

# Bayesian Compressive Sensing for Ultra-Wideband Channel Models

Mehmet Özgör, Serhat Erköç, and Hakan Ali Çırpan

**Abstract**—Considering the sparse structure of ultra-wideband (UWB) channels, compressive sensing (CS) is suitable for UWB channel estimation. Among various implementations of CS, the inclusion of Bayesian framework has shown potential to improve signal recovery as statistical information related to signal parameters is considered. In this paper, we study the channel estimation performance of Bayesian CS (BCS) for various UWB channel models and noise conditions. Specifically, we investigate the effects of (i) sparse structure of standardized IEEE 802.15.4a channel models, (ii) signal-to-noise ratio (SNR) regions, and (iii) number of measurements on the BCS channel estimation performance, and compare them to the results of  $l_1$ -norm minimization based estimation, which is widely used for sparse channel estimation. The study shows that BCS exhibits superior performance at higher SNR regions only for adequate number of measurements and sparser channel models (e.g., CM1 and CM2). Based on the results of this study, BCS method or the  $l_1$ -norm minimization method can be preferred over the other for different system implementation conditions.

**Keywords**—Bayesian compressive sensing (BCS), IEEE 802.15.4a channel models,  $l_1$ -norm minimization, ultra-wideband (UWB) channel estimation.

## I. INTRODUCTION

ULTRA-WIDEBAND (UWB) impulse radio (IR) [1] is an emerging technology for wireless communications. Owing to distinguishing properties such as having low transmit power, low-cost simple structure, immunity to flat fading and capability of resolving multipath components individually with good time resolution, UWB-IR systems have received great interest from both academia and industry [2]. Considering these properties, UWB-IRs have been selected as the physical layer structure of Wireless Personal Area Network (WPAN) standard IEEE 802.15.4a for location and ranging, and low data rate applications [3]. In the implementation of UWB-IRs, one of the main challenges is the channel estimation. Due to ultra-wide bandwidth of UWB-IRs, the main disadvantage of implementing the conventional maximum likelihood (ML) channel estimator is that very high sampling rates, i.e., very high speed A/D converters are required for precise channel estimation.

In order to overcome the high-rate sampling problem, compressive sensing (CS) proposed in [4], [5] can be considered for UWB channel estimation. CS is a promising paradigm in signal processing, where a signal that is sparse in a known

transform domain can be recovered with high probability from a set of random linear projections with much fewer measurements than usually required by the dimensions of this domain. As the received consecutive UWB pulses arrive with a considerable time delay and can be resolved individually at the receiver, sparse structure assumption is widely accepted for UWB multipath channels. Accordingly, CS has been exploited for UWB channel estimation [6], [7], where the conventional  $l_1$ -norm minimization method has been used to estimate UWB channel coefficients.

Among various implementations of CS, one approach has been to include the Bayesian model. Considering the sparse Bayesian model in [8], a Bayesian framework has been defined for CS in [9]. In [10], a hierarchical form of Laplace priors on signal coefficients is taken into consideration for Bayesian CS (BCS). Both of the frameworks have shown potential to improve signal recovery as the posterior density function over the associated sparse channel coefficients is considered. In [11], a Turbo BCS algorithm for sparse signal reconstruction through exploiting and integrating spatial and temporal redundancies in multiple sparse signal reconstruction is proposed. In [12], a Laplace prior based BCS algorithm in [10] has been modified for joint reconstruction of received sparse signals and channel parameters for multiuser UWB communications. In [13], the proposed approach in [9] is considered for UWB channel estimation, where BCS estimation results are compared to the  $l_1$ -norm minimization results. However, the authors have not considered the effects of UWB channel models (i.e., sparsity condition) or additive noise level (i.e., Bayesian approach depends on the statistical information about channel parameters and additive noise) on the channel estimation performance.

In this paper, motivated by investigating the factors that affect the performance of BCS in realistic UWB channels, we study the effects of standardized IEEE 802.15.4a channel models, signal-to-noise ratio (SNR) regions, and number of measurements on the channel estimation performance. These factors are important to analyze as sparsity, noise level and measurements directly affect the BCS model. Accordingly, BCS channel estimation performance for various scenarios is compared to the  $l_1$ -norm minimization based estimation, which is a method widely used for sparse channel estimation. In addition computation time of both methods is discussed. The comparison results provided are important in order to define the conditions where BCS may be preferred over the conventional  $l_1$ -norm minimization method.

