w_2x_{m2} & w_3x_{m3} & . & . & w_nx_{mn} \end{bmatrix}$$

*Step* 3. Determine the positive-ideal and the negative-ideal solutions: The "positive-ideal" donated as $A^*$, and the "negative-ideal" donated as $A^-$ alternatives (or solutions) are defined as follows

$$A^*=\{(\max_i v_{ij}\,|\,j\in J),(\min_i v_{ij}\,|\,j\in J'),i=1,2,3,\ldots,m\} \tag{8}$$

$$A^*=\{v_{1*},v_{2*},\ldots,v_{n*}\}$$
$$A^-=\{(\min_i v_{ij}\,|\,j\in J),(\max_i v_{ij}\,|\,j\in J'),i=1,2,3,\ldots,m\}$$
$$A^-=\{v_{1-},v_{2-},\ldots,v_{n-}\} \tag{9}$$

where, $J=\{j=1,2,3,\ldots,n\}$ and $J'=\{j=1,2,3,\ldots,n\}$

From the previous definitions, it follows that alternative $A^*$ indicates the most preferable alternative or the positive-ideal solution. Similarly, alternative $A^-$ indicates the least preferable alternative or the negative-ideal solution.

*Step* 4. Calculate the separation measure: The $n$-dimensional Euclidean distance method is next applied to measure the separation distances of each alternative from the positive-ideal solution and the negative-ideal solution. Thus, for the distances from the positive-ideal solution we have:

$$S_{i^*}=\sqrt{\sum_{j=1}^{n}(v_{ij}-v_{j^*})^2}\ \text{ for } i=1,2,3,\ldots m \tag{10}$$

where $S_{i^*}$ is the distance of each alternative from the positive-ideal solution.

$$S_{i-}=\sqrt{\sum_{j=1}^{n}(v_{ij}-v_{j-})^2}\ \text{ for } i=1,2,3,\ldots m, \tag{11}$$

where $S_{i-}$ is the distance of each alternative from the negative-ideal solution.

*Step* 5. Calculate the relative closeness to the ideal solution: The relative closeness of an alternative $A_i$ with respect to the ideal solution $A^*$ is defined as follows: $C_{i^*}=S_{i-}/S_{i^*}+S_{i-}$, where

$$1 \geq C_{i*} \geq 0, \text{ and } i=1,2,3,\ldots,m. \quad (12)$$

Apparently, $C_{i^*}=1$, if $A_i=A^*$, and $C_{i-}=0$, if $A_i=A^-$

*Step* 6. Rank the preference order: The best alternative can be now decided according to the preference rank order of $C_{i^*}$. Therefore, the best alternative is the one that has the shortest distance to the ideal solution.

## 3. Case study

In this section, a case study was realized in a company which designs and manufacturers various kinds of cutting tools, to prove its applicability and validity. Based on the customer demand rising from national and international markets, the company made a decision on increasing the production volume of cutting tools holders by investing a special kind of CNC machine tool. Then, they first set up a cross-functional team to do a list of possible alternatives in market and determine the evaluation criteria for the analysis as given in Table 1. After the initial screening process by the cross-functional team, the number of the alternatives was reduced to a reasonable level, 3 as A1, A2, and A3. Here the problem was now to determine the priorities of the machine tool alternatives and find out the best satisfying one.

**Table 1**
List of selection criteria with their attributes for machine tool selection.

| # | Selection criteria | Evaluation attributes |
|---|---|---|
| 1 | Productivity (PR) | Spindle speed, power, cutting feed, traverse speed |
| 2 | Flexibility (FL) | Number of tools, rotary table |
| 3 | Space (SP) | Machine dimensions |
| 4 | Adaptability (AD) | CNC type, number of taper |
| 5 | Precision (PC) | Repeatability, thermal deformation |
| 6 | Reliability (RE) | Bearing failure rate, reliability of drive system |
| 7 | Safety and environment (SE) | Mist collector, safety door, fire extinguisher |
| 8 | Maintenance and Service (MS) | Repair service, regular maintenance |

