

Channel Estimation in Underwater Cooperative OFDM System with Amplify-and-Forward Relaying

Habib Şenol*, Erdal Panayırçı*, Mustafa Erdoğan* and Murat Uysal†

*Department of Electrical-Electronics Engineering, Kadir Has University, 34083, Istanbul, Turkey
Email: {hsenol, eepanay, mustafa.erdogan}@khas.edu.tr

†Department of Electrical-Electronics Engineering, Özyeğin University, 34794, Istanbul, Turkey
Email: murat.uysal@ozyegin.edu.tr

Abstract—This paper is concerned with a challenging problem of channel estimation for amplify-and-forward cooperative relay based orthogonal frequency division multiplexing (OFDM) systems in the presence of sparse underwater acoustic channels and of the correlative non-Gaussian noise. We exploit the sparse structure of the channel impulse response to improve the performance of the channel estimation algorithm, due to the reduced number of taps to be estimated. The resulting novel algorithm initially estimates the overall sparse channel taps from the source to the destination as well as their locations using the matching pursuit (MP) approach. The correlated non-Gaussian effective noise is modeled as a Gaussian mixture. Based on the Gaussian mixture model, an efficient and low complexity algorithm is developed based on the combinations of the MP and the space-alternating generalized expectation-maximization (SAGE) technique, to improve the estimates of the channel taps and their location as well as the noise distribution parameters in an iterative way. The proposed SAGE algorithm is designed in such a way that, by choosing the admissible hidden data properly on which the SAGE algorithm relies, a subset of parameters is updated for analytical tractability and the remaining parameters for faster convergence. Computer simulations show that underwater acoustic (UWA) channel is estimated very effectively and the proposed algorithm has excellent symbol error rate and channel estimation performance.

I. INTRODUCTION

Underwater wireless communication has received a growing attention and research has been active for over a decade on designing the methods for underwater applications. It has been of critical importance to provide high-speed wireless links with high link reliability in various underwater applications such as offshore oil field exploration/monitoring, oceanographic data collection, maritime archaeology, seismic observations, environmental monitoring, port and border security among many others.

Underwater wireless communication can be achieved through radio, optical, or sound (acoustic) waves. Among the three methods, acoustic transmission is the most practical and commonly employed method due to favorable propagation characteristics of sound waves in the underwater environments and research efforts therefore have focused on this area. However, an underwater acoustic channel presents a communication system designer with many challenges. The three distinguishing characteristics of this channel are frequency-dependent propagation loss, severe multipath with much longer delay spreads [1], and low speed of sound propagation. None of these characteristics are nearly as pronounced in land-based radio channels, the fact that makes underwater wireless communication extremely difficult, and necessitates dedicated system design. Relay-assisted cooperative diversity presents a viable solution for underwater acoustic communication to extend transmission range and mitigate the

degrading effects of fading. Cooperative diversity also named as user cooperation is a transmission method which extracts spatial diversity advantages through the use of relays [2]. The concept of cooperative diversity has been recently applied to underwater acoustic (UWA) communication and the number of current studies in this area is very limited [3], [4], [5], [6]. Mainly, Decode and Forward (DF) and Amplify and Forward (AF) relays have been adopted in practice for cooperative diversity systems. As remarked in [7] that the AF operation mode puts less processing burden on the relay and that AF relay actually outperforms DF relays under certain conditions.

The orthogonal frequency division multiplexing (OFDM)-based cooperative communication systems in underwater acoustic channels assuming various cooperation protocols are promising and seem to be a primary candidate for next generation UWA systems, due to their robustness to large multipath spreads [8], [3], [9]. The fundamental performance bounds of such systems are determined by the inherent characteristics of the underwater channel and by the reliable channel state information (CSI) available at the destination, to enable high transmission speeds and high link reliability. However, almost all the existing works assume that the perfect channel knowledge is available and there are only few results exists on channel estimation for the relay networks suggested under quite nonrealistic assumptions [7], [10]. Given sufficiently wide transmission bandwidth, the impulse response of the underwater acoustic channel is often sparse as the multipath arrivals becomes resolvable [1]. Furthermore, the effective noise entering the system between the source and the destination through the relay is correlated and non Gaussian. The combination of sparse structure and correlated non-Gaussian noise type creates a challenging channel estimation problem for relay based corporation diversity UWA systems. To the best of our knowledge, the problem of channel estimation for underwater AF relay channels has not been addressed satisfactorily in the literature and this motivated our present work.

In this paper we provide a new pilot assisted channel estimation technique for relay networks that employ the AF transmission scheme. Our main contribution in this work is two folds. First, we exploit the sparse structure of the channel impulse response to improve the performance of the channel estimation algorithm, due to the reduced number of taps to be estimated. The resulting algorithm initially estimates the overall sparse channel taps from the source to the destination as well as their locations using the matching pursuit (MP) approach [11]. The correlated non-Gaussian effective noise is modeled as a Gaussian mixture. Second, based on the Gaussian mixture model we develop an efficient and low complexity novel algorithm by combining the MP and the SAGE techniques, called the

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MP-SAGE algorithm which relies on the concept of the admissible hidden data, to improve the estimates of the channel taps and their location as well as the noise distribution parameters in an iterative way. We demonstrate that by suitably choosing the admissible hidden data on which the SAGE algorithm relies, a subset of parameters is updated for analytical tractability and the remaining parameters for faster convergence [12].

The remainder of the paper is organized as follows. Section II presents system model for an OFDM-based underwater cooperative wireless communication system and describes the main parameters of the UWA channel. Section III proposes the new channel estimation algorithm including a computational complexity analysis. Section IV provides the performance results while Section V contains concluding remarks.

