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Broadband impedance matching via lossless unsymmetrical lattice networks

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ABSTRACT

In this paper, a broadband impedance matching network (equalizer) design algorithm has been proposed. In the equalizer, a lossless unsymmetrical lattice network has been utilized. The branch impedances of the lattice network are considered as singly terminated lossless LC networks, since it is not desired to dissipate power in the equalizer. After giving the algorithm, its usage has been illustrated via an example.

1. Introduction

Design of broadband matching networks is an essential problem for microwave communication system engineers [1]. So analytic theory of broadband matching [2], [3] and computer-aided-design (CAD) methods are essential tools for the designers [4–6]. But it is well known that analytic theory is difficult to utilize even if the source and load impedances are simple. Therefore, it is always attractive to use CAD techniques. All the CAD techniques optimize the matched system performance. But performance optimization is highly nonlinear with respect to element values and needs very good initials [7]. As a result, initial element values are vital for successful optimization.

In matching network design problems, ladder networks are used, since these structures have very low sensitivity [8]. In addition they are preferable since they are unbalanced, thus all the shunt branches can be grounded. If non-minimum-phase transfer characteristics are desired, the network complexity increases. This added complexity is the result of right-half plane zeros of a transfer function, which can be realized only by a signal-cancellation process. This requires more than one path of transmission between the input and output ports. In ladder networks, this can be realized by parallel or bridged structures. Without the common ground between the input and output ports, right half-plane zeros can be realized by a bridge structure of the form shown in Fig. 1.

Therefore, in this paper, two-port bridge structures are employed in matching networks. In the following sections, two-port bridge networks and real frequency broadband matching will be summarized. Then, the proposed design algorithm and an example will be presented.

2. Bridge networks

When a voltage source of frequency ω_0 with the resistance R_S is applied to the input port of a bridge network (Fig. 1) and the branch impedances are related such that $[Z_1(j\omega_0)Z_4(j\omega_0) = Z_2(j\omega_0)Z_3(j\omega_0)]$, the voltage drop across the load impedance Z_L is zero, resulting in a zero of transmission or a zero of the voltage transfer ratio at $(p = j\omega_0)$. A bridge network satisfying these conditions is said to be balanced. If opposite arms of the bridge have the same impedances $[Z_A(p) = Z_1(p) = Z_4(p)]$ and $[Z_B(p) = Z_2(p) = Z_3(p)]$, the network is said to be symmetrical. If the bridge leads are twisted, the configuration seen in Fig. 2 is obtained, which is known as a unsymmetrical lattice network. Although any physically realizable transfer function can be synthesized by using unsymmetrical RLC lattice network, a symmetrical lattice restricts the range of realizable functions. Especially, certain physically realizable transfer functions having poles on the $j\omega$-axis cannot be synthesized by using symmetrical lattice structures [9].

3. Real frequency broadband matching

Let us consider the classical single matching problem (resistive source impedance and complex load impedance) shown in Fig. 2.

The matching conditions of the complex load Z_L to the resistive generator R_S can be formulated in terms of the normalized reflection coefficients at ports 1 and 2. The input reflection coefficients f_{Norm} can be defined by

$$\rho 1 = \frac{Z_{in} - R_S}{Z_{in} + R_S} \qquad (1)$$

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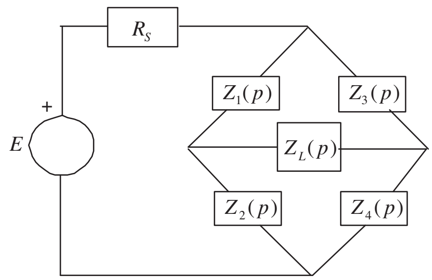

Fig. 1. General two-port bridge network.