**Table 2**
Criteria pairwise comparison matrix using TFNs ($\tilde{1},\tilde{3},\tilde{5},\tilde{7},\tilde{9}$).

| Criteria | PR | FL | SP | AD | PC | RE | SE | MS |
|---|---|---|---|---|---|---|---|---|
| PR | 1 | $\tilde{3}$ | $\tilde{5}$ | $\tilde{1}$ | $\tilde{7}$ | $\tilde{9}$ | $\tilde{7}$ | $\tilde{9}$ |
| FL | $\tilde{3}^{-1}$ | 1 | $\tilde{3}$ | $\tilde{3}$ | $\tilde{5}$ | $\tilde{5}$ | $\tilde{3}$ | $\tilde{7}$ |
| SP | $\tilde{5}^{-1}$ | $\tilde{3}^{-1}$ | 1 | $\tilde{1}$ | $\tilde{3}$ | $\tilde{1}$ | $\tilde{3}$ | $\tilde{7}$ |
| AD | $\tilde{1}^{-1}$ | $\tilde{3}^{-1}$ | $\tilde{1}^{-1}$ | 1 | $\tilde{1}$ | $\tilde{3}$ | $\tilde{5}$ | $\tilde{9}$ |
| PC | $\tilde{7}^{-1}$ | $\tilde{5}^{-1}$ | $\tilde{3}^{-1}$ | $\tilde{1}^{-1}$ | 1 | $\tilde{3}$ | $\tilde{1}$ | $\tilde{3}$ |
| RE | $\tilde{9}^{-1}$ | $\tilde{5}^{-1}$ | $\tilde{1}^{-1}$ | $\tilde{3}^{-1}$ | $\tilde{3}^{-1}$ | 1 | $\tilde{1}$ | $\tilde{3}$ |
| SE | $\tilde{7}^{-1}$ | $\tilde{3}^{-1}$ | $\tilde{3}^{-1}$ | $\tilde{5}^{-1}$ | $\tilde{1}^{-1}$ | $\tilde{1}^{-1}$ | 1 | $\tilde{3}$ |
| MS | $\tilde{9}^{-1}$ | $\tilde{7}^{-1}$ | $\tilde{7}^{-1}$ | $\tilde{9}^{-1}$ | $\tilde{3}^{-1}$ | $\tilde{3}^{-1}$ | $\tilde{3}^{-1}$ | 1 |

We carried out the fuzzy ANP study using the five triangular fuzzy numbers ($\tilde{1},\tilde{3},\tilde{5},\tilde{7},\tilde{9}$) to express the preference in the pair wise comparisons. Then, we obtained the fuzzy comparison matrix for the relative importance of the criteria as shown in Table 2. Then, the lower limit and upper limit of the fuzzy numbers with respect to the α were defined as follows by applying Eq. (2), as shown in Table 3;

$$\tilde{1}_\alpha=[1,3-2\alpha],\quad \tilde{3}_\alpha=[1+2\alpha,5-2\alpha].\quad \tilde{3}_\alpha^{-1}=\left[\frac{1}{5-2\alpha},\frac{1}{1+2\alpha}\right],$$

$$\tilde{5}_\alpha=[3+2\alpha,7-2\alpha].\quad \tilde{5}_\alpha^{-1}=\left[\frac{1}{7-2\alpha},\frac{1}{3+2\alpha}\right],$$

$$\tilde{7}_\alpha=[5+2\alpha,9-2\alpha].\quad \tilde{7}_\alpha^{-1}=\left[\frac{1}{9-2\alpha},\frac{1}{5+2\alpha}\right],$$

$$\tilde{9}_\alpha=[7+2\alpha,11-2\alpha].\quad \tilde{9}_\alpha^{-1}=\left[\frac{1}{11-2\alpha},\frac{1}{7+2\alpha}\right]$$