The rest of the paper is organized as follows. In Section II, IEEE 802.15.4a channel models that are widely used in UWB research are explained. In Section III, the overview of CS theory,  $l_1$ -norm minimization, Bayesian model and their applications to UWB channel estimation are presented. In

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Section IV, simulation results for performance comparison are provided. Concluding remarks are given in Section V.

## II. UWB CHANNEL MODEL

In this section, the discrete-time equivalent UWB channel model and the standardized IEEE 802.15.4a channel models are presented, respectively.

In order to obtain the discrete-time channel model, the general channel impulse response (CIR) should be presented first. Accordingly, the continuous-time channel  $h(t)$  can be modeled as

$$h(t) = \sum_{k=1}^{L_r} h_k \delta(t - \tau_k), \quad (1)$$

where  $h_k$  represents the  $k$ th multipath gain coefficient,  $\tau_k$  is the delay of the  $k$ th multipath component,  $\delta(\cdot)$  is the Dirac delta function and  $L_r$  is the number of resolvable multipaths.

The continuous-time CIR given in (1) assumes that multipaths may arrive any time. This is referred to as the  $\tau$ -spaced channel model [14]. If a pulse is  $T_s$ -seconds duration, then an approximate equivalent channel model can be obtained for practical purposes [7]. Hence, the equivalent  $T_s$ -spaced channel model can be expressed as

$$h(t) = \sum_{n=1}^N c_n \delta(t - nT_s), \quad (2)$$

where  $T_c = NT_s$  is the channel length and  $\{c_n\}$  are the resulting new channel coefficients. Using (2), the discrete-time equivalent channel can be written as

$$\mathbf{h} = [c_1, c_2, \dots, c_N]^T, \quad (3)$$

where the channel resolution is  $T_s$ . Assuming that  $\mathbf{h}$  has  $K$  nonzero coefficients, the sparsity assumption of (3) is valid if  $K \ll N$ .

Based on the discrete-time equivalent channel model above, the UWB channels are widely accepted as having a sparse structure. This assumption for UWB channels plays an important role in CS based UWB channel estimation. However, the channel environment should be inspected to prove this assumption. In [15], a comprehensive model for UWB propagation channels, which was accepted as the standardized channel model for IEEE 802.15.4a, has been developed considering various channel environments and conducting different measurement campaigns. These environments include indoor residential, indoor office, outdoor, industrial environments, agricultural areas and body area networks with having either a line-of-sight (LOS) or a non-LOS (NLOS) transmitter-receiver connection. In [7], the sparsity assumption of UWB channels has been discussed over the channel models CM1 (LOS residential indoor), CM2 (NLOS residential indoor), CM5 (LOS outdoor) and CM8 (NLOS industrial). In order to investigate the effects of channel sparsity on the BCS channel estimation performance, we will consider the same channel models in the current study. More details on the channel models CM1, CM2, CM5 and CM8 can be found in [7] and [15].

## III. CS FOR UWB CHANNEL ESTIMATION

Assuming that the UWB channels are sparse, CS can be employed for UWB channel estimation in order to overcome the high-rate sampling problem. In the following, we will present the overview of CS theory and its application to UWB channel estimation, and the Bayesian CS model, respectively.

### A. Overview of Compressive Sensing

Consider the problem of reconstructing a discrete-time signal  $\mathbf{x} \in \mathfrak{R}^N$  which can be represented in an arbitrary basis  $\Psi \in \mathfrak{R}^{N \times N}$  with the weighting coefficients  $\boldsymbol{\theta} \in \mathfrak{R}^N$  as

$$\mathbf{x} = \sum_{n=1}^N \psi_n \theta_n = \Psi \boldsymbol{\theta}. \quad (4)$$