where Z_{in} is the input impedance seen at port 1 when port 2 is terminated by the load Z_L. Similarly the reflection coefficient at port 2 can be defined by

$$\rho 2 = \frac{Z_{out} - Z_L^*}{Z_{out} + Z_L} \tag{2}$$

where Z_{out} is the impedance seen at port 2 when port 1 is terminated by the source resistance R_S, and the upper asterisk denotes complex conjugation. Here, ρ_2 is the normalized reflection coefficient at port 2. Since the two-port is considered as lossless, we have on the imaginary axis of the complex frequency plane

$$|\rho_1|^2 = |\rho_2|^2. \tag{3}$$

Then, the transducer power gain (TPG) at real frequencies can be expressed as

$$\mathrm{TPG}(\omega) = 1 - |\rho_1|^2 = 1 - |\rho_2|^2. \tag{4}$$

The goal in broadband matching is to design the lossless network N, which consists of the arm impedances $Z_1(p)$, $Z_2(p)$, $Z_3(p)$ and $Z_4(p)$, such that TPG given by Eq. (4) is maximized inside a desired frequency band. Obviously, maximizing TPG(ω) means to minimizing the modulus of the reflection coefficients $|\rho_1|$ or $|\rho_2|$. In this context, the matching problem is reduced to the determination of a realizable impedance function Z_{in} or Z_{out}.

Let the equalizer input impedance Z_{in} be expressed in terms of its real and imaginary parts on the real frequency axis as

$$Z_{in}(j\omega) = R_{in}(\omega) + jX_{in}(\omega). \tag{5}$$

By using Eq. (5), Eq. (4) and Eq. (1) we obtain transducer power gain (TPG) in terms of the real and imaginary parts of the input impedance Z_{in} of the equalizer N and the source resistance R_S as follows:

$$\mathrm{TPG}(\omega) = \frac{4R_S R_{in}(\omega)}{(R_S + R_{in}(\omega))^2 + (X_{in}(\omega))^2}. \tag{6}$$

Namely the matching problem consists of finding $Z_{in}(j\omega)$ such that TPG(ω) is maximized inside a desired frequency band. Once Z_{in} is determined properly, the equalizer network N can be synthesized directly by using the obtained impedance or the corresponding reflection coefficient.

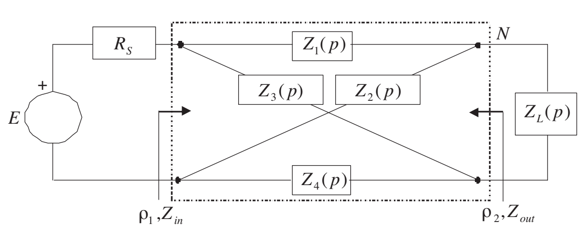

Fig. 2. Unsymmetrical lattice network.

4. Rationale of the matching procedure

For a lossless two-port like the one depicted in Fig. 2, the canonic form of the scattering matrix is given by [10], [11]

$$S(p) = \begin{bmatrix} S_{11}(p) & S_{12}(p) \\ S_{21}(p) & S_{22}(p) \end{bmatrix} = \frac{1}{g(p)} \begin{bmatrix} h(p) & \mu f(-p) \\ f(p) & -\mu h(-p) \end{bmatrix} \tag{7}$$

where $p = \sigma + j\omega$ is the complex frequency variable, and $\mu = \pm 1$ is a unimodular constant. If the two-port is reciprocal, then the polynomial $f(p)$ is either even or odd. In this case, $\mu = +1$ if $f(p)$ is even, and $\mu = -1$ if $f(p)$ is odd. Thus, for a lossless, reciprocal two-port

$$\mu = \frac{f(-p)}{f(p)} = \pm 1. \tag{8}$$

For a lossless two-port with resistive termination, energy conversation requires that

$$S(p)S^T(-p) = I, \tag{9}$$

where I is the identity matrix. The explicit form of Eq. (9) is known as the Feldtkeller equation and given as

$$g(p)g(-p) = h(p)h(-p) + f(p)f(-p). \tag{10}$$

In Eqs. (7) and (10), $g(p)$ is a strictly Hurwitz polynomial of nth degree with real coefficients, and $h(p)$ is a polynomial of nth degree with real coefficients. The polynomial function $f(p)$ includes all transmission zeros of the two-port.