II. SYSTEM MODEL

We consider an UWA cooperative wireless communication scenario where the source node S transmits information to the destination node D with the assistance of relay node R each of which is equipped with a single pair of transmit and receive antenna. The cooperation is based on the receive diversity (RD) protocol [13] with a single-relay amplify-and-forward (AF) relaying with half-duplex nodes. In our work, we assume that the relay node does not perform the channel estimation to keep its complexity as low as possible. As shown in Fig. 1, in the broadcasting phase, the source node transmits to the destination and the relay nodes. In the relaying phase, the relay node forwards a scaled noisy version of the signals received from the source. The channel between each node pair is assumed to be quasi-static frequency-selective Rician fading. The channel impulse responses (CIRs) for $S \rightarrow R$, $R \rightarrow D$ and $S \rightarrow D$ links are sparse and denoted by $\tilde{\mathbf{h}}^{SR}$, $\tilde{\mathbf{h}}^{RD}$ and $\tilde{\mathbf{h}}^{SD}$ having maximum discrete-valued multipath delays \tilde{L}_{SR} , \tilde{L}_{RD} and \tilde{L}_{SD} , respectively. $L_{SR} \ll \tilde{L}_{SR}$, $L_{RD} \ll \tilde{L}_{RD}$ and $L_{SD} \ll \tilde{L}_{SD}$ denote the number of non-zero elements of the multipath channels. Channel coefficients (taps) on each link is a complex Gaussian random variable with independent real and imaginary parts with mean $\mu_\ell/\sqrt{2}$ and the variance $\sigma_\ell^2/2$. Let $\Omega_\ell = E\{|h_\ell|^2\} = \mu_\ell^2 + \sigma_\ell^2$ denotes the power profile of the relevant Rician multipath channel and $\sum_{\ell=1}^L \Omega_\ell = 1$, $L \in \{L_{SR}, L_{RD}, L_{SD}\}$. Moreover, Rician κ -factor for ℓ th tap is the ratio of the power in the mean component to the power in the diffuse component, i.e. $\kappa_\ell = \mu_\ell^2/\sigma_\ell^2$. Therefore, each channel tap is given by

$$h_\ell = \sqrt{\frac{\kappa_\ell \Omega_\ell}{\kappa_\ell + 1}} \left(\frac{1+j}{\sqrt{2}} \right) + \sqrt{\frac{\Omega_\ell}{\kappa_\ell + 1}} \check{h}_\ell, \quad \ell = 1, 2, \dots, \check{L}$$

and $\check{L} \in \{\check{L}_{SR}, \check{L}_{RD}, \check{L}_{SD}\}$, (1)

where \check{h}_ℓ is a complex Gaussian random variable with zero mean and unit variance.

The additive ambient noise, generated by underwater acoustic channels has several distinct physical origins each corresponding to particular frequency range [14]. In this paper, we assume that power spectral density of the ambient noise is modeled in 10 - 100 KHz band as a function of frequency in Hz as

$$N(f) = \frac{f_0 \sigma_v^2}{\pi(f^2 + f_0^2)}, \quad (2)$$

where σ_v^2 is the noise variance, and f_0 is chosen as a model parameter of the colored noise autocorrelation function ($f_0 T_s = 0.01, 0.05, 0.1, \text{etc.}$). Note that the autocorrelation function of the

ambient noise can be obtained from (2) as

$$\rho(n - n') = \sigma_v^2 e^{-2\pi|n-n'|f_0 T_s}, \quad (3)$$

where T_s is the sampling period. Consequently, the complex-valued additive Gaussian ambient noises on the links $S \rightarrow R$, $R \rightarrow D$ and $S \rightarrow D$ are denoted by $\mathbf{v}^{SR} = [v_0^{SR}, v_1^{SR}, \dots, v_{N-1}^{SR}]^T$, $\mathbf{v}^{RD} = [v_0^{RD}, v_1^{RD}, \dots, v_{N-1}^{RD}]^T$ and $\mathbf{v}^{SD} = [v_0^{SD}, v_1^{SD}, \dots, v_{N-1}^{SD}]^T$ respectively. We assume that CIRs remain constant over a period of one block transmission and vary independently from block to block.

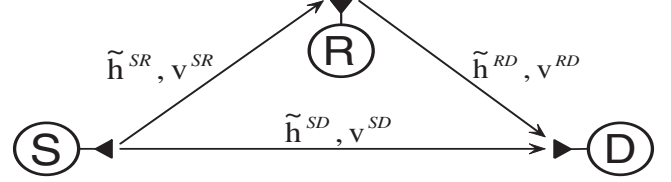


Fig. 1. Single-relay transmission model

We now consider an OFDM based UWA relay system with N subcarriers. To avoid inter-symbol interference (ISI) a cyclic prefix is added between adjacent OFDM blocks. After FFT and removing the cyclic prefix, time domain received data block in the broadcasting phase (1st time slot) at the relay and the destination nodes are given as

$$y_{1,n}^R = \sum_{l=1}^{\tilde{L}_{SR}} \tilde{h}_l^{SR} s_{n-l} + v_n^{SR}, \quad (4)$$

$$y_{1,n}^D = \sum_{l=1}^{\tilde{L}_{SD}} \tilde{h}_l^{SD} s_{n-l} + v_n^{SD}, \quad (5)$$

respectively, where $s_n = \frac{1}{N} \sum_{k=1}^N d_k e^{j2\pi nk/N}$ is the time-domain signal sample transmitted from the S node at n th discrete time and d_k is the data symbol transmitted over the k th subchannel. In the relaying phase (2nd time slot), the time domain received signal at the destination node is

$$y_{2,n}^D = \frac{1}{\gamma} \sum_{l=1}^{\tilde{L}_{RD}} \tilde{h}_l^{RD} y_{1,n-l}^R + v_n^{RD} \quad (6)$$

where $\gamma = \sqrt{\sum_{n=1}^N E\{|y_{1,n}^R|^2\}}$ is the normalization factor. To ensure that the power budget is not violated, the relay node normalizes the receive signal $y_{1,n}^R$, $n = 1, 2, \dots, N$ by γ . Inserting (4) into (6), the vector form of (6) can be expressed as