Suppose that  $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_N]^T$  has only  $K$  nonzero coefficients, where  $K \ll N$  and  $\Psi = [\psi_1, \psi_2, \dots, \psi_N]$ . As  $\mathbf{x}$  is a linear combination of only  $K$  basis vectors, it can be called a  $K$ -sparse signal and can be expressed as

$$\mathbf{x} = \sum_{i=1}^K \psi_{n_i} \theta_{n_i}, \quad (5)$$

where  $\{n_i\}$  are the indices that correspond to nonzero coefficients. By projecting  $\mathbf{x}$  onto a random measurement matrix  $\Phi \in \mathfrak{R}^{M \times N}$ , a set of measurements  $\mathbf{y} \in \mathfrak{R}^M$  can be obtained as

$$\mathbf{y} = \Phi \Psi \boldsymbol{\theta}, \quad (6)$$

where  $M \ll N$ . Here, the measurement matrix should be incoherent with the basis in addition to the sparsity condition for accurate weighting coefficients estimation. The incoherency is usually achieved by random matrices with independent identically distributed (i.i.d) elements from Gaussian or Bernoulli distributions [16]. Instead of using the  $N$ -sample  $\mathbf{x}$  to estimate the weighting coefficients  $\boldsymbol{\theta}$ , the  $M$ -sample measurement vector  $\mathbf{y}$  can be used. Accordingly,  $\boldsymbol{\theta}$  can be estimated as

$$\hat{\boldsymbol{\theta}} = \min \|\boldsymbol{\theta}\|_1 \quad \text{subject to} \quad \mathbf{y} = \Phi \Psi \boldsymbol{\theta}, \quad (7)$$

where  $l_p$ -norm is denoted as  $\|\boldsymbol{\theta}\|_p = \left( \sum_{n=1}^N |\theta_n|^p \right)^{\frac{1}{p}}$ . The reconstruction problem hence becomes an  $l_1$ -norm optimization problem, and estimating  $\boldsymbol{\theta}$  from the vector  $\mathbf{y}$  instead of  $\mathbf{x}$  corresponds to a lower sampling rate at the receiver.

The CS theory explained in (4)-(7) can be employed to UWB channel estimation. Suppose that  $\mathbf{g} \in \mathfrak{R}^N$  is the discrete-time representation of the received signal given as

$$\mathbf{g} = \mathbf{P} \mathbf{h} + \mathbf{n}, \quad (8)$$

where  $\mathbf{P} \in \mathfrak{R}^{N \times N}$  is a scalar matrix representing the time-shifted pulses,  $\mathbf{h} = [c_1, c_2, \dots, c_N]^T$  are the channel gain coefficients, and  $\mathbf{n}$  are the additive white Gaussian noise (AWGN) terms. Since the UWB channel structure is sparse,  $\mathbf{h}$  has only  $K$  nonzero coefficients. Similar to (6), the received signal  $\mathbf{g}$  can be projected onto a random measurement matrix  $\Phi \in \mathfrak{R}^{M \times N}$  so as to obtain  $\mathbf{y} \in \mathfrak{R}^M$  as

$$\begin{aligned} \mathbf{y} &= \Phi \mathbf{P} \mathbf{h} + \Phi \mathbf{n} \\ &= \mathbf{A} \mathbf{h} + \mathbf{z}. \end{aligned} \quad (9)$$

Due to the presence of the noise term  $\mathbf{z}$ , the channel  $\mathbf{h}$  can be estimated as

$$\hat{\mathbf{h}} = \min \|\mathbf{h}\|_1 \quad \text{subject to} \quad \|\mathbf{A}\mathbf{h} - \mathbf{y}\|_2 \leq \epsilon, \quad (10)$$

where  $\epsilon$  is related to the noise term as  $\epsilon \geq \|\mathbf{z}\|_2$ . The  $l_1$ -norm minimization problem in (10) can be recast as a second-order cone program (SOCP) and solved\* with a generic log-barrier algorithm.