Consider the bridge network seen in Fig. 1 or Fig. 2. Since it is not desired to dissipate any power in the impedances $Z_1(p)$, $Z_2(p)$, $Z_3(p)$ and $Z_4(p)$, they must contain only inductors and capacitors. Also their terminations must be either short or open, not resistive terminations. So these impedances are singly terminated lossless LC networks [12].

In [12], it has been shown that for a lossless singly terminated network, the input reflection coefficient can be expressed as

$$S_{11}(p) = \pm\frac{g(-p)}{g(p)} = \alpha\frac{g(-p)}{g(p)}, \tag{11}$$

where $\alpha = +1$ and $\alpha = -1$ corresponds to an open and short termination, respectively. So the procedure proposed in [12] can be used to design the impedances $Z_1(p)$, $Z_2(p)$, $Z_3(p)$ and $Z_4(p)$. Then the following algorithm is proposed to design the broadband impedance matching network by using unsymmetrical lattice networks.

4.1. Algorithm

4.1.1. Inputs

- $\omega_{i(\mathrm{actual})} = 2\pi f_{i(\mathrm{actual})}$; $i = 1, 2, \ldots, N_\omega$: measurement or calculation frequencies selected arbitrarily.
- N_ω: Total number of measurement or calculation frequencies.
- $Z_{L(\mathrm{actual})}(j\omega_i) = R_{L(\mathrm{actual})}(\omega_i) + jX_{L(\mathrm{actual})}(\omega_i)$; $i = 1, 2, \ldots, N_\omega$: measured or calculated load impedance data at N_ω frequency points.
- R_S: Given source resistance.
- f_{Norm}: Normalization frequency.
- R_0: Normalization resistance, usually 50 Ω.
- n_k; $k = 1, 2, 3, 4$: Desired number of elements in the arms of the bridge network.
- $\alpha_k = \pm 1$; $k = 1, 2, 3, 4$: Desired termination type of the arms of the bridge network.
- $g_k(p)$; $k = 1, 2, 3, 4$: Initialized polynomial $g(p)$ describing the arm impedances of the bridge network.
- T_0: Desired flat transducer power gain level.
- δ: The stopping criteria for the sum of the square errors. For many practical problems, it is sufficient to choose $\delta = 10^{-3}$.

4.2. Computational steps

Step 1: If the given load impedance and frequencies are actual values, not normalized, then normalize the frequencies with respect to f_{Norm} and set all the normalized angular frequencies

$$\omega_i = \frac{f_{i(\text{actual})}}{f_{\text{Norm}}}$$

Normalize the load impedance with respect to normalization resistance R_0 over the entire frequency band as

$$R_L = \frac{R_{L(\text{actual})}}{R_0}, X_L = \frac{X_{L(\text{actual})}}{R_0}.$$

It should be noted that if the load is specified as admittance data, then the normalization resistance R_0 multiplies the real and imaginary parts of the admittance data (i.e., $G_L = G_{L(\text{actual})}R_0$, $B_L = B_{L(\text{actual})}R_0$).

But if the given load impedance and frequencies are already normalized, then go to the next step directly without any normalization process.

Step 2: Calculate the input impedance values of the arms of the bridge network as

$$Z_{(k)}(j\omega_i) = \frac{1 + S_{(k),11}(j\omega_i)}{1 - S_{(k),11}(j\omega_i)}, \quad (12)$$

$k = 1, 2, 3, 4$ *and* $i = 1, 2, \ldots, N_\omega$

where $S_{(k),11}(j\omega_i) = \alpha_k \frac{g_k(-j\omega_i)}{g_k(j\omega_i)}$ is the input reflection coefficient of the arms of the bridge network.