Later, we substituted the values, $\alpha=0.5$ and $\mu=0.5$ above expression into fuzzy comparison matrix, and obtained the entire $\alpha-cuts$ fuzzy comparison matrix (Eq. (3) was used to calculate eigenvector for pair wise comparison matrix). And finally we also calculated the *CI* and the *CR* values using Eqs. (4) and (5) as follows (Table 4). First we calculated eigenvalue of the matrix *A* by solving the characteristic equation of *A*, $\det(A-\lambda I)=0$ and found out all λ values for A ($\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6,\lambda_7,\lambda_8$). The largest eigenvalue of pair wise matrix, $\lambda_{\max}$ was calculated to be 8.751. The dimension of the matrix, *n* is 8 and the random index, $RI(n)$ is 1.41 (RI-function of the number of attributes (Saaty, 1981).

$$CI=\frac{\lambda_{\max}-n}{n-1}=\frac{8.751-8}{7}=0.107,\quad CR=\frac{CI}{RI}=\frac{0.107}{1.41}=0.079<0.100$$

If the independence relations among the criteria are thought, the team determined the impact of the all criteria on each through pairwise comparison. Totally, seven pairwise comparison matrices using TFNs were developed. Their normalized eigenvectors were calculated and shown as seven columns in Table 5.

The relative importance of the criteria considering interdependence can be obtained by synthesizing the results of Tables 4 and 5, through Eq. (6) (Table 6).

Then, the team was asked to construct the decision criteria and all the members were asked to give a set of crisp values within from 1 to 10 to represent the performance of each alternative with respect to each criterion (*R*). After the decision matrix was determined, we normalized the matrix using Eq. (7) (Table 7) and calculated the weighted normalized matrix ($V=R^*w_{criteria}$) using the results from fuzzy ANP, as given in Table 8. Then, using Eqs. (8) and (9) we determined the positive and negative ideal solutions and indicated them using * and - symbols in Table 8.

Later, the separation measures ($S_{i^*},S_{i-}$) are calculated using Eqs. (10) and (11) and shown in Table 9.

**Table 3**
$\alpha-cuts$ fuzzy comparison matrix ($\alpha=0.5$).

| Criteria | PR | FL | SP | AD | PC | RE | SE | MS |
|---|---|---|---|---|---|---|---|---|
| PR | 1 | [2,4] | [4,6] | [1,2] | [6,8] | [8,10] | [6,8] | [8,10] |
| FL | [1/4, 1/2] | 1 | [2,4] | [2,4] | [4,6] | [4,6] | [2,4] | [6,8] |
| SP | [1/6, 1/4] | [1/4, 1/2] | 1 | [1,2] | [2,4] | [1,2] | [2,4] | [6,8] |
| AD | [1/2, 1] | [1/4, 1/2] | [1/2, 1] | 1 | [1,2] | [2,4] | [4,6] | [8,10] |
| PC | [1/8, 1/6] | [1/6, 1/4] | [1/4, 1/2] | [1/2, 1] | 1 | [2,4] | [1,2] | [2,4] |
| RE | [1/10, 1/8] | [1/6, 1/4] | [1/2, 1] | [1/4, 1/2] | [1/4, 1/2] | 1 | [1,2] | [2,4] |
| SE | [1/8, 1/6] | [1/4, 1/2] | [1/4, 1/2] | [1/6, 1/4] | [1/2, 1] | [1/2, 1] | 1 | [2,4] |
| MS | [1/10, 1/8] | [1/8, 1/6] | [1/8, 1/6] | [1/10, 1/8] | [1/4, 1/2] | [1/4, 1/2] | [1/4, 1/2] | 1 |