### B. Bayesian Compressive Sensing

In this section, the CS problem will be presented from a Bayesian perspective for UWB channel estimation. In the BCS framework proposed in [8], [9], the statistical information about the compressible signal and the additive noise is considered, where  $l_1$ -norm minimization does not consider these factors. Considering sparsity prior of  $\mathbf{h}$  and the noise model assumption together with the signal model in (9), BCS<sup>†</sup> can be used for UWB channel estimation. Taking into consideration (9), the full posterior distribution over all unknowns of interest for the problem at hand becomes

$$p(\mathbf{h}, \boldsymbol{\beta}, \sigma^2 | \mathbf{y}) = \frac{p(\mathbf{y} | \mathbf{h}, \boldsymbol{\beta}, \sigma^2) p(\mathbf{h}, \boldsymbol{\beta}, \sigma^2)}{p(\mathbf{y})} \quad (11)$$

where  $\boldsymbol{\beta}$  represents hyperparameters that control the inverse variance of each channel coefficient, and  $\sigma^2$  is variance of each noise term in  $\mathbf{z}$ . Unfortunately, this full posterior term is not tractable since the integral

$$p(\mathbf{y}) = \int \int \int p(\mathbf{y} | \mathbf{h}, \boldsymbol{\beta}, \sigma^2) p(\mathbf{h}, \boldsymbol{\beta}, \sigma^2) d\mathbf{h} d\boldsymbol{\beta} d\sigma^2 \quad (12)$$

cannot be computed analytically. Hence, we decompose the full posterior distribution as

$$p(\mathbf{h}, \boldsymbol{\beta}, \sigma^2 | \mathbf{y}) \equiv p(\mathbf{h} | \mathbf{y}, \boldsymbol{\beta}, \sigma^2) p(\boldsymbol{\beta}, \sigma^2 | \mathbf{y}). \quad (13)$$

In (9), the noise term  $\mathbf{z}$  can be modelled probabilistically as independent zero-mean Gaussian random variables:

$$p(\mathbf{z}) = \prod_{m=1}^M \mathcal{N}(z_m | 0, \sigma^2). \quad (14)$$

This noise model infers Gaussian likelihood for observation  $\mathbf{y}$ :

$$p(\mathbf{y} | \mathbf{h}, \sigma^2) = (2\pi\sigma^2)^{-M/2} \exp\left(-\frac{\|\mathbf{y} - \boldsymbol{\Phi}\mathbf{h}\|^2}{2\sigma^2}\right). \quad (15)$$

Suppose that a zero-mean Gaussian prior distribution is defined on channel coefficients with  $\beta_n$ :

$$\begin{aligned} p(\mathbf{h} | \boldsymbol{\beta}) &= \prod_{n=1}^N \mathcal{N}(h_n | 0, \beta_n^{-1}) \\ &= (2\pi)^{-N/2} \prod_{n=1}^N \beta_n^{1/2} \exp\left(-\frac{\beta_n h_n^2}{2}\right). \end{aligned} \quad (16)$$

\*For the implementation of (10), the codes provided by Romberg and Candes publicly available at <http://users.ece.gatech.edu/justin/11magic/> are used.

<sup>†</sup>For the implementation of BCS, the codes provided by Shihao Ji publicly available at <http://people.ee.duke.edu/lcarin/BCS.html> are used.

$\{\beta_n\}$ 's are independent hyperparameters that consist of  $\boldsymbol{\beta} = [\beta_1, \dots, \beta_N]^T$ , and control the strength of the prior over associated channel coefficients individually.

The first term of (13),  $p(\mathbf{h} | \mathbf{y}, \boldsymbol{\beta}, \sigma^2)$ , the posterior distribution over the channel coefficients, can be expressed via Bayes' rule as

$$p(\mathbf{h} | \mathbf{y}, \boldsymbol{\beta}, \sigma^2) = \frac{p(\mathbf{y} | \mathbf{h}, \sigma^2) p(\mathbf{h} | \boldsymbol{\beta})}{p(\mathbf{y} | \boldsymbol{\beta}, \sigma^2)}. \quad (17)$$