Step 3: Calculate the input impedance of the bridge network when port 2 is terminated by the load impedance Z_L via the following equation (see Fig. 2)

$$Z_{in} = \frac{N}{D}$$

where

$$N = Z_1(Z_4Z_L + Z_3Z_L + Z_2Z_3 + Z_3Z_4 + Z_2Z_4) + Z_2Z_4Z_L + Z_2Z_3Z_L + Z_2Z_3Z_4$$

and

$$D = Z_1(Z_L + Z_2 + Z_4) + Z_2Z_3 + Z_2Z_L + Z_4Z_L + Z_3Z_4 + Z_3Z_L.$$

The input impedance expression has been obtained by using $\Delta - \text{to} - Y$ transformation equations [13].

Step 4: Calculate transducer power gain via Eq. (6) as follows

$$\text{TPG}(\omega_i) = \frac{4R_S R_{in}(\omega_i)}{(R_S + R_{in}(\omega_i))^2 + (X_{in}(\omega_i))^2}$$

where $R_{in}(\omega_i) = \text{Real}(Z_{in}(j\omega_i))$ and $X_{in}(\omega_i) = \text{Imag}(Z_{in}(j\omega_i))$.

Step 5: Calculate the sum of the squared error via

$$\delta_c = \sum_{i=i}^{N_\omega} \left|\varepsilon(j\omega_i)\right|^2$$

where $\varepsilon(j\omega_i) = T_0 - TPG(\omega_i)$.

Step 6: If $\delta_c \le \delta$, synthesize $S_{(k),11}(p) = \alpha_k \frac{g_k(-p)}{g_k(p)}$ and obtain the arm networks of the bridge, then stop. Otherwise, change α_k (termination types) and $g_k(p)$ (initialized polynomials) via any optimization routine and go to Step 2.

5. Example

In this section, an example will be given to illustrate the proposed algorithm. Here all the calculations will be made by using normalized values. After designing the matching network, all components can be de-normalized by using the given normalization frequency (f_{Norm}) and resistance (R_0).

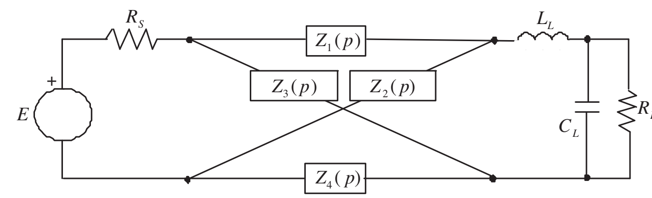

Fig. 3. The source and load terminations, $R_S = 1$, $L_L = 1$, $C_L = 3$, $R_L = 1$ (normalized).

Table 1
Calculated load impedance data (normalized).

ω	R_L	X_L
0.1	0.9174	−0.1752
0.2	0.7353	−0.2412
0.3	0.5525	−0.1972
0.4	0.4098	−0.0918
0.5	0.3077	0.0385
0.6	0.2358	0.1755
0.7	0.1848	0.3118
0.8	0.1474	0.4450
0.9	0.1206	0.5743
1.0	0.1000	0.7000

The source resistance and the load impedance which is selected as a series inductor and a parallel connection of a capacitor and a resistor in normalized values can be seen in Fig. 3.

Since the given source and load terminations have normalized element values, there is no need a normalization process. In Table 1, the calculated load impedance values are given.

The selected initial coefficients of the polynomials ($g_k(p)$), the alpha constants (α_k) and the desired flat transducer power gain level (T_0) are as follows, $g_1 = [\,4 \quad 2 \quad 3\,]$, $g_2 = [\,2 \quad 4 \quad 3\,]$, $g_3 = [\,3 \quad 5 \quad 2\,]$, $g_4 = [\,1 \quad 2 \quad 4\,]$, $\alpha_1 = +1$, $\alpha_2 = -1$, $\alpha_3 = -1$, $\alpha_4 = -1$, and $T_0 = 0.7$, respectively.

After running the proposed algorithm, the following polynomial coefficients and alpha constants are obtained, $g_1 = [\,6.0437 \quad 23.1923 \quad 3.1920\,]$, $g_2 = [\,6.3061 \quad 7.7312 \quad 0.2542\,]$, $g_3 = [\,13.1356 \quad 6.4255 \quad 0.0907\,]$, $g_4 = [\,1.3511 \quad 13.3529 \quad 12.2343\,]$, $\alpha_1 = +1$, $\alpha_2 = -1$, $\alpha_3 = -1$, $\alpha_4 = -1$, respectively.