**Table 4**
Pair wise comparison matrix for the relative importance of the criteria ($\alpha$=0.5 and $\mu$=0.5).

| Criteria | PR | FL | SP | AD | PC | RE | SE | MS | e-Vector (w) |
|---|---|---|---|---|---|---|---|---|---|
| PR | 1.000 | 3.000 | 5.000 | 1.500 | 7.000 | 9.000 | 7.000 | 9.000 | 0.346 |
| FL | 0.375 | 1.000 | 3.000 | 3.000 | 5.000 | 5.000 | 3.000 | 7.000 | 0.213 |
| SP | 0.208 | 0.375 | 1.000 | 1.500 | 3.000 | 1.500 | 3.000 | 7.000 | 0.116 |
| AD | 0.750 | 0.375 | 0.750 | 1.000 | 1.500 | 3.000 | 5.000 | 9.000 | 0.145 |
| PC | 0.146 | 0.208 | 0.375 | 0.750 | 1.000 | 3.000 | 1.500 | 3.000 | 0.066 |
| RE | 0.113 | 0.208 | 0.750 | 0.375 | 0.375 | 1.000 | 1.500 | 3.000 | 0.048 |
| SE | 0.146 | 0.375 | 0.375 | 0.208 | 0.750 | 0.750 | 1.000 | 3.000 | 0.045 |
| MS | 0.113 | 0.146 | 0.146 | 0.113 | 0.375 | 0.375 | 0.375 | 1.000 | 0.021 |
| | | | | | | | | $\lambda_{max}$ | *8.751* |
| | | | | | | | | *CI* | *0.107* |
| | | | | | | | | *RI* | *1.41* |
| | | | | | | | | *CR* | *0.079 < 0.100 ok.* |

**Table 5**
Degree of relative impact for evaluation criteria (S).

| | PR | FL | SP | AD | PC | RE | SE | MS |
|---|---|---|---|---|---|---|---|---|
| PR | 0.000 | 0.745 | 0.000 | 0.739 | 0.519 | 0.739 | 0.000 | 0.760 |
| FL | 0.507 | 0.000 | 0.000 | 0.261 | 0.296 | 0.000 | 0.000 | 0.000 |
| SP | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| AD | 0.216 | 0.182 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| PC | 0.130 | 0.074 | 0.000 | 0.000 | 0.000 | 0.153 | 0.000 | 0.145 |
| RE | 0.086 | 0.000 | 0.000 | 0.000 | 0.105 | 0.000 | 0.000 | 0.095 |
| SE | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 |
| MS | 0.061 | 0.000 | 0.000 | 0.000 | 0.079 | 0.108 | 0.000 | 0.000 |

**Table 6**
Weights of the evaluation criteria ($w_{criteria}=S^{*}w^{T}$).

| | PR | FL | SP | AD | PC | RE | SE | MS |
|---|---|---|---|---|---|---|---|---|
| $w_{criteria}$ | 0.352 | 0.233 | 0.116 | 0.114 | 0.071 | 0.039 | 0.045 | 0.032 |

**Table 7**
Normalized decision matrix based 1–10 scale (R).

| | PR | FL | SP | AD | PC | RE | SE | MS |
|---|---|---|---|---|---|---|---|---|
| A1 | 0.200 | 0.167 | 0.091 | 0.222 | 0.091 | 0.250 | 0.278 | 0.500 |
| A2 | 0.300 | 0.333 | 0.273 | 0.278 | 0.182 | 0.333 | 0.333 | 0.167 |
| A3 | 0.500 | 0.500 | 0.636 | 0.500 | 0.727 | 0.417 | 0.389 | 0.333 |

**Table 8**
Weighted normalized decision matrix ($V=R^{*}w_{criteria}$).

| | PR | FL | SP | AD | PC | RE | SE | MS |
|---|---|---|---|---|---|---|---|---|
| A1 | $0.070^{-}$ | $0.039^{-}$ | $0.011^{-}$ | $0.025^{-}$ | $0.006^{-}$ | $0.010^{-}$ | $0.013^{-}$ | $0.016^{*}$ |
| A2 | 0.106 | 0.078 | 0.032 | 0.032 | 0.013 | 0.013 | 0.015 | $0.005^{-}$ |
| A3 | $0.176^{*}$ | $0.117^{*}$ | $0.074^{*}$ | $0.057^{*}$ | $0.052^{*}$ | $0.016^{*}$ | $0.018^{*}$ | 0.011 |