$$\mathbf{y}_2^D = \Gamma \tilde{\mathbf{h}} + \mathbf{v}, \quad (7)$$

where $\mathbf{y}_2^D = [y_{2,0}^D, y_{2,1}^D, \dots, y_{2,N-1}^D]^T$ is the time-domain received vector on the destination node in the relaying phase, $\Gamma = \frac{1}{\gamma} \mathbf{F}^{-1} \mathbf{D} \mathbf{F}$, with \mathbf{F} being the FFT matrix whose k th row and n th column entry $[\mathbf{F}]_{k,n} = e^{-j2\pi nk/N}$ and \mathbf{D} is a diagonal matrix having the data symbols $\{d_k\}_{k=0}^{N-1}$ on its main diagonal. $\tilde{\mathbf{h}} = \tilde{\mathbf{h}}^{SR} \circledast \tilde{\mathbf{h}}^{RD}$ and

$$\begin{aligned} \mathbf{v} &= \tilde{\mathbf{h}}^{RD} \circledast \mathbf{v}^{SR} + \mathbf{v}^{RD} \\ &= \frac{1}{\gamma} \mathbf{F}^{-1} \mathbf{D}_{\tilde{\mathbf{H}}^{RD}} \mathbf{F} \mathbf{v}^{SR} + \mathbf{v}^{RD} \end{aligned} \quad (8)$$

denote the cascaded sparse multipath channel and additive noise on $S \rightarrow R \rightarrow D$ link, respectively, where \circledast is the N -sample circular convolution operator and $\mathbf{D}_{\tilde{\mathbf{H}}^{RD}}$ represents a diagonal matrix whose main diagonal vector is $\tilde{\mathbf{H}}^{RD} = \mathbf{F} \tilde{\mathbf{h}}^{RD}$.

It is obvious from (8) that the ambient noise \mathbf{v} is non-Gaussian and colored. Thus, without going further toward the channel estimation

step, the observation model in (7) can be reduced to the one with additive white non-Gaussian noise by the use of a noise-whitening filter, based on the singular value decomposition (SVD) of the covariance matrix of \mathbf{v} , $\Sigma_{\mathbf{v}} = \mathbf{U}\mathbf{\Upsilon}\mathbf{U}^\dagger$, where \mathbf{U} is an $N \times N$ complex valued unitary transformation matrix, $\mathbf{\Upsilon}$ is an $N \times N$ diagonal matrix with positive real entries and $(\cdot)^\dagger$ denotes the conjugate transpose operator. Consequently, the colored noise can be transformed into a white noise through the linear transformation $\Psi\mathbf{v} = \mathbf{w}$, where $\mathbf{w} = [w_1, w_2, \dots, w_N]^T$ is a non-Gaussian white noise vector with identity covariance matrix and $\Psi = \mathbf{\Upsilon}^{-1/2}\mathbf{U}^\dagger$ is termed as whitening matrix. Multiplying (7) by Ψ from the left we obtain the following observation model

$$\mathbf{y} = \mathbf{A}\tilde{\mathbf{h}} + \mathbf{w} \in \mathcal{C}^{N \times 1}, \quad (9)$$

where $\mathbf{y} = \Psi\mathbf{y}_2^D$ and $\mathbf{A} = \Psi\mathbf{\Gamma} \in \mathcal{C}^{N \times N}$ is the convolution matrix generated from data symbols.

In this work, we are mainly interested in estimation of $\tilde{\mathbf{h}}$ in (9) where $\tilde{\mathbf{h}} \in \mathcal{C}^N$ is a complex valued, sparse multipath channel vector with non-zero entries, h_1, h_2, \dots, h_L ($L \ll N$) and the associated random channel tap positions, $\eta_1, \eta_2, \dots, \eta_L$. The received signal in (9) can be rewritten as

$$\mathbf{y} = \sum_{\ell=1}^L \mathbf{a}_{\eta_\ell} h_\ell + \mathbf{w}, \quad (10)$$

where, \mathbf{a}_{η_ℓ} is the η_ℓ th column vector of the matrix \mathbf{A} corresponding to the ℓ th multipath channel tap position. Note that the matrix \mathbf{A} is known by the receiver completely since it contains only pilot symbols during the training phase in a given frame as shown in Fig.2. We

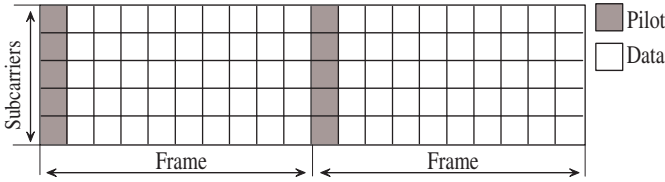


Fig. 2. Pilot Scheme of the UWA-OFDM system

model the white, non-Gaussian noise samples $w_n, n = 1, 2, \dots, N$ in (9) as an identically independent distributed (i.i.d.), M -term Gaussian mixture as follows

$$p(w_n) = \sum_{m=1}^M p(w_n | \nu_n = m) p(\nu_n = m) = \sum_{m=1}^M \frac{\lambda_m}{\pi \sigma_m^2} e^{-|w_n|^2 / \sigma_m^2}, \quad (11)$$

where $p(w_n | \nu_n = m) \triangleq \frac{1}{\pi \sigma_m^2} e^{-|w_n|^2 / \sigma_m^2}$, $\nu_n \in \{1, 2, \dots, M\}$ is the n th random mixture index that identifies which term in Gaussian mixture pdf in (11) produced the additive noise sample w_n and $p(\nu_n = m) = \lambda_m$ is the probability that w_n is chosen from the m th term in the mixture pdf, with $\sum_{m=1}^M \lambda_m = 1$. In (11), σ_m^2 denotes the variance of the m th Gaussian mixture.

