New Trellis Code Design for Spatial Modulation

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***Abstract*—Spatial modulation (SM), in which multiple antennas are used to convey information besides the conventional M-ary signal constellations, is a new multiple-input multiple-output (MIMO) transmission technique, which has recently been proposed as an alternative to V-BLAST (vertical Bell Labs layered space-time). In this paper, a novel MIMO transmission scheme, called *spatial modulation with trellis coding* (SM-TC), is proposed. Similar to the conventional trellis coded modulation (TCM), in this scheme, a trellis encoder and an SM mapper are jointly designed to take advantage of the benefits of both. A soft decision Viterbi decoder, which is fed with the soft information supplied by the optimal SM decoder, is used at the receiver. A pairwise error probability (PEP) upper bound is derived for the SM-TC scheme in uncorrelated quasi-static Rayleigh fading channels. From the PEP upper bound, code design criteria are given and then used to obtain new 4-, 8- and 16-state SM-TC schemes using quadrature phase-shift keying (QPSK) and 8-ary phase-shift keying (8-PSK) modulations for $2, 3$ and 4 bits/s/Hz spectral efficiencies. It is shown via computer simulations and also supported by a theoretical error performance analysis that the proposed SM-TC schemes achieve significantly better error performance than the classical space-time trellis codes and coded V-BLAST systems at the same spectral efficiency, yet with reduced decoding complexity.**

***Index Terms*—Trellis coding, trellis coded modulation, MIMO systems, spatial modulation.**

I. INTRODUCTION

EMERGING generations of wireless communication systems mostly rely on the use of multiple-input multiple-output (MIMO) transmission technologies, which offer significant improvements in channel capacity and reliability compared to single antenna transmission systems [1]. Therefore, MIMO transmission techniques have attracted considerable attention during the past decade and several MIMO transmission schemes have been proposed. One of the most encouraging MIMO techniques is V-BLAST (vertical Bell Labs layered space-time) [2] whose basic principle is the multiplexing of the input data stream onto the transmit antennas of a MIMO link. Despite the high spectral efficiency provided by V-BLAST, a major drawback is its very high maximum likelihood (ML) decoding complexity which is caused by inter-channel interference (ICI) at the receiver. Instead of a high complexity ML decoder, one can use a low complexity suboptimum decoder such as a minimum mean square error (MMSE) decoder; however this results in a significant degradation in error performance. As an alternative to V-BLAST transmission, a novel MIMO transmission scheme known as spatial modulation (SM) has been introduced by Mesleh *et al.* in [3,4] to remove the ICI completely between the transmit antennas of a MIMO link. The basic principle of SM is to use the indices of multiple antennas to convey information in addition to the conventional two dimensional signal constellations such as *M*-ary phase shift keying (*M*-PSK) and *M*-ary quadrature amplitude modulation (*M*-QAM), where *M* is the constellation size. Consequently, the task of an optimal SM decoder [5] is to jointly search for all of the *M*-ary constellation points and transmit antennas to decide on both the transmitted symbol and the index of the transmit antenna over which this symbol is transmitted. SM provides some advantages compared to classical MIMO transmission systems in which all antennas transmit simultaneously. Since only one transmit antenna is active during each symbol transmission, ICI is completely eliminated in SM and this results in much lower (linear) decoding complexity. Furthermore, SM does not require synchronization between the transmit antennas of the MIMO link and only one radio frequency (RF) chain is needed at the transmitter.

In recent studies, inspired by SM, a space-shift keying (SSK) modulation scheme in which only antenna indices are used to transmit information has been proposed [6]. In addition to the advantages of SM over V-BLAST, SM has been combined with space-time block coding (STBC) [7] to provide transmit diversity.

The inventors of SM have proposed a trellis coded spatial modulation scheme in [8,9], where the key idea of trellis coded modulation (TCM) [10] is partially applied to SM to improve its performance in correlated channels. In this scheme, a group of information bits is first split into two sequences, where the second sequence directly enters the SM mapper while the first sequence enters the SM mapper after passing through a four-state convolutional encoder and then a random block interleaver. The SM mapper chooses the active transmit antenna by modulating the coded bits of the first sequence and the constellation symbol by modulating the uncoded bits of the second sequence. In other words, the TCM technique is used in conjunction with SM to partition the transmit antennas into subsets by maximizing the spacing between antennas of

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the same subset and only the information bits that determine the transmit antenna number are convolutionally encoded. At the receiver, with an optimal SM decoder, a hard decision Viterbi decoder is employed for the coded bits; then combining with the demodulated uncoded bits gives an estimate of the original information bit sequence. It has been shown in [8] that this scheme does not provide any error performance advantage compared to uncoded SM in uncorrelated channel conditions; on the other hand, the scheme of [8] does exhibit improved performance in correlated channels. The reason for this behavior can be explained by the trellis coding gain which does not have an impact on the performance when all the channel paths are uncorrelated. Here, we propose a different design method to construct a trellis coded SM scheme which benefits from the advantages of trellis coding in both uncorrelated and correlated channels.

In this paper, a novel MIMO transmission scheme, called *spatial modulation with trellis coding* (SM-TC), which directly combines trellis coding and SM, is proposed. Similarly to conventional TCM, the trellis encoder and the SM mapper are jointly designed and a soft decision Viterbi decoder which is fed with the soft information supplied by the optimal SM decoder, is used at the receiver. The SM-TC mechanism, which switches between transmit antennas of a MIMO link, provides a type of virtual interleaving and offers an additional diversity gain, known as time diversity [11]. First, we derive the general conditional pairwise error probability (CPEP) for SM-TC and then, for quasi-static Rayleigh fading channels, by averaging over channel coefficients, we obtain the unconditional pairwise error probability (UPEP) of SM-TC for error events with path lengths two and three. Code design criteria are given for the SM-TC scheme, which are then used to obtain the best codes with optimized distance spectra for error events with path lengths two and three. New SM-TC schemes with 4, 8 and 16 states are proposed for $2, 3$ and 4 bits/s/Hz spectral efficiencies. It is shown via computer simulations that the proposed SM-TC schemes for uncorrelated and correlated Rayleigh fading channels provide significant error performance improvements over space-time trellis codes (STTCs), coded V-BLAST systems and the scheme given in [8] in terms of bit error rate (BER) and frame error rate (FER) yet with a lower decoding complexity. In addition to this, from an implementation point of view, unlike the STTCs, our scheme requires only one RF chain at the transmitter, even if we have a higher number of transmit antennas, and requires no synchronization between them.

The organization of the paper is as follows. In Section II, we give our system model and introduce the new SM-TC scheme. In Section III, the PEP upper bound for the SM-TC scheme is derived. Design criteria and design examples for SM-TC are presented in Section IV. Bit error probability (BEP) analysis of the new scheme is given in Section V. Simulation results and performance comparisons are given in Section VI. Finally, Section VII includes the main conclusions of the paper.

Notation: Bold, lowercase and capital letters are used for column vectors and matrices, respectively. $(.)^*$, $(.)^T$ and $(.)^H$ denote complex conjugation, transposition and Hermitian transposition, respectively. $\mathbf{A}(p,q)$ represents the entry on the pth row and qth column of $\mathbf{A}$. $\det(\mathbf{A})$ and $\operatorname{rank}(\mathbf{A})$ denote the determinant and rank of $\mathbf{A}$, respectively. $C(\mathbf{A})$ represents the column space of $\mathbf{A}$ and $\lambda_i^{\mathbf{A}}$ denotes ith eigenvalue of $\mathbf{A}$. $\mathbb{R}$ and $\mathbb{C}$ denote the fields of real and imaginary numbers, respectively. For a complex variable x, $\Re\{x\}$ denotes the real part of x. The probability of an event is denoted by $\Pr(\cdot)$. The probability density function (p.d.f.) of a random variable (r.v.) X is denoted by $f(x)$. $E\{\cdot\}$ stands for expectation. $X \sim \mathcal{N}(m_X, \sigma_X^2)$ denotes that the real r.v. X has the Gaussian distribution with mean m_X and variance σ_X^2. $X \sim \mathcal{CN}(0, \sigma_X^2)$ represents the distribution of a circularly symmetric complex Gaussian r.v. X. $Q(\cdot)$ denotes the tail probability of the standard Gaussian distribution. The number of elements in a set η is denoted as $n(\eta)$. χ represents a complex signal constellation of size M.