Considering Gaussian likelihood together with Gaussian prior, this posterior distribution is also  $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$  where

$$\begin{aligned} \boldsymbol{\Sigma} &= (\boldsymbol{\Lambda} + \sigma^{-2} \boldsymbol{\Phi}^T \boldsymbol{\Phi})^{-1}, \\ \boldsymbol{\mu} &= \sigma^{-2} \boldsymbol{\Sigma} \boldsymbol{\Phi}^T \mathbf{y}, \end{aligned} \quad (18)$$

with  $\boldsymbol{\Lambda} = \text{diag}(\beta_1, \beta_2, \dots, \beta_N)$  and is analytically tractable. To compute the full posterior distribution approximately, the second term, hyperparameter posterior,  $p(\boldsymbol{\beta}, \sigma^2 | \mathbf{y})$  in (13) needs to be approximated. This approximation is provided by type-II maximum likelihood procedure. According to the Bayes' theorem, hyperparameter posterior  $p(\boldsymbol{\beta}, \sigma^2 | \mathbf{y})$  can be expressed as:

$$p(\boldsymbol{\beta}, \sigma^2 | \mathbf{y}) \propto p(\mathbf{y} | \boldsymbol{\beta}, \sigma^2) p(\boldsymbol{\beta}, \sigma^2). \quad (19)$$

Using appropriately selected uniform hyperpriors for  $\boldsymbol{\beta}$  and  $\sigma^2$  (i.e.,  $p(\boldsymbol{\beta}, \sigma^2 | \mathbf{y}) \propto p(\mathbf{y} | \boldsymbol{\beta}, \sigma^2)$ ), the estimates of  $\boldsymbol{\beta}$  and  $\sigma^2$  can be found by maximizing marginal likelihood function (LF)  $p(\mathbf{y} | \boldsymbol{\beta}, \sigma^2)$  as a consequence of type-II maximum likelihood procedure. The marginal LF can be obtained by integrating over the channel coefficients  $\mathbf{h}$  as:

$$p(\mathbf{y} | \boldsymbol{\beta}, \sigma^2) = \int_{-\infty}^{\infty} p(\mathbf{y} | \mathbf{h}, \sigma^2) p(\mathbf{h} | \boldsymbol{\beta}) d\mathbf{h}. \quad (20)$$

Maximization of the marginal LF with respect to  $\boldsymbol{\beta}$  or equivalently, its logarithm can be expressed as:

$$\begin{aligned} \mathcal{L}(\boldsymbol{\beta}, \sigma^2) &= \log p(\mathbf{y} | \boldsymbol{\beta}, \sigma^2) \\ &= \log \int_{-\infty}^{\infty} p(\mathbf{y} | \mathbf{h}, \sigma^2) p(\mathbf{h} | \boldsymbol{\beta}) d\mathbf{h} \\ &= -\frac{1}{2} [M \log(2\pi) + \log |\mathbf{C}| + \mathbf{y}^T \mathbf{C}^{-1} \mathbf{y}] \end{aligned} \quad (21)$$

where  $\mathbf{C} = \sigma^2 \mathbf{I} + \boldsymbol{\Phi} \boldsymbol{\Lambda}^{-1} \boldsymbol{\Phi}^T$  and  $\mathbf{I} \in \Re^{M \times M}$  is an identity matrix. Differentiating  $\mathcal{L}(\boldsymbol{\beta}, \sigma^2)$  with respect to  $\boldsymbol{\beta}$  and  $\sigma^2$ , and equating it to zero yields the following expressions which can be employed iteratively:

$$\beta_n^{\text{new}} = \frac{\gamma_n}{\mu_n^2}, \quad \sigma^{2\text{new}} = \frac{\|\mathbf{y} - \boldsymbol{\Phi} \boldsymbol{\mu}\|_2^2}{M - \sum_{n=1}^N \gamma_n}, \quad (22)$$

where  $\gamma_n \in [0, 1]$  is defined as  $\gamma_n = 1 - \beta_n \sum_{nn}$  with  $\sum_{nn}$  being the  $n$ th diagonal element of the posterior coefficient covariance from (18) and  $\mu_n$  is the  $n$ th posterior coefficient mean from (18).

By employing re-estimates of hyperparameters, an iterative systematic approach is used to determine which basis vectors should be included in the model and which should be removed to promote sparsity [9]. Further details and steps of the BCS algorithm can be found in [8].