III. SPARSE MULTIPATH CHANNEL ESTIMATION WITH MP-SAGE ALGORITHM

We now propose a new iterative algorithm, called the MP-SAGE algorithm, based on the SAGE and the MP techniques for channel estimation employing the signal model given by (9). The SAGE algorithm, proposed by Fessler et al. [15], is a twofold generalization of the so-called "expectation maximization" (EM) algorithm that provides updated estimates for an unknown parameter set Θ . First, rather than updating all parameters simultaneously at iteration (i) , only a subset of Θ_S indexed by $S = S[i]$ is updated while keeping

the parameters in the complement set $\Theta_{\bar{S}}$ fixed; and second, the concept of the complete data χ is extended to that of the so-called admissible hidden data χ_S to which the observed signal \mathcal{R} is related by means of a possibly nondeterministic mapping. The convergence rate of the SAGE algorithm is usually higher than that of the EM algorithm, because the conditional Fisher information matrix of given for each set of parameters is likely smaller than that of the complete data, given for the entire space. At the i th iteration, the expectation-step (E-step) of the SAGE algorithm is defined

$$Q_S(\Theta_S | \Theta^{[i]}) = E \left\{ p \left(\chi_S, \Theta_S | \Theta_{\bar{S}}^{[i]} \right) | \mathcal{R}, \Theta^{[i]} \right\}.$$

In the maximization step (M-step), only Θ_S is updated, i.e.,

$$\Theta_S^{[i+1]} = \arg \max_{\Theta_S} Q_S(\Theta_S | \Theta^{[i]})$$

$$\Theta_{\bar{S}}^{[i+1]} = \Theta_{\bar{S}}^{[i]}.$$

The MP algorithm is an iterative procedure which can sequentially identify the dominant channel taps and estimate the associated tap coefficients by choosing the the column \mathbf{a}_{η_ℓ} of \mathbf{A} in (9) which best aligned with the residual vector until all the taps are identified. The detail description of the MP algorithm is given in Sec. III-C. Finally our proposed MP-SAGE algorithm implements the MP algorithm at each SAGE iteration step by updating, all the dominant channel taps and the associated tap coefficients sequentially. The details of the MP-SAGE algorithm is presented below:

The unknown parameter set to be estimated in our problem is

$$\Phi = \{\mathbf{h}, \boldsymbol{\eta}, \boldsymbol{\alpha}\}, \quad (12)$$

where $\mathbf{h} = [h_1, h_2, \dots, h_L]^T$, $\boldsymbol{\eta} = [\eta_1, \eta_2, \dots, \eta_L]^T$ and $\boldsymbol{\alpha} = \{\lambda_1, \dots, \lambda_M, \sigma_1^2, \dots, \sigma_M^2\}$.

The first step in deriving the MP-SAGE algorithm for estimating Φ based on the received vector \mathbf{y} is the specifications of "complete data" and "admissible hidden data" sets whose pdfs are characterized by the common parameters set Φ . To obtain a receiver architecture that iterates between soft-data and channel estimation in the MP-SAGE algorithm, we decompose Φ into $L + 1$ subsets, representing the parameters, \mathbf{h} , $\boldsymbol{\eta}$ and $\boldsymbol{\alpha}$, as follows.

- The first L subsets of Φ are chosen as $\Phi_\ell = \{h_\ell, \eta_\ell\}$, $\ell = 1, 2, \dots, L$. For each subset we define $\bar{\Phi}_\ell = \Phi \setminus \Phi_\ell = \{\bar{\mathbf{h}}_\ell, \bar{\boldsymbol{\eta}}_\ell, \boldsymbol{\alpha}\}$, $\bar{\mathbf{h}}_\ell = \mathbf{h} \setminus h_\ell$ and $\bar{\boldsymbol{\eta}}_\ell = \boldsymbol{\eta} \setminus \eta_\ell$, where \setminus denotes the exclusion operator.
- The $(L + 1)$ st subset of Φ is chosen as by $\Phi_{L+1} = \boldsymbol{\alpha}$ and $\bar{\Phi}_{L+1} \triangleq \Phi \setminus \Phi_{L+1} = \Phi \setminus \boldsymbol{\alpha} = \{\mathbf{h}, \boldsymbol{\eta}\}$

At the SAGE iteration (i) , only the parameters in one set are updated, whereas the other parameters are kept fixed, and this process is repeated until all parameters are updated. According to the above parameter subset definitions, each iteration of the SAGE algorithm for our problem has two steps:

- 1) $\Phi_\ell, \ell = 1, 2, \dots, L$ is updated with the MP-SAGE algorithm while $\bar{\Phi}_{L+1}$ is fixed.
- 2) Φ_{L+1} is updated with the SAGE algorithm while $\bar{\Phi}_\ell, \ell = 1, 2, \dots, L$ is fixed.

We now derive the MP-SAGE algorithm below by also specifying the corresponding admissible hidden data and complete data sets.

A. Estimation of $\Phi_\ell = \{h_\ell, \eta_\ell\}, \ell = 1, 2, \dots, L$

A suitable approach for applying the MP-SAGE algorithm for estimation of Φ_ℓ is to decompose the n th sample of the receive signal in (10) into the sum

$$y_n = x_n^{(\ell)} + \bar{x}_n^{(\ell)}, \quad (13)$$

where

$$x_n^{(\ell)} = a_{n, \eta_\ell} h_\ell + w_n \quad \text{and} \quad \bar{x}_n^{(\ell)} = \sum_{\ell'=1, \ell' \neq \ell}^L a_{n, \eta_{\ell'}} h_{\ell'} \quad (14)$$

and a_{n, η_ℓ} denotes n th element of the \mathbf{a}_{η_ℓ} . We define the admissible hidden data as $\chi_\ell = \{\mathbf{x}^{(\ell)}, \boldsymbol{\nu}\}$, where $\mathbf{x}^{(\ell)} = [x_1^{(\ell)}, x_2^{(\ell)}, \dots, x_N^{(\ell)}]^T$ and $\boldsymbol{\nu} = [\nu_1, \nu_2, \dots, \nu_N]^T$.