II. System Model

The considered SM-TC system model is shown in Fig. 1. The independent and identically distributed (i.i.d.) binary information sequence $\mathbf{u}$ is encoded by a rate $R = k/n$ trellis (convolutional) encoder whose output sequence $\mathbf{v}$ enters the SM mapper. The spatial modulator is designed in conjunction with the trellis encoder to transmit n coded bits in a transmission interval by means of the symbols selected from an M-level signal constellation such as M-ary phase-shift keying (M-PSK), M-ary quadrature amplitude modulation (M-QAM), etc., and of the antenna selected from a set of n_T transmit antennas such that $n = \log_2(Mn_T)$. The SM mapper first specifies the identity of the transmit antenna determined by the first $\log_2 n_T$ bits of the coded sequence $\mathbf{v}$. It than maps the remaining $\log_2 M$ bits of the coded sequence onto the signal constellation employed for transmission of the data symbols. Due to trellis coding, the overall spectral efficiency of the SM-TC would be k bits/s/Hz. The new signal generated by the SM is denoted by $x = (i, s)$ where $s \in \chi$ is the data symbol transmitted over the antenna labeled by $i \in \{1, 2, \cdots, n_T\}$. That is, the spatial modulator generates an $1 \times n_T$ signal vector $\begin{bmatrix} 0 & 0 & \cdots & s & 0 & \cdots & 0 \end{bmatrix}$ whose ith entry is s at the output of the n_T transmit antennas for transmission. The MIMO channel over which the spatially modulated symbols are transmitted, is characterized by an $n_T \times n_R$ matrix $\mathbf{H}$, whose entries are i.i.d. r.v.'s having the $\mathcal{CN}(0,1)$ distribution, where n_R denotes the number of receive antennas. We assume that $\mathbf{H}$ remains constant during the transmission of a frame and takes independent values from one frame to another. We further assume that $\mathbf{H}$ is perfectly known at the receiver, but is not known at the transmitter. The transmitted signal is corrupted by an n_R-dimensional additive complex Gaussian noise vector with i.i.d. entries distributed as $\mathcal{CN}(0, N_0)$. At the receiver, a soft decision Viterbi decoder, which is fed with the soft information supplied by the optimal SM decoder, is employed to provide an estimate $\hat{\mathbf{u}}$ of the input bit sequence.

Let us introduce the concept of SM-TC by an example for $k = 2$ bits/s/Hz with $n_T = 4$. Consider an $R = 2/4$ trellis encoder [12] with the octal generator matrix $\left[\begin{smallmatrix} 0 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \end{smallmatrix}\right]$, followed by the SM mapper. At each coding step, the first two coded bits determine the active transmit antenna over which the quadrature phase-shift keying (QPSK) symbol determined by the last two coded bits is transmitted. The corresponding trellis diagram is depicted in Fig. 2, where each branch is labeled

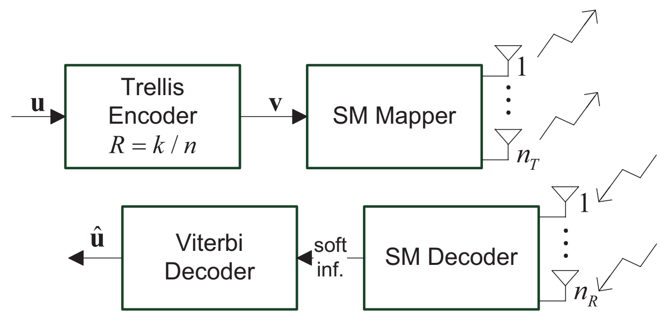

Fig. 1. SM-TC system model.

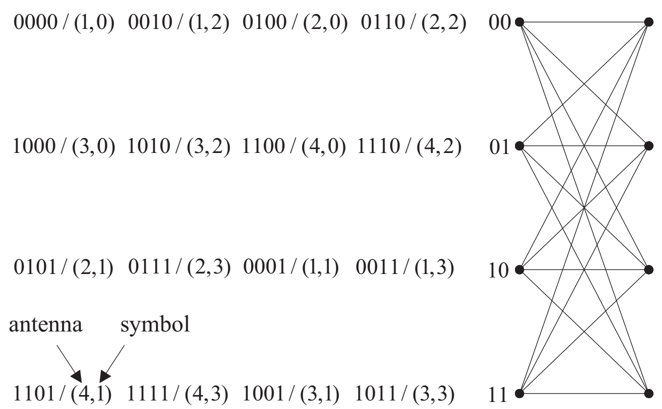

Fig. 2. Trellis diagram of the SM-TC scheme with $R = 2/4$ trellis encoder, four transmit antennas and QPSK symbols given by $\exp(j2\pi s/4)$.

by the corresponding output bits and SM symbol (i, s), where $i \in \{1, 2, 3, 4\}$ and $s \in \{0, 1, 2, 3\}$. Although this scheme can be considered as a generalization of [8], it differs from that of [8] in three basic ways. Firstly, to provide coding as well as diversity gain, all information bits are convolutionally encoded unlike in [8], in which only the information bits determining the corresponding antenna index are encoded. Thus, our joint encoding allows the operation of an optimum soft decoder at the receiver, and consequently improves the error performance of the system significantly. Secondly, an interleaver is not included in our scheme; however, we benefit from the SM-TC mechanism which acts as a virtual interleaver by switching between transmit antennas of a MIMO link to provide additional time diversity. Finally, a soft decision Viterbi decoder is employed at the receiver opposite to the hard decision Viterbi decoder of [8]. From these major differences in the operation of two schemes, we conclude that our scheme can be considered as being more directly inspired by Ungerboeck's TCM, in which the conventional M-PSK or M-QAM mapper of TCM is replaced by an SM mapper.

III. Pairwise-Error Probability (PEP) Derivation of the SM-TC Scheme

In this section, first, the CPEP of the SM-TC scheme is derived, and then for quasi-static Rayleigh fading channels, by averaging over channel fading coefficients, the UPEP of the SM-TC scheme is obtained for error events with path lengths two and three. For the sake of simplicity, one receive antenna is assumed; however, all results can be easily extended to any number of receive antennas. A pairwise error event of length N occurs when the Viterbi decoder decides in favor of the spatially modulated symbol sequence $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N)$ when $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ is transmitted, where $x_n = (i_n, s_n)$, $s_n \in \chi$ is the transmitted symbol over the i_nth antenna $(1 \leq i_n \leq n_T)$ at the nth transmission interval.

Let the received signal is given by

$$y_n = \alpha_n s_n + w_n \tag{1}$$

for $1 \leq n \leq N$, where α_n is the complex fading coefficient from the i_nth transmit antenna to the receiver at the nth transmission interval, and w_n is the noise sample with the $\mathcal{CN}(0, N_0)$ distribution. Let $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_N)$ and $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_N)$ denote the sequences of fading coefficients corresponding to transmitted and erroneously detected SM symbol sequences, $\mathbf{x}$ and $\hat{\mathbf{x}}$, respectively. The CPEP for this case is given by

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}} | \boldsymbol{\alpha}, \boldsymbol{\beta}) = \Pr\{ m(\mathbf{y}, \hat{\mathbf{x}}; \boldsymbol{\beta}) \geq m(\mathbf{y}, \mathbf{x}; \boldsymbol{\alpha}) | \mathbf{x}\} \tag{2}$$

where $m(\mathbf{y}, \mathbf{x}; \boldsymbol{\alpha}) = \sum_{n=1}^{N} m(y_n, s_n; \alpha_n) = -\sum_{n=1}^{N} |y_n - \alpha_n s_n|^2$ is the decision metric for $\mathbf{x}$, since y_n is Gaussian when conditioned on α_n and s_n. Then, (2) can be expressed as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}} | \boldsymbol{\alpha}, \boldsymbol{\beta}) = \Pr\left\{ \sum_{n=1}^{N} |y_n - \alpha_n s_n|^2 \geq \sum_{n=1}^{N} |y_n - \beta_n \hat{s}_n|^2 \middle| \mathbf{x} \right\}. \tag{3}$$

With simple manipulation (3) takes the form

$$\begin{aligned} \Pr(\mathbf{x} \to \hat{\mathbf{x}} | \boldsymbol{\alpha}, \boldsymbol{\beta}) &= \Pr\left\{ \sum_{n=1}^{N} |\alpha_n|^2 |s_n|^2 - 2\Re\{y_n \alpha_n^* s_n^*\} \geq \right. \\ &\quad \left. \sum_{n=1}^{N} |\beta_n|^2 |\hat{s}_n|^2 - 2\Re\{y_n \beta_n^* \hat{s}_n^*\} \middle| \mathbf{x} \right\} \\ &= \Pr\left\{ \sum_{n=1}^{N} -|\alpha_n s_n - \beta_n \hat{s}_n|^2 + 2\Re\{\tilde{w}_n\} \geq 0 \middle| \mathbf{x} \right\} \end{aligned} \tag{4}$$