#### IV. SIMULATION RESULTS

In this section, we investigate the effects of number of measurements, SNR regions, and the IEEE 802.15.4a channel models on the BCS channel estimation performance, and compare the results to the performance of the  $l_1$ -norm minimization results. As the performance measure, we evaluate the mean-square error (MSE) of the estimated channel vector. To remove the path loss effect and to treat each channel model fairly, we normalize the channel coefficients as  $\sum_{n=1}^N c_n^2 = 1$ . For the simulations, the channel length and resolution are fixed to  $T_c = 250\text{ns}$  and  $T_s = 0.25\text{ns}$ , respectively, resulting in the discrete-time channel length  $N = T_c/T_s = 1000$ . The performances are evaluated for  $M = \{250, 500, 750\}$  measurements in the  $[0, 30]\text{dB}$  SNR region. Here,  $M/N$  can be regarded as the compression ratio. The elements of the measurement matrix  $\Phi$  are obtained from the  $\mathcal{N}(0, 1)$  distribution, and the basis where the channel vector is sparse is defined as  $\Psi = \mathbf{I}$  in our simulations.

In Figs. 1, 2, 3 and 4, the channel estimation performances of BCS and  $l_1$ -norm minimization are compared for various number of measurements and SNR values for the channel models CM1, CM2, CM5, and CM8, respectively. The best channel estimation performance for both methods is obtained for CM1, as it exhibits the most sparse structure among these channel models [7]. BCS outperforms  $l_1$ -norm minimization in the sparser channel models CM1 and CM2 for SNR values greater than 12-13dB for all measurements considered. This can be explained as for the higher SNR regions posterior density function over the channel coefficients and noise is beneficial to the channel coefficient estimation, whereas for lower SNR regions the uncertainty in the estimation is higher. As for CM5, which is a less sparse channel, the number of measurements should be greater than  $M = 500$  in order for BCS to have a superior performance at higher SNR regions. As for CM8, which is not a sparse channel model, as the multipaths arrive almost in every time bin, the BCS performs inferior compared to the  $l_1$ -norm minimization for almost all conditions. In summary, BCS can be an effective channel estimation method for sparser channel models at high SNR regions. This is mainly due to BCS considering the channel and noise statistics and providing a posterior density function over noise and the channel coefficients, whereas the  $l_1$ -norm minimization method not utilizing such statistics.

Finally, a short discussion on the computation time of both methods is provided. Although a fair comparison of the computation complexities is more desired (currently under investigation), we compare the average computation time of a channel estimator realization for both methods based on the publicly available codes, where their main structures are not modified but adapted to IEEE 802.15.4a channel estimation. In Tables I, II, III and IV, the computation times of both methods are provided for different number of measurements at CM1, CM2, CM5, and CM8, respectively. The simulations were run on a computer that has a 3.4 GHz Intel Core i7 CPU and a 3.88 GB RAM. It can be observed that the computation time of BCS is significantly shorter than the  $l_1$ -norm minimization for every channel model and number of observations. While the computation time of the  $l_1$ -norm minimization does not change much with sparsity or the number of measurements,

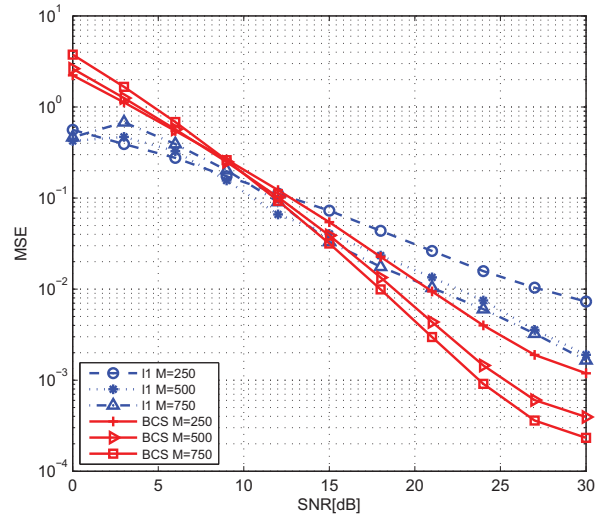


Fig. 1. MSE performance comparison of Bayesian CS and  $l_1$ -norm minimization for CM1.