To perform the *E-Step* of the MP-SAGE algorithm, the conditional expectation is taken over χ_ℓ given the observation \mathbf{y} and given that Φ equals its estimate calculated at i th iteration:

$$\begin{aligned} Q_\ell(\Phi_\ell | \Phi^{(i)}) &= E\{\log p(\chi_\ell | \Phi_\ell, \bar{\Phi}^{(i)}) | \mathbf{y}, \Phi^{(i)}\} \\ &= E\{\log p(\mathbf{x}^{(\ell)}, \boldsymbol{\nu} | h_\ell, \eta_\ell, \bar{\mathbf{h}}_\ell^{(i)}, \bar{\boldsymbol{\eta}}_\ell^{(i)}, \boldsymbol{\alpha}^{(i)}) | \mathbf{y}, \mathbf{h}^{(i)}, \boldsymbol{\eta}^{(i)}, \boldsymbol{\alpha}^{(i)}\}, \end{aligned} \quad (15)$$

where

$$\begin{aligned} \log p(\mathbf{x}^{(\ell)}, \boldsymbol{\nu} | h_\ell, \eta_\ell, \bar{\mathbf{h}}_\ell^{(i)}, \bar{\boldsymbol{\eta}}_\ell^{(i)}, \boldsymbol{\alpha}^{(i)}) &= \log p(\boldsymbol{\nu} | \boldsymbol{\alpha}^{(i)}) \\ &\quad + \log p(\mathbf{x}^{(\ell)} | \boldsymbol{\nu}, h_\ell, \eta_\ell, \boldsymbol{\alpha}^{(i)}) \\ &\sim - \sum_{n=1}^N \frac{|x_n^{(\ell)} - a_{n, \eta_\ell} h_\ell|^2}{(\sigma_{\nu_n}^2)^{(i)}}. \end{aligned} \quad (16)$$

Inserting (16) in (15) we obtain

$$Q_\ell(\Phi_\ell | \Phi^{(i)}) = \sum_{n=1}^N \delta_n^{(i)} \left(2\Re\{\widehat{x}_n^{(\ell)} a_{n, \eta_\ell}^* h_\ell^*\} - |a_{n, \eta_\ell} h_\ell|^2 \right), \quad (17)$$

where $\Re(\cdot)$ and $(\cdot)^*$ denote the real part and the conjugate operators, respectively, and $\widehat{x}_n^{(\ell)}$ is defined as

$$\widehat{x}_n^{(\ell)} \triangleq E\{x_n^{(\ell)} | y_n, \mathbf{h}^{(i)}, \boldsymbol{\eta}^{(i)}, \boldsymbol{\alpha}^{(i)}\}.$$

Recalling (13) it follows that

$$\widehat{x}_n^{(\ell)} = y_n - \sum_{\ell'=1, \ell' \neq \ell}^L a_{n, \eta_{\ell'}} h_{\ell'}, \quad (18)$$

and $\delta_n^{(i)}$ in (17) is defined as

$$\begin{aligned} \delta_n^{(i)} &\triangleq E_{\{\nu_n | y_n, \mathbf{h}^{(i)}, \boldsymbol{\eta}^{(i)}, \boldsymbol{\alpha}^{(i)}\}} \left\{ \frac{1}{(\sigma_{\nu_n}^2)^{(i)}} \right\} \\ &= \sum_{m=1}^M \frac{1}{(\sigma_m^2)^{(i)}} p_{\nu_n}^{(i)}(m), \quad n = 1, 2, \dots, N. \end{aligned} \quad (19)$$

Keeping in mind $p(\nu_n = m | \boldsymbol{\alpha}^{(i)}) = \lambda_m^{(i)}$, the posterior probability density function of the random mixture index ν_n at i th iteration, $p_{\nu_n}^{(i)}(m)$, is evaluated as follows

$$\begin{aligned} p_{\nu_n}^{(i)}(m) &\triangleq p(\nu_n = m | y_n, \mathbf{h}^{(i)}, \boldsymbol{\eta}^{(i)}, \boldsymbol{\alpha}^{(i)}) \\ &\sim \lambda_m^{(i)} e^{-|y_n - \sum_{\ell=1}^L a_{n, \eta_\ell} h_\ell^{(i)}|^2 / (\sigma_m^2)^{(i)}} / (\pi(\sigma_m^2)^{(i)}) \\ &= \frac{\lambda_m^{(i)} e^{-|y_n - \sum_{\ell=1}^L a_{n, \eta_\ell} h_\ell^{(i)}|^2 / (\sigma_m^2)^{(i)}} / (\pi(\sigma_m^2)^{(i)})}{\sum_{m'=1}^M \lambda_{m'}^{(i)} e^{-|y_n - \sum_{\ell=1}^L a_{n, \eta_\ell} h_\ell^{(i)}|^2 / (\sigma_{m'}^2)^{(i)}} / (\pi(\sigma_{m'}^2)^{(i)})}. \end{aligned} \quad (20)$$

The vector form of (17) can be written as follows

$$Q_\ell(\Phi_\ell | \Phi^{(i)}) = 2\Re\{\mathbf{a}_{\eta_\ell}^\dagger \mathbf{D}_\delta^{(i)} \widehat{\mathbf{x}}^{(\ell)} h_\ell^*\} - \mathbf{a}_{\eta_\ell}^\dagger \mathbf{D}_\delta^{(i)} \mathbf{a}_{\eta_\ell} |h_\ell|^2, \quad (21)$$

where from (18) $\widehat{\mathbf{x}}^{(\ell)} = [\widehat{x}_1^{(\ell)}, \dots, \widehat{x}_N^{(\ell)}]^T = \mathbf{y} - \sum_{p=1, p \neq \ell}^L \mathbf{a}_{\eta_p} h_p^{(i)}$ and $\mathbf{D}_\delta^{(i)}$ is a diagonal matrix with entries $\delta_1^{(i)}, \delta_2^{(i)}, \dots, \delta_N^{(i)}$ that are calculated from (19).