where $\tilde{w}_n = w_n(\beta_n^* \hat{s}_n^* - \alpha_n^* s_n^*)$. Denoting the nth term of the sum in (4) by d_n, we obtain $\Pr(\mathbf{x} \to \hat{\mathbf{x}} | \boldsymbol{\alpha}, \boldsymbol{\beta}) = \Pr\left\{ \sum_{n=1}^{N} d_n \geq 0 \middle| \mathbf{x} \right\}$. Let $d = \sum_{n=1}^{N} d_n$ be the decision variable to be compared with the zero threshold. Since, $\tilde{w}_n$ is Gaussian with distribution $\mathcal{CN}\left(0, N_0 |\beta_n^* \hat{s}_n^* - \alpha_n^* s_n^*|^2\right)$, it is straightforward to show that d is also Gaussian with distribution $\mathcal{N}(m_d, \sigma_d^2)$ where, $m_d = -\sum_{n=1}^{N} |\alpha_n s_n - \beta_n \hat{s}_n|^2$ and $\sigma_d^2 = 2N_0 \sum_{n=1}^{N} |\alpha_n s_n - \beta_n \hat{s}_n|^2$. Finally, the CPEP of the SM-TC scheme is calculated from (4) as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}} | \boldsymbol{\alpha}, \boldsymbol{\beta}) = Q\left(\frac{-m_d}{\sigma_d}\right) = Q\left(\sqrt{\frac{\sum_{n=1}^{N} A_n}{2N_0}}\right) \tag{5}$$

where $A_n = |\alpha_n s_n - \beta_n \hat{s}_n|^2$. An error event with path length N is defined as an error event satisfying $A_n \neq 0$ and $A_n \neq A_m$ for $1 \leq n, m \leq N$, where it is possible to have $\alpha_n = \beta_n$ or $s_n = \hat{s}_n$ for some values of n due to the use of SM. Note that, during the SM-TC code design, for considered N values, we allow only the error events satisfying these conditions. This constraint, which will be explained in the sequel, is crucial for good code design. Using the bound $Q(x) \leq \frac{1}{2} e^{-x^2/2}$, the

CPEP of the SM-TC scheme can be upper bounded by

$$\Pr\left(\mathbf{x} \to \hat{\mathbf{x}} \middle| \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leq \frac{1}{2} \exp\left(-\frac{\gamma}{4} \sum\nolimits_{n=1}^{N} \left|\alpha_n s_n - \beta_n \hat{s}_n\right|^2\right) \tag{6}$$

where $\gamma = E_s/N_0 = 1/N_0$ is the average received signal-to-noise ratio (SNR). Note that, if $\alpha_n = \beta_n$ for all n, $1 \leq n \leq N$, the term in the sum of (6) reduces to $|\alpha_n|^2 |s_n - \hat{s}_n|^2$, which yields the CPEP of the conventional TCM scheme [13]. For an interleaver with infinite depth which transforms the quasi-static fading channel into a fast fading channel, the UPEP of TCM can be easily derived by averaging over the p.d.f. of $|\alpha_n|^2$, and the resulting design criteria for TCM are to maximize the effective length and the product distance of the TCM code [14]. However, the derivation of the UPEP for the considered SM-TC scheme in which an interleaver is not included, is quite complicated because of the varying statistical dependence between $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ through error events of path length N.

The CPEP upper bound of the SM-TC scheme, which is given in (6), can be alternatively rewritten in matrix form as

$$\Pr\left(\mathbf{x} \to \hat{\mathbf{x}} \middle| \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leq \frac{1}{2} \exp\left(-\frac{\gamma}{4} \mathbf{h}^H \mathbf{S} \mathbf{h}\right) \tag{7}$$

where $\mathbf{h} = \left[\, h_1 \;\, h_2 \;\, \cdots \;\, h_{n_T} \,\right]^T$ is the $n_T \times 1$ channel vector with $h_i, i = 1, 2, \cdots, n_T$ representing the channel fading coefficient from ith transmit antenna to the receiver, which is assumed to be constant through the error event. $\mathbf{S} = \sum_{n=1}^{N} \mathbf{S}_n$ where $\mathbf{S}_n$ is an $n_T \times n_T$ Hermitian matrix representing a realization of α_n and β_n which are related to the channel coefficients as $\alpha_n = h_{i_n}$, $\beta_n = h_{j_n}$, i_n and $j_n \in \{1, 2, \cdots, n_T\}$ being the transmitted and detected antenna indices, respectively. The entries of the matrix $\mathbf{S}_n$, $n = 1, 2, \cdots, N$ are given as follows:
For $i_n \neq j_n$

$$\mathbf{S}_n(p, q) = \begin{cases} |s_n|^2, & \text{if } p = q = i_n \\ |\hat{s}_n|^2, & \text{if } p = q = j_n \\ -s_n^* \hat{s}_n, & \text{if } p = i_n, q = j_n \\ -s_n \hat{s}_n^*, & \text{if } p = j_n, q = i_n \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

and for $i_n = j_n$

$$\mathbf{S}_n(p, q) = \begin{cases} d_{E_n}^2, & \text{if } p = q = i_n \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

where $d_{E_n}^2 = |s_n - \hat{s}_n|^2$. As an example, for $n_T = 4$ with $\alpha_n = h_1$ and $\beta_n = h_3$ (i.e., $i_n = 1$ and $j_n = 3$) $\mathbf{S}_n$ is obtained as

$$\mathbf{S}_n = \begin{bmatrix} |s_n|^2 & 0 & -s_n^* \hat{s}_n & 0 \\ 0 & 0 & 0 & 0 \\ -s_n \hat{s}_n^* & 0 & |\hat{s}_n|^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{10}$$

In order to obtain the UPEP of the SM-TC scheme, (7) should be averaged over the multivariate complex Gaussian p.d.f. of $\mathbf{h}$ which is given as [15] $f(\mathbf{h}) = (1/\pi^{n_T})\, e^{-\mathbf{h}^H \mathbf{h}}$ since the entries of $\mathbf{h}$ are i.i.d. with p.d.f. $\mathcal{CN}(0, 1)$. The UPEP upper bound of the SM-TC is calculated from (7) as

$$\begin{aligned} \Pr\left(\mathbf{x} \to \hat{\mathbf{x}}\right) &\leq \frac{1}{2} \int_{\mathbf{h}} \pi^{-n_T} \exp\left(-\frac{\gamma}{4} \mathbf{h}^H \mathbf{S} \mathbf{h}\right) \exp\left(-\mathbf{h}^H \mathbf{h}\right) d\mathbf{h} \\ &= \frac{1}{2} \int_{\mathbf{h}} \pi^{-n_T} \exp\left(-\mathbf{h}^H \boldsymbol{\Sigma}^{-1} \mathbf{h}\right) d\mathbf{h} \end{aligned} \tag{11}$$

where $\boldsymbol{\Sigma}^{-1} = \left[\frac{\gamma}{4}\mathbf{S} + \mathbf{I}\right]$ and $\mathbf{I}$ is the $n_T \times n_T$ identity matrix. Since $\boldsymbol{\Sigma}$ is a Hermitian and positive definite complex covariance matrix, the integrand of the multivariate complex Gaussian p.d.f. given in (11) yields the following result [15]:

$$\Pr\left(\mathbf{x} \to \hat{\mathbf{x}}\right) \leq \frac{1}{2} \det\left(\boldsymbol{\Sigma}\right) = \frac{1}{2 \det\left(\frac{\gamma}{4}\mathbf{S} + \mathbf{I}\right)}. \tag{12}$$

Although (12) gives an effective and simple way to evaluate the UPEP upper bound of the SM-TC scheme in closed form, for an error event with path length N, the matrix $\mathbf{S}$ has $(n_T)^{2N}$ possible realizations which correspond to all of the possible transmitted and detected antenna indices along this error event. However, as we will show in the sequel, due to the special structure of $\mathbf{S}$, these $(n_T)^{2N}$ possible realizations can be grouped into a small number of distinct types having the same UPEP upper bound, and the resulting upper bound calculated from (12) is mainly determined by the number of degrees of freedom of the error event which is defined as follows:

Definition 1: For an error event with path length N, the number of degrees of freedom (DOF) is defined as the total number of different channel fading coefficients in $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ sequences. It can be easily shown that DOF $\leq 2N$.

For example, for $N = 2$, $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ and $\boldsymbol{\beta} = (\beta_1, \beta_2)$, DOF $= 3$ if $\alpha_1 = \beta_1 \neq \alpha_2 \neq \beta_2$. Besides the DOF, a second fact, which is explained as follows, determines the result of (12). Let us rewrite (6) as

$$\begin{aligned} \Pr\left(\mathbf{x} \to \hat{\mathbf{x}} \middle| \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leq \frac{1}{2} \exp\Bigg(-\frac{\gamma}{4} \Big[\sum\nolimits_{\eta} |\alpha_n|^2 |s_n - \hat{s}_n|^2 \\ + \sum\nolimits_{\tilde{\eta}} |\alpha_n s_n - \beta_n \hat{s}_n|^2\Big]\Bigg) \end{aligned} \tag{13}$$

where η and $\tilde{\eta}$ are the sets of all n for which $\alpha_n = \beta_n$ and $\alpha_n \neq \beta_n$, respectively, and $n(\eta) + n(\tilde{\eta}) = N$. The first term in (13) corresponds to the TCM term while the second term corresponds to the SM term. Note that in some cases, the same DOF value can be supported with different $n(\eta)$ and $n(\tilde{\eta})$ values, and this also affects the result of (12).