**Table 9**
Calculating the separation measures ($S_{i^*}$,$S_{i-}$).

| | $S_{i^*}$ | $S_{i-}$ |
|---|---|---|
| A1 | 0.156 | 0.011 |
| A2 | 0.103 | 0.057 |
| A3 | 0.005 | 0.583 |

Finally, the relative closeness to the ideal solution of each alternative was calculated using Eq. (12) and alternatives are ranked based on their relative closeness to the ideal solution values ($C_{i^*}$).

**Table 10**
Final ranking of machine tool alternatives.

| Rank | Alternative | Closeness coefficient ($C_{i^*}$) |
|---|---|---|
| 1 | A3 | 0.991 |
| 2 | A2 | 0.359 |
| 3 | A1 | 0.064 |

The results are shown in Table 10. As seen in Table 10, the best machine tool alternative is found as the 3rd alternative, A3.

## 4. Conclusions

In this paper, we proposed an integrated approach using the modified TOPSIS and fuzzy ANP in order to carry out the following tasks: the fuzzy ANP method is used determine the relative weights of a set of criteria, as the modified TOPSIS method is utilized to rank competing conceptual design alternatives in terms of their overall performance in order to reach to the best satisfying one.

Furthermore, in this work, instead of using all the steps of the fuzzy ANP method to determine the final machine tool alternative, we integrated the fuzzy ANP with the modified TOPSIS to eliminate the time-consuming fuzzy calculations of the fuzzy ANP method. Only some steps of the fuzzy ANP method to reflect the interdependences among the criteria is used to determine the weights of the evaluation criteria, required for the modified TOPSIS method. We used both methods because machine tool selection problem has been critically important for most companies for a long time. Making wrong decision in selecting the proper machine tool might put a company into risk in terms of losing market share, cost and time.

The ANP method used here arrives at a synthetic score, which may be quite useful for decision-maker. The ANP methodology powered by fuzzy logic is a robust multiple criteria method to find out weights of the alternatives. It is also effective as both quantitative and qualitative characteristics can be considered simultaneously without sacrificing their relationships.

The fuzzy logic is also used to model the vagueness and uncertainty of the decision-makers. Use of fuzzy logic provides to get more reliably judgments of the decision makers than the crisp-based methods. The strengths of fuzzy models are their ability to approximate very complex, multi-dimensional processes and their insensitivity to noisy data. Their identification is computationally intensive but, once established, they provide quick responses. Unfortunately, fuzzy logic calculations require a considerably time to construct and process the pairwise comparisons, if especially the number of alternatives and criteria are more. If so, software like Super-Decisions Software should be used. In addition, prescreening process could be good way of narrowing down the size of the problem. However, the approach proposed here does not consider all the possible factors and criteria associated with machine tool selection problem. The elements presented in the framework are specific to a special manufacturing organization. The proposed methodology can easily be adapted to different situations by adjusting the different levels of the hierarchy and their related attributes.

On the other hand, this proposed approach can be used for any researcher and practitioner in different fields, who need a reliable tool to evaluate a set of alternatives in terms of evaluation criteria which can be determined based on what kind of the problem is deal with. In this study, the proposed methodology was also applied for a company to evaluate CNC machine tools in terms of a set of technical characteristics thought as evaluation criteria. In future research, a knowledge-based system (KBS) or expert system (ES) can be adapted to this approach to interpret the outputs automatically via a user interface. A KBS or ES creates a rule-based database to interpret the analysis results, and makes its comments using an inference engine, and presents them to the user whenever needed.

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