In the *M-step* of the MP-SAGE algorithm, the estimates of $\Phi_\ell = \{h_\ell, \eta_\ell\}$ are updated at the $(i+1)$ st iteration according to

$$\Phi_\ell^{(i+1)} = \arg \max_{\Phi_\ell} Q_\ell(\Phi_\ell | \Phi^{(i)}), \quad (22)$$

where $Q_\ell(\Phi_\ell | \Phi^{(i)})$ is given by (21). So, taking the derivative of $Q_\ell(\Phi_\ell | \Phi^{(i)})$ with respect to h_ℓ^* and equating to zero, we find the final SAGE estimates of (η_ℓ, h_ℓ) at $(i+1)$ st iteration as follows:

$$\begin{aligned} \eta_\ell^{(i+1)} &= \arg \max_{\eta_\ell} \frac{|\mathbf{a}_{\eta_\ell}^\dagger \mathbf{D}_\delta^{(i)} \widehat{\mathbf{x}}^{(\ell)}|^2}{\mathbf{a}_{\eta_\ell}^\dagger \mathbf{D}_\delta^{(i)} \mathbf{a}_{\eta_\ell}}, \quad \eta_\ell \in \{1, 2, \dots, N\} \\ &\quad \eta_\ell \notin \{\eta_1^{(i+1)}, \dots, \eta_{\ell-1}^{(i+1)}\} \\ h_\ell^{(i+1)} &= \frac{\mathbf{a}_{\eta_\ell^{(i+1)}}^\dagger \mathbf{D}_\delta^{(i)} \widehat{\mathbf{x}}^{(\ell)}}{\mathbf{a}_{\eta_\ell^{(i+1)}}^\dagger \mathbf{D}_\delta^{(i)} \mathbf{a}_{\eta_\ell^{(i+1)}}}. \end{aligned} \quad (23)$$

Based on the above result, $\{\eta_\ell, h_\ell\}$ can be sequentially estimated for $\ell = 1, 2, \dots, L$, incorporating the previous estimates in the MP-SAGE mode as follows:

Step 1) For $i = 0$, determine the initial estimates $\{\eta_\ell^{(0)}, h_\ell^{(0)}\}, \ell = 1, 2, \dots, L$, from the MP algorithm as described in Sec. III-C.

Step 2) For $i \leftarrow (i+1)$, and $\ell = 1, 2, \dots, L$, compute $\{\eta_\ell^{(i+1)}, h_\ell^{(i+1)}\}$ from (23), replacing $\widehat{\mathbf{x}}^{(\ell)}$ with the residual vector $\mathbf{r}_\ell^{(i)}$ of the MP algorithm. It can be shown that, the residual vector can be computed recursively as

$$\mathbf{r}_\ell^{(i)} = \mathbf{r}_{\ell-1}^{(i)} - (\mathbf{a}_{\eta_\ell^{(i)}} h_\ell^{(i)} - \mathbf{a}_{\eta_{\ell-1}^{(i)}} h_{\ell-1}^{(i+1)}) \quad (24)$$

where $\mathbf{r}_0^{(i)} = \widehat{\mathbf{x}}^{(1)}$ and $\mathbf{a}_{\eta_0^{(i)}} = 0, h_0^{(i)} = 0$ for all (i) .

Step 3) If $\ell = L$ go to the next SAGE iteration step.

Step 4) continue the SAGE iterations until convergence. END

B. Estimation of $\Phi_{L+1} = \boldsymbol{\alpha} = \{\lambda_1, \dots, \lambda_M, \sigma_1^2, \dots, \sigma_M^2\}$

We define the complete data as $\chi_{L+1} = \{\mathbf{y}, \boldsymbol{\nu}\}$ to estimate the mixture parameters $\boldsymbol{\alpha} = \{\lambda_1, \dots, \lambda_M, \sigma_1^2, \dots, \sigma_M^2\}$. Now, let us derive the MP-SAGE algorithm.

To perform the *E-Step* of the MP-SAGE algorithm, the conditional expectation is taken over χ_{L+1} given the observation \mathbf{y} and given that Φ equals its estimate calculated at i th iteration:

$$\begin{aligned} Q_{L+1}(\Phi_{L+1} | \Phi^{(i)}) &= E\{\log p(\chi_{L+1} | \Phi_{L+1}, \bar{\Phi}^{(i)}) | \mathbf{y}, \Phi^{(i)}\} \\ &= E\{\log p(\mathbf{y}, \boldsymbol{\nu} | \boldsymbol{\alpha}, \mathbf{h}^{(i)}, \boldsymbol{\eta}^{(i)}) | \mathbf{y}, \mathbf{h}^{(i)}, \boldsymbol{\eta}^{(i)}, \boldsymbol{\alpha}^{(i)}\}, \end{aligned} \quad (25)$$

where

$$\begin{aligned} \log p(\mathbf{y}, \boldsymbol{\nu} | \boldsymbol{\alpha}, \mathbf{h}^{(i)}, \boldsymbol{\eta}^{(i)}) &= \log p(\boldsymbol{\nu} | \boldsymbol{\alpha}) + \log p(\mathbf{y} | \boldsymbol{\nu}, \boldsymbol{\alpha}, \mathbf{h}^{(i)}, \boldsymbol{\eta}^{(i)}) \\ &\sim \sum_{n=1}^N \left[\log(\lambda_{\nu_n}) - \log(\sigma_{\nu_n}^2) - \frac{1}{\sigma_{\nu_n}^2} \left| y_n - \sum_{\ell=1}^L a_{n, \eta_\ell} h_\ell^{(i)} \right|^2 \right]. \end{aligned} \quad (26)$$

Inserting (26) in (25) we obtain

$$Q_{L+1}(\Phi_{L+1}|\Phi^{(i)}) = \sum_{n=1}^N \sum_{m=1}^M p_{\nu_n}^{(i)}(m) \left[\log\left(\frac{\lambda_m}{\sigma_m^2}\right) - \frac{1}{\sigma_m^2} \left| y_n - \sum_{\ell=1}^L a_{n,\eta_\ell^{(i)}} h_\ell^{(i)} \right|^2 \right], \quad (27)$$

where $p_{\nu_n}^{(i)}(m)$ is given in (20).