In the following, for the aforementioned distinct cases, we calculate the UPEP upper bound of the SM-TC scheme from (12), for error event path lengths $N = 2$ and 3. For the sake of simplicity, we assume a constant envelope M-PSK constellation such that $|s_n|^2 = |\hat{s}_n|^2 = 1$; however, all results can be easily extended to varying envelope constellations.

A. Error Events with Path Length Two

We consider the distinct types of $\mathbf{S}$ for $N = 2$ and we present the UPEP upper bound of the SM-TC from (12) for the following seven different types of error events. Without loss of generality, we assume $n_T = 4$ which supports the maximum DOF value for $N = 2$. However, all results are also valid for $n_T > 4$.

Type 1: DOF $= 1$, $n(\eta) = 2, n(\tilde{\eta}) = 0$. For this type, the matrix $\mathbf{S}$ has only one non-zero element $\mathbf{S}(i_1, i_1) = d_{E_1}^2 + d_{E_2}^2$, and the resulting UPEP is calculated from (12) as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}})_1 \leq \frac{2}{4 + \gamma\left(d_{E_1}^2 + d_{E_2}^2\right)}. \tag{14}$$

Type 2: DOF $= 2$ and $n(\eta) = 2, n(\tilde{\eta}) = 0$. For this type, the matrix $\mathbf{S}$ has two non-zero elements $\mathbf{S}(i_1, i_1) = d_{E_1}^2$ and $\mathbf{S}(i_2, i_2) = d_{E_2}^2$, and the resulting UPEP is calculated from (12) as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}})_2 \leq \frac{8}{\left(4 + \gamma d_{E_1}^2\right)\left(4 + \gamma d_{E_2}^2\right)}. \tag{15}$$

Type 3: DOF $= 2$ and $n(\eta) = 1, n(\tilde{\eta}) = 1$. For representational simplicity, without loss of generality, let us assume $i_1 = j_1 = i_2 = 1, j_2 = 2$. Then, we obtain

$$\mathbf{S} = \begin{bmatrix} 1 + d_{E_1}^2 & -s_2^* \hat{s}_2 & 0 & 0 \\ -s_2 \hat{s}_2^* & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{16}$$

Simple manipulation gives the UPEP upper bound of SM-TC from (12) as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}})_3 \leq \frac{8}{16 + 4\left(2 + d_{E_1}^2\right)\gamma + d_{E_1}^2 \gamma^2} \tag{17}$$

which can easily shown to be independent of the values of i_1, j_1, i_2 and j_2 if $i_1 = j_1 = i_2 \neq j_2$ or $i_1 = j_1 = j_2 \neq i_2$ due to the special structure of $\mathbf{S}$. Note that, for $i_2 = j_2 = i_1 \neq j_1$ or $i_2 = j_2 = j_1 \neq i_1$, $d_{E_1}^2$ should be replaced by $d_{E_2}^2$ in (17). The matrix of (16) is only one of 48 possible realizations of $\mathbf{S}$ for this type of error events, however, since DOF and $n(\eta)$ values remains unchanged for all realizations, the resulting UPEP bound is unique and given by (17). It is straightforward to show that for the case of $n_T > 4$ the UPEP bound remains unchanged for the same type of error events due to the structure of $\mathbf{S}$.

Type 4: DOF $= 3$ and $n(\eta) = 1, n(\tilde{\eta}) = 1$. Without loss of generality, let us assume $i_1 = j_1 = 1, i_2 = 2, j_2 = 3$. For this case, we obtain

$$\mathbf{S} = \begin{bmatrix} d_{E_1}^2 & 0 & 0 & 0 \\ 0 & 1 & -s_2^* \hat{s}_2 & 0 \\ 0 & -s_2 \hat{s}_2^* & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{18}$$

With simple manipulation, we obtain the UPEP upper bound of SM-TC from (12) as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}})_4 \leq \frac{4}{8 + 2\left(2 + d_{E_1}^2\right)\gamma + d_{E_1}^2 \gamma^2} \tag{19}$$

which is the generic UPEP bound for 48 possible realizations of $\mathbf{S}$ for this type of error event.

Type 5: DOF $= 2$, $n(\eta) = 0, n(\tilde{\eta}) = 2$ and we always have $i_n \neq j_n$ for $n = 1, 2$. With the assumption $i_1, j_1, i_2, j_2 \in \{1, 2\}$, the entries of the matrix $\mathbf{S}$ are obtained as follows: $\mathbf{S}(1,1) = \mathbf{S}(2,2) = 2$,

$$\mathbf{S}(1,2) = \begin{cases} -s_1^* \hat{s}_1 - s_2^* \hat{s}_2 & \text{if } i_1 < j_1 \text{ and } i_2 < j_2 \\ -s_1^* \hat{s}_1 - s_2 \hat{s}_2^* & \text{if } i_1 < j_1 \text{ and } i_2 > j_2 \\ -s_1 \hat{s}_1^* - s_2^* \hat{s}_2 & \text{if } i_1 > j_1 \text{ and } i_2 < j_2 \\ -s_1 \hat{s}_1^* - s_2 \hat{s}_2^* & \text{if } i_1 > j_1 \text{ and } i_2 > j_2. \end{cases} \tag{20}$$

and $\mathbf{S}(2,1) = \mathbf{S}^*(1,2)$. After much simplification, we obtain the UPEP upper bound from (12) as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}})_5 \leq \frac{4}{8 + 8\gamma + (1 - \cos\theta)\gamma^2} \tag{21}$$

where $\theta = \pm\Delta\theta_1 \pm \Delta\theta_2, \Delta\theta_n = \theta_n - \hat{\theta}_n, n = 1, 2$ and $s_1 = e^{j\theta_1}, \hat{s}_1 = e^{j\hat{\theta}_1}, s_2 = e^{j\theta_2}, \hat{s}_2 = e^{j\hat{\theta}_2}$, with $\theta_1, \hat{\theta}_1, \theta_2, \hat{\theta}_2 \in \left\{\frac{2\pi r}{M}, r = 0, \cdots, M-1\right\}$. Interestingly, we observe from (21) that the UPEP bound is dependent on the transmitted and detected M-PSK symbols along the error event. This can be explained by the cross terms coming from $\mathbf{S}(1,2)$ and $\mathbf{S}(2,1)$ which results from this type of error events, of which there are a total of 24. Note that our previous error event definition ensures that $\cos\theta \neq 1$.

Type 6: DOF $= 3$ and $n(\eta) = 0, n(\tilde{\eta}) = 2$. If we assume that $i_1 = i_2 = 1, j_1 = 2$ and $j_2 = 3$, we have

$$\mathbf{S} = \begin{bmatrix} 2 & -s_1^* \hat{s}_1 & -s_2^* \hat{s}_2 & 0 \\ -s_1 \hat{s}_1^* & 1 & 0 & 0 \\ -s_2 \hat{s}_2^* & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{22}$$

Simple manipulation gives the UPEP bound for this type error events from (12) as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}})_6 \leq \frac{8}{16 + 16\gamma + 3\gamma^2} \tag{23}$$

which is the generic UPEP upper bound for 96 different realizations of $\mathbf{S}$ for this case.

Type 7: DOF $= 4$ and $n(\eta) = 0, n(\tilde{\eta}) = 2$. Let us assume $i_1 = 1, j_1 = 2, i_2 = 3$ and $j_2 = 4$, then we obtain

$$\mathbf{S} = \begin{bmatrix} 1 & -s_1^* \hat{s}_1 & 0 & 0 \\ -s_1 \hat{s}_1^* & 1 & 0 & 0 \\ 0 & 0 & 1 & -s_2^* \hat{s}_2 \\ 0 & 0 & -s_2 \hat{s}_2^* & 1 \end{bmatrix}. \tag{24}$$

The UPEP upper bound for this type of error events is found from (12) as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}})_7 \leq \frac{2}{4 + 4\gamma + \gamma^2} \tag{25}$$

which is the generic UPEP upper bound for 24 different realizations of $\mathbf{S}$ for this case.