After synthesizing the corresponding reflection coefficients $\left(S_{(k),11}(p) = \alpha_k \frac{g_k(-p)}{g_k(p)}\right)$ the bridge network seen in Fig. 4 is reached. The obtained transducer power gain curve is given in Fig. 5.

Actual element values can be obtained by de-normalization. In this case, actual element values are given by

$$\text{Actual Capacitor} = \frac{(\text{Normalized Capacitor}/2\pi f_{\text{Norm}})}{R_0},$$

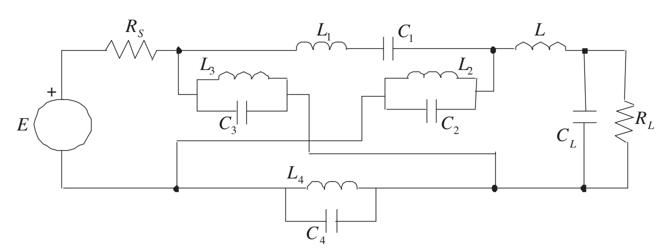

Fig. 4. Designed matching network, $L_1 = 0.2606$, $C_1 = 7.2658$, $L_2 = 30.4138$, $C_2 = 0.8157$, $L_3 = 70.8434$, $C_3 = 2.0443$, $L_4 = 1.0914$, $C_4 = 0.1012$ (normalized).

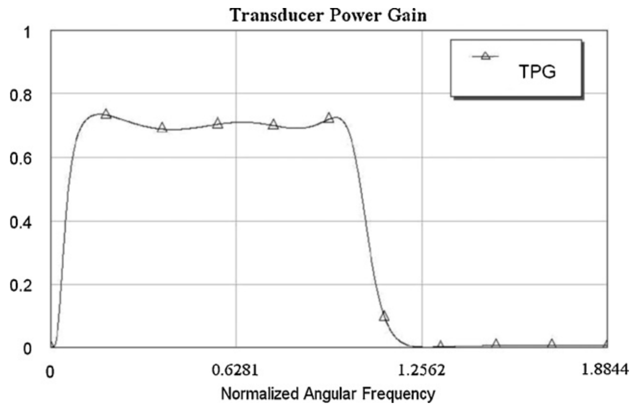

Fig. 5. Transducer power gain.

$$\text{Actual Inductor} = \left(\frac{\text{Normalized Inductor}}{2\pi f_{\text{Norm}}}\right) R_0,$$

$$\text{Actual Resistor} = (\text{Normalized Resistor}) R_0.$$

Since the matching network is designed by using normalized values, the cutoff frequency of the network is $\omega = 1$ (see Fig. 5). After de-normalization process, it shifts to the given normalization frequency, since $f_{i(\text{actual})} = \omega_i f_{\text{Norm}}$.

As can be seen in Fig. 5, a nearly flat transducer power gain curve is obtained within the required frequency band at the desired flat gain level ($T_0 = 0.7$).

6. Results and conclusion

An algorithm has been proposed to design broadband impedance matching networks via lossless unsymmetrical lattice networks. Since it is not desired to dissipate power in the equalizer, the arm impedances of the lattice network are selected as singly terminated lossless LC sections. In the paper, single matching problem (resistive source impedance and complex load impedance) has been considered. But the same procedure can be used easily for double matching problems (complex source impedance and complex load impedance).

In the example, the desired flat transducer power gain level is selected as 0.7. As can be seen from the transducer power gain graph, a nearly flat gain curve around this level has been obtained.

It is shown that the proposed method generates very good initials to improve the matched system performance by optimizing the element values. Therefore, it is expected that the proposed algorithm can be used as a front-end for the commercially available CAD tools to design broadband matching networks for communication systems.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.aeue.2011.05.005.

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