In the M -step of the SAGE algorithm, the estimates $\Phi_{L+1} = \alpha$ are updated at the $(i+1)$ st iteration according to the following constraint maximization problem:

$$\Phi_{L+1}^{(i+1)} = \arg \max_{\Phi_{L+1}} Q_{L+1}(\Phi_{L+1}|\Phi^{(i)}) \quad (28)$$

subject to :

$$\sum_{m=1}^M \lambda_m = 1, \quad \lambda_{m'} \geq 0, \quad m' = 1, 2, \dots, M.$$

The optimization problem in (28) can be decoupled into two minimization problems. The first one is a convex minimization problem with a constraint and the other is a simple minimization problem without constraint. These problems are given as

1)

$$\min_{\lambda_1, \dots, \lambda_M} - \sum_{n=1}^N \sum_{m=1}^M p_{\nu_n}^{(i)}(m) \log(\lambda_m) \quad (29)$$

subject to :

$$\sum_{m=1}^M \lambda_m = 1, \quad \lambda_m \geq 0, \quad m = 1, 2, \dots, M$$

2)

$$\min_{\sigma_1^2, \dots, \sigma_M^2} \sum_{n=1}^N \sum_{m=1}^M p_{\nu_n}^{(i)}(m) \left[\log(\sigma_m^2) + \frac{1}{\sigma_m^2} \left| y_n - \sum_{\ell=1}^L a_{n,\eta_\ell^{(i)}} h_\ell^{(i)} \right|^2 \right] \quad (30)$$

Solving the first problem in (29) using a convex optimization technique with lagrangian we have

$$\lambda_m^{(i+1)} = \frac{1}{N} \sum_{n=1}^N p_{\nu_n}^{(i)}(m), \quad m = 1, 2, \dots, M. \quad (31)$$

where $p_{\nu_n}^{(i)}(m)$ is given in (20). Keeping in mind (31), it is straightforward to show that the solution of the minimization problem in (30) is

$$(\sigma_m^2)^{(i+1)} = \frac{1}{N \lambda_m^{(i+1)}} \left(\mathbf{y} - \sum_{\ell=1}^L \mathbf{a}_{\eta_\ell^{(i)}} h_\ell^{(i)} \right)^\dagger \mathbf{D}_P^{(i)}(m) \left(\mathbf{y} - \sum_{\ell=1}^L \mathbf{a}_{\eta_\ell^{(i)}} h_\ell^{(i)} \right), \quad (32)$$

where $\mathbf{D}_P^{(i)}(m)$ is a diagonal matrix with entries $p_{\nu_1}^{(i)}(m), p_{\nu_2}^{(i)}(m), \dots, p_{\nu_N}^{(i)}(m)$.

Note that, as seen from (23), the positions of the dominant channel taps are identified and the associated channel tap coefficients are estimated sequentially during the estimation of $\Phi_\ell = \{h_\ell, \eta_\ell\}$, $\ell = 1, 2, \dots, L$, of the SAGE algorithm within the MP framework. More clearly, at $(i+1)$ st step of the SAGE algorithm, the columns of \mathbf{A}

C. Initialization of the Algorithm

1) *Initialization of $\Phi^{(0)} = \{\eta_\ell^{(0)}, h_\ell^{(0)}, \ell = 1, 2, \dots, L\}$* : We apply the matching pursuit (MP) algorithm to determine $\Phi^{(0)}$ considering the observation model in (9). As a first step in the MP algorithm, the column in the matrix $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{N-1}]$ which is best aligned with the residue vector $\mathbf{r}_0 = \mathbf{y}$ is found and denoted \mathbf{a}_{η_1} . Then the projection of \mathbf{r}_0 along this direction is removed from \mathbf{r}_0 and the residual \mathbf{r}_1 is obtained. The algorithm proceeds by sequentially choosing the column which is the best matches until termination criterion is met. At the ℓ th iteration, the index of the vector from \mathbf{A} most closely aligned with the residual vector $\mathbf{r}_{\ell-1}$ is obtained as follows

$$\eta_\ell^{(0)} = \arg \max_j \frac{|\mathbf{a}_j^\dagger \mathbf{r}_{\ell-1}|^2}{\mathbf{a}_j^\dagger \mathbf{a}_j}, \quad j = 1, 2, \dots, N \quad \text{and } j \notin \{\eta_1^{(0)}, \eta_2^{(0)}, \dots, \eta_{\ell-1}^{(0)}\}, \quad (33)$$

and the channel tap at position $\eta_\ell^{(0)}$ is

$$h_\ell^{(0)} = \frac{\mathbf{a}_{\eta_\ell^{(0)}}^\dagger \mathbf{r}_{\ell-1}}{\mathbf{a}_{\eta_\ell^{(0)}}^\dagger \mathbf{a}_{\eta_\ell^{(0)}}}. \quad (34)$$

The new residual vector is computed as $\mathbf{r}_\ell = \mathbf{r}_{\ell-1} - h_\ell^{(0)} \mathbf{a}_{\eta_\ell^{(0)}}$.

2) *Initializations of the Gaussian-mixture parameters $\{\lambda_m^{(0)}, (\sigma_m^2)^{(0)}, m = 1, 2, \dots, M\}$* : The empirical pdf of the Gaussian mixture noise in (11) is obtained first by means of the samples generated from the Gaussian distribution of the random vectors $\mathbf{v}^{SR}, \mathbf{v}^{RD}, \mathbf{h}^{RD}$, representing the additive Gaussian noise and the channel impulse response on the links $S \rightarrow R \rightarrow D$, having known covariance matrices. The Gaussian-mixture parameters are then determined by solving the following constrained optimization problem numerically.

$$J(\lambda_1, \dots, \lambda_M, \sigma_1^2, \dots, \sigma_M^2) = \sum_{j=1}^{N_s} \left| p_{emp}(w_j) - \sum_{m=1}^M \frac{\lambda_m}{\pi \sigma_m^2} e^{-|w_j|^2 / \sigma_m^2} \right|^2$$

with constraints $\sum_{m=1}^M \lambda_m = 1$ and $\forall m, \lambda_m > 0$, where N_s is the number of noise samples.