The seven different types of error events presented above cover all possible 256 realizations of $\mathbf{S}$ for $N = 2$ and $n_T = 4$. We observe from the UPEP bounds obtained above that for given $n(\eta)$ and $n(\tilde{\eta})$ values, the UPEP bound decreases with increasing DOF, which should be considered in SM-TC code design. We also observe that, for small θ values, Type 5 provides the worst UPEP bound for DOF $= 2$, while we have to avoid the error events of Type 1 to maintain the diversity order of the SM-TC scheme, which is two for $N = 2$ when

TABLE I
UPEP VALUES FOR ERROR EVENTS WITH PATH LENGTH THREE

$n(\eta)$	DOF	PEP
0	2^*	$\frac{4}{8+12\gamma+(3-c)\gamma^2}$
0	3	$\frac{16}{32+48\gamma+18\gamma^2+\left(1-\cos\tilde{\theta}\right)\gamma^3}$
0	3^*	$\frac{16}{32+48\gamma+4(4-\cos\theta)\gamma^2+(1-\cos\theta)\gamma^3}$
0	4	$\frac{8}{16+24\gamma+9\gamma^2+\gamma^3}$
0	4^*	$\frac{8}{16+24\gamma+2(5-\cos\theta)\gamma^2+(1-\cos\theta)\gamma^3}$
0	5	$\frac{16}{32+48\gamma+22\gamma^2+3\gamma^3}$
0	6	$\frac{4}{8+12\gamma+6\gamma^2+\gamma^3}$
1	2^*	$\frac{4}{8+2\left(4+d_{E_n}^2\right)\gamma+\left(1+d_{E_n}^2-\cos\theta\right)\gamma^2}$
1	3	$\frac{32}{64+16\left(4+d_{E_n}^2\right)\gamma+4\left(3+2d_{E_n}^2\right)\gamma^2+d_{E_n}^2\gamma^3}$
1	3^*	$\frac{16}{32+8\left(4+d_{E_n}^2\right)\gamma+4\left(1+2d_{E_n}^2-\cos\theta\right)\gamma^2+(1-\cos\theta)d_{E_n}^2\gamma^3}$
1	4	$\frac{32}{64+16\left(4+d_{E_n}^2\right)\gamma+4\left(3+4d_{E_n}^2\right)\gamma^2+3d_{E_n}^2\gamma^3}$
1	5	$\frac{8}{16+4\left(4+d_{E_n}^2\right)\gamma+4\left(1+d_{E_n}^2\right)\gamma^2+d_{E_n}^2\gamma^3}$
2	2	$\frac{8}{16+8\gamma+4d_1\gamma+d_1\gamma^2}$
2	3	$\frac{32}{64+32\gamma+16d_1\gamma+4\left(d_{E_n}^2+2d_{E_m}^2+d_{E_n}^2d_{E_m}^2\right)\gamma^2+d_{E_n}^2d_{E_m}^2\gamma^3}$
2	4	$\frac{16}{32+16\gamma+8d_1\gamma+4d_1\gamma^2+3d_{E_n}^2d_{E_m}^2\gamma^3}$
3	1	$\frac{2}{4+d_2\gamma}$
3	2	$\frac{8}{16+4d_2\gamma+d_{E_3}^2\left(d_{E_1}^2+d_{E_2}^2\right)\gamma^2}$
3	3	$\frac{32}{64+16d_2\gamma+4\left(d_{E_1}^2d_{E_2}^2+d_{E_1}^2d_{E_3}^2+d_{E_2}^2d_{E_3}^2\right)\gamma^2+d_{E_1}^2d_{E_2}^2d_{E_3}^2\gamma^3}$

DOF ≥ 2. This could be simply verified from the UPEP bounds given above in which UPEP is proportional to γ^{-2} for $\gamma \gg 1$.

B. Error Events with Path Length Three

In this subsection, we deal with error events with path length $N = 3$. Considering DOF with $n(\eta)$ and $n(\tilde{\eta})$ values, we observe 18 different types of error events. In Table I, we give resulting UPEP upper bounds of SM-TC from (12), for these different types where $c = \cos(\pm\Delta\theta_1 \pm \Delta\theta_2) + \cos(\pm\Delta\theta_1 \pm \Delta\theta_3) + \cos(\pm\Delta\theta_2 \pm \Delta\theta_3)$, $\tilde{\theta} = \pm\Delta\theta_1 \pm \Delta\theta_2 \pm \Delta\theta_3$, $d_1 = d_{E_n}^2 + d_{E_m}^2$, $d_2 = d_{E_1}^2 + d_{E_2}^2 + d_{E_3}^2$ and $n, m \in \{1,2,3\}$, $n \neq m$. The asterisk for DOF values means the considered error event includes a sub-error event with length two of Type 5, and therefore, the resulting UPEP upper bound is dependent on θ which is defined in (21). As seen from Table I, a diversity order of three is achieved by the SM-TC scheme when DOF ≥ 3. We give the following theorem, which generalizes the concepts developed above.

Theorem 1: In case of an error event with path length N, in order to achieve a diversity order of N (a UPEP upper bound of a/γ^N for $\gamma \gg 1$ and $a \in \mathbb{R}^+$), a necessary condition is DOF $\geq N$.

Proof: Let us define $\mathbf{A} = \mathbf{\Sigma}^{-1} = \left[\frac{\gamma}{4}\mathbf{S} + \mathbf{I}\right] = [\mathbf{B} + \mathbf{I}]$; then from (12), we obtain $\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \leq 1/(2\det(\mathbf{A}))$. Since $\mathbf{A}$ is a positive definite Hermitian matrix with all real and positive eigenvalues, $\det(\mathbf{A}) = \prod_{i=1}^{n_T} \lambda_i^{\mathbf{A}}$. However, $\mathbf{B}$ is not generally of full rank, i.e., $\operatorname{rank}(\mathbf{B}) = b \leq n_T$. Since $\mathbf{A} = \mathbf{B} + \mathbf{I}$, from the properties of eigenvalues, we obtain $\lambda_i^{\mathbf{A}} = \lambda_i^{\mathbf{B}} + 1$, for $i = 1, \cdots, b$ and $\lambda_i^{\mathbf{A}} = 1$, for $i > b$, which yields $\det(\mathbf{A}) = \prod_{i=1}^{b}\left(\lambda_i^{\mathbf{B}} + 1\right) = \prod_{i=1}^{b}\left(\frac{\gamma}{4}\lambda_i^{\mathbf{S}} + 1\right)$. Ignoring the second term at high SNR values, we obtain the UPEP upper bound of SM-TC as $\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \leq \left(2\left(\frac{\gamma}{4}\right)^b \prod_{i=1}^{b} \lambda_i^{\mathbf{S}}\right)^{-1}$ which states that the diversity order of the system is determined by the rank of $\mathbf{S}$ since γ decays with $-b$. According to the subadditivity of rank [16], $b = \operatorname{rank}(\mathbf{S}) \leq \sum_{i=1}^{N} \operatorname{rank}(\mathbf{S}_i) = N$, since $\operatorname{rank}(\mathbf{S}_i) = 1$ for all i. Since DOF is the total number of columns (or rows) of $\mathbf{S}$ with not all zero entries, we also have $b \leq$ DOF. In order to prove the theorem, we have to show that $b = N$ only for DOF $\geq N$. To satisfy $b = N$, we have to show that the rank subadditivity inequality given above holds. It can be shown that only for DOF $\geq N$, can the intersection of column (or equivalently row) spaces of $\tilde{\mathbf{S}} = \sum_{i=1}^{M} \mathbf{S}_i$ and $\mathbf{S}_j$, $j = M+1, \cdots, N$ be the null space, i.e., $C(\tilde{\mathbf{S}}) \cap C(\mathbf{S}_j) = \emptyset$ for all $M < N$ (see the example below), which is a sufficient condition for rank additivity [16]. This completes the proof. ■

Note that if an error event of path length N includes a critical sub-error event, which is defined as an error event with path length $N' \leq N$ and DOF$' \leq$ DOF such that $n(\tilde{\eta}) = N' =$ DOF$'$, then the UPEP is upper bounded by $a/[(1-\cos\tilde{\theta})\gamma^N]$ for $\gamma \gg 1$ where $\tilde{\theta} = \pm\Delta\theta_1 \pm \Delta\theta_2 \pm \cdots \pm \Delta\theta_{N'}$. Therefore, for the values of $\tilde{\theta} = 0$, the diversity order b becomes smaller than N (which can be shown to be $b = N - 1$) since the rank additivity does not hold. This is why the condition in Theorem 1 is only necessary but not sufficient. Type 5 of Section III.A and the second type given in Table I are the critical error events for $N = 2$ and $N = 3$, respectively. Our code design ensures $\tilde{\theta} \neq 0$ for $N' = 2$ and 3 since we deal with error events with path lengths $N = 2$ and 3, while it is rare to have such error events including critical sub-error events with higher N' and N values, for which even if $\tilde{\theta} = 0$ could not destroy the overall diversity order of the SM-TC system. Therefore, in our code design, we always aim to guarantee DOF $\geq N$.

According to Theorem 1, to achieve a diversity order of N, DOF could not be smaller than N. The following example illustrates this fact.