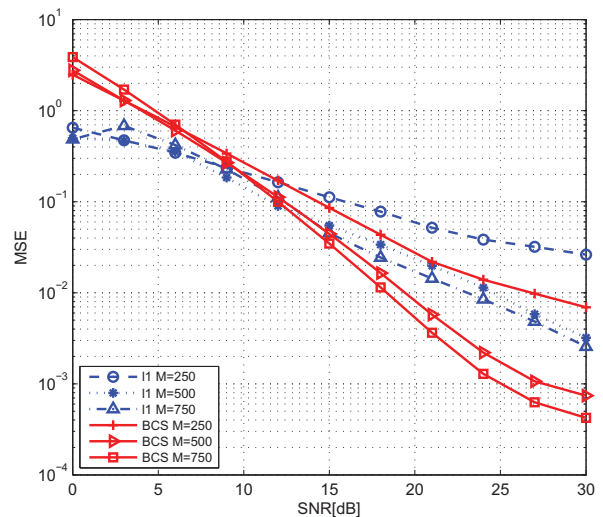


Fig. 2. MSE performance comparison of Bayesian CS and  $l_1$ -norm minimization for CM2.

TABLE I  
COMPUTATION TIME OF BOTH METHODS FOR CM1

Number of measurements	$l_1$ -norm minimization	Bayesian CS
M=250	3.5911 secs	0.13607 secs
M=500	3.6684 secs	0.2892 secs
M=750	3.5778 secs	0.76564 secs

TABLE II  
COMPUTATION TIME OF BOTH METHODS FOR CM2

Number of measurements	$l_1$ -norm minimization	Bayesian CS
M=250	3.6073 secs	0.15767 secs
M=500	3.627 secs	0.31896 secs
M=750	3.4591 secs	0.82328 secs

it increases for BCS when the channel is less sparse and the number of measurements is increased.

## V. CONCLUSION

In this paper, we considered the application of Bayesian CS to UWB channel estimation, and studied its channel estimation performance for various UWB channel models and noise conditions. Specifically, we investigated the effects of the sparse structure of standardized IEEE 802.15.4a channel models, SNR regions, and number of measurements on the BCS channel estimation performance, and compared them to the results of the conventional  $l_1$ -norm minimization based estimation. The simulation results show that BCS exhibits superior performance at sparser channel models and higher SNR regions as it utilizes the statistics of channel coefficients and noise. Moreover, the computation time of BCS has been found to be shorter for the cases considered, and the evaluation of computation complexity is under further investigation. Based on the results of this study, the implementation conditions of BCS can be determined for practical cases.

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## REFERENCES

- [1] M. Z. Win and R. A. Scholtz, "Ultra-widebandwidth time-hopping spread-spectrum impulse radio for wireless multiple-access communications," *IEEE Trans. Commun.*, vol. 48, pp. 679–691, Apr. 2000.
- [2] D. Porcino and W. Hirt, "Ultra-wideband radio technology: potential and challenges ahead," *IEEE Commun. Mag.*, vol. 41, no. 7, pp. 66–74, Jul. 2003.
- [3] IEEE Std 802.15.4a-2007, "Part 15.4: Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for Low-Rate Wireless Personal Area Networks (WPANs)," 2007.
- [4] E. J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, pp. 489–509, Feb. 2006.
- [5] D. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, pp. 1289–1306, Apr. 2006.
- [6] J. Parades, G. R. Arce, and Z. Wang, "Ultra-wideband compressed sensing: channel estimation," *IEEE J. Sel. Topics Signal Process.*, vol. 1, pp. 383–395, Oct. 2007.
- [7] M. Bařaran, S. Erkućuk, and H. A. Çırpan, "The effect of channel models on compressed sensing based UWB channel estimation," *IEEE Proc. ICUBW'11* pp. 375–379, Sep. 2011.
- [8] M. E. Tipping and A. C. Faul, "Fast marginal likelihood maximization for sparse Bayesian models," *Proc. 9th Intl. Workshop Artificial Intelligence and Stats.*, 2003.
- [9] S. Ji, Y. Xue, and L. Carin, "Bayesian compressive sensing," *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2346–2355, June 2008.
- [10] S. D. Babacan, R. Molina, and A. K. Katsaggelos, "Fast Bayesian compressive sensing using Laplace priors," *IEEE Proc. ICASSP'09*, pp. 2873–2876, Apr. 2009.
- [11] D. Yang, H. Li, and G. D. Peterson, "Decentralized Turbo Bayesian compressed sensing with application to UWB systems," *EURASIP Jour. on Adv. in Signal Process.*, vol. 2011, article ID 817947, Mar. 2011.
- [12] L. Shi, Z. Zhou, and L. Tang, "Ultra-wideband channel estimation based on distributed Bayesian compressive sensing," *JDCTA Intl. Jour. of Digital Content Tech. and Appl.*, vol. 5, no. 2, pp. 1–8, Feb. 2011.
- [13] L. Shi, Z. Zhou, L. Tang, H. Yao, and J. Zhang, "Ultra-wideband channel estimation based on Bayesian compressive sensing," *IEEE Proc. ISCIT'10*, pp. 779–782, Oct. 2010.
- [14] S. Erkućuk, D. I. Kim, and K. S. Kwak, "Effects of channel models and Rake receiving process on UWB-IR system performance," *IEEE Proc. ICC'07*, pp. 4896–4901, June 2007.
- [15] A. F. Molisch et. al., "A comprehensive standardized model for ultrawideband propagation channels," *IEEE Trans. Antennas Propag.*, vol. 54, pp. 3151–3166, Nov. 2006.
- [16] E. J. Candes and M. B. Walkin, "An introduction to compressive sampling," *IEEE Signal Process. Mag.*, vol. 25, pp. 21–30, Mar. 2008.