D. Complexity Analysis

The computational complexity of the algorithm is presented in Table-I. Note that, the initial values, $\{\lambda_m^{(0)}, (\sigma_m^2)^{(0)}, m = 1, 2, \dots, M\}$, are precomputed as mentioned in Section III-C2. The initial values, $\eta_\ell^{(0)}, h_\ell^{(0)}$ in (33) and (34), are obtained by the MP algorithm and require approximately $N^2 L$ complex multiplications (CMs) and $N^2 L$ complex additions (CAs) as given in Table-I. In each iteration of the MP-SAGE algorithm, $\eta_\ell^{(i+1)}, h_\ell^{(i+1)}, \mathbf{r}_\ell^{(i+1)}, \{p_{\nu_n}^{(i)}(m)\}_{m=1}^M, n = 1, \dots, N$, and $\{\lambda_m^{(i+1)}, \sigma_m^2^{(i+1)}\}_{m=1}^M$ are updated and need approximately $N^2 L + 2MN(L+1)$ CMs and $N^2 L + MN(L+3)$ CAs. As a result, it follows from Table-I that the total computational complexity per iteration of the MP-SAGE channel estimation algorithm presented in this work is approximately $2N^2 L + 2MN(L+1)$ CMs and $2N^2 L + MN(L+3)$ CAs $\sim \mathcal{O}(N^2 L)$. Consequently the complexity of the algorithm is in the order of $\mathcal{O}(NL)$ per OFDM subcarrier.

IV. SIMULATION RESULTS

In this section, we present computer simulation results to assess the performance of OFDM-based cooperative communication systems in UWA channels with the proposed channel estimation algorithm. Simulation parameters are chosen as in Table-II. The initial estimates

TABLE I
COMPUTATIONAL COMPLEXITY DETAILS

| INITIALIZATION | | | |
|-------------------|--|--------------------|--------------------|
| Eq. No | Variable | CMs | CAs |
| (33) and (34) | $\eta_\ell^{(0)}, h_\ell^{(0)}, \mathbf{r}_\ell$ | $\approx N^2 L$ | $\approx N^2 L$ |
| MP-SAGE ITERATION | | | |
| (23) and (24) | $\eta_\ell^{(i+1)}, h_\ell^{(i+1)}, \mathbf{r}_\ell^{(i+1)}$ | $\approx N^2(L+1)$ | $\approx N^2(L+1)$ |
| (20) | $\{p_{\nu_n}^{(i)}(m)\}_{m=1}^M$ $n=1, \dots, N$ | $\approx 2MN(L+5)$ | $\approx MN(L+2)$ |
| (31) and (32) | $\{\lambda_m^{(i+1)}, \sigma_m^2{}^{(i+1)}\}_{m=1}^M$ | $\approx 2MN$ | $\approx MN$ |

of the multipath channel positions and taps used in MP-SAGE algorithm are performed by the reduced complexity MP algorithm. The initial estimates of the multipath channel positions is used in the BLUE estimator as well.

Figs. 3 and 4 show MSE and SER performance curves of the MP, BLUE and MP-SAGE algorithms for Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift Keying (QPSK) signaling format. As seen from these curves, MP-SAGE algorithm having excellent channel estimation performance and symbol error rate outperforms MP and BLUE estimators. We conclude from these curves that our novel MP-SAGE algorithm exhibits a good performance in the estimation of channel tap positions, tap coefficients, and the Gaussian mixture parameters used to model the pdf of the correlative non-Gaussian noise. Particularly, as seen from Fig.4 that our estimation algorithm has approximately 3 dB performance gain over the BLUE estimator at SER= 10^{-3} when QPSK signaling is employed.

TABLE II
SIMULATION PARAMETERS

| | |
|-------------------------------------|---------------------------------|
| Number of Subcarriers (N) | 256 |
| Channel Bandwidth (BW) | 3 KHz |
| Sampling Frequency (f_s) | BW |
| Multipath Delays (τ) | $[0 \ 21 \ 34 \ 52] \times T_s$ |
| Multipath Powers (Ω) | $[0.25 \ 0.5 \ 0.15 \ 0.1]$ |
| Rician Factor (κ) | 2 dB |
| $f_o T_s$ | 0.01 |
| Number of Gaussian Mixtures (M) | 10 |
| Number of OFDM Frame Length | 10 |
| Number of iterations | 10 |

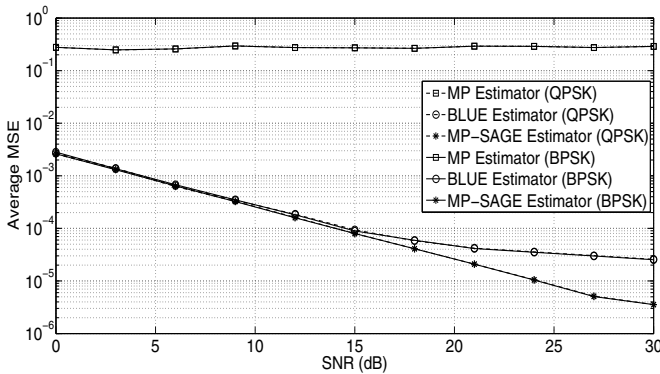


Fig. 3. MSE performance comparisons

V. CONCLUSIONS

In this work we have presented a novel channel estimation algorithm for AF cooperative relay based OFDM systems in the presence of sparse underwater acoustic channels and of the correlative non-Gaussian noise modeled with a finite Gaussian mixture pdf. The proposed algorithm is based on the SAGE and the MP techniques. The MP algorithm was combined with the SAGE algorithm in such a way that at each SAGE iteration step, the nonzero taps and the locations of

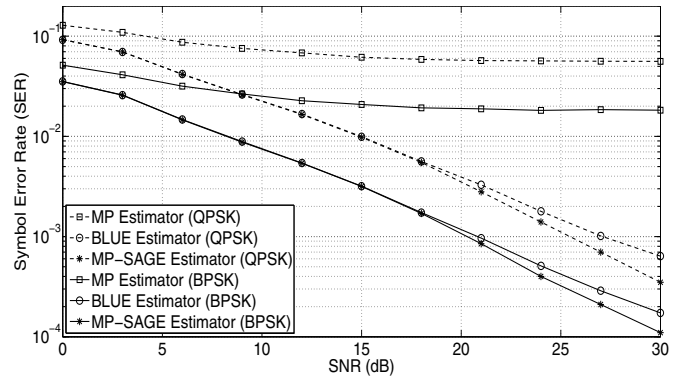


Fig. 4. SER performance comparisons

the sparse channel taps were determined and associated channel taps estimated by the MP algorithm. Finally, the computer simulations have shown that UWA channel is estimated very effectively and the proposed algorithm has excellent symbol error rate and channel estimation performance.

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