Example 1: Suppose we have $\mathbf{S}_1$ and $\mathbf{S}_2$ for $N = 2$ with DOF $= 2$ and $i_1, j_1, i_2, j_2 \in \{1,2\}$. Since $\operatorname{rank}(\mathbf{S}_1) = \operatorname{rank}(\mathbf{S}_2) = 1$, the columns of $\mathbf{S}_1$ and $\mathbf{S}_2$ span two disjoint subspaces of $\mathbb{C}^{n_T}$ with dimension one (when $\theta \neq 0$ if this error event belongs to Type 5). Consequently, rank$(\tilde{\mathbf{S}}) = 2$, where $\tilde{\mathbf{S}} = \mathbf{S}_1 + \mathbf{S}_2$, and the columns of $\tilde{\mathbf{S}}$ span a subspace of $\mathbb{C}^{n_T}$ such that the vectors in this column space all have the form $(x, y, 0, \cdots, 0)^T \in \mathbb{C}^{n_T}$. Therefore, to achieve a diversity order of three, we not only need a third matrix $\mathbf{S}_3$, but also this matrix must satisfy DOF ≥ 3 by using at least one new row (and column) in $\tilde{\tilde{\mathbf{S}}}$ where $\tilde{\tilde{\mathbf{S}}} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$, and in this way, we ensure $C(\mathbf{S}_3) \cap C(\tilde{\mathbf{S}}) = \emptyset$ and according to the rank additivity we obtain rank$(\tilde{\tilde{\mathbf{S}}}) = \sum_{i=1}^{3} \operatorname{rank}(\mathbf{S}_i) = 3$. If we assume that DOF $= 3$ with $\mathbf{S}_3$, the same procedure is applied for $\mathbf{S}_4$, and so on. On the other hand, if we have DOF $= 4$ with $\mathbf{S}_3$, with the matrix $\mathbf{S}_4$, we not need to increase DOF, since the dimension of $C(\tilde{\tilde{\mathbf{S}}})$ is again equal to three and therefore, $C(\mathbf{S}_4) \cap C(\tilde{\tilde{\mathbf{S}}}) = \emptyset$ when $C(\mathbf{S}_4) \cap C(\tilde{\mathbf{S}}) = \emptyset$ and rank additivity holds.

The following remark generalizes the UPEP upper bound of SM-TC for any number of receive antennas.

Remark 1: For n_R receive antennas, with simple manipulation, it can be shown that for $\gamma \gg 1$, the UPEP upper bound of SM-TC is given as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \leq \frac{1}{2\left(\frac{\gamma}{4}\right)^{bn_R}\left(\prod_{i=1}^{b} \lambda_i^{\mathbf{S}}\right)^{n_R}} \tag{26}$$

which implies that the diversity order of the SM-TC is bn_R as expected.

For the analysis of the SM-TC scheme under correlated channel conditions, we consider a spatial correlation (SC) channel model [17] in which the correlated channel matrix can be determined by $\mathbf{H}_{corr} = \mathbf{R}_t^{1/2}\mathbf{H}\mathbf{R}_r^{1/2}$ where $\mathbf{R}_t = [r_{ij}]_{n_T \times n_T}$, $\mathbf{R}_r = [r_{ij}]_{n_R \times n_R}$ are the SC matrices at the transmitter and the receiver, respectively, and $\mathbf{H}$ is the MIMO channel coefficient matrix described earlier. Note that in our computer simulations, we have taken an exponential correlation matrix model in which $r_{ij} = r_{ji}^* = r^{|j-i|}$ and $|r| < 1$ where r is the amount of correlation. The UPEP of the SM-TC scheme can be obtained for the correlated case, by averaging (7) over the correlated complex Gaussian p.d.f. of $\mathbf{h}$ given by $f(\mathbf{h}) = (\pi^{-n_T}/\det(\mathbf{K}))\, e^{-\mathbf{h}^H\mathbf{K}^{-1}\mathbf{h}}$, where $\mathbf{K} = E\{\mathbf{h}\mathbf{h}^H\}$ is the full-rank channel correlation matrix, assuming one receive antenna. Then, similar to the derivations in (11-12), after some algebra, the UPEP of the SM-TC is obtained as follows:

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \leq \frac{1}{2\det\left(\frac{\gamma}{4}\mathbf{K}\mathbf{S} + \mathbf{I}\right)}. \tag{27}$$

Note that, $\text{rank}(\mathbf{KS}) = \text{rank}(\mathbf{S})$, since the rank of $\mathbf{S}$ is not changed upon multiplication by a nonsingular matrix. Consequently, we conclude that spatial correlation does not have any effect on the diversity order of the SM-TC scheme. However, it effects the asymptotic coding gain through eigenvalues of the matrix $\mathbf{KS}$.

IV. SM-TC Code Design Criteria and Design Examples

In this section, we give design criteria for SM-TC and provide some code design examples for spectral efficiencies $k = 2, 3$ and 4 bits/s/Hz.

By considering the UPEP analysis of the previous section, the following design criteria are derived for the SM-TC scheme:

1) *Diversity gain criterion*: For a trellis code with minimum error event length N, to achieve a diversity order of N, DOF must be greater than or equal to N for all error events with path length greater than or equal to N.
2) *Coding gain criterion*: After ensuring maximum diversity gain, the distance spectrum of the SM-TC should be optimized by considering the UPEP values calculated from (12).

As an application of the SM-TC design criteria, we consider two 4-state trellis codes for $k = 2$ bits/s/Hz. The first code, called *Code A*, is chosen as the trellis code shown in Fig. 2 with the generator matrix $\left[\begin{smallmatrix}0&3&0&1\\1&0&2&0\end{smallmatrix}\right]$. This code has 96 different error events with $N = 2$. The second code, called *Code B*, has the generator matrix $\left[\begin{smallmatrix}3&1&1&0\\1&2&0&1\end{smallmatrix}\right]$. The distance spectra of these two codes are given in Table II for $N = 2$ for high SNR values $(\gamma \gg 1)$ with corresponding multiplicity values. Both of the codes are designed to optimize the UPEP of the error events with $N = 2$. However, they cannot guarantee to satisfy DOF ≥ 3 for $N = 3$, while they always ensure DOF ≥ 2. In Fig. 3, the BER and FER performance of these two codes, obtained by Monte Carlo simulations, is compared for a frame length of 40 bits with respect to the received SNR for one and two receive antennas. As seen from Fig. 3, although the worst case UPEP of *Code A* is higher than that of *Code B*, there is substantial error performance gap between these two codes in favor of *Code A*, due to its improved distance spectrum.

TABLE II
Distance Spectra of Two SM-TC Schemes with $N = 2$ $(\gamma \gg 1)$

	$1/\gamma^2$	$2/\gamma^2$	$8/3\gamma^2$	$4/\gamma^2$
Code A	16	32	32	16
Code B	0	0	96	0

TABLE III
Octal Generator Matrices of SM-TC Schemes with Different Numbers of States

State	$k = 2$ bits/s/Hz	$k = 3$ bits/s/Hz	$k = 4$ bits/s/Hz
4	$\begin{bmatrix}0&3&0&1\\1&0&2&0\end{bmatrix}$	-	-
8	$\begin{bmatrix}0&2&4&2\\3&4&0&1\end{bmatrix}$	$\begin{bmatrix}0&2&1&0&1&0\\0&1&2&0&0&1\\1&0&0&2&0&0\end{bmatrix}$	-
16	$\begin{bmatrix}5&1&3&0\\1&4&0&3\end{bmatrix}$	$\begin{bmatrix}0&4&2&0&2&0\\0&2&0&4&0&2\\3&0&5&0&1&1\end{bmatrix}$	$\begin{bmatrix}0&2&0&1&0&0\\1&0&2&0&0&0\\0&1&0&0&2&0\\0&0&1&0&0&2\end{bmatrix}$

In Table III, we give the octal generator matrices of the SM-TC codes for $k = 2, 3$ and 4 bits/s/Hz spectral efficiencies and different numbers of states. All codes are designed manually according to the SM-TC design criteria given above. For 2 bits/s/Hz transmission, we use four transmit antennas with a QPSK constellation, while for 3 and 4 bits/s/Hz, we use eight transmit antennas with an 8-PSK constellation since, $R = 2/4$, $R = 3/6$ and $R = 4/6$ trellis encoders are employed to obtain $k = 2, 3$ and 4 bits/s/Hz, respectively. For 2 bits/s/Hz, we extended our 4-state code to eight states and optimized its distance spectrum to maximize the number of error events with DOF > 2. On the other hand, our 16-state code is designed such that the error events with $N \geq 3$ ensure DOF ≥ 3, and therefore, diversity order of three is achieved. For 3 bits/s/Hz, in the same manner as performed previously, we optimized the distance spectra of 8- and 16-state codes, guaranteed DOF ≥ 2 and maximized the number of error events with DOF > 2. Finally for 4 bits/s/Hz, we constructed a 16-state code that also guarantees DOF ≥ 2. In all of our SM-TC constructions, we assigned SM symbols to the branches of the trellis in such a way that a catastrophic code is avoided. We also guarantee

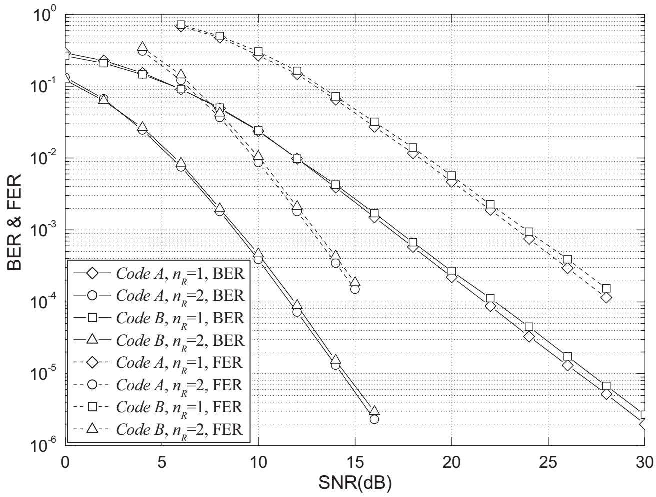

Fig. 3. Error performances of two 4-state SM-TC schemes ($k = 2$ bits/s/Hz).