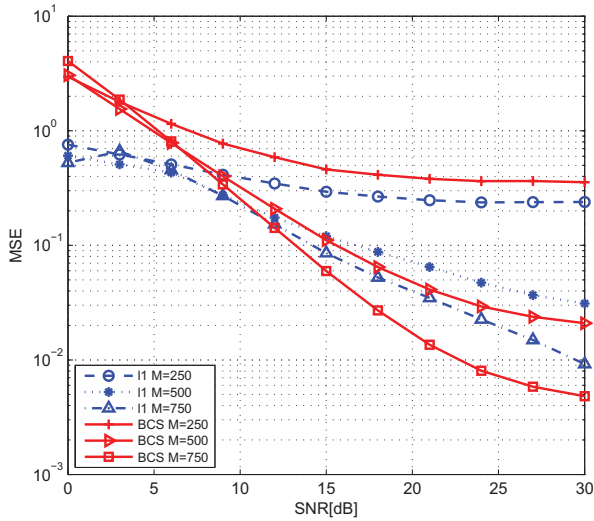


Fig. 3. MSE performance comparison of Bayesian CS and  $l_1$ -norm minimization for CM5.

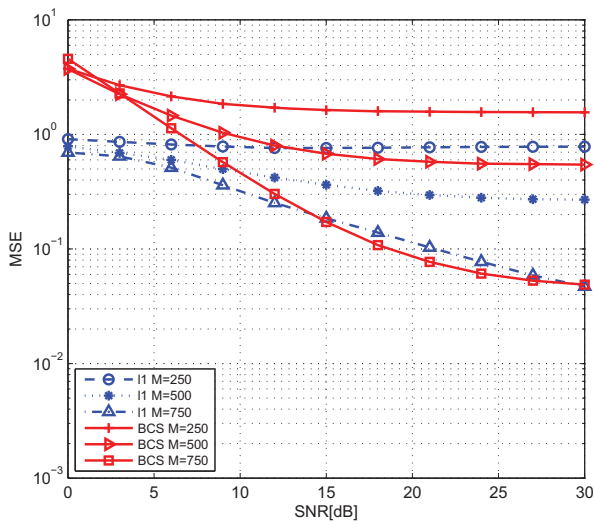


Fig. 4. MSE performance comparison of Bayesian CS and  $l_1$ -norm minimization for CM8.

TABLE III  
COMPUTATION TIME OF BOTH METHODS FOR CM5

Number of measurements	$l_1$ -norm minimization	Bayesian CS
M=250	3.748 secs	0.22791 secs
M=500	3.5745 secs	0.47146 secs
M=750	3.2783 secs	1.1099 secs

TABLE IV  
COMPUTATION TIME OF BOTH METHODS FOR CM8

Number of measurements	$l_1$ -norm minimization	Bayesian CS
M=250	3.8257 secs	0.27791 secs
M=500	4.0806 secs	0.84026 secs
M=750	3.6359 secs	1.9952 secs