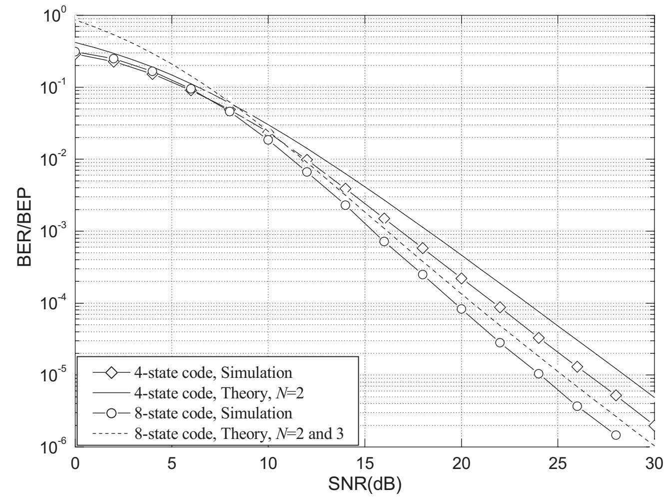

Fig. 4. Comparison of theoretical BEP curves with simulation results for 4- and 8-state codes at 2 bits/s/Hz.

DOF ≥ 2 for higher values of N to maintain the diversity order of the system.

V. Evaluation of Bit Error Probability Performance

In this section, we evaluate the approximate BEP performance of our SM-TC scheme. Due to the dependence between the terms of (6), the use of transfer function based upper bounding techniques is not feasible for SM-TC. Therefore, instead of using the transfer function technique which considers error events with all lengths, we are restricted to an approximation of the average BEP, which considers the error events with lengths up to a finite value, given as [19]

$$P_b \approx \frac{1}{c}\sum\nolimits_{\mathbf{x}}\left[\frac{1}{k}\sum\nolimits_{\substack{\hat{\mathbf{x}}\\ \hat{\mathbf{x}}\neq\mathbf{x}}} e\left(\mathbf{x},\hat{\mathbf{x}}\right)\Pr\left(\mathbf{x}\rightarrow\hat{\mathbf{x}}\right)\right] \quad (28)$$

where k is the number of input bits per trellis transition, $e\left(\mathbf{x},\hat{\mathbf{x}}\right)$ is the number of bit errors associated with each error event, and c is the total number of different realizations of $\mathbf{x}$. Because of the non-uniformity of SM-TC codewords (i.e., the UPEP is dependent on the transmitted codeword) (28) takes into account all possible codewords for a given error event path length. As an example, consider the *Code A* of the previous section. Due to the design symmetry, we consider only the path pairs originating from the first state. If we consider $N = 2$, there are 16 error events of Types 3 and 6, and 8 error events of Types 4 and 5, respectively. Each path-pair corresponding to the error events of Types 3, 4 and 5 contributes one bit error, while this value is equal to two for Type 6. From (28), we obtain

$$P_b \approx \frac{1}{32}\big[16\Pr\left(\mathbf{x}\rightarrow\hat{\mathbf{x}}\right)_3 + 8\Pr\left(\mathbf{x}\rightarrow\hat{\mathbf{x}}\right)_4 + 8\Pr\left(\mathbf{x}\rightarrow\hat{\mathbf{x}}\right)_5 + 32\Pr\left(\mathbf{x}\rightarrow\hat{\mathbf{x}}\right)_6\big] \quad (29)$$

which is plotted in Fig. 4 and compared with the simulation results for one receive antenna. In the same figure, we also show the BEP approximation for an 8-state code at 2 bits/s/Hz for $N = 2$ and 3. As seen from Fig. 4, the BEP expressions provide a reasonable approximation to the actual BEP of the SM-TC schemes, and we conclude that (28) can be used as an effective tool to investigate the approximate BEP performance behavior of our scheme for different configurations.

VI. Simulation Results and Comparisons

In this section, we present simulation results for the SM-TC scheme with different configurations and make comparisons with corresponding reference systems. The BER and FER performance of these schemes was evaluated via Monte Carlo simulations for various spectral efficiencies and numbers of states as a function of the average SNR per receive antenna. In all cases, we assumed that channel fading coefficients remain unchanged for 20 consecutive transmissions, which corresponds to a frame length of $20k$ bits for both the SM-TC and the reference systems operating at $k = 2, 3$ and 4 bits/s/Hz spectral efficiencies.

A. Comparisons with the STTCs

In this subsection, we compare via computer simulations, the error performance of the proposed SM-TC scheme with that of STTCs for 2 and 3 bits/s/Hz spectral efficiencies. Note that STTCs have been designed and optimized for quasi-static fading channels and can be considered as TCM schemes for MIMO systems [21]. We first performed simulations for 2 bits/s/Hz transmission with one and two receive antennas. In Figs. 5 and 6, the BER and FER performance of the proposed 4-, 8- and 16-state SM-TC schemes is compared with 4-, 8- and 16-state optimal QPSK STTCs with two transmit antennas, respectively. As seen from Figs. 5 and 6, the SM-TC schemes provide significant improvements in both BER and FER performance compared to the STTCs. It is also observed that with increasing number of states, the performance gap between SM-TC and STTC schemes increases since SM-TC provides higher coding gains with increasing complexity. Note that our 16-state code offers the highest improvement in both BER and FER performance over the 16-state STTC due to its third order time diversity compared to the second order diversity of the STTC. It can be seen from Fig. 6 that a substantial amount of improvement is achieved in the FER performance especially for 8 and 16 states. Although the

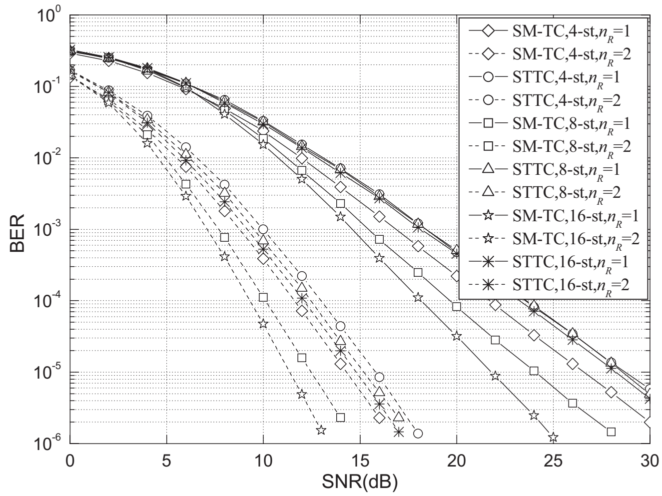

Fig. 5. BER performance for 4-,8- and 16-state SM-TC and STTC schemes at 2 bits/s/Hz.

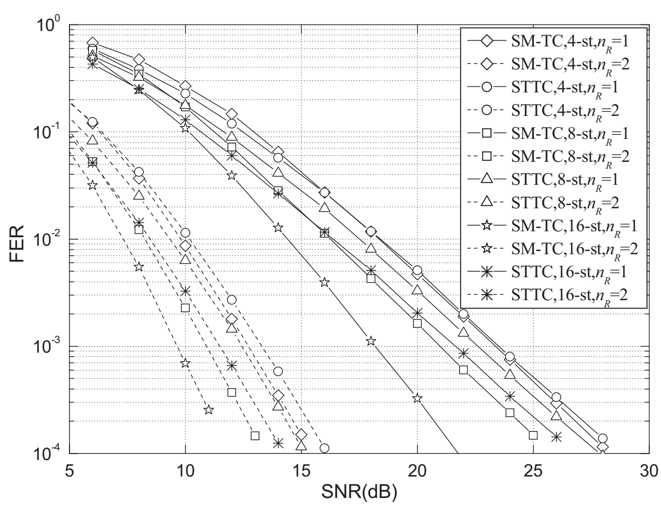

Fig. 6. FER performance for 4-,8- and 16-state SM-TC and STTC schemes at 2 bits/s/Hz.

worst case UPEP values of STTCs given in [20], are lower than those of SM-TC schemes for 4 and 8 states, the better FER performance of the SM-TC may be explained by two facts. First, we have optimized the distance spectra of SM-TC schemes; then, we conclude from Theorem 1 that error events with higher lengths can contribute UPEP values at higher time diversity orders.

Second, we carried out our simulations for 3 bits/s/Hz transmission. In Fig. 7, the BER performance of the proposed 8- and 16-state SM-TC schemes is compared with that of 8- and 16-state optimal 8-PSK STTCs for two transmit antennas. From Theorem 1 it follows that, for 3 bits/s/Hz transmission, the error performance gap between SM-TC schemes and STTCs becomes greater than that of the 2 bits/s/Hz case in favor of the SM-TC, since we employ eight transmit antennas in this case.

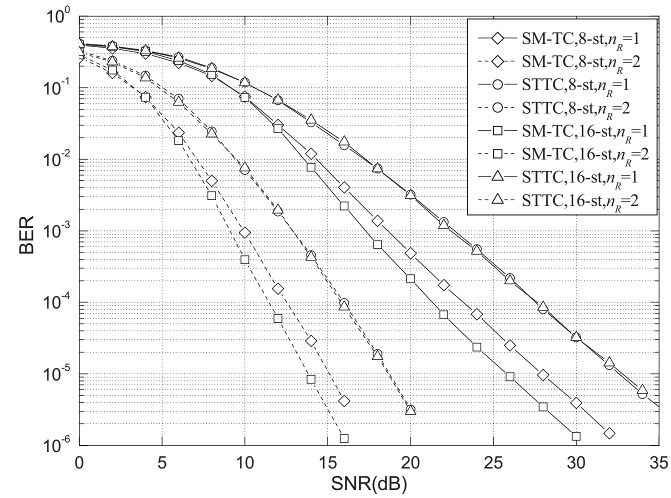

Fig. 7. BER performance for 8- and 16-state SM-TC and STTC schemes at 3 bits/s/Hz.

B. Comparisons with the SM, coded V-BLAST and the scheme of [8]

In this subsection, we compare the BER performance of the proposed SM-TC scheme with that of uncoded SM, coded V-BLAST and the scheme of [8]. We have considered two types of coded V-BLAST systems. The first system, *coded V-BLAST-I*, was the vertically encoded (single coded) V-BLAST [22], where the incoming bits were first encoded by a single convolutional encoder, then the coded bits were demultiplexed into n_T parallel substreams, and each substream was interleaved and finally modulated. At the receiver, the transmitted symbols for all layers were detected, demodulated and deinterleaved. The bits of all layers were then multiplexed and given to the hard decision Viterbi decoder. In the second system, *coded V-BLAST-II*, we replaced the SM mapper block of our SM-TC scheme by a V-BLAST encoder. Similar to STTCs, a soft decision Viterbi decoder, which performs ML decoding by using the corresponding branch metrics, was considered. The scheme of [8] and coded V-BLAST-I employ $R = 1/2$ convolutional codes with generator sequences $[5, 2]$ and $[5, 7]$, respectively, while coded V-BLAST-II systems use the same convolutional codes as that of SM-TC schemes. In all simulations, we considered four receive antennas and the size of interleavers was chosen equal to 10000. For the simulations under correlated channel conditions, the SC model given in Section III was considered.

In Fig. 8, simulation results are given for 3 bits/s/Hz transmission. In order to reach this efficiency, the SM employs four transmit antennas with binary phase-shift keying (BPSK), coded V-BLAST-I and II systems employ three transmit antennas with QPSK, and the scheme of [8] employs four transmit antennas with QPSK. As seen from Fig. 8, when compared with the reference systems, our 8-state SM-TC scheme achieves the best BER performance for both uncorrelated ($r = 0$) and correlated ($r = 0.7$) channels due to its high diversity and coding gains. Note that spatial correlation only affects the asymptotic coding gain of our scheme which is consistent with the result of (27). As reported in [8,9], the scheme of [8] outperforms coded V-BLAST-I under correlated

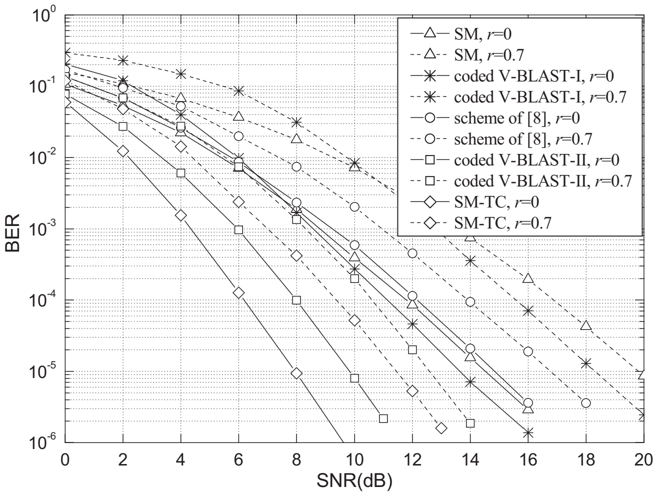

Fig. 8. BER comparison at 3 bits/s/Hz for uncorrelated and correlated channels.

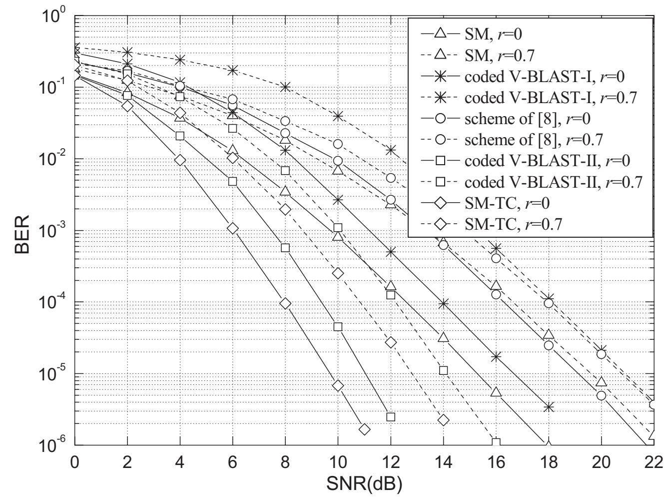

Fig. 9. BER comparison at 4 bits/s/Hz for uncorrelated and correlated channels.

channel conditions, however it is outperformed by coded V-BLAST-I and SM for uncorrelated case. Simulation results for 4 bits/s/Hz transmission are depicted in Fig. 9. For this spectral efficiency, the SM and coded V-BLAST-I employ four transmit antennas with QPSK, coded V-BLAST-II uses three transmit antennas with QPSK, and the scheme of [8] uses four transmit antennas with 8-QAM. Similarly to the 3 bits/s/Hz case, our new 16-state code outperforms all of the reference systems. From Figs. 8 and 9, we conclude that our scheme and coded V-BLAST-II provide time diversity when compared with the SM, the scheme of [8] and coded V-BLAST-I, at the price of increased complexity.

For a given spectral efficiency and number of trellis states, it is observed that the number of metric calculations performed by the soft decision Viterbi decoder is the same as SM-TC codes and STTCs. However, since only one transmit antenna is active in our scheme, contrary to the reference STTCs with the same trellis structure in which two antennas transmit simultaneously, SM-TC provides 25% and 33% reductions in the number of real multiplications and real additions per single branch metric calculation of the Viterbi decoder, respectively, for 2 bits/s/Hz, and these values increase to 30% and 37.5% for 3 bits/s/Hz. Similarly, the trellis decoding complexities of our scheme and coded V-BLAST-II are the same; however, per single metric calculation, our scheme provides 30% and 37.5% reductions in the number of real multiplications and real additions, respectively, for 3 and 4 bits/s/Hz. On the other hand, comparison of our scheme with that of [8] in terms of the computational complexity does not seem to be fair, since as our scheme employs a soft decision decoder, whereas [8] employs a hard decision decoder. As far as the Viterbi decoding complexity is concerned, the complexity of [8] is lower than our scheme because of the fact that the uncoded bits are employed in [8] along with the coded bits, whereas all the bits are encoded in our scheme. However, as seen from the BER performance results shown in Figs. 8-9, this complexity difference is compensated by a significant performance improvement provided by our scheme.

VII. Conclusions

In this paper, we have introduced a novel coded MIMO transmission scheme that directly combines trellis coding and SM. Although one transmit antenna is active during transmission, for quasi-static fading channels, we benefit from the time diversity provided by the SM-TC mechanism, which is forced by our code design criteria to create a kind of virtual interleaving by switching between the transmit antennas of a MIMO link. We have derived a general expression for the CPEP of the SM-TC scheme, and then, UPEP bounds for the SM-TC scheme have been obtained and resulting design criteria have been derived for quasi-static Rayleigh fading channels. According to the SM-TC design criteria, we have proposed some new SM-TC codes which offer significant error performance improvements over alternatives while having a lower decoding complexity for 2, 3 and 4 bits/s/Hz transmissions.

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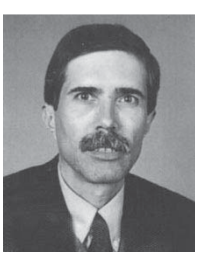

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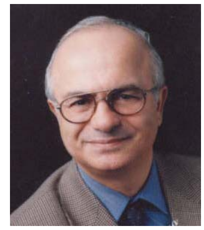

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